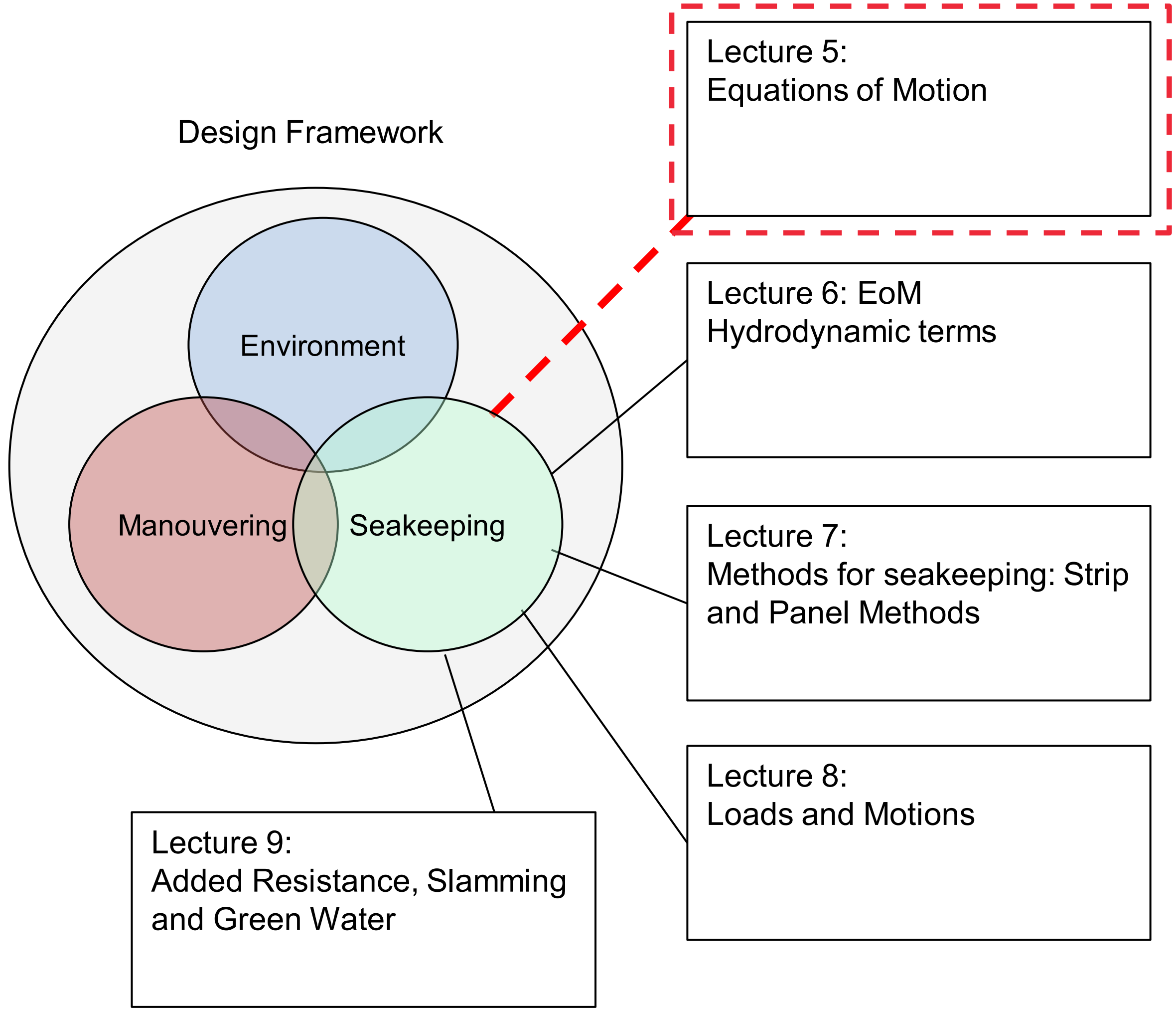
Aalto University
School of Engineering

MEC-E2004 Ship Dynamics (L)

Lecture 5 – Equations of Motion (Part I)

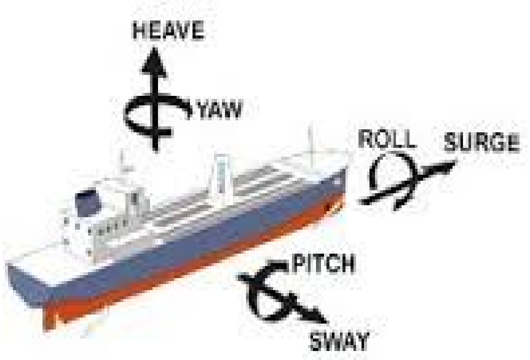
Where is this lecture on the course?

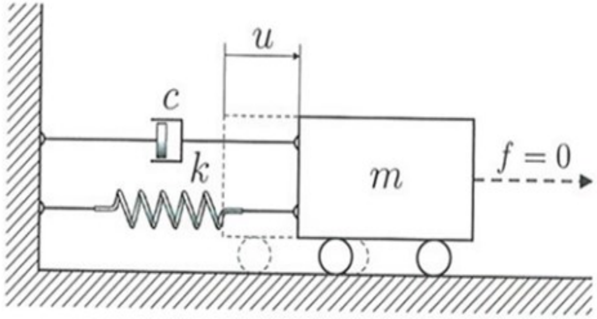


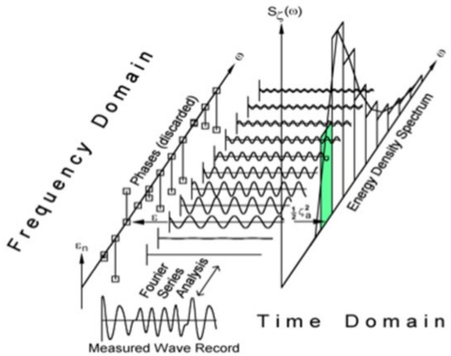
Contents

- **Aim :** To introduce the equations of motion and how these are formed using basic rigid body dynamics.

- <u>Literature</u>
  - Journee, J.M.J., ”Introduction to Ship Hydromechanics”
  - Lloyd, A.R.J.M, ”Seakeeping – Ship Behavior in Rough Weather”, John Wiley & Sons
  - Bertram, V., ”Practical Ship Hydrodynamics”, Butterworth-Heinemann, Ch. 4.
  - Matusiak, J., ”Ship Dynamics”, Aalto University
  - Lewis, E. V. Principles of Naval Architecture. Vol. 3, ”Motions in waves and controllability”
  - Rawson, K. J., ”Basic Ship Theory. Volume 2, Ship dynamics and design - ch.12 Seakeeping & ch.13 Manoeuvrability”.







Motivation

- Ship motions are affected by numerous factors such as :
  - Sea state
  - Propulsive equipment (rudders, propulsors etc.)
  - Cargo movement
  - Special general arrangement features

- Practical and well validated methods and procedures that are suitable for ship design are essential.

- Classic methods are based on linear ship dynamics (potential flow analysis methods).
  - They allow us to use spectral techniques and statistics
  - They can be updated with correction factors to account non-linearities

- Non linear methods become useful when ship motions are excessive or we model extreme events. Approaches exist in time-domain for specific sea states, time-frames and using different time histories. CFD approaches also emerge.

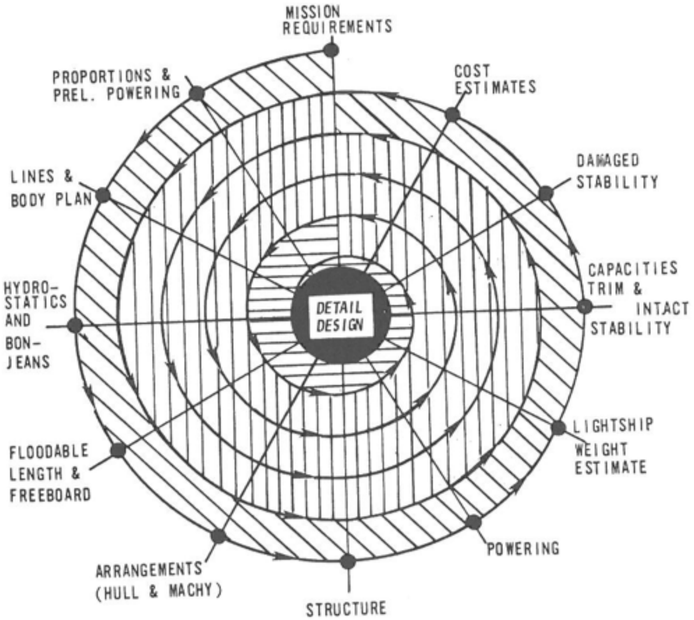
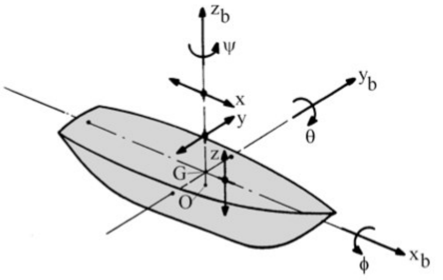


