PHYS-C0252 - Quantum Mechanics Part 7
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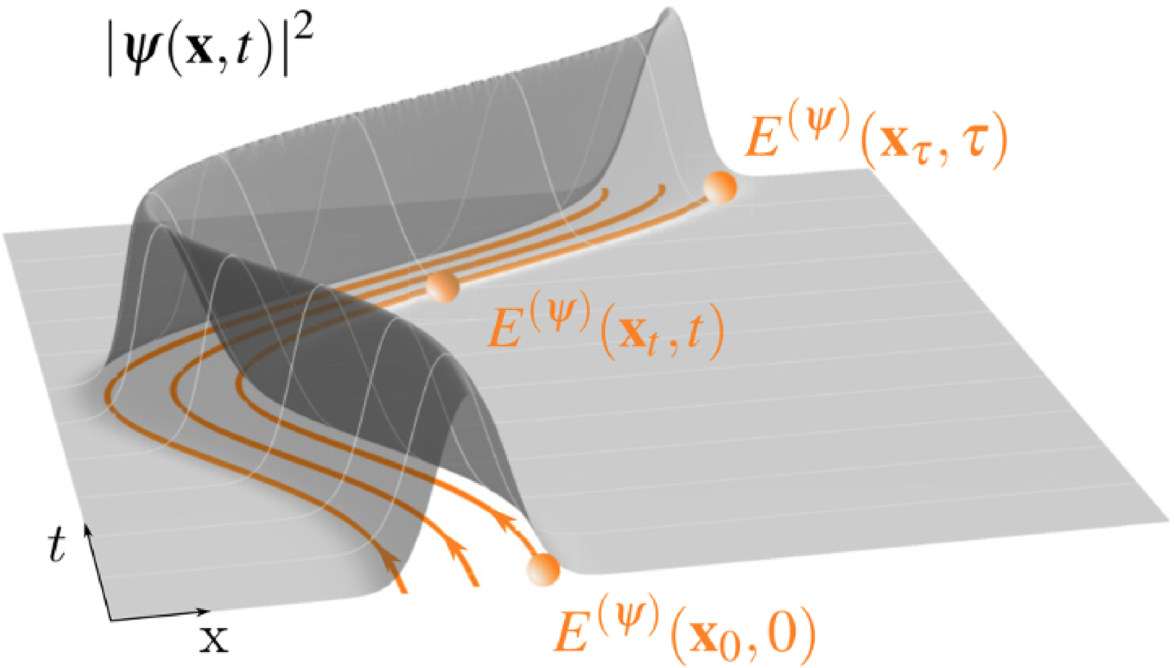
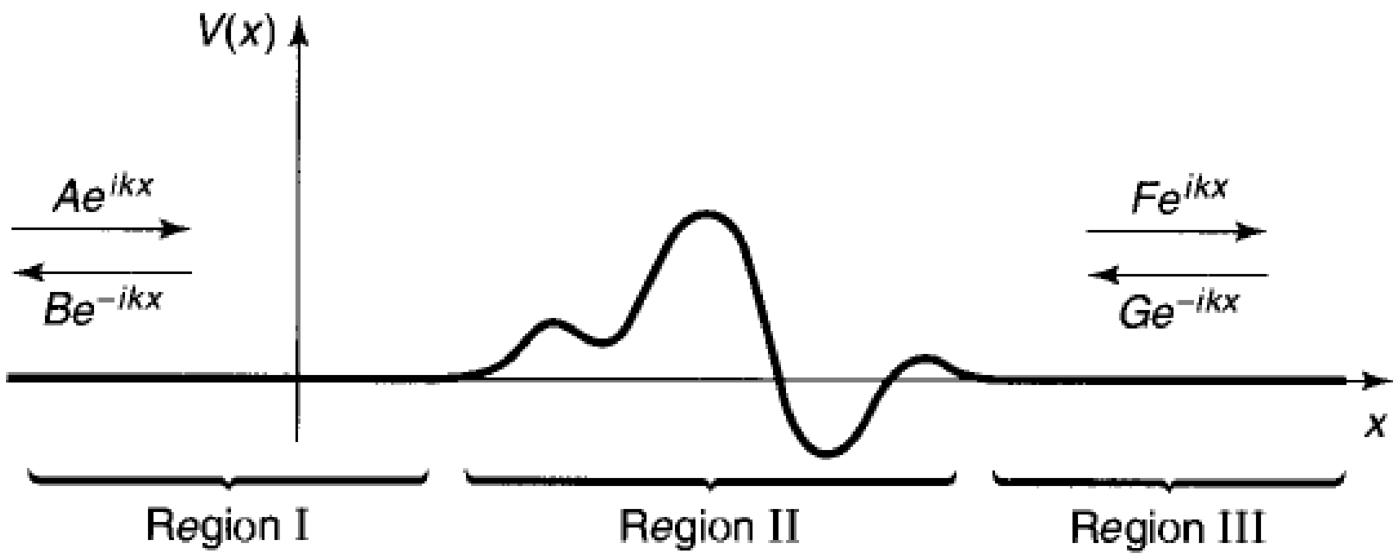


