## Ten Lectures on Ship Dynamics (1st Edition)



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#### **Preface**

The concept of ship dynamics has a very wide meaning, embracing the fundamentals of both deterministic and performance-based methods with ship safety. As such, the subject is of paramount importance for its wide implications in the design and operation of ships and floating offshore installations. Contemporary developments in this specific field tend to be collected and thoroughly debated especially considering uncertainties associated with multi-physics modeling and simulation as well as the emergence of modern technologies. Teaching ship dynamics involves understanding the ocean waves, seakeeping, stability, wave loads and dynamic response, validation methods (e.g. full scale measurements, model tests), dynamic stability in waves, added resistance, maneuvering and directional control. Within this context principles of basic fluid mechanics, structural dynamics, potential flow nonlinear hydrodynamics, CFD methods and parameter system identification interplay in a way that is fundamentally challenging.

With the above in mind, this textbook briefly outlines principles of Ship dynamics along the lines of the synonymous MSc course (MEC-E2004) I have been delivering at Aalto University since January 2018. The lecture notes do not cover in detail the whole spectrum of the subject. They should be considered auxiliary to series of presentations and tutorials delivered at class and intend to help early post graduate students to comprehend the importance of the subject within the context of naval architecture.

I am grateful to my Teaching Assistants Mr. Zeiad Abdelghafor and Mr. Hassan Yosri for helping me to conceptualize this set of lecture notes.

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### Contents

Preface	9	2
Lecture	e 1: Introduction to Ship Dynamics	6
1.1.	Key definitions	7
1.2.	Specialist phenomena	9
1.3.	Engineering practice	11
1.4.	References	14
Lecture 2 : Controlling Ship Dynamics		15
2.1	Elements of ship resistance	15
2.1.1 Introduction		15
2.	1.2 Key definitions	15
2.2	The dynamics of resistance and thrust	19
2.2.1	1 Ship resistance	19
2.2.2	2 Ship thrust	20
2.2.3	3 Ship rudders	21
2	2.4 The influence of slow speed maneuverability	25
2.3	Stabilisation systems	27
2.4	References	29
Lecture 3: Wave mechanics		30
3.1	Wave formation	30
3.2	Brief overview of wave theories	31
3.3 The regular wave		33
Lecture 4: Irregular waves		37
4.1 T	The statistical representation of irregular waves	37
4.2	Wave superposition and Fourier analysis	40
4.3	Wave energy spectrum	42
4.4	Idealized wave spectra	45
4.5	The statistics of sea states	47
4.6	Extreme waves	49
4.7	References	50
Lecture 5 : Introduction to Ship motions		51
5.1 S	5.1 Ship motions	
5.2 The ship encounter frequency		
5.3 A	Axis of reference	54

5.3 Review of rigid body dynamics	55
5.2.1 Free undamped vibration of 1- DOF system	56
5.2.2 Free damped vibration of single DOF system	57
5.2.3 Forced vibration of 1- DOF system	58
5.2.3 Quasi static, dynamic and resonant responses	61
5.3 References	63
Lecture 6: Ship motions in regular waves	
6.1 Uncoupled heave motion	64
6.2 Uncoupled pitch motion	67
6.3 Coupled heave and pitch motions	67
6.4 Roll in small amplitudes	68
6.5 Roll in large amplitudes	70
6.6 Idealisation of responses in regular waves	71
6.7 References	74
Lecture 7 : Seakeeping methods	75
7.1 Evaluation of hydrodynamic forces	75
7.2 Equations of motion in 6-DOF	77
7.2 Linear seakeeping analysis methods	78
7.2.1 Strip theory	78
7.2.2 Pulsating source Green Function method	81
7.3 Non-linear seakeeping analysis methods – a brief reference	83
7.4 References	86
Lecture 8 : Wave Loads	87
8.1 Classification of wave loads	87
8.2 Murray's method	90
8.3 Wave induced responses	91
8.4 The importance of hydrodynamic actions	92
8.5 Response in irregular waves	94
8.6 Hydroelasticity of ships	96
8.8 References	101
Lecture 9 Additional seakeeping topics	102
9.1 Seakeeping criteria	102
9.2 Experimental methods for seakeeping and wave loads	104
9.2.1 Model scale facilities	104