Fig. 1 Basic design spiral

Assignment 3

- **Grades 1-3:**

  - ✓ Select a book-chapter related to the ship equations of motion and read it
  - ✓ Identify the main components associated to equations of motion of your ship. How and why they relate with the ship's mission (think in operational safety terms) ?
  - ✓ Discuss how the general arrangement, hull form and operational profile of your ship affect the equations of motion (think in design for safety terms).
  - ✓ Start getting familiar with motions and loads design software (e.g. MaxSURF, Napa, etc.) and reflect the software use to the theory learned



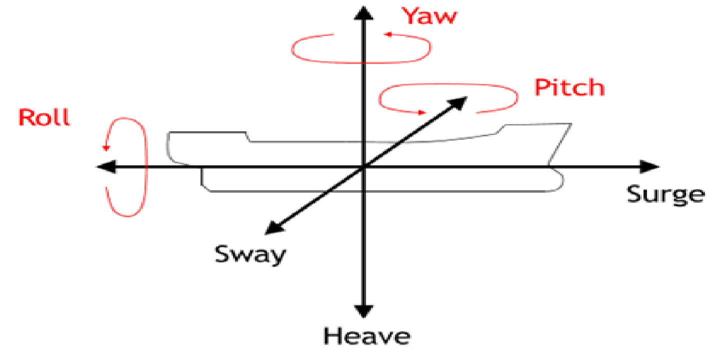
- **Grades 4-5:**

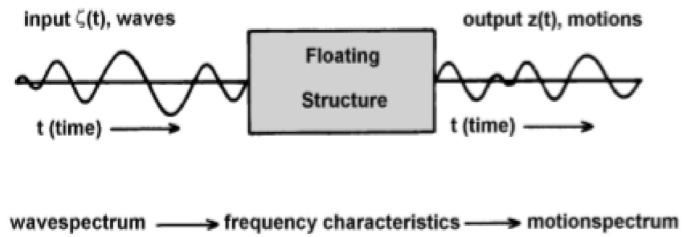
  - ✓ Read 1-2 scientific journal articles related to Ship Equations of Motion
  - ✓ Reflect these in relation to knowledge from books and lecture slides

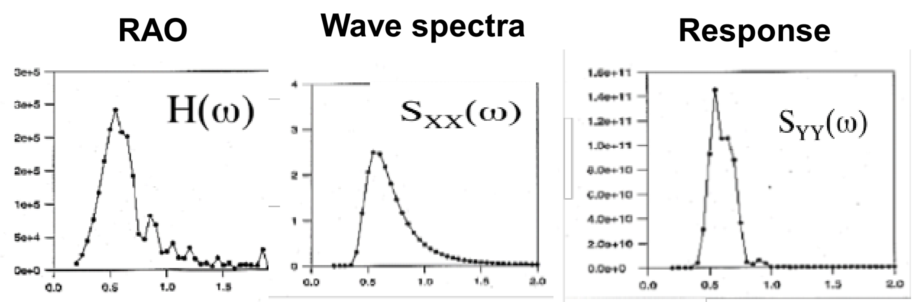
- Report and discuss the work.

Ship Motions - Introduction

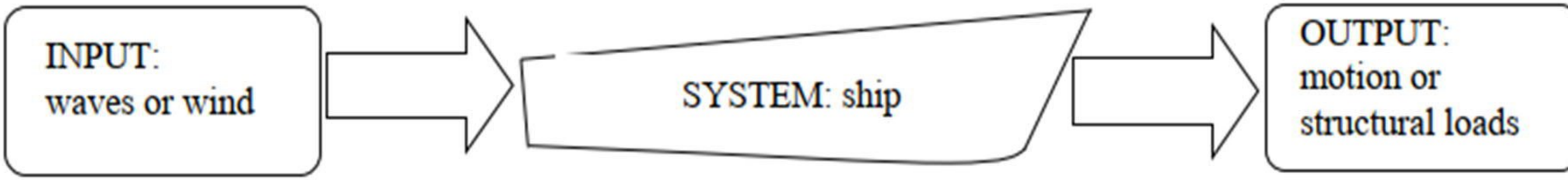
- A rigid ship moves in waves in 6 degrees of freedom (DOF)
- This means that for arbitrarily-shaped ship we will have
  - ✓ 6 equations of motion
  - ✓ 6 unknowns
- These must be solved simultaneously
- For port-starboard-symmetry these equations reduce to two sets of uncoupled EoM containing 3 unknows namely :
  - ✓ surge, heave, pitch
  - ✓ sway, yaw, roll
- We approximate the response by superposition of elementary waves progressing in :
  - ✓ Different lengths
  - ✓ Different directions

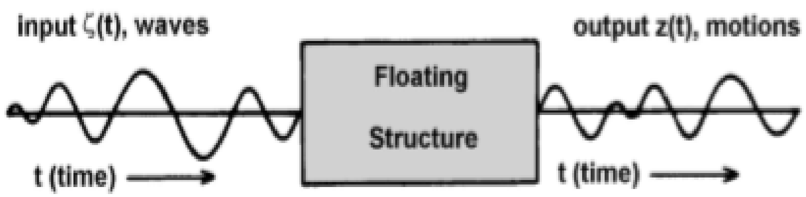




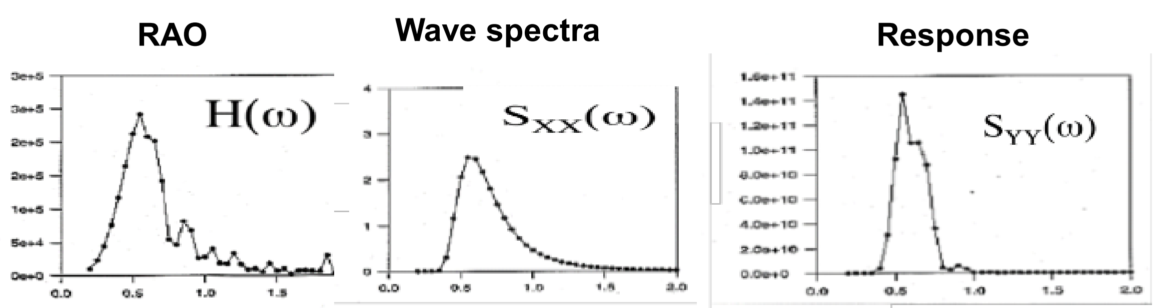


Ship Motions - Introduction

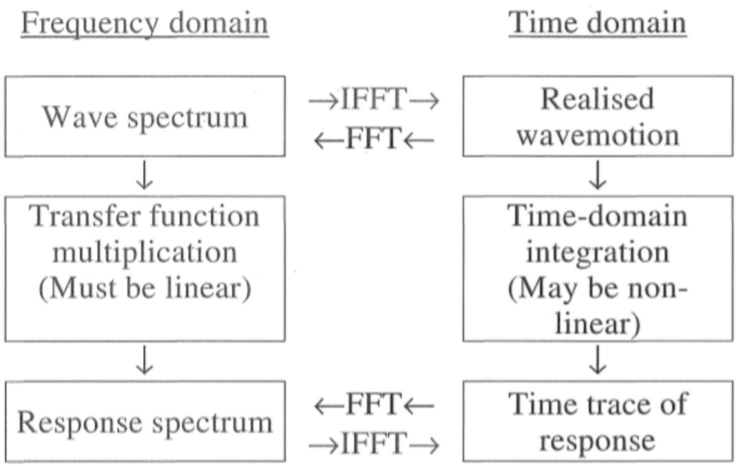


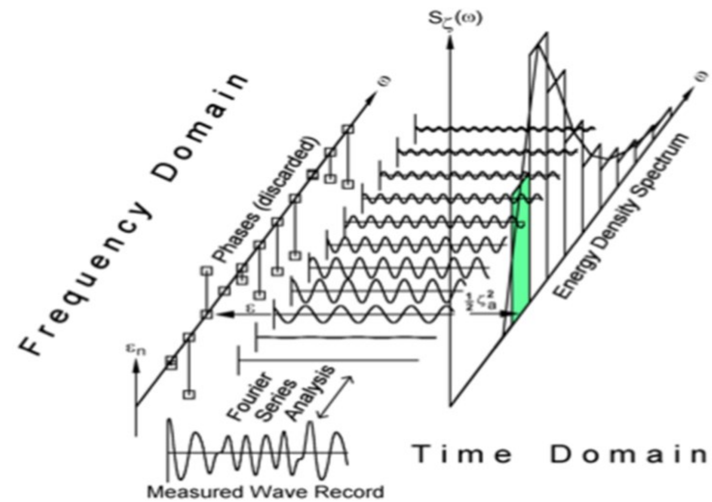


wavespectrum → frequency characteristics → motionspectrum



Frequency vs Tome domain - Revision



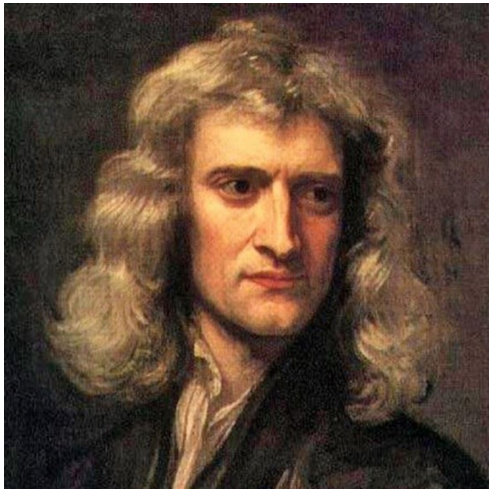


The Fourier Transform of a function g(t) is defined by: $$\mathscr{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t}\, dt \quad (1)$$

