Aalto-yliopisto
Teknillinen korkeakoulu

EXERCISE 5

ELEC-E8409 HIGH VOLTAGE ENGINEERING

Arslan.bashir@aalto.fi
Tuas 3532

Question 1

- Derive the probability that an insulator will pass the 15/2 test (2 flashovers in 15 impulses). What is the probability when the test voltage is 125 kV, 50% breakdown voltage is 130 kV and standard deviation of breakdown is 3% ?

15/2 test:

$n = 15$ (*number of impulses in test series*)
$k = 0...2$ (*number of breakdowns in test series*)

Binomial distribution = *discrete probability distribution of the number of successes in a sequence of n independent "yes/no" experiments, each of which yields success ("yes") with probability p.*

Probability mass function:

p = probability that the given voltage will result in breakdown

$$P(k) = \boxed{\binom{n}{k}} p^k (1-p)^{n-k}$$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$$

where k! = k(k-1)(k-2) ... ·1

$$P(0) = P(0\ \text{breakdown}) = \binom{15}{0} p^0 (1-p)^{15} = \boxed{(1-p)^{15}}$$

$$P(1) = P(1\ \text{breakdown}) = \binom{15}{1} p^1 (1-p)^{14} = \boxed{15p(1-p)^{14}}$$

$$P(2) = P(2\ \text{breakdown}) = \binom{15}{2} p^2 (1-p)^{13} = \boxed{105p^2(1-p)^{13}}$$

In order to pass the test, only 2 breakdowns are allowed in the 15 measurement series

$$P(\max 2\ \text{breakdown}) = P(0) + P(1) + P(2)$$

Cumulative distribution function:

$$P = \sum_{i=0}^{2} \binom{15}{i} p^i (1-p)^{15-i} = \boxed{(1-p)^{13}(1+13p+91p^2)}$$

Determine value of probability *p*:

P(λ) = *p*

where $\lambda = \dfrac{x-\mu}{\sigma}$

$x = U_t = 125$ kV

$\mu = U_{50} = 130$ kV

$\sigma = 3\% = 0.03 \cdot 130$ kV

λ = -1.28

P(-1.28) = (1 – **0.799**)/2 = 0.10 = *p*

<u>Insert value *p* = 0.10 into P</u>:

P = 0.9^{13}(1 + 1.3 + 0.91) = 0.8159

Probability that the test object passes the 15/2 test with maximum 2 allowable breakdowns in a 15 impulse series when test voltage U_t = 125 kV is

82%

Question 2

- In order to determine an insulator's 50% breakdown voltage, a series of tests were conducted where voltage was increased steadily from 200 kV until breakdown occurred. The following breakdown values were obtained (in kV): 478, 487, 503, 499, 481, 518, 530, 512, 495, 480, 471, 535, 505, 507, 491, 498, 506, 521, 482, 493. Determine the insulator's 50% breakdown voltage and its standard deviation using the probability sheet. Also, calculate the mean and experimental standard deviation using the measured data.

Measured breakdown values (in kV):

478, 487, 503, 499, 481, 518, 530, 512, 495, 480, 471, 535, 505, 507, 491, 498, 506, 521, 482, 493

1. Arrange the measured values in order of **magnitude**
2. Calculate the respective **cumulative probability p_c** for each value

