ELEC-E8116 Model-based control systems /exercises 10

1. Consider a SISO system and a state feedback control

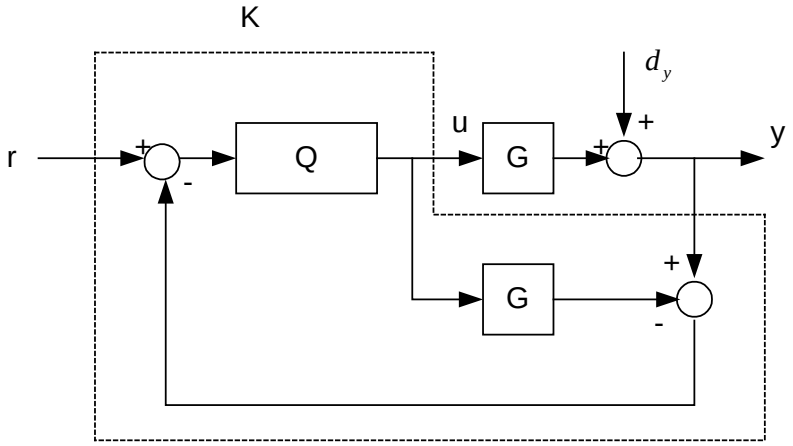
$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$u(t) = -Lx(t)$$

where L is chosen as a solution to the infinite time optimal (LQ) horizon problem.

- **a.** Prove that the loop gain is $H(s) = L(sI - A)^{-1}B$
- **b.** Prove that $|1 + H(i\omega)| \geq 1$
- **c.** Show that for the LQ controller
  - phase margin is at least 60 degrees
  - gain margin is infinite
  - the magnitude of the sensitivity function is less than 1
  - the magnitude of the complementary sensitivity function is less than 2.

2. Consider the IMC control structure, which is used to control a stable and minimum phase SISO process G.



Note that in addition to the reference r a disturbance signal d_y is modelled to enter at the output of the process. By using the IMC design discussed in the lectures analyse the response to step inputs at r and d_y.