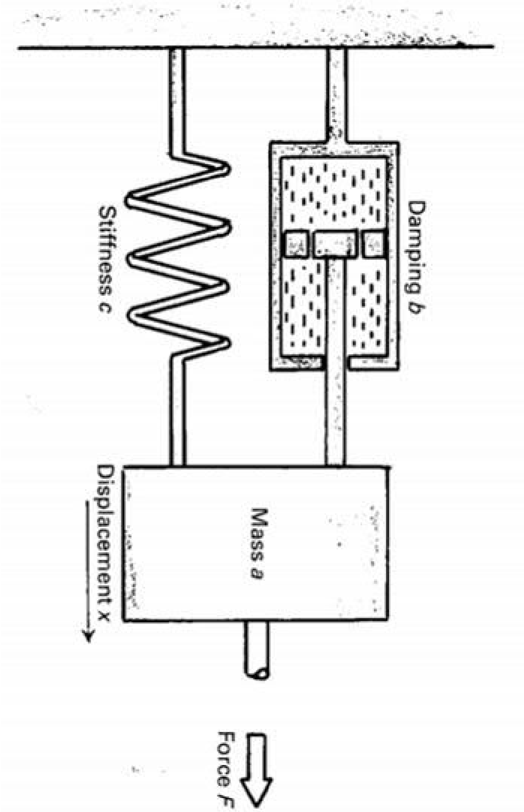
The result is a function of f, or frequency. As a result, G(f) gives how much power g(t) contains at the frequency f. **G(f) is often called the spectrum of g**. In addition, g can be obtained from G via the inverse Fourier Transform (convolution integral) as :

$$\mathscr{F}^{-1}\{G(f)\} = \int_{-\infty}^{\infty} G(f) e^{2\pi i f t}\, df = g(t) \quad (2)$$

Dynamics of rigid bodies - revision



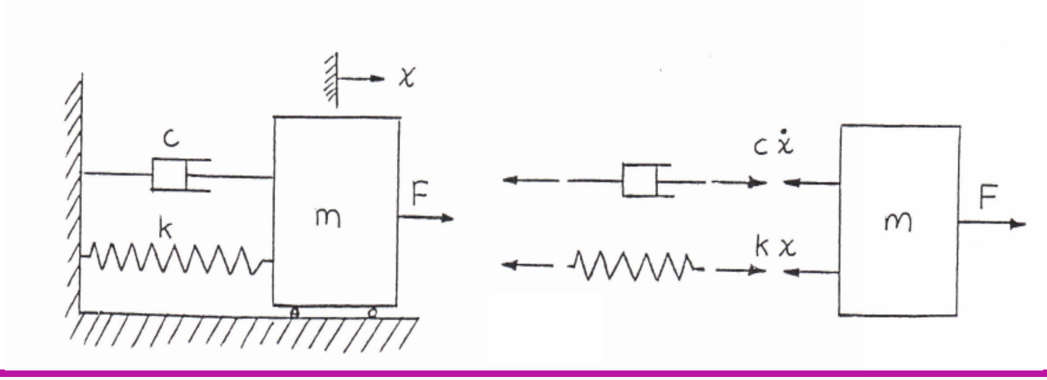
$$\sum \vec{F} = m\ddot{x}$$



Newton's 2nd Law

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

- If we set F(t) = 0 then we obtain the complementary function; i.e. the function expressing the response of the system when we have free vibration
- F(t) is also known as the particular integral ; i.e. a function expressing the excitation and affecting the frequency response function
- In ship dynamic terms this means that dynamic response may be simply affected by the complementary function or her combination with the particular integral
- For ships the mass (m), stiffness (k) and damping (c) terms should include both wet and dry parts



Case 1 : Undamped free vibration (1 - DOF)

- Assume the system is conservative and the vibration is free. The equation of motion reduces to:

$$m\ddot{x} + kx = 0$$

- Assume sinusoidal solution $x = e^{\lambda t}$

Natural frequency of the system

$$\lambda^2 m + k = 0,\ \lambda = \pm\sqrt{\frac{k}{m}} = \pm j\omega_n$$

- The response is defined as :

$$x = A_1 e^{j\omega_n t} + B_1 e^{-j\omega_n t} = A\,sin(\ \ \omega t) + B\,cos(\ \ \omega t) = X\,sin(\ \ \omega t + \phi)$$

- The amplitude and phase are defined as :

$$X = \sqrt{A^2 + B^2} \qquad \phi = \tan^{-1}(B / A)$$

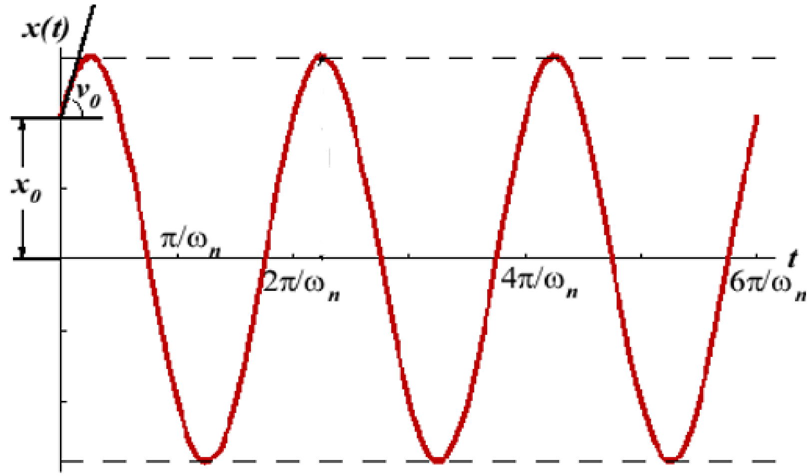
Case 1 : Undamped free vibration (1 dof)

- If we assume the initial conditions: $x(t=0) = x_0, \dot{x}(t=0) = v_0$

$$X = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \qquad \phi = tan^{-1}\left(\frac{\omega_n x_0}{v_0} \right)$$

- Therefore the final solution of this system is defined as:

$$x(t) = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} sin\left(\omega t + \phi \right)$$



Case 2 : Damped free vibration (1- DOF)

- The amplitude of oscillation of the spring, mass, damper system will reduce with time due to damping effects. The damper works by dissipating the energy of the system to zero. For this case Newton's equation becomes :

$$m\ddot{x} + c\dot{x} + kx = 0$$

- Assume sinusoidal solution $x = e^{\lambda t}$

$$m\lambda^2 + c\lambda + k = 0, \qquad \lambda_{1,2} = \frac{-c + \sqrt{c^2 - 4mk}}{2m}$$

- There are three solutions to the above differential equation that link to three different types of motions:

  1. If $\lambda_{1,2}$ are real ($c^2 - 4mk > 0$ (corresponding to **overdamped** case.
  2. If $\lambda_{1,2}$ are imaginary ($c^2 - 4mk < 0$ (corresponding to **underdamped** case.
  3. If $\lambda_1 = \lambda_2$ are real ($c^2 - 4mk = 0$ (leading to $c_{cr} = \sqrt{4mk} = 2m\omega_n$ that corresponds to **critically damped** case (i.ethe system overshoots and comes back to rest).

Case 2 : Damped free vibration (1- DOF)

- Another approach to solve Newton's equation is the damping ratio (ζ). This is the ratio of the damping coefficient of the system to the critical damping coefficient:

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}} \rightarrow c = 2m\omega_n\zeta$$

$$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]$$

- The three types of motions can then be defined by the damping ratio as:

    1. $\zeta > 1$ (for overdamped case);
    2. $\zeta < 1$ (for underdamped case) and
    3. $\zeta = 1$ for the critically damped case.