4.4 Scattering Matrix

- The results based on specific potentials can be generalized to incoming and outgoing waves for any potential in Region II:



$V = 0$ $\qquad$ $V(x) \neq 0$ $\qquad$ $V = 0$

For the scattering sites $E > 0$

$$\psi''(x) = -k^2\psi(x), \;\; k = \frac{\sqrt{2mE}}{\hbar}$$

and the general solutions for $V = 0$ are (as before)

$$\psi(x) = Ae^{\imath kx} + Be^{-\imath kx}$$

and

$$\psi(x) = Fe^{\imath kx} + Ge^{-\imath kx}$$

The general solution in between has to be (why?)

$$\psi(x) = Cf(x) + Dg(x)$$

where $f(x)$ and $g(x)$ are any two linearly independent solutions of the (static) SE for the given $V(x)$

There are four BCs that can be used to give B and F in terms of A and G:

$$B = S_{11}A + S_{12}G, \;\; F = S_{21}A + S_{22}G$$

It is suggestive to build up a 2 x 2 matrix

$$\mathbf{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

which is the *scattering (S) matrix* $\begin{pmatrix} B \\ F \end{pmatrix} = \mathbf{S} \begin{pmatrix} A \\ G \end{pmatrix}$

For scattering from the left, $G = 0$:

$$R_l = \left. \frac{|B|^2}{|A|^2} \right|_{G=0} = |S_{11}|^2, \quad T_l = \left. \frac{|F|^2}{|A|^2} \right|_{G=0} = |S_{21}|^2$$

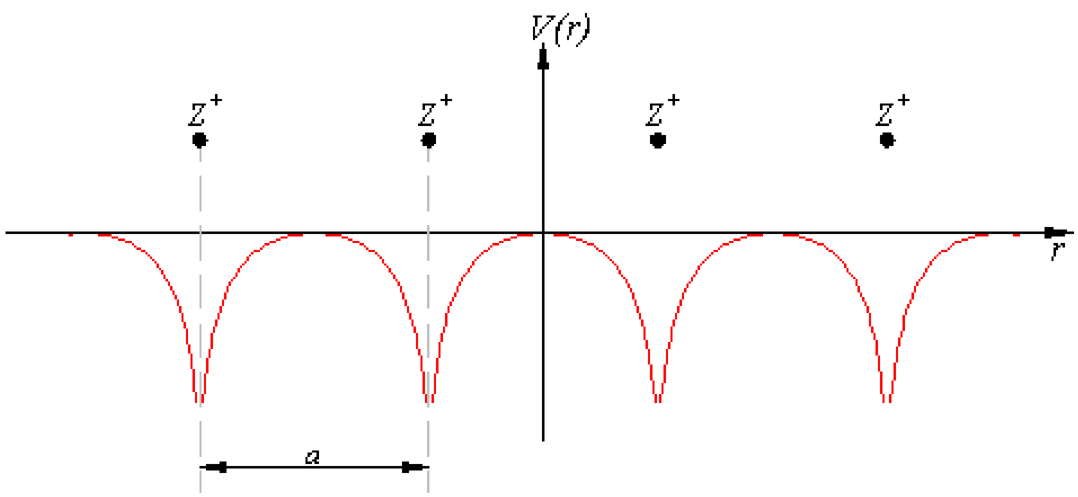
and from the right, $A = 0$:

