

Aalto University

School of Engineering

MEC-E2004 Ship Dynamics (L)

Lecture 3 – Ocean waves

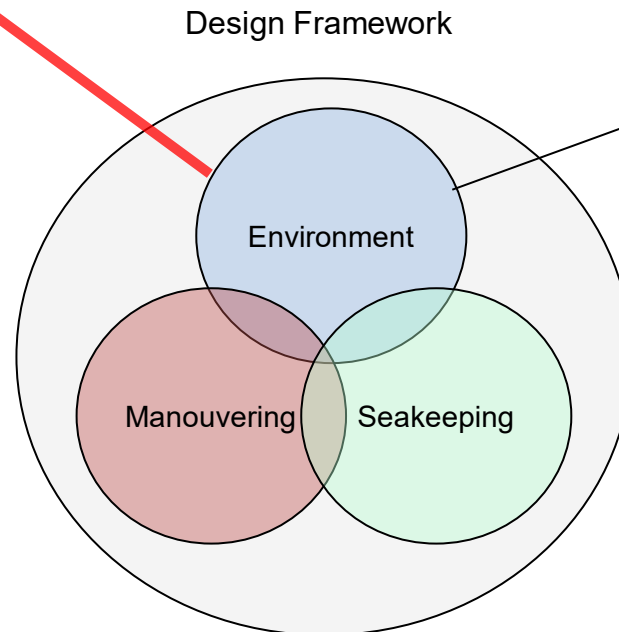
Where is this lecture on the course?

Fluid mechanics

Lecture 3:
Ocean Waves

Random Loads and Processes

Lecture 4:
Wave Spectra and statistics



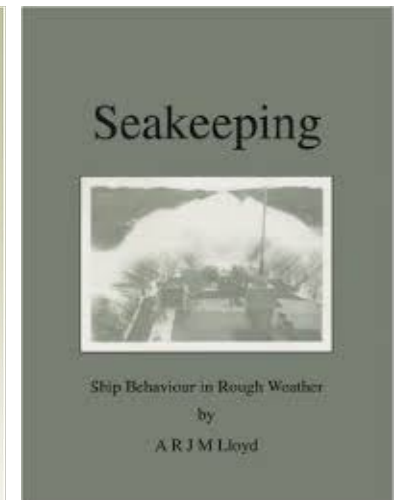
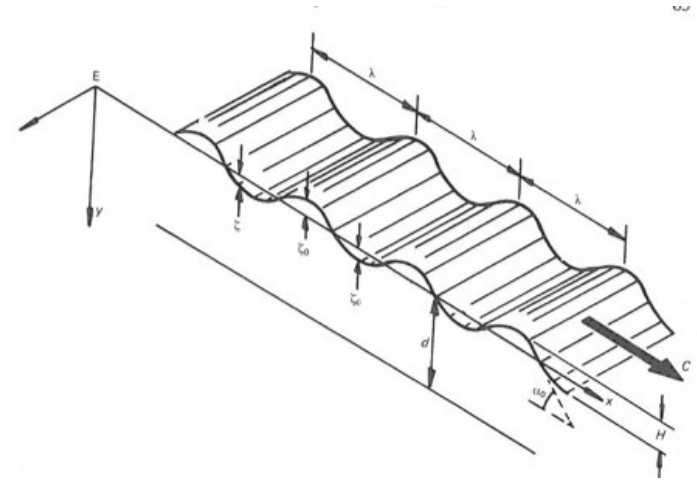
Contents

Aims :

- To explain how sea surface waves and ocean waves form and develop
- To introduce the background to potential flow wave formation theories. Specific topics include :
 - ✓ Regular and irregular waves
 - ✓ Influence of water depth
 - ✓ Energy contents of waves
- To highlight the importance of non-linear wave model idealisations

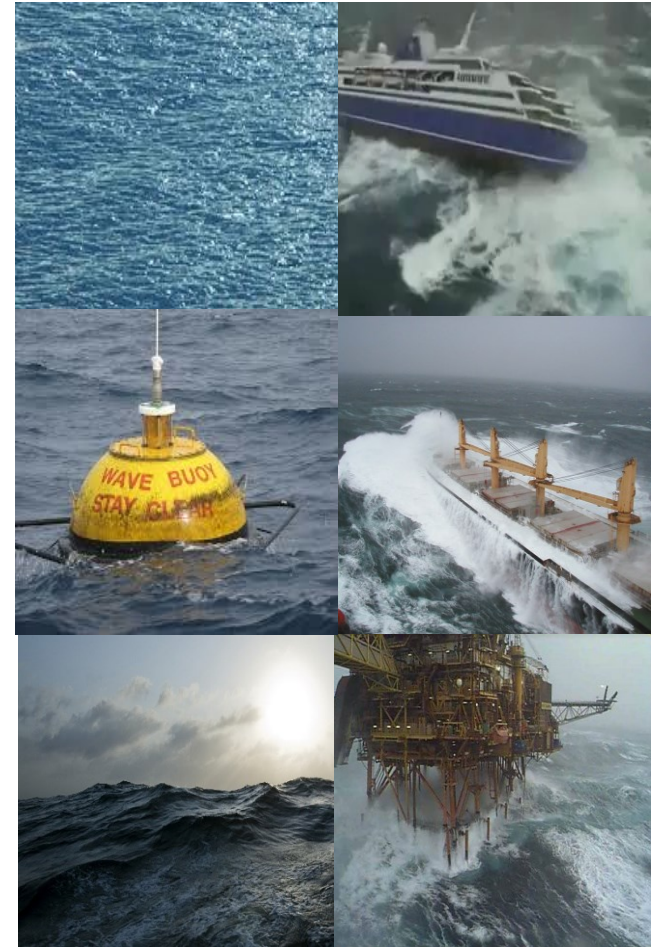
Literature :

- Lloyd, A.J.M.R., “Seakeeping, Ship Behavior in Rough Weather”, Chapters 3 & 4
- Ochi, M., “Ocean Waves - The Stochastic Approach”, Cambridge Series, Ocean Technology, 6, Chapter 1
- Lewis, E. V. “Principles of Naval Architecture - Motions in waves and controllability”, Vol. 3, Society of Naval Architects and Marine Engineers, Chapter 8
- DNV, “Environmental Conditions and Environmental Loads”, Recommended Practice DNV-RP-C205



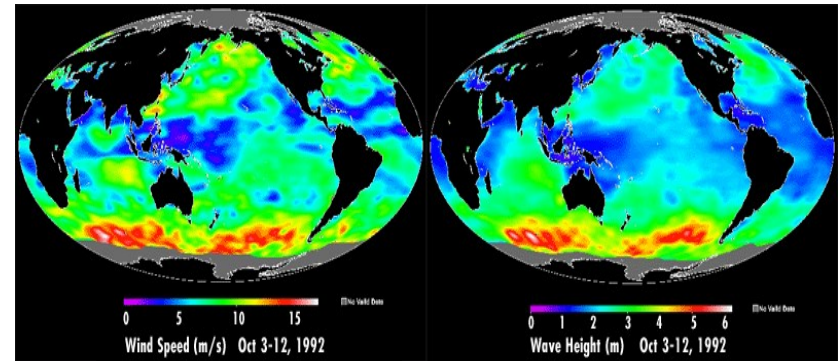
Motivation

- Ships and offshore structures are exposed to environmental effects leading to loads and motions. Those may amplify their dynamic response or their ability or manoeuvre
- The sea surface is highly nonlinear/irregular even for short time windows of very few hours. Over longer periods of time this irregularity becomes even more obvious
- Wave record measurements (e.g. measurements from wave buoys or satellite images) can help us support this visual finding
- Practically we need to assess ship motion responses and loads for these varying conditions and for this reason we have to develop our understanding on ocean waves and model wave kinematics. Understanding of both linear and nonlinear methods may be key for design and operational assesment.

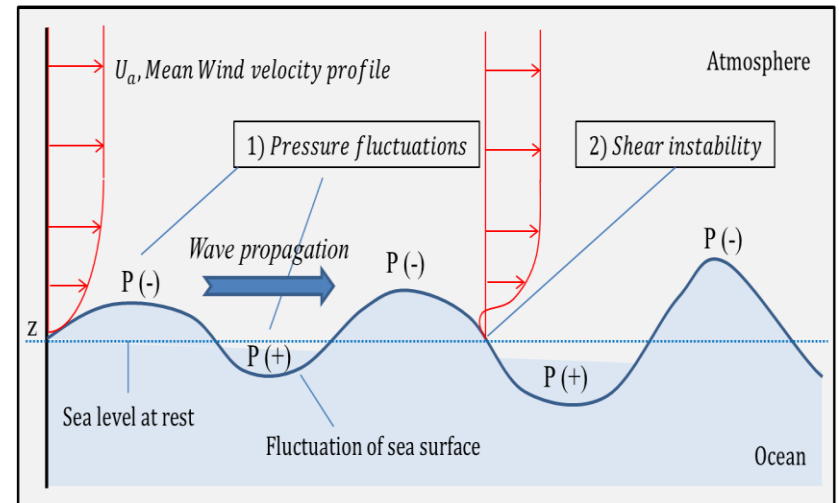


Wave Formation

- Waves are typically generated by
 - wind
 - earthquakes etc.
- Two mechanisms for wind-generated waves
 - Pressure fluctuations in the sea surface [Phillips, Phillips, O. M.: 1957, “On the Generation of Waves by Turbulent Wind”, J. Fluid Mech. 2, 417–445] <https://doi.org/10.1017/S0022112057000233>
 - Shear force in the interface of water and air [Miles, J. W.: 1957, “On the Generation of Surface Waves by Shear Flows”, J. Fluid Mech. 3, 185–204] <https://doi.org/10.1017/S0022112057000567>
- Today we accept that usually the formation of waves starts from pressure fluctuations
 - waves are enlarged by shear forces and then
 - waves interact forming longer waves

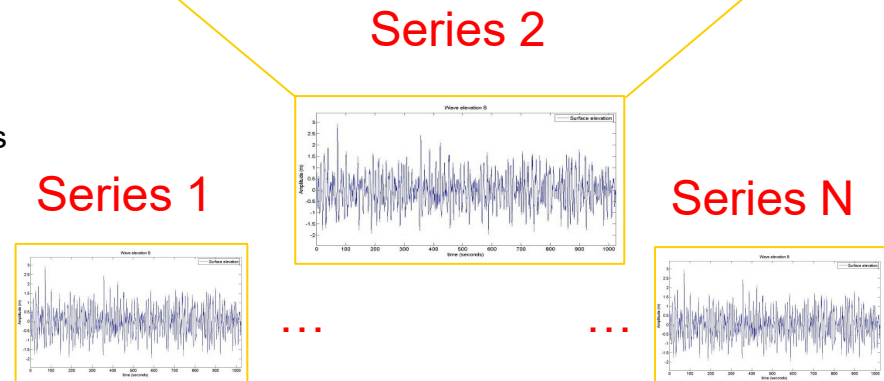
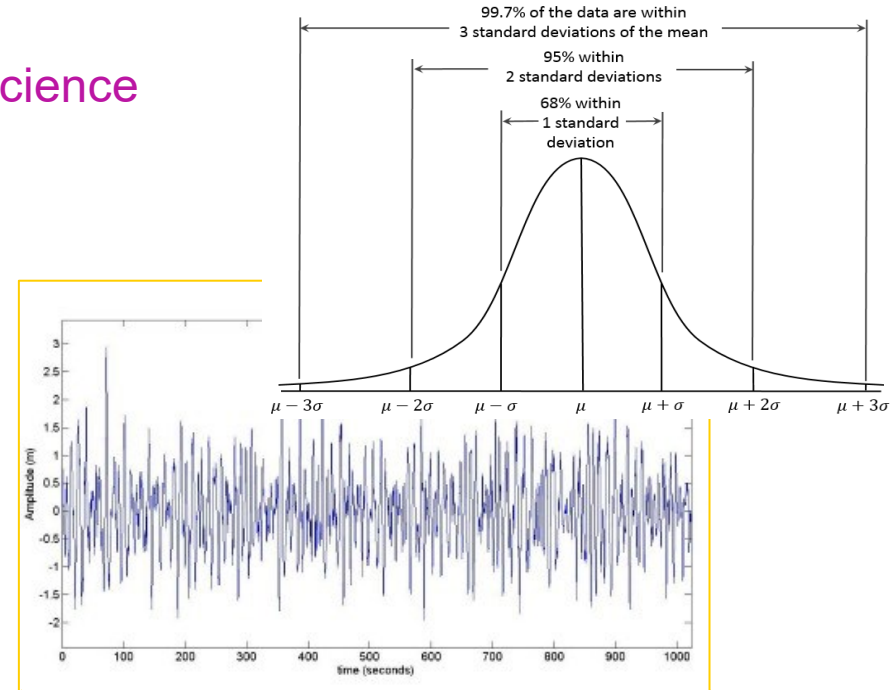


https://en.wikipedia.org/wiki/Wind_wave



Ocean Waves – background science

- In terms of statistics of wave elevation the sea state may be considered stationary (i.e. statistically steady) for short period of 30min to 3 hrs. This in theory means that :
 - Ocean waves can be described by Normal or Rayleigh probability distributions
 - Statistical properties (e.g. mean standard deviation) are constant
 - Lifetime predictions are sequence/aggregation of short term responses (within the context of linearity)
- The tools that idealise waves combine principles of
 - Fluid mechanics
 - Principles of stochastic processes / mechanics
 - Probability theory



Assignment 2

Grades 1-3:

- ✓ Select a book-chapter related to ocean waves
- ✓ Define the water depths for your ship's route and seasonal variations of wave conditions
- ✓ Based on potential flow theory, sketch what kind of waves you can encounter during typical journey (deep water, shallow water)
- ✓ Identify and select the most suitable wave spectra for your ship - Justify the selection.
- ✓ Discuss the aspects (e.g. likelihood) to consider in case of extreme events from viewpoint of operational area

Grades 4-5:

- ✓ Read 1-2 scientific journal articles related to ship dynamics
- ✓ Reflect these in relation to knowledge from books and lecture slides

- Report and discuss the work.