- The response of the system in terms of these two roots is defined as:

$$x(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t}$$

Case 2 : Damped free vibration (1- DOF)

For the underdamped case where the damping ratio range is $0 < \zeta < 1$ this leads to:

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \omega_d j$$

$$\omega_d = \sqrt{1 - \zeta^2}$$

$$x(t) = Ae^{-\zeta\omega_n t} sin(\quad \omega_d t + \phi)$$

$$= \sqrt{\frac{\left(v_0 + x_0\zeta\omega_n\right)^2 + \left(x_0\omega_d\right)^2}{\omega_d^2}} e^{-\zeta\omega_n t} sin(\quad \omega_d t + tan^{-1}(\quad \frac{x_0\omega_d}{v_0 + x_0\zeta\omega_n}) \Bigg)$$

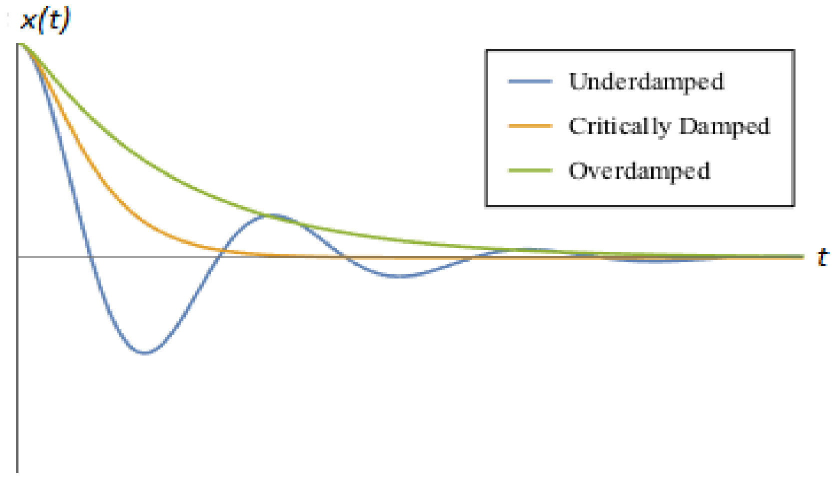
Case 2 : Damped free vibration (1 dof)

- If we follow the same procedure, the solution of overdamped case is given by:

$$x(t) = a_3 e^{(-\zeta\omega_n + \omega_d)t} + a_4 e^{(-\zeta\omega_n - \omega_d)t}$$

- Similarly, for the critically damped case, the solution is given by:

$$x(t) = [x_0 + (v_0 + \omega_n x_0)t]e^{-\omega_n t}$$



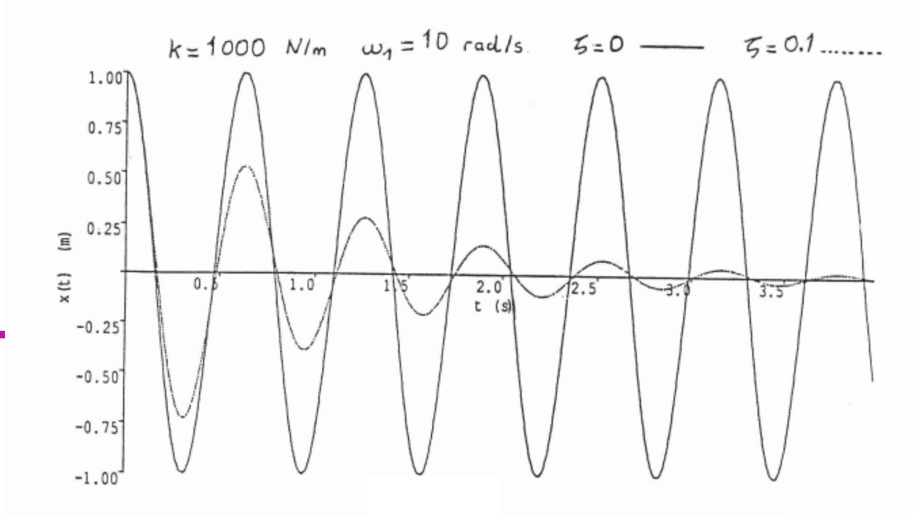
How can we practically assess damping ?

A practical way to assess damping that is broadly applicable in the area of ship hydrodynamics is the damping decay test. This can be mathematically expressed using the log decrement that is the natural logarithm of the ratio of two successive amplitudes.

$$\delta = ln\frac{X_1}{X_2} = ln\frac{Ae^{-\zeta\omega_n t_1}}{Ae^{-\zeta\omega_n(t_1+T_d)}} = ln\, e^{\zeta\omega_n T_d} = \zeta\omega_n T_d$$

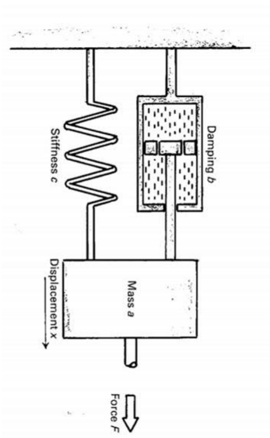
$$\because T_d = 2\pi/\omega_d \rightarrow \therefore \delta = \frac{2\pi\zeta\omega_n}{\omega_d} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Since the damping ratio is very small in that case, the log decrement can be approximated by: $\delta = 2\pi\zeta$



Case 3 : Forced Vibration – 1 DOF

Consider adding harmonic excitation to the vibration system where $F(t)$ varies in sinusoidal manner instead of being arbitrary function in time:



$$m\ddot{x} + c\dot{x} + kx = F(t) = F_0\, cos(\omega t)$$

$$\rightarrow \ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = f_0 \cos(\omega t)$$

$$f_0 = F_0 / m$$

This is a differential equation of the 2nd order. Accordingly, it is prone to a general and particular solution which when combined together they may give the response function of the system.

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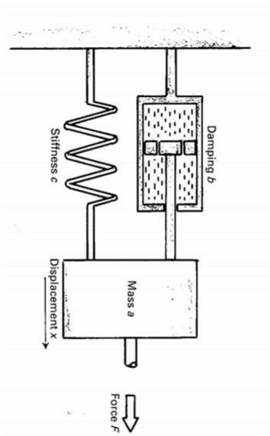

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This is a differential equation of the 2nd order. Accordingly, it is prone to a general and particular solution which when combined together they may give the response function of the system. The **general solution** is given when the left-hand side of the equation is equal to zero.

$$\ddot{x}_g(t) + 2\zeta\omega_n\dot{x}_g(t) + \omega_n^2 x_g(t) = 0$$

$$x_g(t) = Ae^{-\zeta\omega_n t}\,sin(\omega_d t + \phi),\ \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Case 3 : Forced Vibration – 1 DOF

- The **particular solution** is defined as : $\ddot{x}_p(t) + 2\zeta\omega_n\dot{x}_p(t) + \omega_n^2 x_p(t) = f_0\cos(\omega t)$

- There are two possible trial solutions to the particular solution namely

$$x_p(t) = A_s\cos(\omega t) + B_s\sin(\omega t) \text{ or } x_p(t) = X\cos(\omega t - \theta)$$

where : $X^2 = A_s^2 + B_s^2,\quad \theta = \tan^{-1}(B_s / A_s)$

- Substituting the trial solution in the equation of motion leads to:

$$(-A_s\omega^2 + \ 2B_s\zeta\omega_n\omega + A_s\omega_n^2 - f_0)\cdot\cos(\omega t) +$$
$$(-B_s\omega^2 - 2A_s\zeta\omega_n\omega + Bs\omega_n^2)\cdot\sin(\omega t) = 0$$

- For this equation to be zero at any time t, the two coefficients multiplied by $sin(\omega t)$ and $sin(\omega t)$ must be zero.