9.2.2 The im	nportance of similarity laws	105
9.2.3 Physic	cal model testing	106
9.2.4 Full sc	cale measurements	109
9.3 Reference	S	111
Lecture 10 : Add	ed Resistance and Maneuvering	112
10.1 Added	Resistance	112
10.1.1 A	dded Resistance in Regular Waves	112
10.1.2 A	dded Resistance in Irregular Waves	112
10.2 Aerody	ynamic Forces	112
10.3 Weath	ner Routing	112
10.4 Maneu	uvering	112
10.4.1 Co	ontrollability and motion stability	112
10.4.2 M	Naneuvering models	112
10.4.3 Se	ea trials	112
Appendix 1- Revi	iew of Probability and Statistics for Marine Applications	112

#### Lecture 1: Introduction to Ship Dynamics

In the broadest sense, the subject of ship dynamics is concerned with all conditions where the inertia forces interplay a role in ship motions. Traditionally, the ship is assumed to behave as a rigid body that is static or slowly moving between positions of equilibrium. Thus, ship dynamic idealizations should account for all operational conditions that differ from the ideal still water condition or the case of a ship that progresses with constant heading and forward speed in regular wave conditions. Forward speed effects and the influence of structural dynamic elastic distortions (known as hydroelasticity effects) on ship dynamic response may also play a prominent role for slender ships with large deck openings (e.g. container ships) or large bow flare (e.g. container ships, LNG ships, cruise liners, high speed boats etc.).

The ocean environment is stochastic. Thus, ship dynamic models comprise of sub-models encompassing the principles of *ocean wave mechanics, seakeeping, maneuvering, structural vibration, and dynamic stability.* Usually, directional stability and control topics consist part of maneuvering. Yet, modelling each of the sub-models within the context of hydrodynamics is prone to simplifying assumptions that should be well evaluated before models are used for design or operational decision support. As an example, traditional seakeeping models are linear and understood within the context of potential flow analysis. In some cases, such linear models may be sufficiently accurate for the prediction of loads and ship motions in small amplitude, regular waves. We can therefore benefit up to a point from the utilization of linear models to derive responses in the frequency domain. However, a major shortcoming in using the linear approach is that it cannot be used for the prediction of some classes of responses. A linear hydrodynamic model cannot predict the pure loss of stability in waves, parametric rolling and the influence of asymmetry of sagging and hogging especially when large amplitude ship motions influence ship dynamic behavior in waves. On the other hand, unlike seakeeping models, maneuvering is usually considered in still water and associated in plane motions (surge, sway and yaw) are evaluated in the time domain. This means that the assumption that heeling may be ignored is not ideal in terms of assuring ship safety.

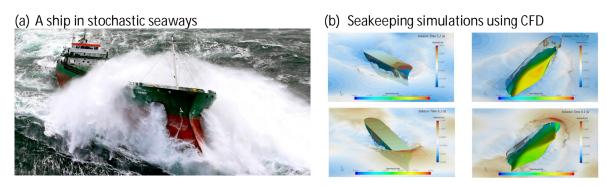


Figure 1.1 A ship in waves (reality vs simulation)

Over the years, the dominance of simplified seakeeping, wave loading and maneuvering models has been associated to poor computer performance. In the last decade higher computing speed allowed for the maturity of non-linear seakeeping and unified seakeeping-maneuvering and hydroelastic models when needed. Whereas understanding the influence of hydroelasticity, hydrodynamic added resistance in waves and ship dynamic stability in waves remain challenging problems there is a clear trend of positive developments with direct application in design and safety. With the advent of supercomputers and Computational Fluid Dynamic (CFD) based methods theoretical advances remain prominent.

#### 1.1. Key definitions

Seakeeping refers to a ship's ability to remain at sea in all conditions and carry her intended mission. Traditionally, topics such as dynamic stability in waves, strength, maneuvering, added resistance are directly linked with seakeeping dynamics. This is because excessive ship motions may have adverse effects on ship design and life cycle operations. They may lead to hull rupture or distortions, discomfort of the passengers and crew, result in less efficient working conditions and bad worker / customer experience. Added water resistance due to ship motions in waves and propellers exposed to heavy conditions may also result in reduced ship efficiency. This is because severe motions and heavy loading on propulsors may lead to voluntary ship speed loss. To control this problem computerized weather routing systems are now fitted on several ships allowing the master greater control of speed and seaworthiness in demanding or extreme conditions. Yet the development of sound decision support criteria encompassing principles of nonlinear hydrodynamics remains a challenge. In traditional seakeeping analysis the ship is modelled as a rigid body moving in six degrees of freedom namely three translations (heave, surge, and sway), and three rotations (roll, pitch, and yaw). For dynamic stability in waves her oscillatory degrees of freedom (i.e. roll, pitch and heave) should be controlled. In heavy seas a ship's bow may dig into waves and water may be driven over the ship's forecastle deck. The phenomenon is known as deck wetness. The main factors affecting this phenomenon is the relative motion of the bow and the sea surface and the freeboard forward. Slamming is another effect associated with local pressure exerted on the bottom and forward regions of the ship due to a sudden change in the vertical acceleration. This is followed by vibration in the ship's girder in its natural frequencies and links with principles of hydroelasticity.