Example

Mediterranean or Baltic Sea

- 9 months in open water
- 3 months in ice

Route: ...

Water depth: ...

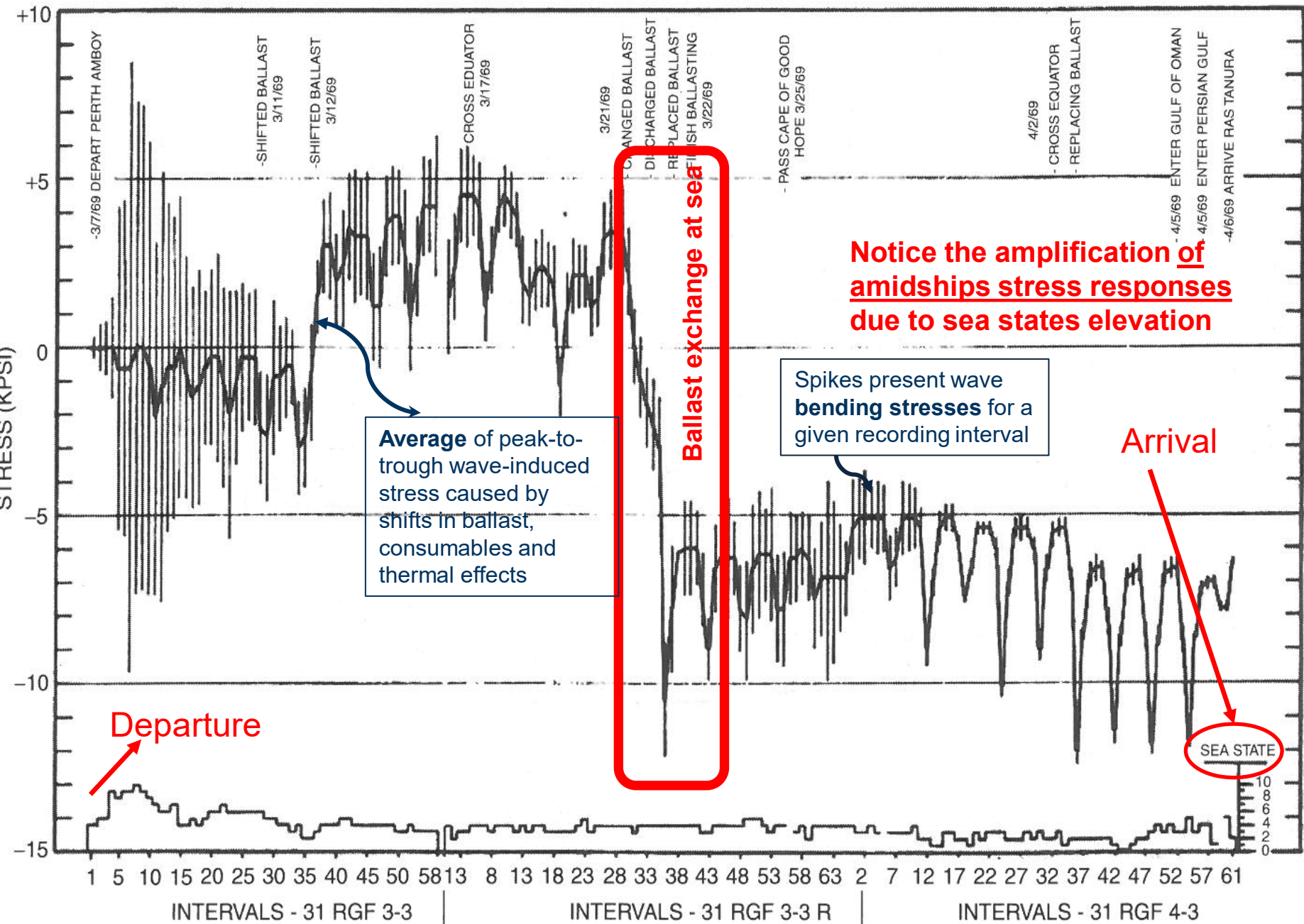


Fig. 4 Typical voyage variation in stresses, *R.G. Follis*, in ballast.

Further observations

To theoretically simulate the observations from the real signal presented we consider:

- Contribution of several components by summing all contributions

$$\zeta(x, y, t) = \sum_i \zeta_i(x, y, t)$$

- When we look at the result we can identify
 - Narrow band process, frequencies focused (useful for the assessment of loading and dynamic response)
 - White noise, frequency range is large

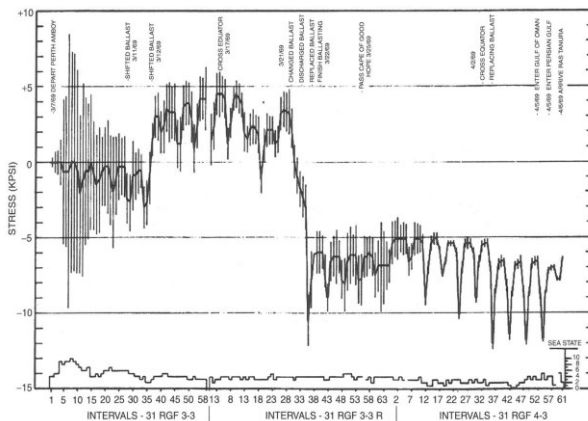
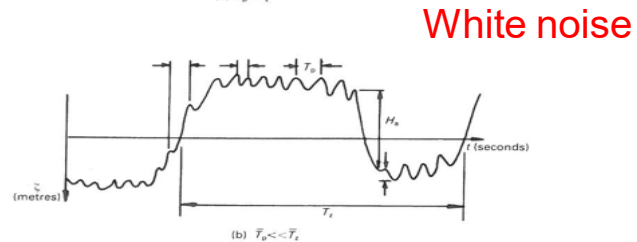
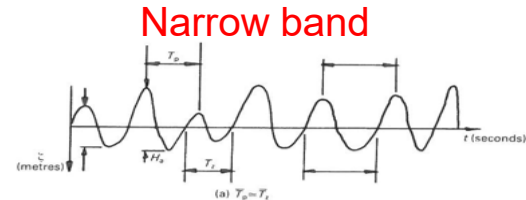
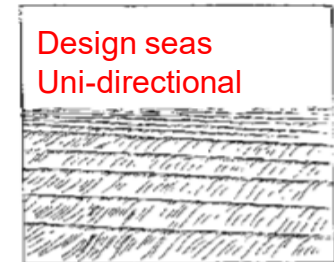
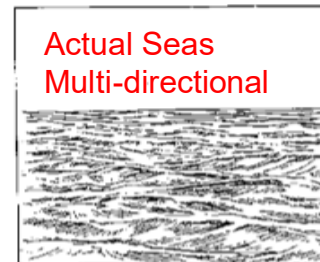
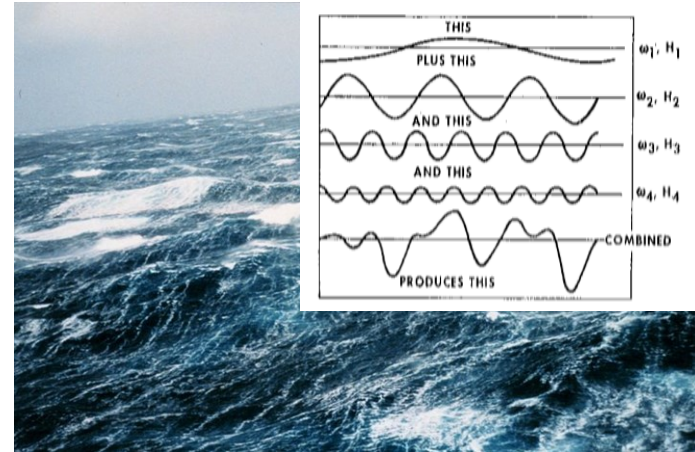


Fig. 4 Typical voyage variation in stresses, R.G. Fitch, in ballast.



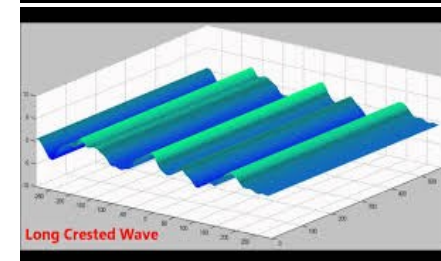
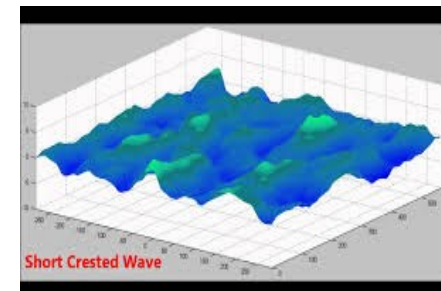
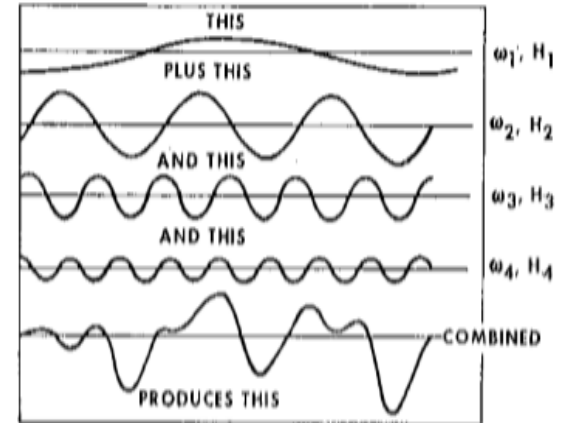
Wave patterns

- In open seas the pattern of waves observed is very complex and for this reason ocean waves are irregular or random. This means that we observe :
 - large number of waves of different length and height
 - different directions for different waves
- In the vicinity of the coastal line, it is observed that the wave pattern exhibits certain regularities
 - prevailing direction of waves progression
 - length of the waves



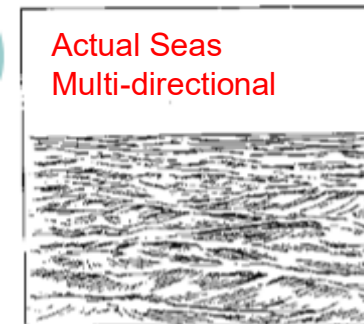
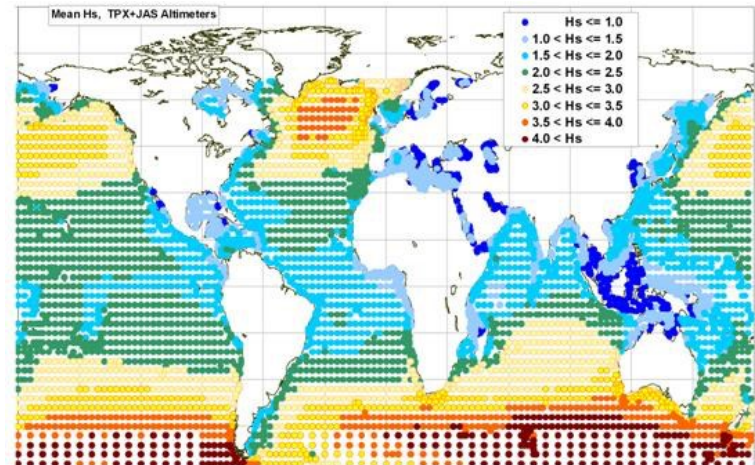
Key definitions

- A **regular wave** (also known as single wave component) has a single frequency, wavelength and amplitude (height)
- **Irregular waves** can be viewed as the superposition of a number of regular waves with different frequencies and amplitudes
- **Long-crested waves** are waves formed toward the same direction; **Short-crested waves** are waves formed toward different directions
- **Short term (ST) wave loads** generally relate with ocean or coastal wave formations over 0.5h-3h. **Long term (LT) wave loads** are assessed over life time (e.g. 20 years for ships) and comprise of a sequence of short term events. LT predictions consider multiple sea areas, routes, weather (see IACS URS11, Rec. 14, BSRA Stats etc.)