Case 3 : Forced Vibration – 1 DOF

- Solving these two equations we can find the two unknowns:

$$A_s = \frac{(\omega_n^2 - \omega^2) f_0}{\left(\omega_n^2 - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2}$$

$$B_s = \frac{2\zeta\omega_n\omega f_0}{\left(\omega_n^2 - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2}$$

- The particular solution after solving the unknowns becomes:

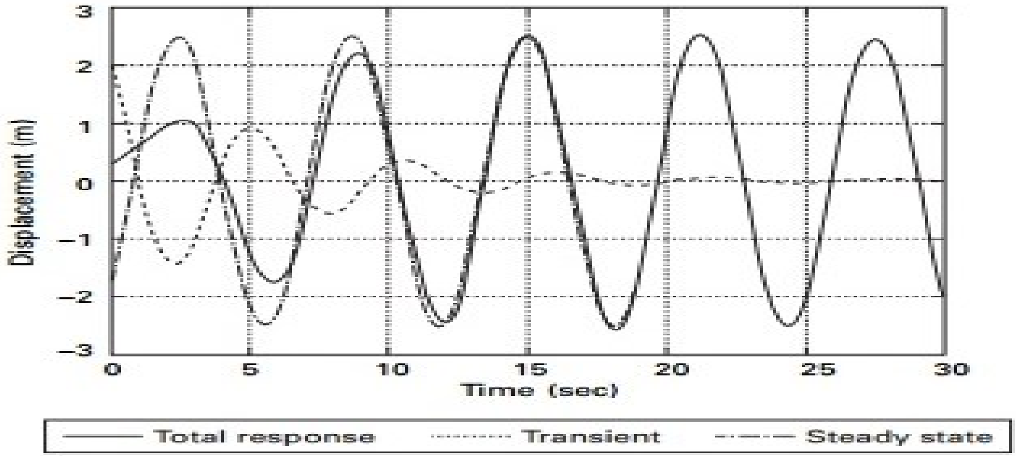
$$x_p(t) = X\,cos(\omega t - \theta)$$
$$= \frac{f_0}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2}} cos\left(\omega t - arctan\left[\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right]\right)$$

Case 3 : Forced Vibration – 1 DOF

The first term in the full solution is the transient solution, which tends to zero as the time goes to infinity, while the second term is the steady oscillatory solution. The second term is of more importance as it is the steady solution. In many cases, we neglect the transient solution. The full solution then reduces to:

$$x_p(t) = X\cos(\omega t - \theta)$$

$$X = \frac{f_0}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2}} \qquad \theta = \tan^{-1}\left[\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right]$$



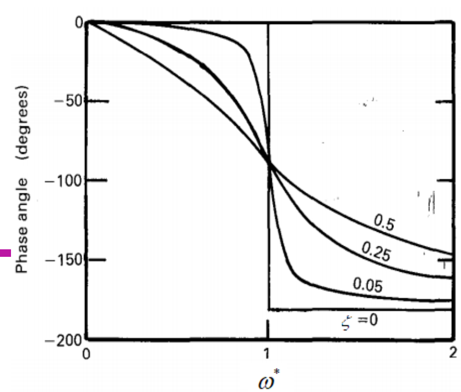
Transient, steady state and total response.

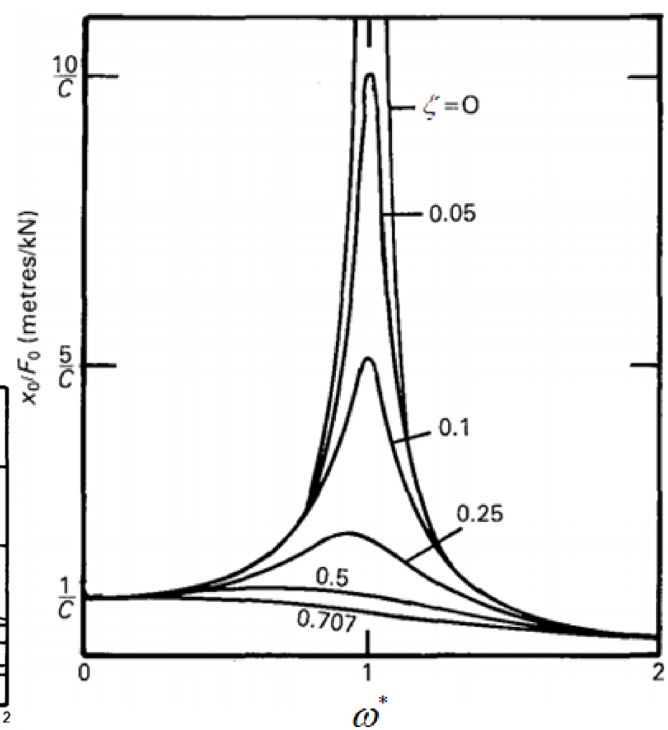
Case 3 : Forced Vibration – 1 DOF

If we rewrite these equations as a function of the frequency ratio $\omega^* = \omega / \omega_n$ we get the expression

$$\frac{Xk}{F_0} = \frac{X\omega_n^2}{f_0} = \frac{1}{\sqrt{\left(1-\omega^{*2}\right)^2 + \left(2\zeta\omega^*\right)^2}} \qquad \theta = tan^{-1}\left(\frac{2\zeta\omega^*}{1-\omega^{*2}}\right)$$

The term in the left-hand side is known as the **amplitude ratio**. When the system is undamped, the amplitude ratio, when the frequency of vibration approaches the natural frequency, gets to extremely significant value, and such case is known by resonance.





Forced Vibrations due to Harmonic Excitation

If we apply a Fourier integral on the excitation force of Newton's equation of motion external loading and response are defined as

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_P(\omega) e^{i\omega t} d\omega \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_x(\omega) e^{i\omega t} d\omega$$

Then Newton's equation of motion becomes :

$$-m\int_{-\infty}^{\infty} A_x(\omega)\omega^2 e^{i\omega t} d\omega + c\int_{-\infty}^{\infty} A_x(\omega) i\omega e^{i\omega t} d\omega + k\int_{-\infty}^{\infty} A_x(\omega) e^{i\omega t} d\omega = \int_{-\infty}^{\infty} A_P(\omega) e^{i\omega t} d\omega$$

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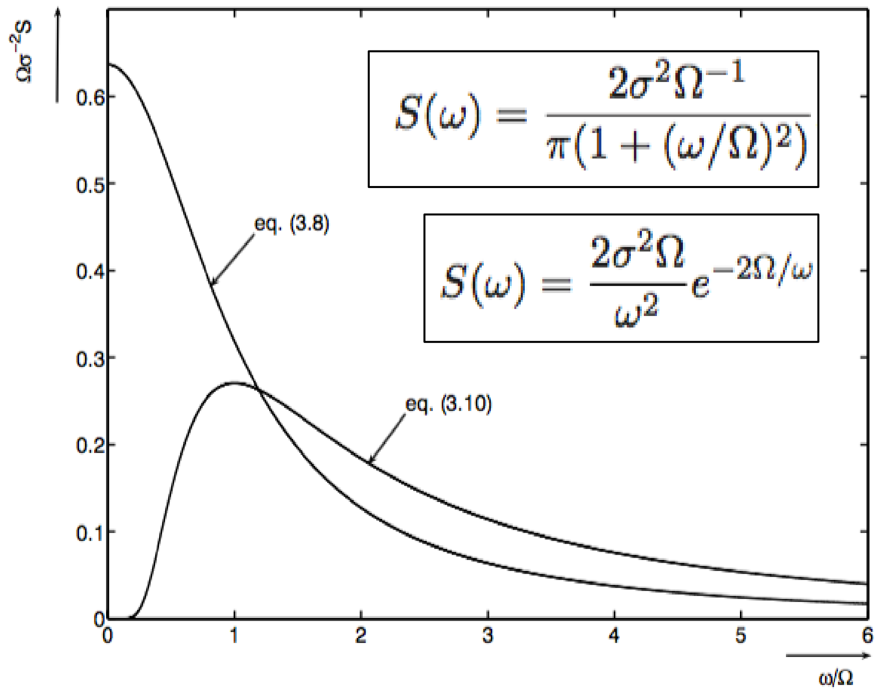
- For $A_x(\omega) = \dfrac{A_P(\omega)}{-m\omega^2 + ci\omega + k}$ ⇨ $A_x(\omega^*) = \dfrac{A_P^n(\omega^*)}{-m\omega^{*2} + \delta i\omega^* + 1}$, where $\begin{cases} A_P^n(\omega^*) = \dfrac{A_P(\omega)}{m\omega_n^2} \\ \delta = \dfrac{c}{m\omega_n} \end{cases}$

Forced Vibrations due to Harmonic Excitation

If we multiply this term with the complex conjugate, we get the spectral density for that system:

$$S_x(\omega) = \lim_{T\to\infty} \frac{1}{\pi T} |A_x(\omega)|^2 = \lim_{T\to\infty} \frac{1}{\pi T} \left| \frac{A_P^n(\omega^*)}{-m\omega^{*2} + \delta i \omega^* + 1} \right|^2 = \frac{S_P^n}{\left(1-\omega^{*2}\right)^2 + \delta^2 \omega^{*2}}$$



Quasi-Static Response

- **At sub-critical case (also known as quasi-static response)** the system can reach high values of spectral density at small frequencies relative to the natural frequency and the stiffness has the major effect on the system
- Only stiffness affects the system response

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2}} \rightarrow S_x \approx S_P^n$$