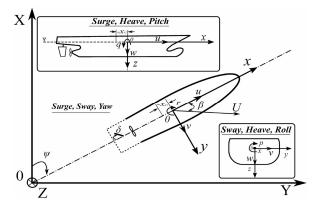
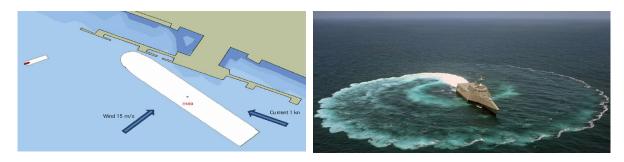


Figure 1.2 Coordinate model representing the 6 degrees of freedom of a rigid ship (Taimuri et al., 2020)

When sailing in congested waterways such as canals or during navigation in harbors, ship control is essential to ensure accurate ship tracking relative to the berth points and safety in relation to other ships in harbor. A ship is said to be directionally stable if a deviation from a set course increases only while an external force or moment is acting to cause the deviation. On the other hand, it is said to be unstable if a course deviation begins or continues even in the absence of an external cause. A directionally unstable ship is easy to maneuver, while a stable ship requires less energy expenditure by her steering gear in maintaining a set course. A compromise between extremes is therefore desirable. Another example is dynamic positioning systems used in offshore vessels or drilling platforms. Such systems help maintain floatability and positioning relative to the seabed. Thus propulsors producing ahead and astern thrust as well as turning moments and thrust have been developed. The later (i.e. turning moments and lateral thrust) are provided using rudders directly positioned behind the main propulsors and in some cases additional lateral thrusters are used where higher maneuvering capability is required (e.g. ship

bow/stern regions). Ship maneuvering and control (positional and directional stability) relate with controlling ship course and speed and primarily involve the investigation of motions (e.g. sway and yaw) due to disturbing forces from the environment and / or control mechanisms such as rudders. The maneuvering characteristics of a vessel are usually defined in still water conditions. The influence of wind, waves, and current must be allowed for in applying the data to practical seagoing conditions. Wind effects on maneuvering characteristics are of concern for ships with large superstructure such as cruise ships and ferries. When a ship operates close to banks or at close proximity to another ship she may experience additional forces and turning moments with significant variations. Thus, the use of time domain hydrodynamic models are a norm in manoeuvring but an option in seakeeping. Manoeuvring is often studied in shallow waters but seakeeping in open seas. Finally, seakeeping is studied by an inertial coordinate system while manoeuvring by a ship fixed system. In any case viscous fluid flow idealisations associate with mathematical difficulties and computational cost attributed to both seakeeping and manoeuvring idealisations and their unification remain a medium to long term challenge (Taimuri et al. 2020).

(a) Idealization of a ship maneuvering at close (b) Littoral combat ship in turning circle manouvre proximity to harbor under wind and current forces



(c)Ship manouvering idealisation using potential flow hydrodynamic model (Taimuri et al., 2020)

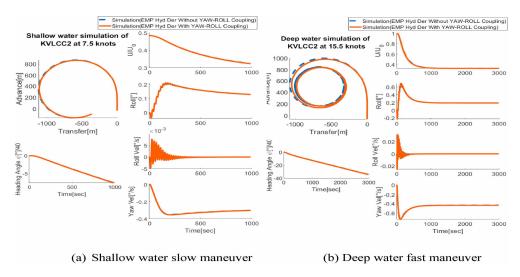


Figure 1.3. Ship manouvering (Taimuri et al., 2020)