Modelling of random processes by probability theory

- A reasonably realistic yet simplified analysis assumes that the sea surface is a linear random process, i.e. a linear stochastic process. This means that
 - Probability distributions may be idealised as Gaussian distributions i.e. a normal distribution with zero mean and a variance sum of component variances (e.g. wave elevation)
 - Fourier analysis/transformations may be used to sum sinusoidal terms etc.
- From modelling perspective we tend to assume small spatial areas and time increments to ensure that variations in statistical properties are minimal



$$\zeta(x, y, t) = \sum_i \zeta_i(x, y, t)$$

$$\langle \zeta(x, y, t) \rangle = 0, \langle \zeta \rangle = \text{mean}$$

$$\langle \zeta^2 \rangle = \sum_i \langle \zeta_i^2 \rangle, \langle \zeta_i^2 \rangle = \text{variance}$$

Wave elevation patterns in 3D are defined by a mean and a variance

Wind Wave Formation

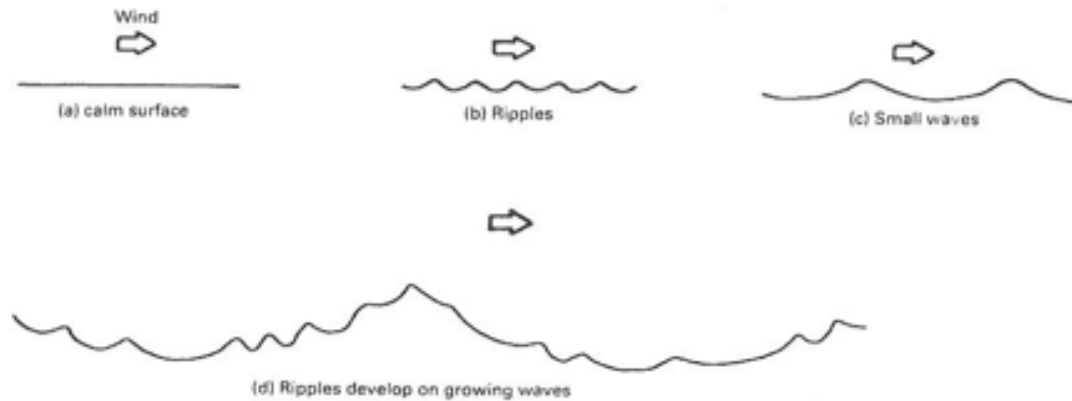
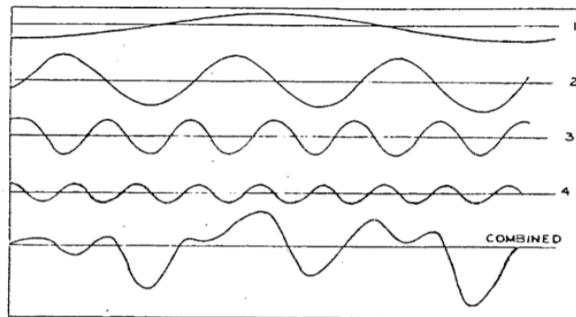
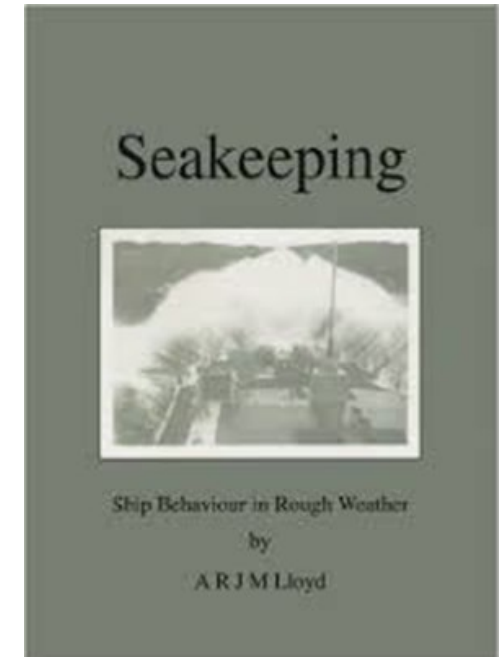


Fig. 4.1 — Wind-generated waves.



Superposition sequence over time



https://www.youtube.com/watch?v=UDyhcxR_90

Wind Wave Formation

Growth of spectrum by component accumulation

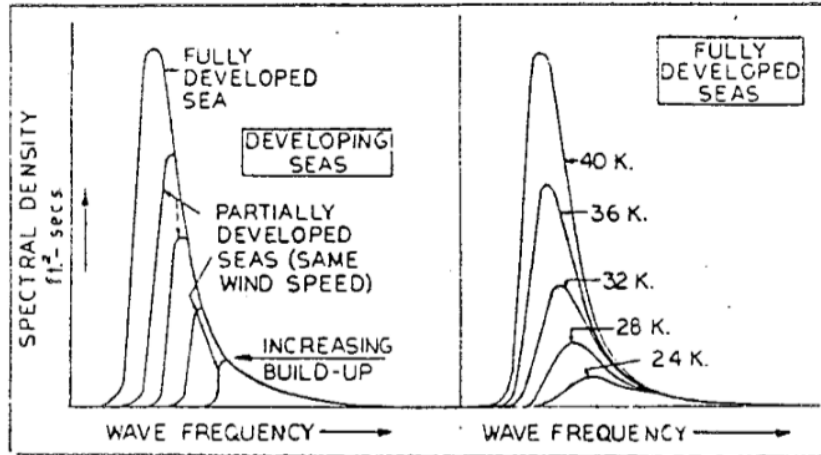


Fig. 4 Growth of spectrum

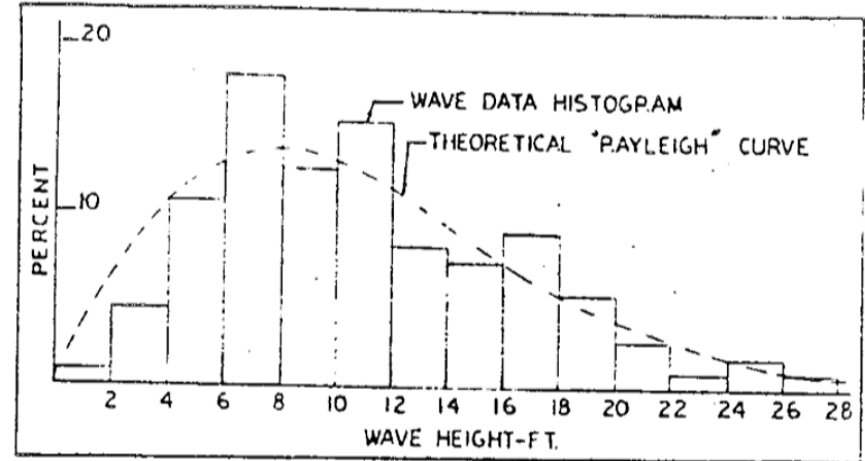


Fig. 5 Histogram of wave height measurements

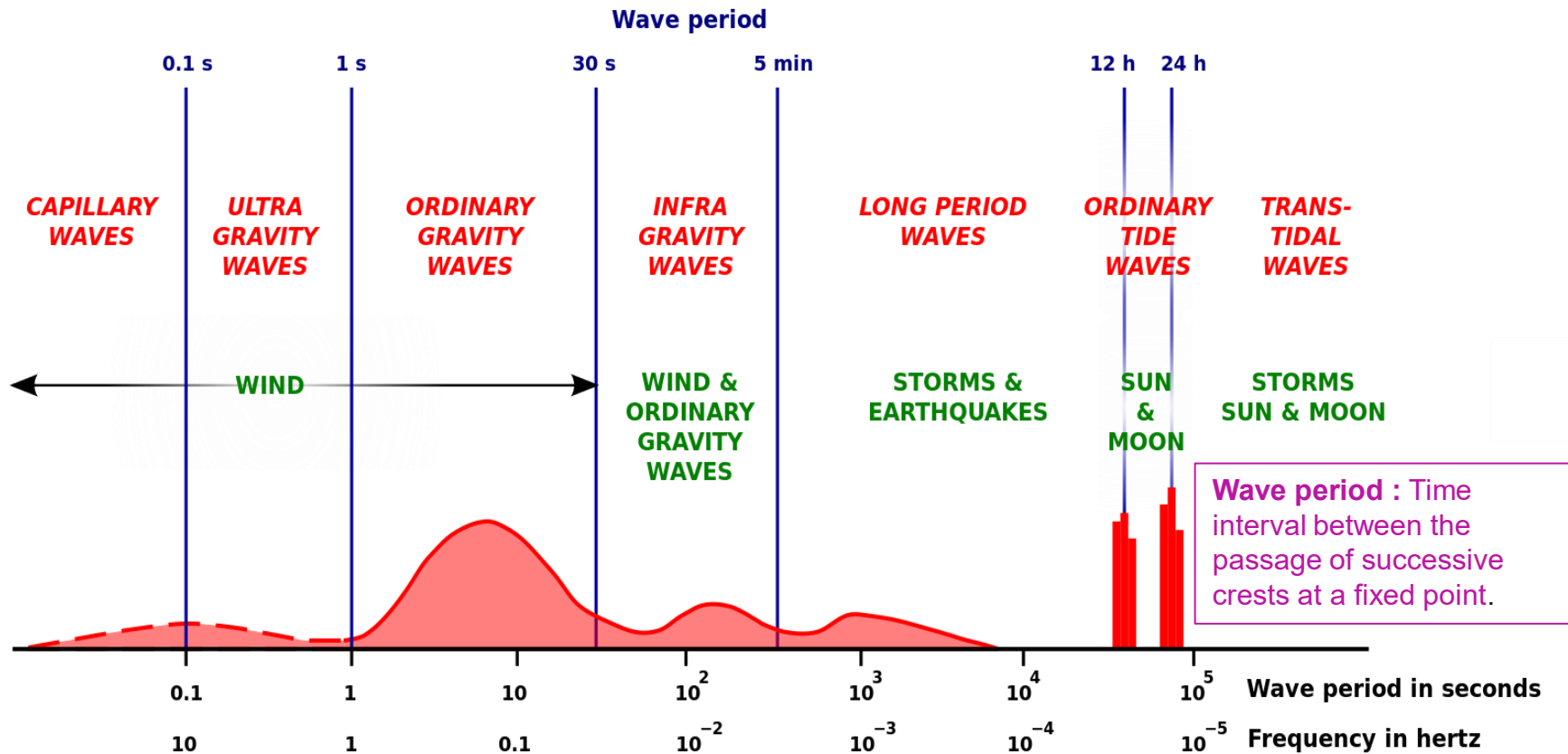
Sea Spectra Simplified

Paper Presented in April 1967 at the Meeting of the Gulf Section of SNAME, USA

By Walter H. Michel¹

A dissertation on the simple wave elements that make up the complex sea, this paper is intended to give the practicing naval architect a clearer view of how regular waves combine into an irregular pattern and how the consequent irregular behavior of a vessel at sea can be predicted on the basis of recent statistical formulations.