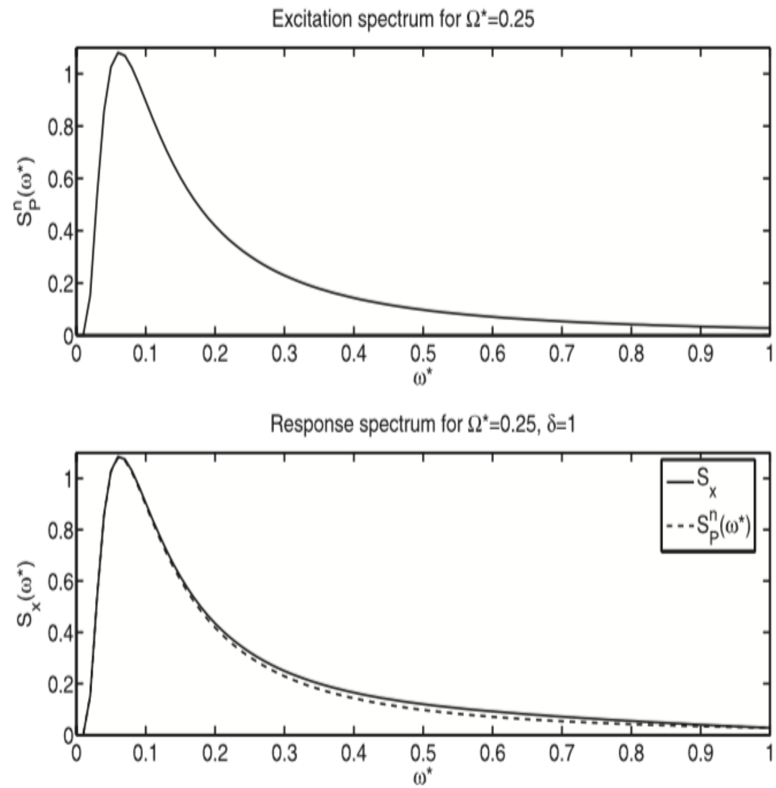


Figure 4.2. Quasi-static or sub-critical response.

Dynamic Response

- At super-critical stage (also known as dynamic response) the highest values of spectral density lie in only high values of frequencies with respect to the natural frequency and damping plays an important role:
- **Only inertia forces affect the system response**

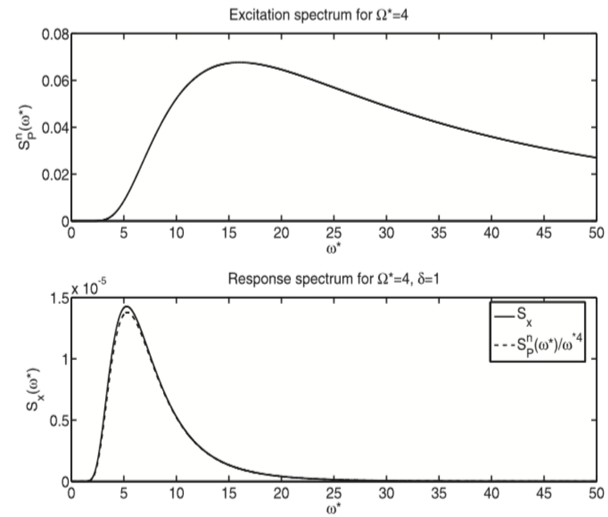


Figure 4.3. Dynamical or super-critical response.

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2}} \rightarrow S_x \approx \frac{S_P^n}{\omega^{*4}}$$

Resonance

- At resonance condition when there is very low damping the frequency ratio ω^*approaches unity. The denominator approaches zero, and the spectral density approaches extremely large value:

- Serious problems which can be controlled only by adjusting damping

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2} \to \approx 0} \to S_x >>>>$$

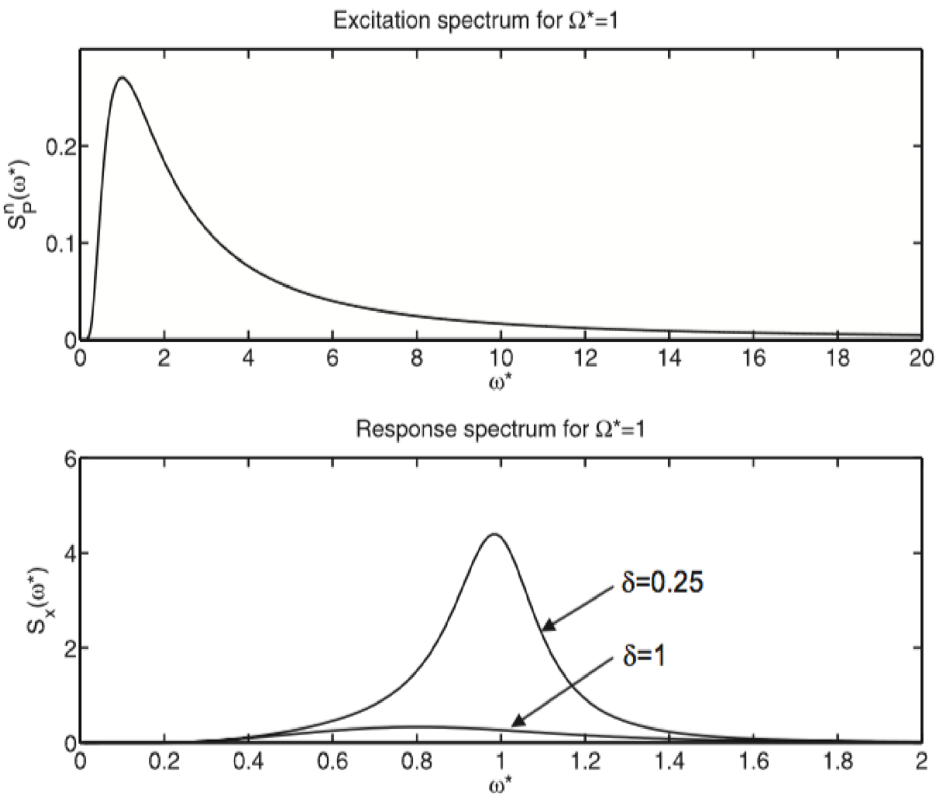
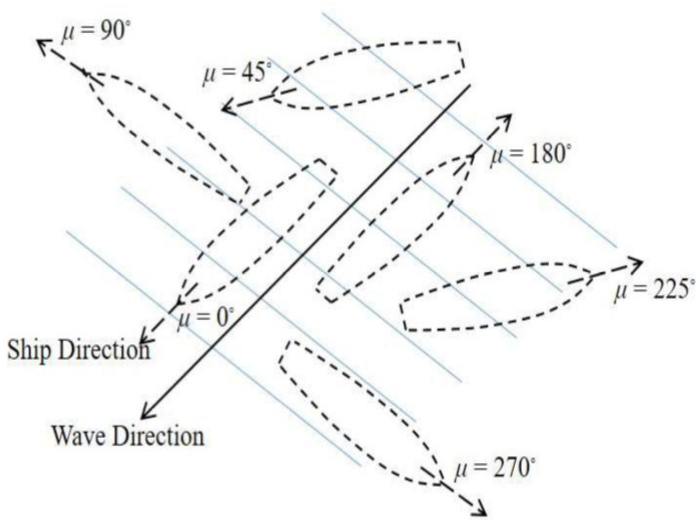
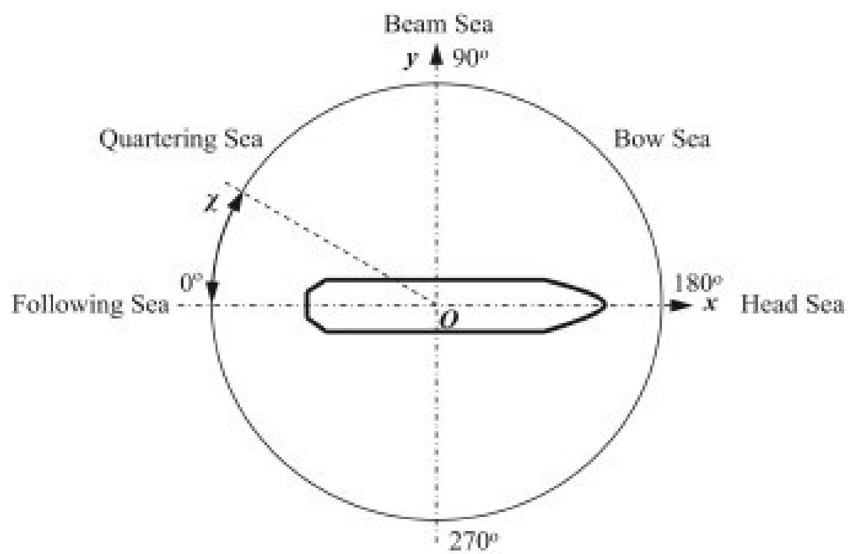


Figure 4.4. Resonant response.

Ship encounter frequency

In ship dynamics the encounter frequency with the waves is used instead of the absolute wave frequency. This is because the ship is moving relative to the waves, and she will meet successive peaks and troughs in a shorter or longer time interval depending upon whether it is advancing into the waves or is travelling in their direction.