$$R_r = \left. \frac{|F|^2}{|G|^2} \right|_{A=0} = |S_{22}|^2, \quad T_r = \left. \frac{|B|^2}{|G|^2} \right|_{A=0} = |S_{12}|^2$$

- The existence of possible bound states is in diverging components of the S-matrix (maybe homework).

4.5 Bloch's Theorem

- An important special case is that of a periodic potential $V(x + a) = V(x)$:

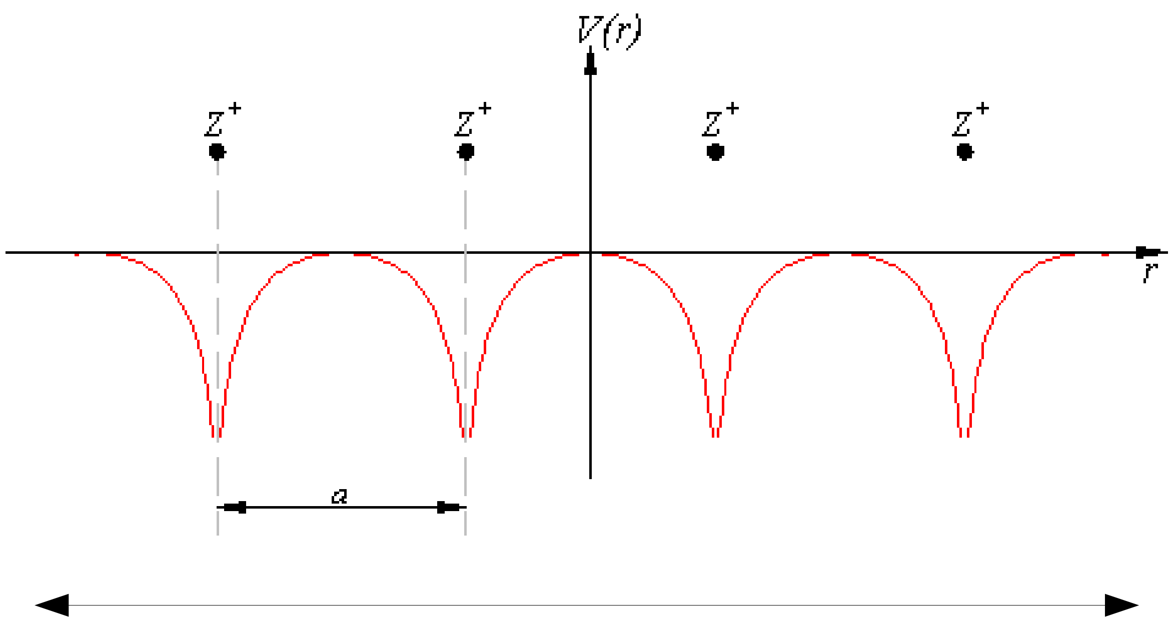


Bloch's theorem: Any wave function that is a solution of the SE in a periodic potential must be of the form

$$\psi(x) = e^{\imath kx}u(x)$$

where $u(x)$ must satisfy $u(x+a) = u(x)$ and the wave vector is quantized as

$$k = \frac{2\pi n}{L}, \ n = 0, \pm 1, ..., \pm \frac{N}{2}$$



$$L = Na$$