Classification of Waves – wave period basis



Classification

Capillary waves
 Ultra-gravity waves
 Ordinary gravity waves
 Infra-gravity waves
 Long-period waves
 Ordinary tides
 Trans-tidal waves

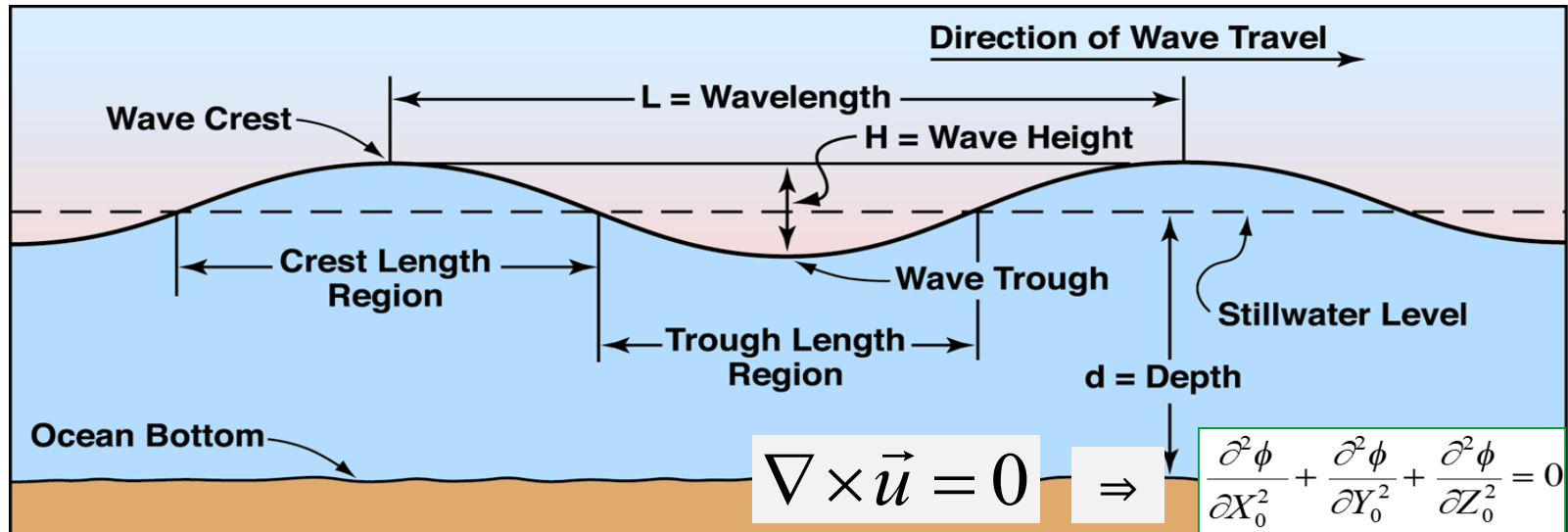
Period

less than 0.1 sec.
 from 0.1 sec. to 1 sec.
 from 1 sec. to 30 sec.
 from 30 sec. to 5 min.
 from 5 min. to 12 hours
 12 hours to 24 hours
 24 hours and up

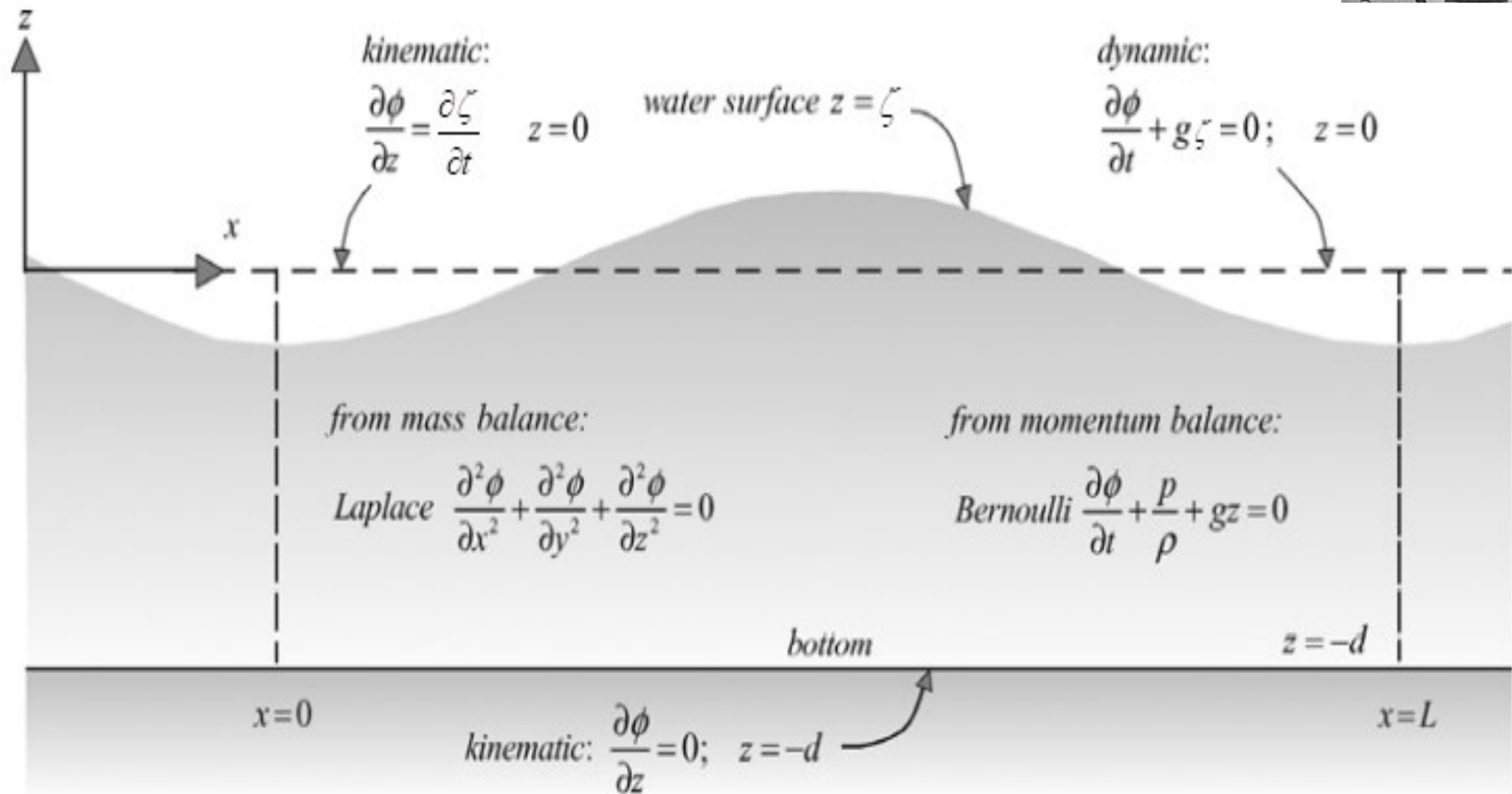
Linear wave - Potential Flow (Airy 1845)



- Single component small amplitude wave
- 2D wave motions do not change with time
- Ideal (i.e. inviscid and irrotational) flow. The water is incompressible and the effects of viscosity, turbulence and surface tension are neglected.
- Laplace velocity potential equation with boundary conditions is used to idealise (1) the sea bed and (2) the deforming sea surface



Linear wave - Potential Flow (Airy 1845)



Linear wave theory- Potential Flow

Regular sinusoidal wave equation $\zeta(X,t) = a \cos(kX - \omega t + \alpha)$ (1)

a : wave amplitude (m)

$h = 2a$: wave height (m)

$\omega = 2\pi/T$: wave frequency (rad/s)

T : wave period (s)

λ : wave length (m)

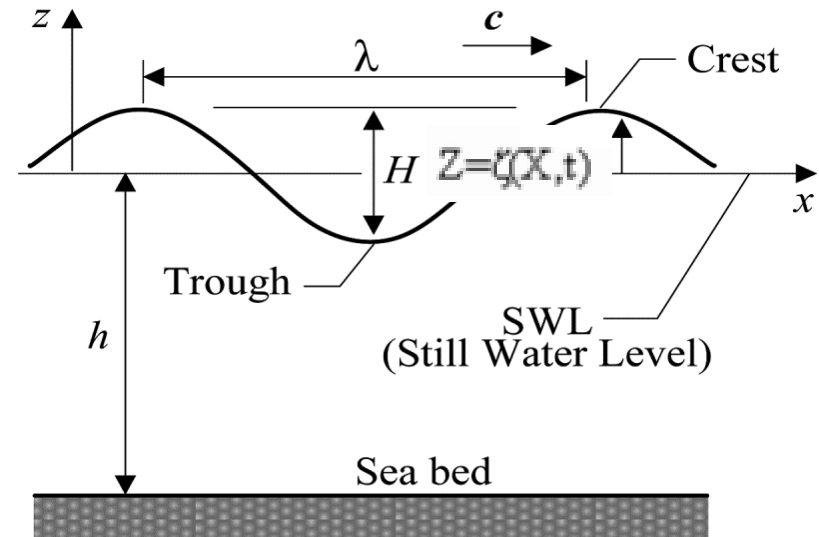
$k = 2\pi/\lambda$: wave number (1/m)

$c = \lambda/T = \omega/k$: wave or phase velocity (m/s)

d : water depth (m)

α : arbitrary phase angle (rad)

β : wave slope



Long crested wave

$$\beta = \frac{d\zeta}{dX} = -ak \sin kX \quad \text{and} \quad |\beta_{\max}| = ak$$

Linear wave theory- Potential Flow

Bernoulli's equation in 2D

- To obtain the regular wave a velocity potential is used – ideal fluid
- Bernoulli's equation takes the form

$$\frac{\partial \Phi}{\partial t} - \frac{1}{2}(u^2 + w^2) - \frac{p}{\rho} - gZ = 0 \quad (2)$$

Velocity potential →

$\frac{\partial \Phi}{\partial t}$: proportional to dynamic pressure

p : pressure (relative to atmospheric)

ρ : fluid density

g : acceleration due to gravity

u, w : fluid particle velocity components

$$u = -\frac{\partial \Phi}{\partial X} \qquad w = -\frac{\partial \Phi}{\partial Z}$$

→ **Wave elevation function**

$$Z = \zeta(X, t)$$

Linear wave theory- Potential Flow

- For regular progressive waves in deep water the velocity potential is defined as

$$\Phi(X, Z, t) = -a \frac{g}{\omega} \frac{\cosh[k(Z + d)]}{\cosh(kd)} \sin(kX - \omega t + \alpha) \quad (3)$$

- For ideal flow condition Laplace's equation should be satisfied $\nabla^2 \Phi = 0$
- The linearised dynamic free surface condition (from Bernoulli's equation after neglecting higher order terms) becomes:

$$\frac{\partial \Phi}{\partial t} - g\zeta = 0 \quad \text{or} \quad \zeta = \frac{1}{g} \frac{\partial \Phi}{\partial t} \quad \text{on } Z = 0 \quad (4)$$

- The kinematic dynamic free surface condition becomes: $\frac{\partial \zeta}{\partial t} - w = 0 \quad \text{on } Z = \zeta = 0$

This combined with Eq.(4) leads to : $\frac{1}{g} \frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial \Phi}{\partial Z} = 0 \quad \text{on } Z = 0 \quad (5)$

Linear wave theory- Potential Flow

- Bottom boundary condition (also no as no flow through BC)

$$\frac{\partial \Phi}{\partial Z} = 0 \quad \text{on } Z = -d$$

- Based on Equation (5) the dynamic free surface BC becomes :

$$\frac{1}{g} \frac{\partial^2 \Phi}{\partial t^2} = -a(-\omega) \sin(kX - \omega t + \alpha)$$

$$\frac{\partial \Phi}{\partial Z} = -a \frac{gk \sinh[k(Z+d)]}{\omega \cosh(kd)} \sin(kX - \omega t + \alpha)$$

$$\frac{\partial \Phi}{\partial Z} = -a \frac{gk}{\omega} \tanh(kd) \sin(kX - \omega t + \alpha) \quad \text{for } Z = 0$$

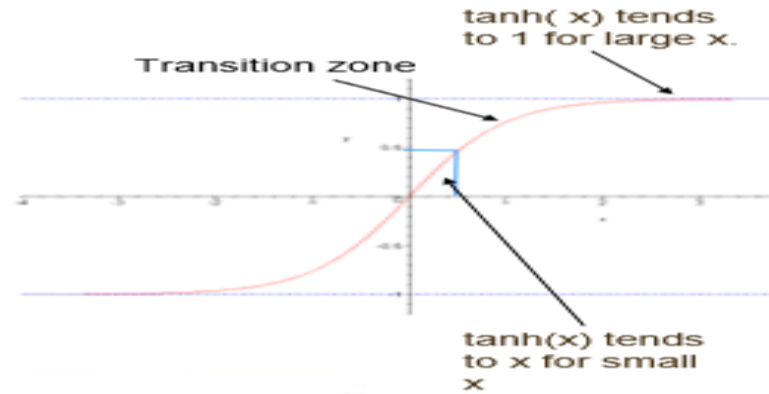
From $\frac{1}{g} \frac{\partial^2 \Phi}{\partial t^2} = -\frac{\partial \Phi}{\partial Z}$ Equation (5)

$$a\omega = \frac{agk}{\omega} \tanh(kd) \quad \text{or} \quad \omega^2 = kg \tanh(kd) \quad (6)$$

Linear wave theory- Potential Flow

- Dispersion relationship (it describes the wave travelling trajectory)

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kd)} = \sqrt{\frac{g}{k} \tanh(2\pi \frac{d}{\lambda})}$$



- Influence of water depth

Deep water

$$d / \lambda \rightarrow \infty \quad \text{or} \quad kd \rightarrow \infty \quad \therefore \tanh(kd) \rightarrow 1 \quad \therefore c = \sqrt{g / k}$$

$$d / \lambda > 0.5 \quad c^2 = \frac{\omega^2}{k^2} = \frac{g}{k} \quad \therefore \omega^2 = kg$$