Ship encounter frequency

- Assuming the waves and ship are on a straight course, the frequency with which the ship will encounter a wave crest depends on the distance between the waves crests (λ — wavelength), the speed of the waves (c — wave celerity that depends on the wavelength), the speed of the ship (U), and the relative angle between the ship heading and the wave heading (μ)

- The encounter period is thus the distance traveled (λ) divided by the speed the ship encounters the waves (c - U cos(μ))

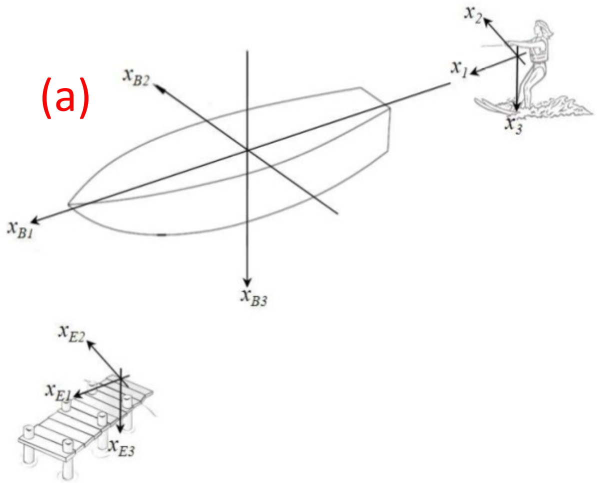
$$\omega_e = \omega - \frac{\omega^2 U}{g} cos(\mu)$$

$$T_e = \frac{\lambda}{c - U \times cos\mu}$$

Coordinate Systems

- We have several coordinate systems for different purposes
  - **Ship CoG or body bound system – xB**
  - **Earth bound system – xE**
  - **Steadily translating system xi**

(a)



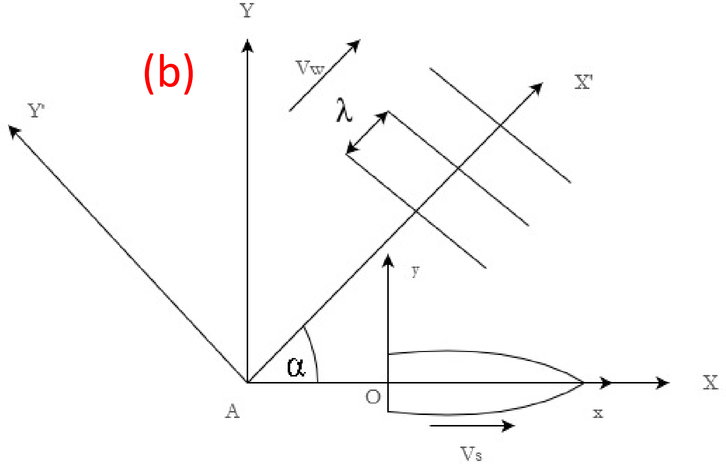
- Considering transformation of coordinates for a regular wave propagating at an angle α (from A X’ Y’ Z’ to A X Y Z) as illustrated in (b)

$$X' = X\cos\alpha + Y\sin\alpha$$

- Then the transformation to the ship’s-fixed coordinate system (oxy) coordinate system:

$$X = x + V_s t,\ \ Y = y$$

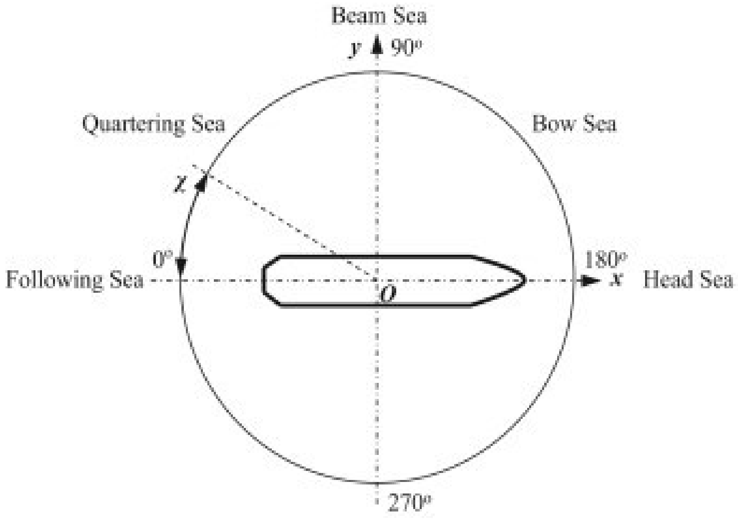
(b)



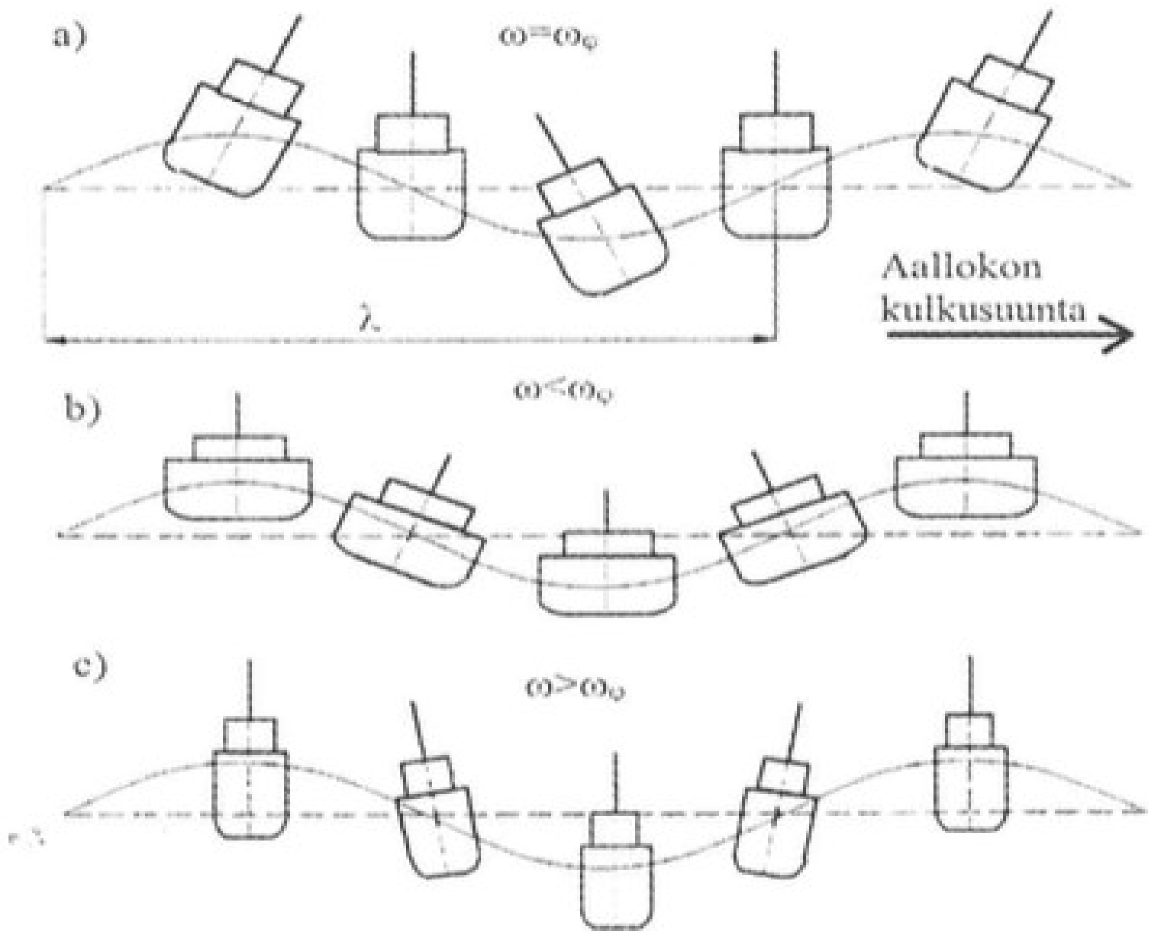
Definition of headings

The heading angle determines the “type” of seas the ship experiences. Heading angles are defined as follows :

- $\mu = 0^0$ – following seas
- $\mu = 180^0$ – head seas
- $\mu = 90^0$ – starboard beam seas
- $\mu = 270^0$ -port beam seas
- $0 \leq \mu \leq 90^0$ – quartering waves on the ship starboard side
- $270^0 \leq \mu \leq 360^0$ – quartering waves on the ship port side
- $90^0 \leq \mu \leq 180^0$ -bow waves on the starboard side
- $180^0 \leq \mu \leq 270^0$ – bow waves on the port side