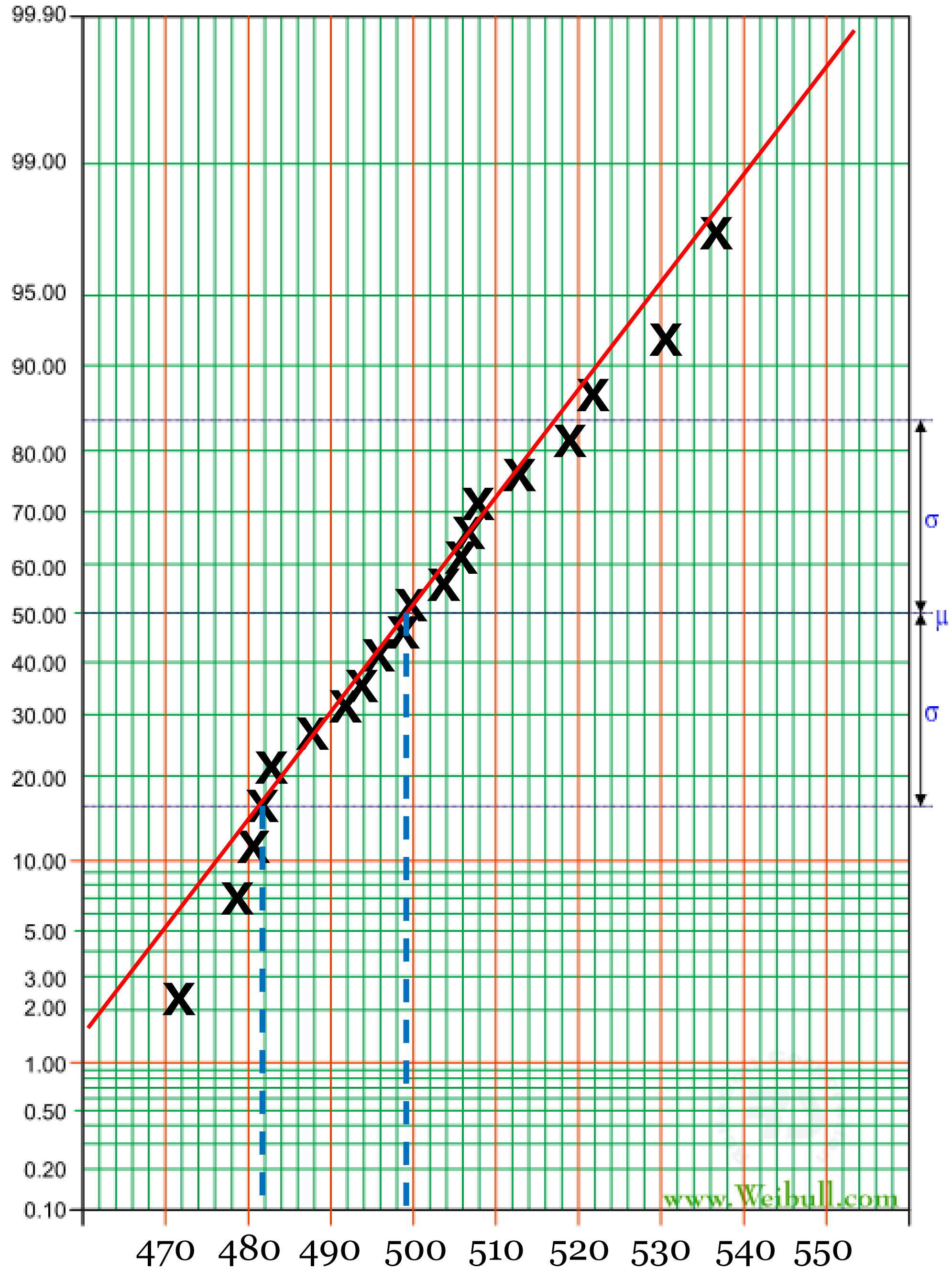
   n = 20 **100% / n = 5 %**

   Start from the midpoint of 5% (2.5%) and increase probability by 5% for each subsequent breakdown voltage

3. **Plot** the values on a probability sheet and draw the trend line

U [kV]	p_c [%]
471	2.5
478	7.5
480	12.5
481	17.5
482	22.5
487	27.5
491	32.5
493	37.5
495	42.5
498	47.5
499	52.5
503	57.5
505	62.5
506	67.5
507	72.5
512	77.5
518	82.5
521	87.5
530	92.5
535	97.5

U [kV]	p_c [%]
471	2.5
478	7.5
480	12.5
481	17.5
482	22.5
487	27.5
491	32.5
493	37.5
495	42.5
498	47.5
499	52.5
503	57.5
505	62.5
506	67.5
507	72.5
512	77.5
518	82.5
521	87.5
530	92.5
535	97.5



The voltage correlating to the 50 % breakdown probability can be observed from the trend line:

U_{50} = 499 kV

Standard deviation can also be observed from the probability sheet:

$$s = U_{50} - U_{16}$$

s = 499 – 482 = 17 kV
(σ = 3.4 %)

$\lambda = 1 \Rightarrow \Phi(\lambda) = 0.6828 = 68\%$
100% - 68% = 32%
32% / 2 = 16%