NB: If the water is not deep it does not mean it is shallow i.e. between deep & shallow water we have to use the dispersion relationship

Shallow water

$$d / \lambda \rightarrow 0 \quad \text{or} \quad kd \rightarrow 0 \quad \therefore \tanh(kd) \rightarrow kd \quad \therefore c = \sqrt{gd}$$

Linear wave theory- The influence of water depth

- Dependence on the distance from free surface in deep water

$$\frac{\cosh[k(Z+d)]}{\cosh(kd)} = \frac{\cosh(kZ)\cosh(kd) + \sinh(kZ)\sinh(kd)}{\cosh(kd)}$$

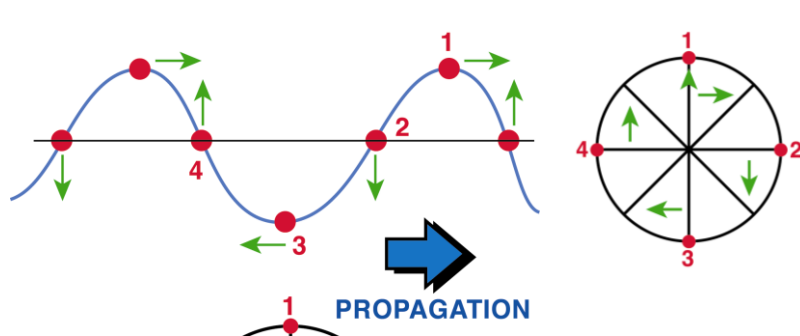
$$= \cosh(kZ) + \sinh(kZ)\tanh(kd)$$

$$= \cosh(kZ) + \sinh(kZ) \quad \text{in deep water}$$

$$= e^{kZ}$$

- Using this exponential variation and Bernoulli's Eq. (ignoring higher order terms) the pressure relative to atmospheric (anywhere (i.e. X,Z) in the fluid is :

$$\Delta p = \rho \left[\frac{\partial \Phi}{\partial t} - gZ \right] = \rho g [e^{kZ} \zeta(X,t) - Z] \quad \text{in deep water (7)}$$



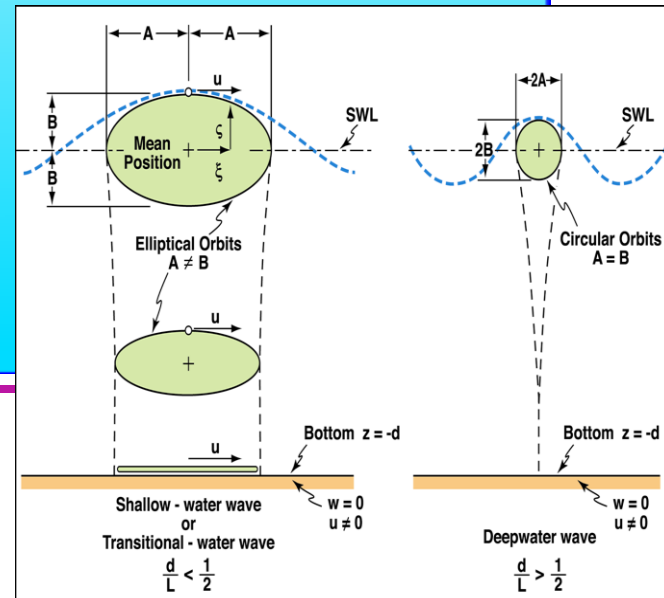
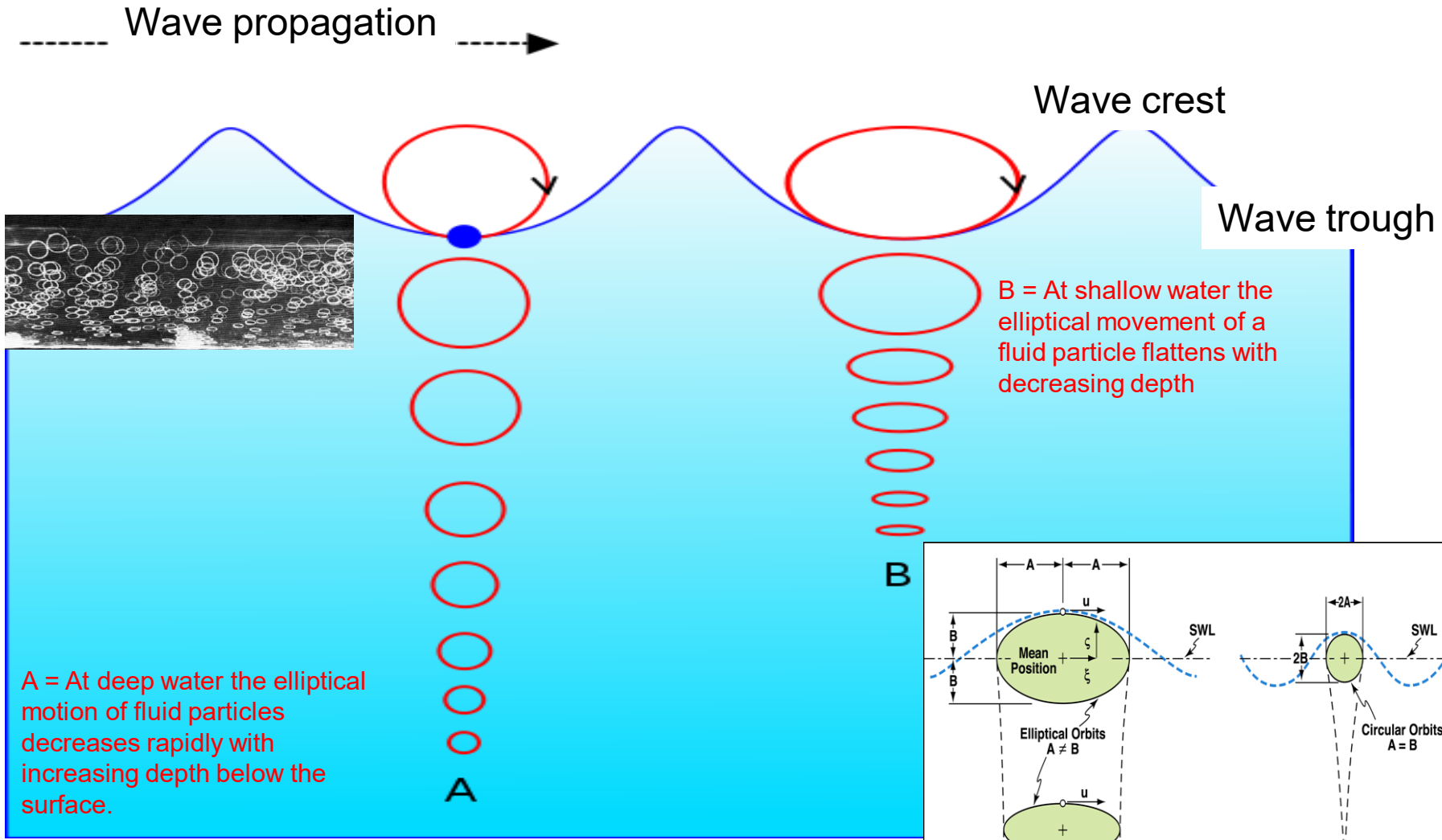
(HORIZONTAL)

$$U = \frac{H}{2} \frac{gT}{L} \frac{\cosh [2\pi(z+d)/L]}{\cosh (2\pi d/L)} \cos \left(\frac{2\pi x}{L} - \frac{2\pi t}{T} \right)$$

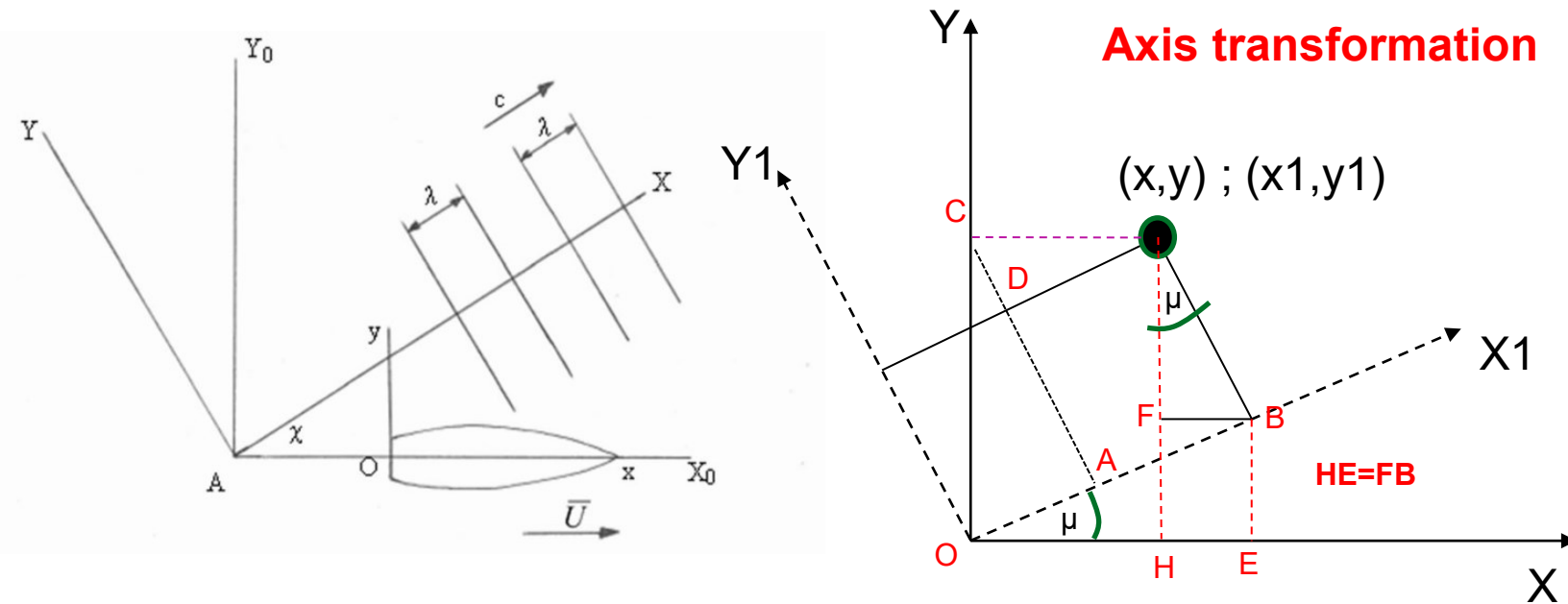
(VERTICAL)

$$W = \frac{H}{2} \frac{gT}{L} \frac{\sinh [2\pi(z+d)/L]}{\cosh (2\pi d/L)} \sin \left(\frac{2\pi x}{L} - \frac{2\pi t}{T} \right)$$

Linear wave theory- The influence of water depth



Linear wave theory- presense of moving ship



$$X_1 = \mathbf{OA+AB} = y \sin \mu + x \cos \mu$$

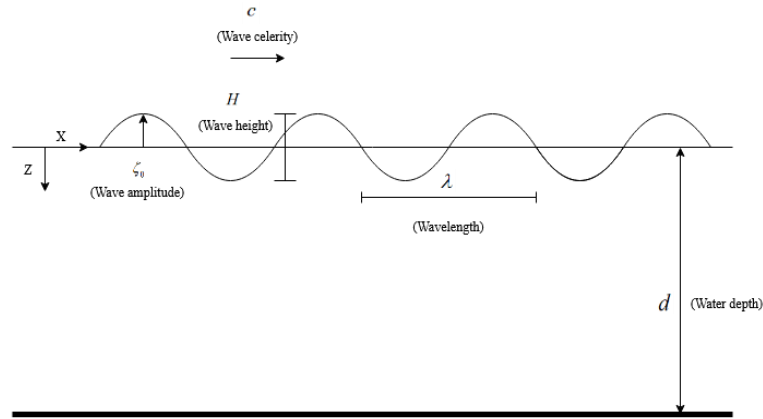
$$Y_1 = \mathbf{AC-DC} = y \cos \mu - x \sin \mu$$

$$X = \mathbf{OE-HE} = x_1 \cos \mu - y_1 \sin \mu$$

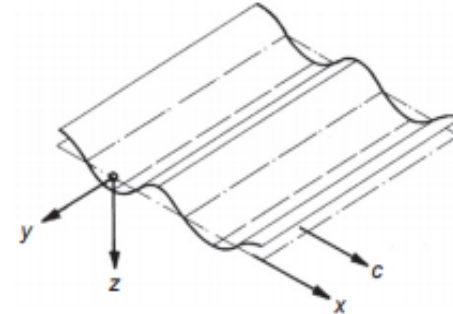
$$Y = \mathbf{EB+FG} = x_1 \sin \mu + y_1 \cos \mu$$