The primary purpose of computing motions and loads of ships in a seaway is to assure the safety of persons on board, the integrity of the ship and, if present, her cargo. They also aim to improve

performance and efficiency (Hirdaris et al., 2014). Excessive motions may cause shift of cargo, damage from loosened deck containers or equipment and dangerously large heel angles and capsizing (Acanfora et al., 2017). Furthermore, ship motions affect the comfort of persons on board, leading to sea sickness or, in extreme cases, to render it impossible for the crew to accomplish the ship's mission. Knowledge of wave-induced loads is necessary to assess the integrity of the ship's structure. Most important for this are vertical and horizontal bending moments, torsional moments, and sometimes shear forces in transverse sections of the hull girder (Hirdaris et al., 2010). Wave-induced local pressure acting on the hull determines the necessary strength of plates, stiffeners, and web frames. Furthermore, steady wave- and wind-induced forces and moments should not prevent the ship from arbitrary course changes and from making some speed ahead.

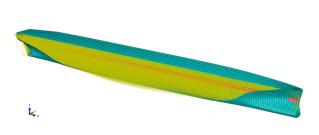
#### 1.2. Specialist phenomena

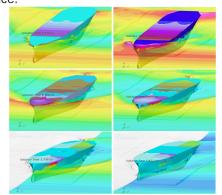
Hydroelasticity is concerned with the interactions of deformable bodies with the water environment in which they operate. Hydroelasticity of ships is concerned with the interaction of the ship modelled as an elastic body with her surrounding fluid. Theoretically, flexible ship dynamics recognize the significant differences in the hydrodynamic, inertia, and elastic forces that may lead to the amplification of wave loads and excessive strains and stresses possibly leading to hull rupture or high fatigue loads. The importance of flexible ship dynamics increased over the last few years as sea transportation and ship sizes expanded. Modern ocean carriers are more flexible, and their structural natural frequencies can fall into the range of the encounter frequencies of the sea spectrum. It is now recognized that hydroelastic effects associated with ship slamming or the antisymmetric (i.e. coupled horizontal bending and torsion) dynamics of ships with large openings may influence wave load predictions. Yet implementation of hydroelasticity models in design work in progress (Lloyd's Register, 2018).

Ship dynamic stability in waves attempts to investigate roll motions alone subject to heeling moments in the irregular seaway. Investigations therefore include include nonlinearities (e.g. roll damping) and provide variation of roll angle in time with the ultimate purpose to investigate whether the ship will capsize. There are also some investigations dealing with the coupled sway-roll-yaw motions; thus bringing together the subjects of directional and dynamic stability. Ship survivability against capsize in heavy seas has become one of the areas of primary concern among ship researchers, designers and regulators in recent years. When a ship is subjected to the effect of large waves it may capsize according to several different scenarios, depending on the magnitude and direction of the wave excitation and the ship's own capability to resist such excitations. Resonant or breaking waves approaching a ship from the ship side ("beam seas") have a potential to excite large rolling which could result in capsize, especially if the intensive oscillation of the ship causes shift of cargo or, if a considerable quantity of green water is shipped on the deck. More dangerous still can be a group of steep and relatively long waves approaching a ship from the stern ("following-seas"). Waves of this kind are known to incur significant reductions in roll restoring capability (i.e. the tendency to return to the upright position) for many types of vessels and they may also instigate dangerous coupled motions. According to a popular classification, in followingseas a ship may capsize in at least three different ways. Pure-loss of stability. This is a sudden, nonoscillatory type capsize taking place around a wave crest due to slow passage from a region of the wave where roll restoring has become negative. Parametric instability is the gradual build-up of excessively large rolling created by a mechanism of internal forcing, the result of a fluctuating restoring that depends on where the ship lies in relation to the wave (ABS, 2019). This phenomenon is related to the periodic change of stability as the ship moves in longitudinal waves at a speed when the ship's wave encounter frequency is approximately twice the rolling natural frequency and the damping of the ship to dissipate the parametric roll energy is insufficient to avoid the onset of a resonant condition. If a ship is in a wave trough, the average waterplane width is significantly greater than in calm water. The flared parts of the bow and stern are more deeply immersed than in calm water and the wall-sided midship is less deep. This makes the mean, instantaneous waterplane wider than in calm water with the result that the metacentric height increases over the calm water value. When the wave crest is located amidships, the waterplane at the immersed portions of the bow and stern are narrower than in calm water. Consequently, the average waterplane is narrower and the metacentric height decreases in comparison to calm water. As a result, the roll restoring moment of the ship changes as a function of the wave's longitudinal position along the ship.

