ELEC-E8116 Model-based control systems
/exercises 9

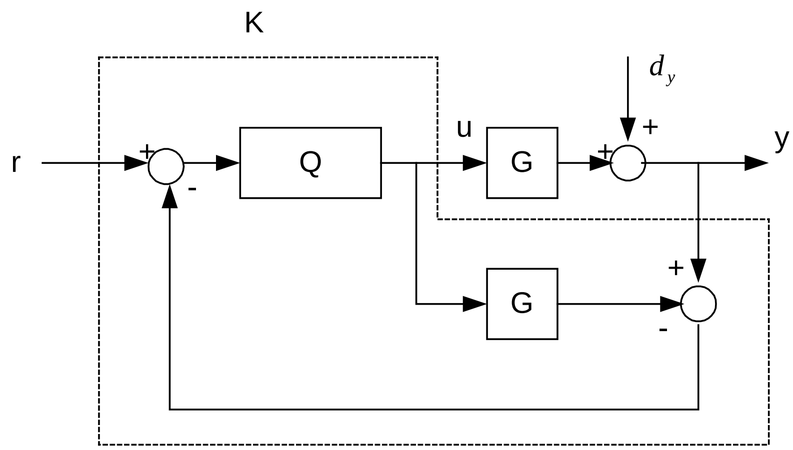
Problem 1. Consider the system

$$\frac{Y(s)}{U(s)} = \frac{s+0.5}{s^2+2s+4} \text{ and the criterion to be minimized } J = \int_0^\infty (3y^2 + 0.5u^2)\,dt .$$

Write a *Matlab m-file* to do the following:

Solve the optimal control law by using the *lqr*-function in Matlab. Calculate the *damping ratio* of the closed loop system. Simulate the system by letting the reference signal be zero (regulator problem) and letting the initial states be non-zero. Then consider the tracking problem. Use a static pre-compensator to set the static gain of the closed-loop system to the value 1. Then simulate the system for a step change in the reference signal.

Problem 2: Consider the following IMC-control configuration, in which the process G is assumed stable.



a. Prove that to study the internal stability, the stability of the transfer functions

$$K(I+GK)^{-1} = Q$$
$$(I+GK)^{-1} = I - GQ$$
$$(I+KG)^{-1} = I - QG$$
$$G(I+KG)^{-1} = G(I-QG)$$

must be investigated. Prove that the system is internally unstable, if either Q or G is unstable.

b. Let a stable controller K be given. How can you characterize those processes, which can be stabilized with this controller? (Hint: Change the roles of the controller and process.)

Problem 3. Consider the control configuration shown in the figure (known as the *Smith-predictor*). Calculate the closed loop transfer function and verify the idea behind this controller. Compare to the *IMC*-controller and prove that the Smith predictor always leads to an internally unstable system, if the process is unstable.