Linear wave theory- presense of moving ship

(a) 2D surface elevation



(b) 3D surface formation



ζ : instantaneous depression of water surface below mean level ($y = 0$)

ζ_a : wave amplitude from mean level ($y = 0$) to a crest or trough

H : wave height (always twice the wave amplitude)

λ : wave length (distance from one crest – or trough – to the next)

c : wave celerity

T : wave period (time interval between successive crests or troughs passing a fixed point)

α : the instantaneous wave slope (gradient of the surface profile)

α_0 : maximum wave slope or wave slope amplitudes

H/λ : wave steepness

Linear wave theory- presense of moving ship

- Considering transformation of coordinates for a regular wave propagating at an angle χ (i.e. from $AXYZ$ to $AX_0Y_0Z_0$ for $Z = Z_0$ axes systems) becomes

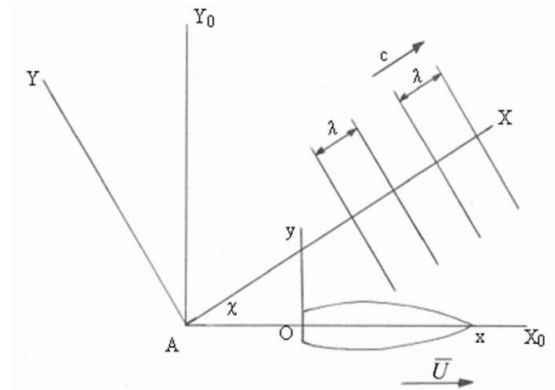
$$X = X_0 \cos \chi + Y_0 \sin \chi$$

- The transformation coordinate to OXZ

$$X_0 = x + \bar{U} t \quad \text{and} \quad y = Y_0$$

- Combining both leads to

$$\begin{aligned} \zeta(x, y, t) &= a \cos(kx \cos \chi + ky \sin \chi + k\bar{U} \cos \chi t - \omega t + \alpha) \\ &= a \cos(kx \cos \chi + ky \sin \chi - \omega_e t + \alpha) \end{aligned}$$

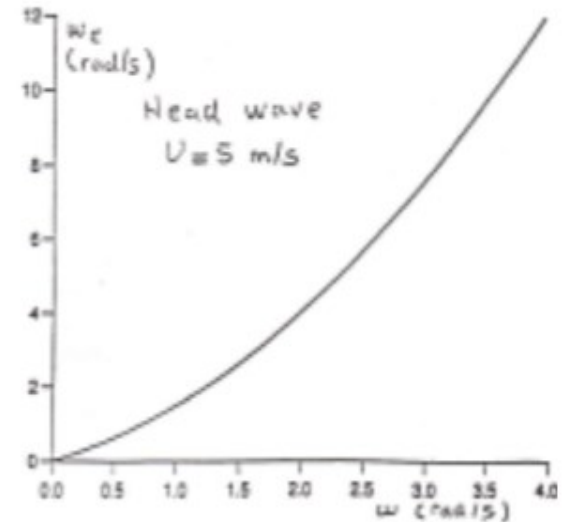


- Where ω_e is the encounter frequency, i.e. the frequency of the wave as seen by an observer of the ship moving in waves

Linear wave theory- encounter frequency

$$\begin{aligned}\omega_e &= \omega - k\bar{U}\cos\chi \\ &= k(c - \bar{U}\cos\chi) \\ \omega_e &= \omega - \frac{\omega^2\bar{U}}{g}\cos\chi \quad \text{in deep water}\end{aligned}$$

$$\text{Head waves } \chi = 180^\circ \therefore \omega_e = \omega + \frac{\omega^2\bar{U}}{g}$$



Quartering (bow) waves, e.g. $\chi = 135^\circ$

Beam waves $\chi = 90^\circ \therefore \omega_e = \omega$

Linear wave theory- encounter frequency

Following waves $\chi = 0^\circ \therefore \omega_e = \omega - \frac{\omega^2 U}{g} = \omega \left(1 - \omega \frac{U}{g} \right)$

$\omega_e = 0$ when $\omega = 0$

$\omega_e = 0$ when $\omega \frac{U}{g} = 1$ (i.e. when $\bar{U} = c$)

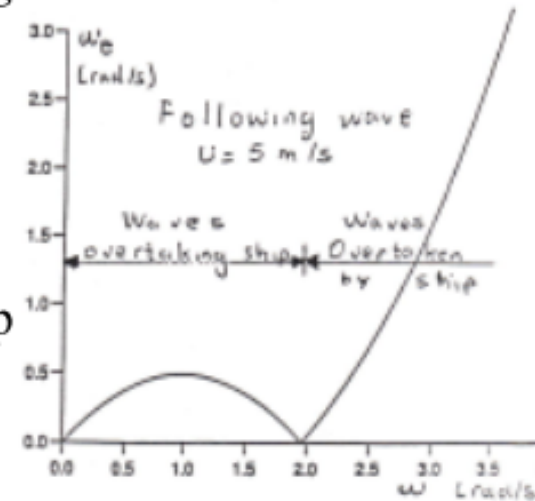
$\omega_e < 0$ when $\omega \frac{U}{g} > 1$ (i.e. when $\bar{U} > c$)

ω_e treated as + ve; following waves overtaken by ship

For $\omega \frac{U}{g} < 1$ following waves overtake ship

ω_e has a maximum value

when $\frac{d\omega_e}{d\omega} = 0 = 1 - 2\omega \frac{U}{g}$ and the max. value is $\frac{g}{4U}$



Same relationship is also valid for any $\chi < 90^\circ$

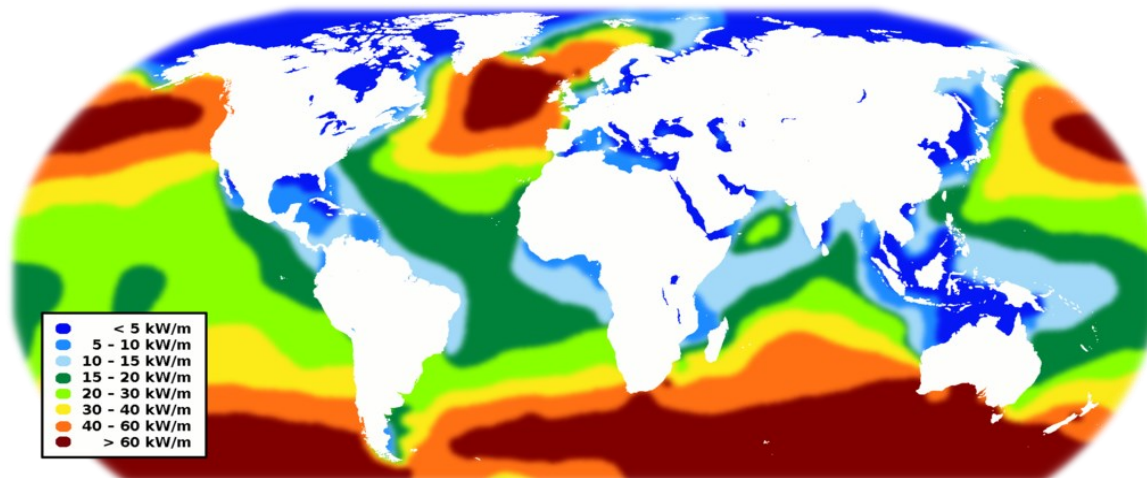
Quartering (stern) waves, e.g. $\chi = 45^\circ$

WAVE ENERGY AND POWER

Kinetic + Potential = Total Energy of Wave System

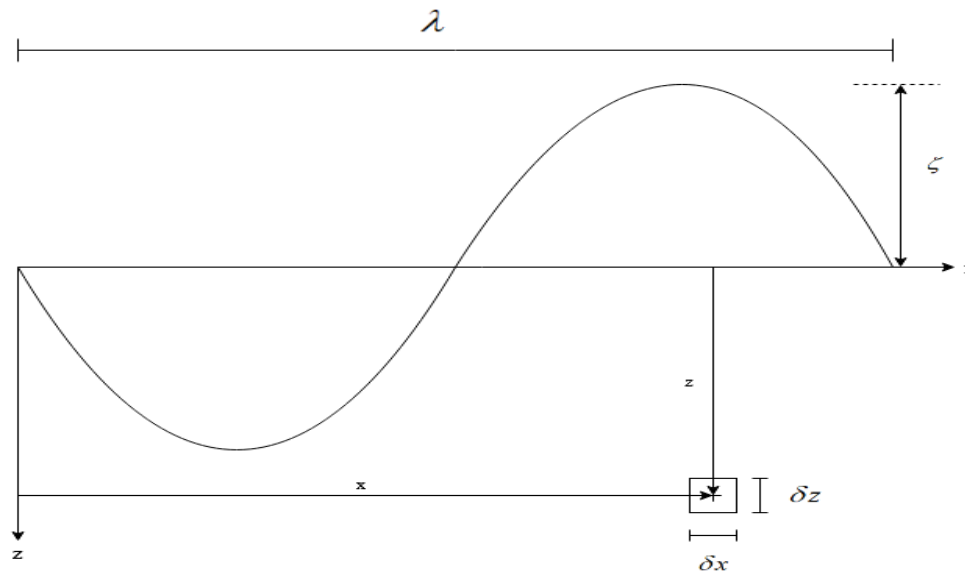
Kinetic: due to H₂O particle velocity

Potential: due to part of fluid mass being above trough. (*i.e.* wave crest)



World map showing wave energy flux in kW per meter wave front

Wave Energy – potential energy idealisation

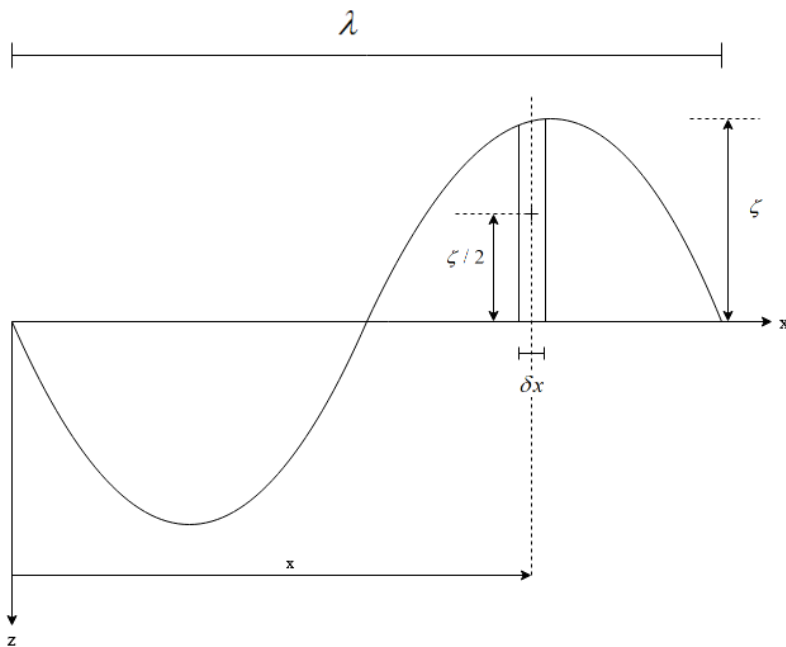


$$P.E. = mgh = (\rho g \delta x) \frac{\zeta}{2} = \frac{\rho g \zeta^2 \delta x}{2}$$