(a) FE model showing the beam (blue) and hull surface connected by the kinematic coupling constraints (yellow)

(b) Time instance of coupled CFD simulations showing bow emerging in and out of the water surface.





(c) Illustration of flexible modes of a container ship

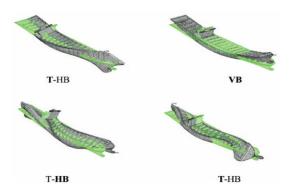
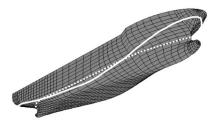
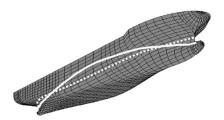


Figure 1.4. Principles of Hydroelasticity modelling (Lakshmynarayanana and Hirdaris, 2020 and Hirdaris et al., 2010).

(a) Profile of Waterline in Wave Trough (Solid) vs. Calm Water (Dotted)

(b) Profile of Waterline in Wave Crest (Solid) vs. Calm Water (Dotted)





Page | 10

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#### Figure 1.5 Ship stability in longitudinal waves (ABS, 2019)

Broaching to (Spyrou, 2011) relates with an unintentional change in the horizontal-plane kinematics of a ship. Broadly, it may be described as the "loss of heading" by an actively steered ship, that is accompanied by an uncontrollable build-up of large deviation from the desired course. Broaching is more commonly arising in waves which come from behind and propagate in a direction forming a small angle, say 10-30 deg, with the longitudinal axis of the ship.

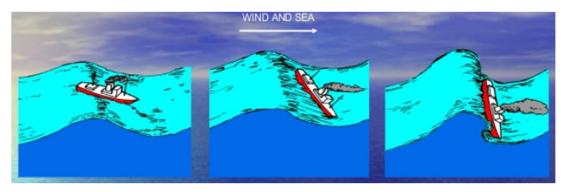


Figure 1.6. Stages of a Broaching to scenario: (a) The ship may run on crest; (b) ship stern gets too high and thus the rudder losses effect; (c) the bow pitches into trough and buries; (d) stern swings round bringing ship abeam to elements; (e) next wave will possibly break over the ship and cause severe damage.

#### 1.3. Engineering practice

Ship safety in design is assured by Classification Society Rules and Design Assessment procedures (e.g. Lloyd's Register, 2018). Class Societies develop rules on ship loads that have to be fulfilled so that ships get license to operate. Traditional Classification rules are based on accident records and experiences with ships in operations as well as theoretical and experimental studies leading to closed form / empirical criteria. Ship safety criteria that relate with maritime operations (e.g. maneuvering and stability requirements) are introduced by the International Maritime Organization (IMO) and developed in association with Flag Administrations, Classification Societies, academia and industry including non-governmental organizations (e.g. IMO, 2017 and IMO, 2013).

The last 20 years computational methods have been used to improve and extend the rules related to wave loads and seakeeping, and to investigate wave responses for newbuilds that differ substantially from those for which the rules were prepared. For reliable load predictions it may be advantageous to apply advanced, possibly costly computations to reduce a ship's scantlings or the probability of structural failures. Regarding ship motions, numerical simulations may help to estimate the probability of excessive motions and accelerations. This may help to extend the safe limits of metacentric height.

Ship dynamics can be assessed by using full-scale measurements, model tests, and numerical methods. Despite advances in theoretical ship hydrodynamics the comprehensive assessment of novel hull forms at preliminary design stage is based on model scale experiments. Development of wave basin models are cut from a plan re-drawn from the hull lines and may be costly unless 3D printing methods are employed. From naval architecture perspective it is imperative to realize that ship models used in model

