Aalto university
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Exercise sheet 6
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 for grading. Deadline Wednesday 11.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as one pdf-file with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 12.11 or Friday 13.11.

(1) Let $\gamma(t) = te^{it}$ for $0 \leq t \leq \pi$ and calculate:
   (a)
$$\int_\gamma \bar{z}\, dz$$
(2p)
   (b)
$$\int_\gamma |z|\, |dz|$$
(2p)
   (c)
$$\int_\gamma z\, dz$$
(2p)

(2) Let $\beta(t) = e^{t+it}$, for $0 \leq t \leq 2\pi$ and $\alpha(t) = (1-t)e^{2\pi} + t$ for $0 \leq t \leq 1$. Let $\gamma = \beta + \alpha$. Evaluate
$$\int_\gamma \frac{1}{z}\, dz.$$
(6p)

(3) Let a and b be real numbers satisfying $a < b$, and let $I(c)$ be defined far any real number c by
$$I(c) = \int_{\gamma_{a,b}(c)} e^{-z^2}\, dz$$
where $\gamma_{a,b}(c)$ is the straight line with initial point $c+ia$ and terminal point $c+ib$. Show that $\lim_{c\to\infty} |I(c)| = 0$ and $\lim_{c\to-\infty} |I(c)| = 0$

(4) Evaluate the integrals (where $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$):

(a)

$$\int_\gamma \frac{1}{(z-2)^2}\ dz$$

(b)

$$\int_\gamma \frac{1}{z^2-4}\ dz$$

(c)

$$\int_\gamma \left(z+\frac{1}{z}\right)^n\ dz$$

where $n = 1, 2, 3, \ldots$.