If we integrate this energy over the entire wavelength, we get the potential energy of the wave per unit width as:

$$E_{PE} = \frac{\rho g \lambda \zeta_0^2}{4}$$

Wave Energy – kinetic energy idealisation



- The wave has also kinetic energy (*K.E.*) and if the total velocity of the segment is q then the kinetic energy of the segment is:

$$K.E. = \frac{1}{2} m q^2 = \frac{\rho q \delta x \delta z}{2}$$

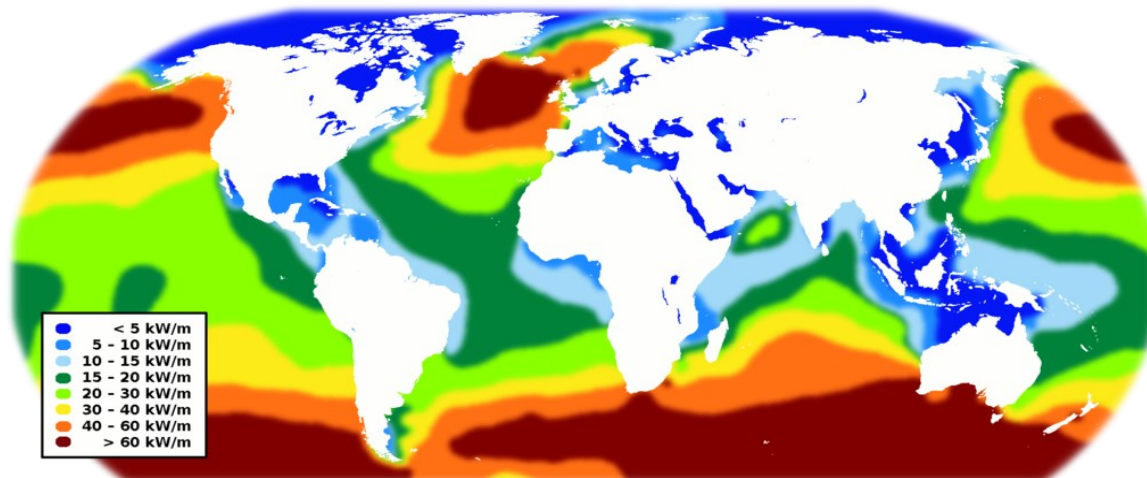
- Consequently, integrating this over the full wave length and by utilizing the relationship between the speed and wavelength gives the total *K.E.* per unit width as:

$$E_{KE} = \frac{\rho g \lambda \zeta_0^2}{4}$$

WAVE ENERGY AND POWER

Kinetic + Potential = Total Energy of Wave System

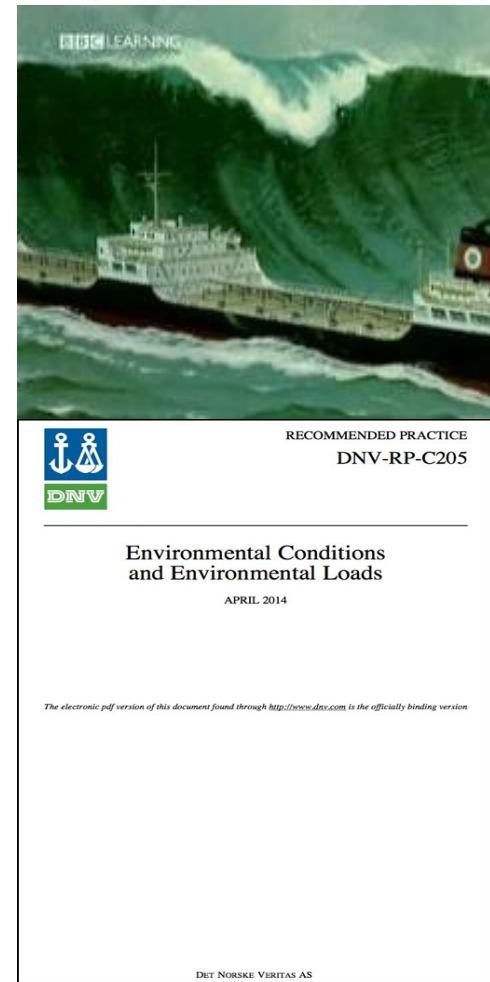
$$E_{TOT} = E_{PE} + E_{KE} = \frac{\rho g \zeta_0^2}{2}$$



World map showing wave energy flux in kW per meter wave front

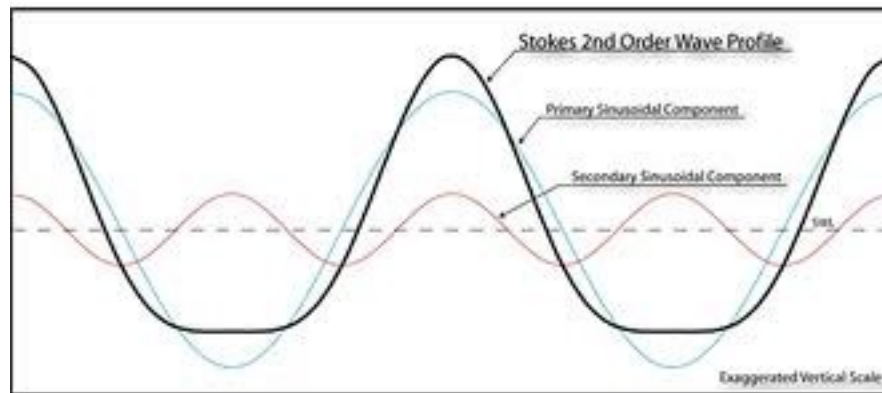
Deviations from Linear Wave Theory

- A linear wave model is very useful in practical engineering work. This is because :
 - It is easy to use,
 - it complies well with the linear modelling of ship responses
 - it enables modelling of the sea by superimposing waves of different lengths and heights
- In some cases, certain non-linear effects have to be considered
 - The information provided by the linear wave model up to the still water level is not sufficient, e.g. we are dealing with local wave pressure loads on ship's side shell
 - An increase of wave steepness results in wave profiles that differ from the ideal cosine form
 - In some cases, certain non-linear effects have to be considered
- Suitably validated NL wave theories and hence NL wave idealisations can provide
 - Better agreement between theoretical and observed wave behavior.
 - Useful in calculating mass transport.

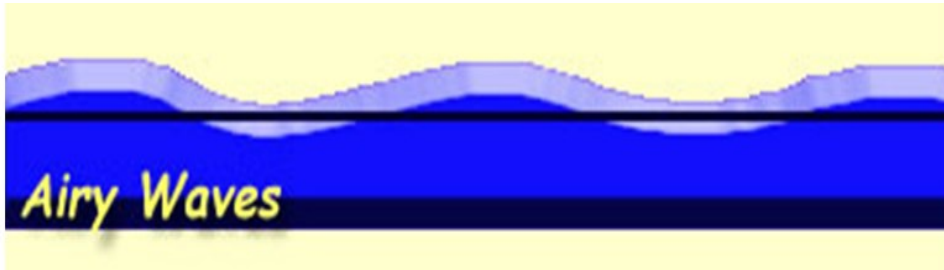


Stokes waves - Introduction

- An elegant and efficient way of dealing with steep regular (inviscid) waves in intermediate and deep water (e.g. coastal and offshore structures)
 - Stokes series expansion is used mathematically (1st, 2nd, 3rd,... order)
 - Cannot handle shallow water waves
- What physically differs them from the linear model is the fact that they attempt to evaluate the pressures up to the actual water surface giving this way better accuracy for local pressures

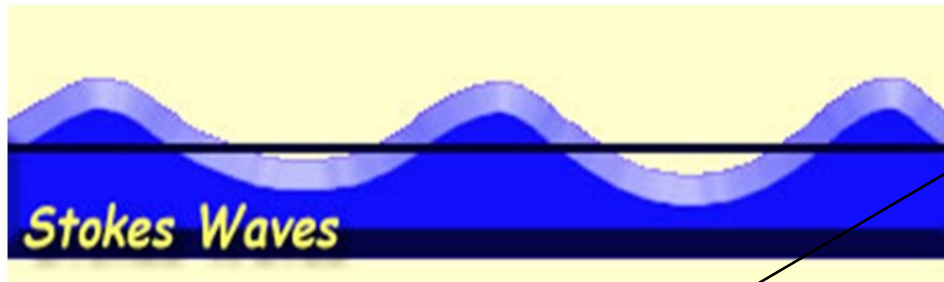


Deviations from Linear Theory – Airy vs Stokes waves



$$\phi = \frac{H}{2} \frac{g}{\omega} \frac{\cosh[k(d+z)]}{\cosh(kd)} \sin(kx - \omega t)$$

$$\eta(x, z, t) = a \sin(kx - \omega t)$$



$$\phi = \frac{H}{2} \frac{g}{\omega} \frac{\cosh[k(d+z)]}{\cosh(kd)} \sin(kx - \omega t) -$$

$$\frac{3}{32} \frac{H^2}{\omega} \frac{\cosh[2k(d+z)]}{\sinh^4(kd)} \sin 2(kx - \omega t)$$

$$\eta(x, z, t) = \frac{H}{L} \sin(kx - \omega t) +$$

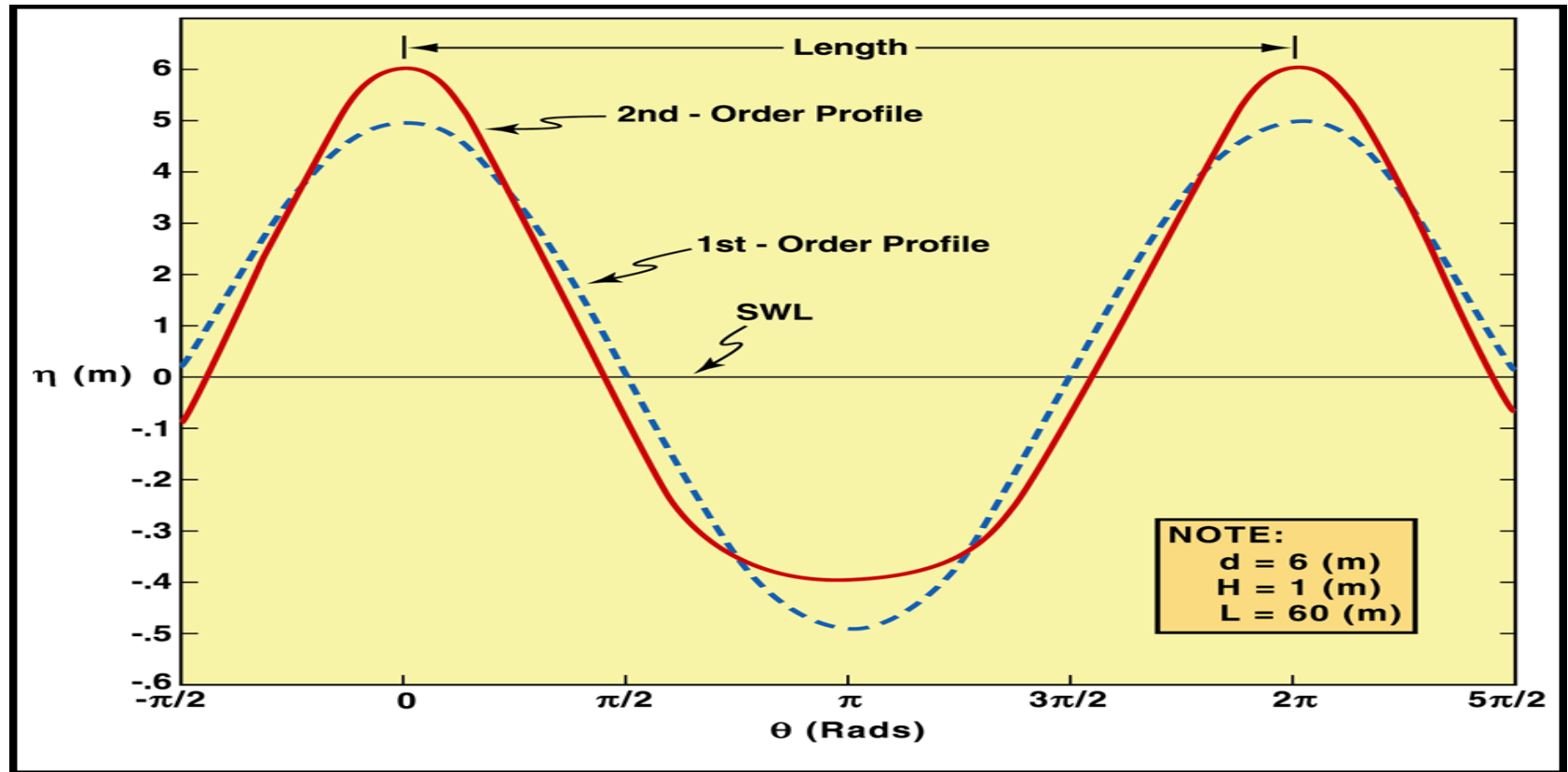
$$\frac{H^2 k}{16} \frac{\cosh kd}{\sinh^3(kd)} (2 + \cosh(2kd) \cos 2(kx - \omega t))$$