tests should be as large as possible to minimize viscosity scale effects. Yet, increased model size should not influence ship dynamics in restricted waters and the size of a stock propeller is to be taken into consideration when the scale for a ship model is selected. The material of which the model is made is not important provided the model is sufficiently rigid. Wood, wax, high density closed cell foam and fiber reinforced plastic are used. Model test results can be converted to full-scale data except for the influence of viscosity, which is small in most cases. More important is the limited size of the model basin, the degree of sophistication of the equipment of the test facility, and cost and time to perform such experiments. In irregular seaways, long test runs are required to obtain representative results. Thus, for seakeeping model, experiments are used today mostly to validate numerical methods. An exception is the sloshing of fluids in tanks, where small-scale effects like wave breaking and the collapse of bubbles may be important for practical questions, but are difficult or impossible to simulate accurately. The range of model tests carried out depend on the type of the analysis or the sub-model by which the ship behavior is investigated. As an example, model tests that aim to predict powering performance of a ship comprises the resistance test, the self-propulsion test and the propeller open-water test. Seakeeping model tests usually employ self-propelled models in narrow towing tanks or broad, rectangular seakeeping basins. The models are sometimes completely free, being kept on course by a rudder operated in remote control or by an autopilot. In other cases, some degrees of freedom are suppressed (e.g. by wires). If internal forces and moments are to be determined, the model is divided into a few sections. The individual watertight sections are coupled to each other by gauges consisting of two rigid frames connected by rather stiff flat springs with strain gauges. Model motions are determined either directly or by measuring the accelerations and integrating them twice in time. Waves and relative motions of ships and waves are measured using two parallel wires penetrating the water surface. The change in the voltage between the wires is then correlated to the depth of submergence in water. The accuracy of ultrasonic devices is slightly worse. The model position in the tank can be determined from the angles between the ship and two or more cameras at the tank side. Either lights or reflectors on the ship give the necessary clear signal.

(a) Segmented model of a container ship (Hirdaris (b) Ship resistance model test et al., 2010)





Figure 1.7. Model tests

Full-scale measurements are possible only if the ship, or a similar one, has been built already. They are expensive; the wave conditions cannot be controlled; and assessing the wave conditions during the measurements with the required accuracy is usually impossible. During full scale measurements Ship motions (with accelerometers and gyros) and sometimes also global and local loads (strain gauges), loss of speed, propeller rpm and torque are all measured. Recording the seaway is difficult in full-scale measurements. This can be done either by recording measurements over many years of operation, or by deducing the maximum values during the lifetime of the ship by extrapolating the recorded

distribution of long-term measurements. The random variation of the actual sea state encountered by the ship introduces considerable inaccuracies for the predicted extreme values even if several years of measurements are available. Whilst model tests and full-scale tests provide accurate measurements of the ship performance; in ship design, the number and range of ship's characteristics modifications are wide. However; model tests and full-scale tests are limited because of the high cost and time consumption. Hence it is not certain whether the elaborated ship hull form together with the designed propeller and appendages will ensure efficient performance of the ship in all conditions. Such possibilities are offered by numerical methods such as computational fluid dynamics and finite element analysis.

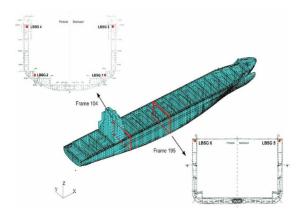


Figure 1.8 Wiring of a containership with strain gauges (Hirdaris et al., 2010)

Linear numerical hydrodynamic models are used to determine motions and structural hull girder loads for ships advancing at constant forward speed in small amplitude regular waves under various combinations of wave frequency and heading. For any seaway described by a wave spectrum, the results are combined to obtain root mean square values of loads extrapolated linearly over wave amplitude. Results for different seaway conditions are then combined to a long-term probability distribution of loads. For suitably selected design conditions, nonlinear corrections to the linear loads can be applied. If more accuracy is required, solvers for Navier-Stokes or Euler equations may be applied, which take into account the water/air interface. Today, using such a code is the obvious choice to compute free-surface waves around the ship including breaking waves, sprays, and air trapping: phenomena that should be considered to predict slamming loads in severe seas accurately. A full understanding and an accurate prediction of hydrodynamic wave body interactions is challenging. The associated nonlinear effects become critical when large-amplitude body motions and/or high surface waves are involved.

- (a) Potential flow hull penalization using Green function potential flow analysis method
- (b) Level of idealization for forward speed hydrodynamic solutions (Numbers 1–6 refer to Levels 1–6 of idealization according to

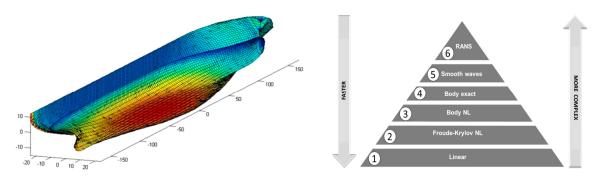


Figure 1.9. Seakeeping methods (Hirdaris et al., 2016).

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