Ships frequency ω_n, in waves of frequency ω_e



Resonance

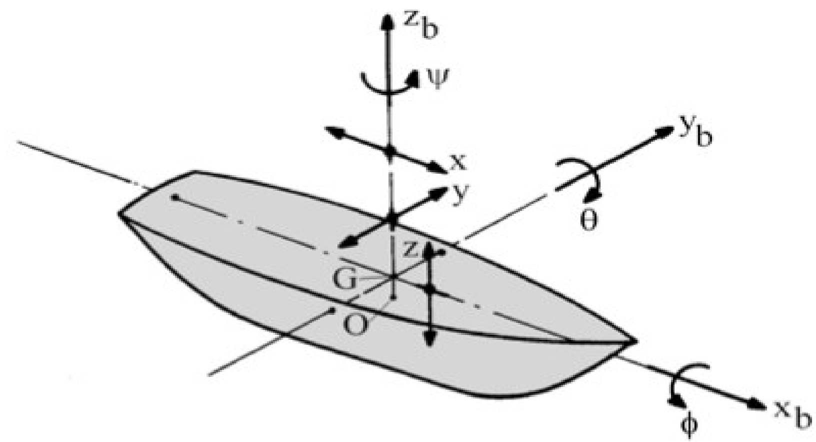
Quasi-static

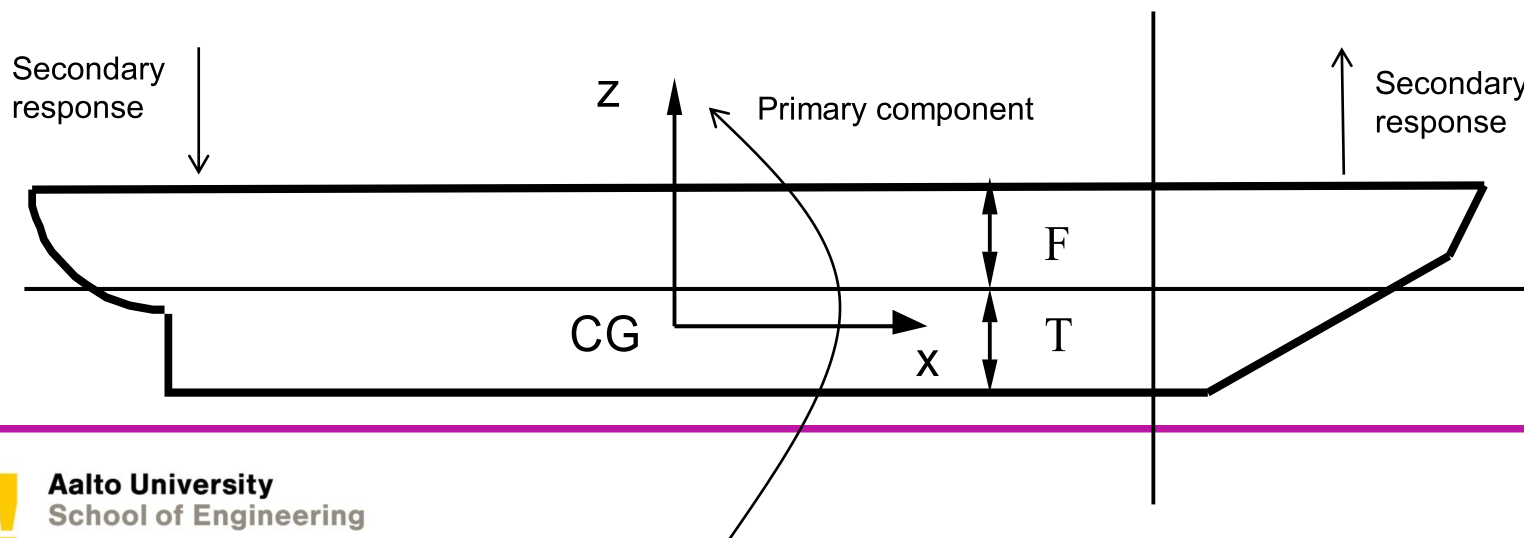
Dynamic

Equation of Motion – steadily translating system

The harmonic motion components of CoG in steadily translating system are (note differences in phase angles) given below. We should know those to evaluate motions.

Surge	:	$x = x_a \cos(\omega_e t + \varepsilon_{x\zeta})$
Sway	:	$y = y_a \cos(\omega_e t + \varepsilon_{y\zeta})$
Heave	:	$z = z_a \cos(\omega_e t + \varepsilon_{z\zeta})$
Roll	:	$\phi = \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta})$
Pitch	:	$\theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta})$
Yaw	:	$\psi = \psi_a \cos(\omega_e t + \varepsilon_{\psi\zeta})$

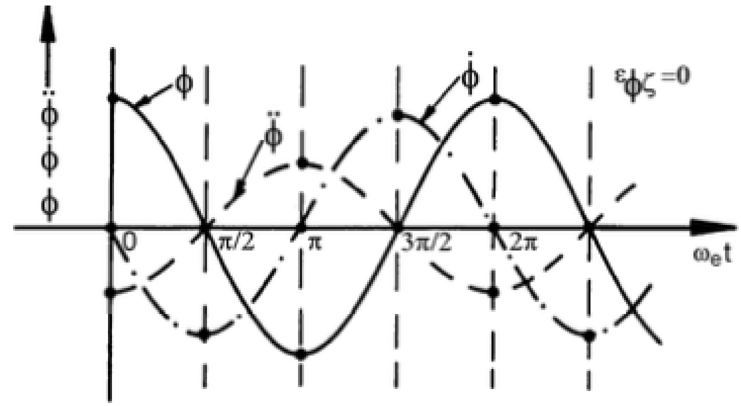




Equation of Motion

As the functions of motion are trigonometric, there is relation between displacement, velocity and acceleration, i.e.

$$\phi = \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta})$$
$$\dot{\phi} = -\omega_e \phi_a \sin(\omega_e t + \varepsilon_{\phi\zeta}) = \omega_e \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta} + \pi/2)$$
$$\ddot{\phi} = -\omega_e^2 \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) = \omega_e^2 \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta} + \pi)$$



With these relations, the equation of motion for all 6 components is given as

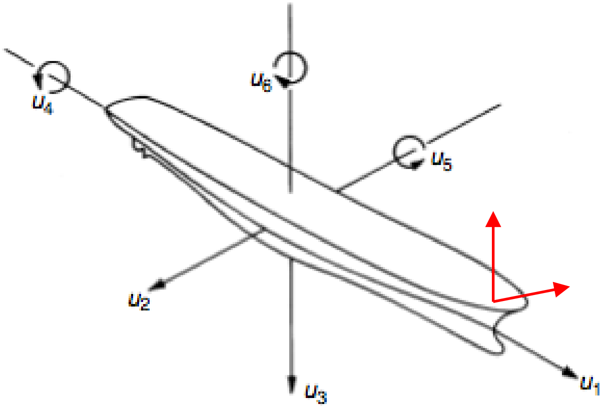
$$[-\omega_e^2(M + A) + i\omega_e N + S]\hat{\vec{u}} = \hat{\vec{F}}_e$$

$$M = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & 0 \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & 0 & -mx_g & 0 \\ 0 & -mz_g & 0 & \theta_{xx} & 0 & -\theta_{xz} \\ mz_g & 0 & -mx_g & 0 & \theta_{yy} & 0 \\ 0 & mx_g & 0 & -\theta_{xz} & 0 & -\theta_{zz} \end{bmatrix} \quad S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g A_w & 0 & -\rho g A_w x_w & 0 \\ 0 & 0 & 0 & gm\overline{GM} & 0 & 0 \\ 0 & 0 & -\rho g A_w x_w & 0 & gm\overline{GM}_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{zz}\omega_g^2 \end{bmatrix}$$

$$\theta_{xx} = \int (y^2 + z^2)\,\mathrm{d}m; \quad \theta_{xz} = \int xz\,\mathrm{d}m; \quad \text{etc.}$$

Summary and next steps

- We have 6 degrees of freedom motion system
  - ➢ Derived from forced, damped vibration
  - ➢ Solution can be derived in time or frequency domain if harmonic excitation is assumed
- Once the motion components in steadily oscillating system are known, the motions at any other point of the rigid body can be derived using superposition
- Next we derive the terms for equation of motion assuming small motions



$$
\begin{array}{lll}
\text{Surge} & : & x = x_a \cos(\omega_e t + \varepsilon_{x\zeta}) \\
\text{Sway} & : & y = y_a \cos(\omega_e t + \varepsilon_{y\zeta}) \\
\text{Heave} & : & z = z_a \cos(\omega_e t + \varepsilon_{z\zeta}) \\
\text{Roll} & : & \phi = \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) \\
\text{Pitch} & : & \theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta}) \\
\text{Yaw} & : & \psi = \psi_a \cos(\omega_e t + \varepsilon_{\psi\zeta})
\end{array}
$$

Thank you !