1. (**Just for fun**) (4p)

   (a) In the lectures, when deriving and using equations of motion in different contexts, we have taken the pressure as a product of density and temperature, $p = nT$. Now, when discussing magnetic confinement of plasma, exactly the same product is used for energy density! Contemplate if this is OK – or has Taina once again made a typo somewhere... (2p)

   (b) The Sun produces energy at the rate of about 3.8×10^{26} W. This is generated by converting mass into energy. How much mass is converted into energy every second? Where does this energy go to? (2p)

2. (**Collapse of a neutron star**) (6p)

   The energy source of a star is nuclear fusion, starting with fusion of hydrogen to helium, and then moving up in the periodic table until reaching the most stable elements around iron. If the star is sufficiently massive, that is. Take such a star at the end of its fusion career, so that it has ran out of exothermal reactions and is cooling down. Then at some point there is not enough kinetic pressure to counteract the gravitational pull and the star will collapse into a so-called *neutron star*. It is called so because, in the process of collapsing, weak interaction turns protons into neutrons. At the end, the only remaining 'force', preventing the whole system from collapsing into a singularity, is the Fermi pressure (i.e., Pauli's exclusion principle). Let's assume that we can and do measure the magnetic field on the surface of the star before and after the collapse, yielding the not-so-unrealistic values of $10\,\mu\mathrm{T}$ for the first measurement and $100\,\mathrm{kT}$ for the latter. Assuming that the plasma in and around this massive star behaves according to *ideal MHD*, what is the radius of the neutron star if the original star had the radius of $R = 1\,000\,000\,\mathrm{km}$?

3. (**Justifying the existence of magnetosphere**) (6p)

   The solar wind introduced in the lectures comes with a pressure exerted on the Earth's magnetic field $\boldsymbol{B}$. This kinetic pressure can be expressed as ρV^2, where ρ is the mass density of the solar wind and V its speed. For Earth to maintain its magnetosphere, the magnetic pressure has to be able to stand up against this external kinetic pressure. Derive a condition for the existence of a magnetosphere around Earth by assigning the magnetosphere an effective radius R_M. Evaluate R_M (in terms of R_E) using typical solar wind parameters $\rho = 5\,\mathrm{u\,cm^{-3}}$ and $V = 400\,\mathrm{km\,s^{-1}}$. Naturally, R_M has to be larger than the Earth radius R_E for the magnetosphere to exist.

4. (**Magnetic dipole field**) (8p)

   Recall from basic electromagnetism: far enough from the magnetic dipole with magnetization M, the field can be expressed as $\boldsymbol{B} = -\nabla\Psi$, where $\Psi = -(\mu_0/(4\pi))\boldsymbol{M} \cdot \nabla(1/r)$.

   (a) Using spherical coordinates and setting the z-axis along the magnetic dipole, show that the magnetic field is
   $$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{(3\boldsymbol{M}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{M}}{r^3}.$$
   (2p)

   (b) Calculate the components of the magnetic field (still in spherical coordinates) as well as the magnitude of the magnetic field. (2p)

   (c) In space and geophysics, instead of the conventional azimuthal angle θ, ranging from 0 (north pole) to π (south pole), one typically uses another angle λ, which measures the azimuthal angle not from the z-axis but, rather, from the (x,y) plane. So, λ ranges from $-\pi/2$ (north pole) to $+\pi/2$ (south pole). Express the magnetic field components in terms of λ. (2p)

   (d) You can follow the field lines by making sure you always keep the same direction as the field line, i.e., $r\frac{d\lambda}{dr} = B_\lambda/B_r$. Using this, show that the distance of a field line from Earth can be expressed as $r = r_0\cos^2\lambda$, where r_0 is the distance of the field line at the equator. (2p)

5. (**Food for thought**)

   During the lectures we learned many things about our very own 'magnetic cage', the dipole field that protects us from being directly hit by the potentially deadly components of the solar wind. What we did not learn is that this field is not permanent but has changed polarity in the course of Earth's history – on a geological time scale. Try to find information about this reversal of the field and contemplate its possible consequences.