NL terms

Deviations from Linear Theory – Airy vs Stokes waves

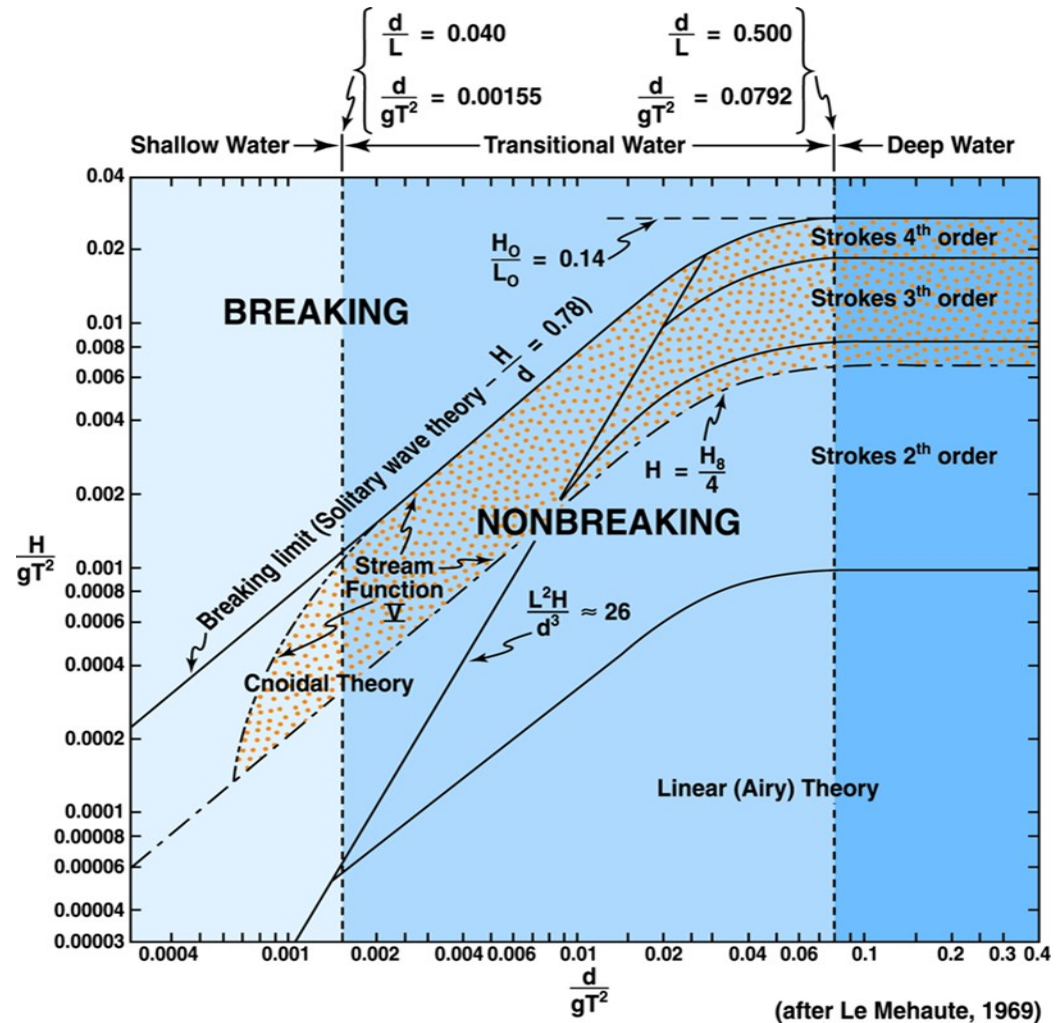
Comparison of second-order Stokes' NL wave profile with linear profile :

Higher order waves are more peaked at the crest, flatter at the trough and with distribution slightly skewed above SWL



Deviations from Linear Theory – Airy vs Stokes waves

- **Linear Wave Theory:** Simple, good approximation for 70-80 % engineering applications.
- **Nonlinear Wave Theory:** Complicated, necessary for about 20-30 % engineering applications.
- Both results are based on the assumption of non-viscous flow.

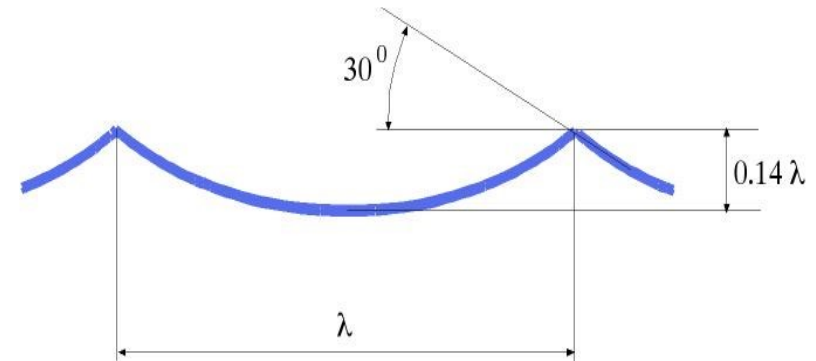
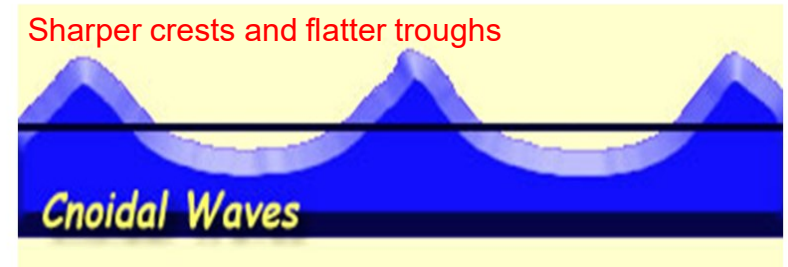


Cnoidal Waves - trochoidal steepest waves

- The steepest possible waves (just before they break, e.g. $H/l=1/7$). They are usually represented by a trochoid as steep wave is not of a cosine form
- An increase of wave steepness is associated with the sharpening of the crests while the troughs get flatter
- We remove the irrotationality assumption from fluid mechanics



Crossing swells, consisting of near-cnoidal wave trains. Photo taken from Phares des Baleines (Whale Lighthouse) at the western point of Île de Ré (Isle of Rhé), France, in the Atlantic Ocean.



Solitary Waves - the waves of translation

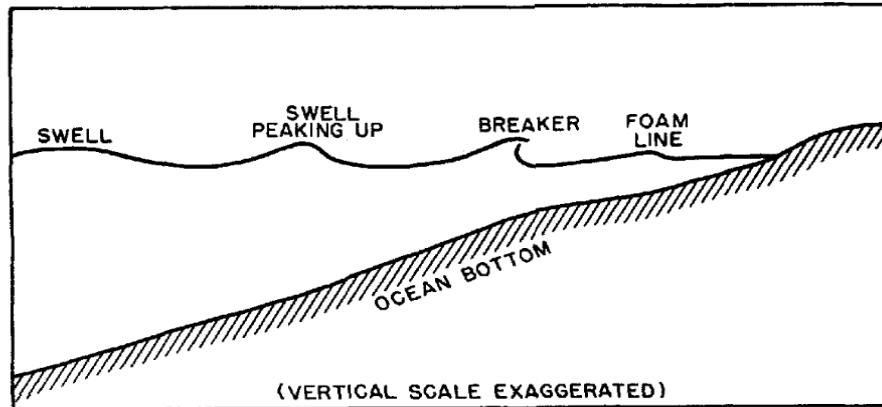
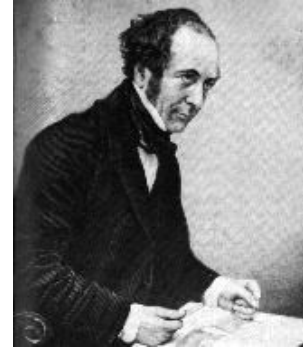


FIGURE 1. Schematic presentation of change in wave shape as wave advances into shallow water.



www.macs.hw.ac.uk/~chris/scott_russell.html

<https://www.youtube.com/watch?v=JD32kkoFU3Y>

THE SOLITARY WAVE THEORY AND ITS APPLICATION TO SURF PROBLEMS*

By WALTER H. MUNK

Scripps Institution of Oceanography and Institute of Geophysics, University of California

Introduction

The purposes of this paper are: (a) to give a summary of useful relationships derived by means of the solitary wave theory, and to plot these relations using dimensionless parameters for the purpose of making the theory accessible to numerical examples;† (b) to review various studies at the Scripps Institution dealing with the application of this theory to surf problems; and (c) to discuss the problem of sand transport in or near the surf zone, in the light of the solitary wave theory.

This investigation represents part of a general project undertaken during the war for the purpose of providing useful wave forecasts for the amphibious forces. By 1943, methods for forecasting sea and swell had been developed^{1,2} and a study of the transformation of waves in shallow water was initiated for the purpose of extending the wave forecasts right into the surf zone. It should be noted that the outer edge of the surf zone (the greatest depth where waves break) is usually the most critical from the point of view of bringing landing craft ashore.

The problem was attacked in three ways: (a) by field observations along the East Coast by the Woods Hole Oceanographic Institution and along the West Coast by the Scripps Institution of Oceanography; (b) by laboratory observations at the Beach Erosion Board wave tank, in Washington, D. C., and later at the Department of Engineering of the University of California in Berkeley, California; (c) by theoretical studies.

A theoretical investigation by Burnside,³ based on the assumptions of constancy of wave periods, conservation of energy, and the linear shallow water (Airy) wave theory, reveals that the waves decrease somewhat in height after entering shallow water, reach a minimum height and then increase.^{4,5} The initial decrease in wave height had been noticed by O'Brien in laboratory investigations. A comparison between the subsequent increase in height as derived from Burnside's equations with that obtained from field and laboratory observations mentioned above, showed the computed increase to be considerably smaller than the observed increase. This discrepancy became increasingly large the nearer one came to the breaking zone, the zone most important for practical forecasts.

One reason for this discrepancy is contained in an assumption underlying the linear Airy theory, namely that the wave height be small compared to

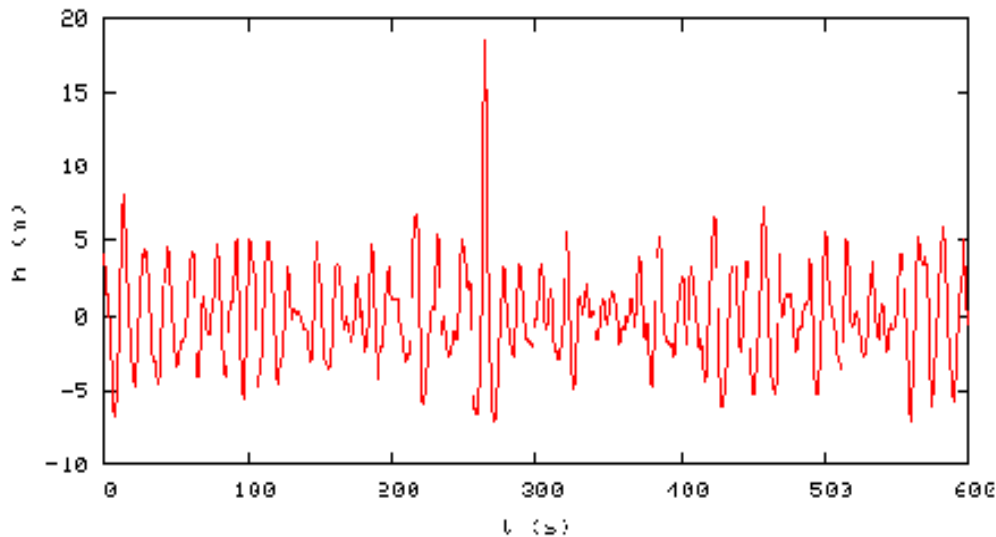
* Contribution from the Scripps Institution of Oceanography, New Series No. 406. This work represents results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department under contract with the University of California.

† PLATES 1-12 at end of the paper.

<http://onlinelibrary.wiley.com/doi/10.1111/j.1749-6632.1949.tb27281.x#abstract>

Freak Waves

Rogue waves (also known as freak waves, monster waves, episodic waves, killer waves, extreme waves and abnormal waves) are large, unexpected and suddenly appearing surface waves.



The Draupner wave, a single giant wave measured on New Year's Day 1995, finally confirmed the existence of freak waves, which had previously been considered near-mythical.

<https://www.youtube.com/watch?v=eMBU1eXDYDc>

Tsunamis

- Seismic sea waves comprising of series of water waves caused by the displacement of a large volume of a body of water, generally in an ocean or a large lake. They are generated by earthquakes, volcanic eruptions and other underwater explosions, landslides, glaciers, meteorite impacts and other disturbances above or below water all have the potential to generate a tsunami
- Tsunami waves do not resemble normal sea waves. They consist of a wavelength that is far longer than usual. They may initially resemble a rapidly rising tide, and for this reason they are often referred to as tidal waves.
- They comprise of series of waves with periods ranging from minutes to hours, arriving in a "wave train" (L=10km, very shallow, Speed 800km/h)



Although the impact of tsunamis is limited to coastal areas their destructive power can be enormous

<https://www.youtube.com/watch?v=oWzdgBNfhQU>

Summary

- The lecture reviewed
 - The wave formation mechanisms
 - The linear theory of surface waves and associated terminology
 - Deviations from the theory
- Generally sea surface can be described as sum of sinusoidal wave components traveling in:
 - Same direction, long-crested waves
 - Different directions, short-crested waves
- When some of wave properties are known, the others can be estimated using the potential flow theory
- For design important components are
 - Energy contents
 - Wave period/length and amplitude
 - The slope of waves
- In some cases non-linearities exist in waves where the potential theory breaks down

