Aalto university
Björn Ivarsson

Exercise sheet 11
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 for grading. Deadline Monday 30.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as one pdf-file with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Tuesday 1.12 or Wednesday 2.12.

(1) Determine the disks of convergence of
   (a) $\sum_{n=1}^{\infty} \sqrt[n]{n}(z-1)^n$ (2p)
   (b) $\sum_{n=1}^{\infty} n^2(z+2)^{2^n}$ (2p)
   (c) $\sum_{n=1}^{\infty} n!(z-i)^{n!}$ (2p)

(2) Find the Taylor series expansion of f about the origin, and identify the largest open disk $\Delta(0,\rho)$ in which the expansion is valid. (*Hint:* Remember the geometric series. Also remember that Taylor series can be differentiated and integrated term-wise in their disks of convergence.)
   (a) $f(z) = (1-z)^{-2}$ (2p)
   (b) $f(z) = (1+z^2)^{-3}$ (2p)
   (c) $f(z) = \operatorname{Log}(1+z^2)$ (2p)

(3) Show that

$$\frac{e^z}{1-z} = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \frac{1}{k!} \right) z^n$$

when $|z| < 1$. (*Hint:* Taylor series can be multiplied in their disks of convergence.)

(4) Assume that f is a non-constant entire function. Assume there is a constant $\lambda \neq 1$ such that $f(\lambda z) = f(z)$ for every $z \in \mathbb{C}$. Show that there must be an integer $m \geq 2$ such that $\lambda^m = 1$. Also show that there is an entire function g such that

$$f(z) = g(z^m)$$

where m is the minimal integer $m \geq 2$ such that $\lambda^m = 1$.