Calculation:

$$\mu = \frac{1}{n}\sum u_i = 499.6\ \text{kV}$$

$$s = \sqrt{\frac{1}{n-1}\sum (u_i - \mu)^2} = 17.6\ \text{kV}$$

$$(\sigma = 3.5\%)$$

Question 3

- During an acceptance test for a 123 kV air insulating device, the impulse test voltage was set at 450 kV. According to the IEC standard, a 450 kV test voltage for an impulse voltage test correlates to 10 % breakdown probability. The test was performed using the up and down method. Test results are documented as follows (x = breakdown, o = no breakdown):

494 kV													x		x									
480 kV									x		o		o		x									
466 kV						x		o		o						x		x				x		
452 kV				x		o		o									o		x		o		o	
438 kV			o		o															o				
424 kV		o																						
410 kV	o																							

Withstand strength is assumed to follow normal distribution when standard deviation σ = 3 %

Did the device pass the test?

<u>Hint</u>: Respective breakdown voltage U_p for breakdown probability p can be estimated using the mean and standard deviation according to the following table.

$U_p = U_{50} - k\sigma$					
p [%]	50	15.9	10	2.3	0.13
k	0	1	1.3	2.0	3.0

In order to pass the test, the 50 % breakdown voltage of the device has to be **larger** than calculated IEC 50% test voltage.

$$\mathbf{U_{50(device)} > U_{50(IEC)}}$$

(the device can withstand more than the imposed test voltage)

According to the IEC standard, a 450 kV test voltage for an impulse voltage test correlates to 10 % breakdown probability.

***p* [%]**	**50**	**15.9**	**10**	**2.3**	**0.13**
k	0	1	1.3	2.0	3.0

$$U_p = U_{50} - k\sigma \qquad U_{10} = U_{50} - 1.3\sigma \qquad \sigma = 3\% = 0.03 \cdot U_{50}$$

$$U_{10} = U_{50} - 1.3(0.03 \cdot U_{50})$$

$$U_{10} = U_{50}\,(1 - 1.3(0.03))$$

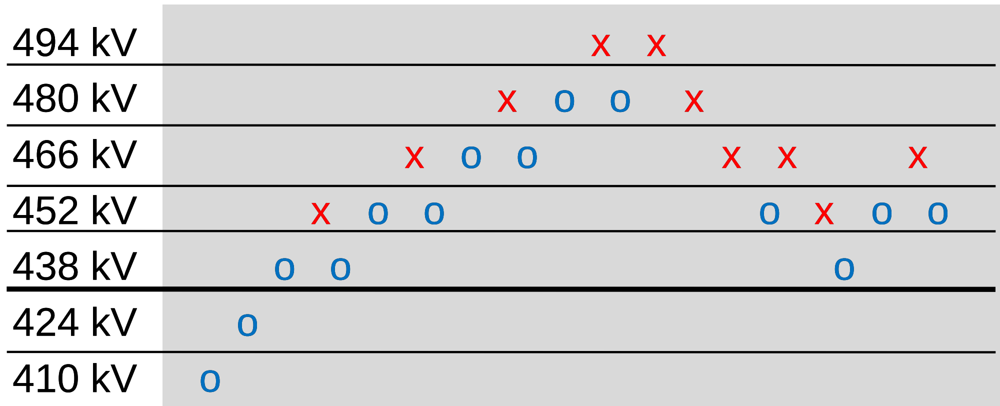
$$U_{50} = U_{10} \,/\, (1 - 1.3(0.03))$$

IEC 50% test voltage: $U_{50(\mathrm{IEC})} = \dfrac{450\,\mathrm{kV}}{1-1.3(0.03)} = \boxed{468.3\,\mathrm{kV}}$

Up and Down test:

$$U_{50} = \frac{\sum n_i U_i}{\sum n_i}$$

where n_i is the number of events at voltage level U_i (only $n_i \geq 2$ levels are considered)



Measure 50% breakdown voltage of device:

$$U_{50(\text{device})} = \frac{(3 \cdot 438) + (7 \cdot 452) + (6 \cdot 466) + (4 \cdot 480) + (2 \cdot 494)}{3 + 7 + 6 + 4 + 2} = 462.8 \text{ kV}$$

$U_{50(\text{device})}$	<	$U_{50(\text{IEC})}$
462.8 kV	<	468.3 kV

The device does **NOT** pass the test. Breakdown strength of the device is less than that specified in the standards.