Aalto University
School of Engineering

MEC-E2004 Ship Dynamics

Mid term assessment and course revision

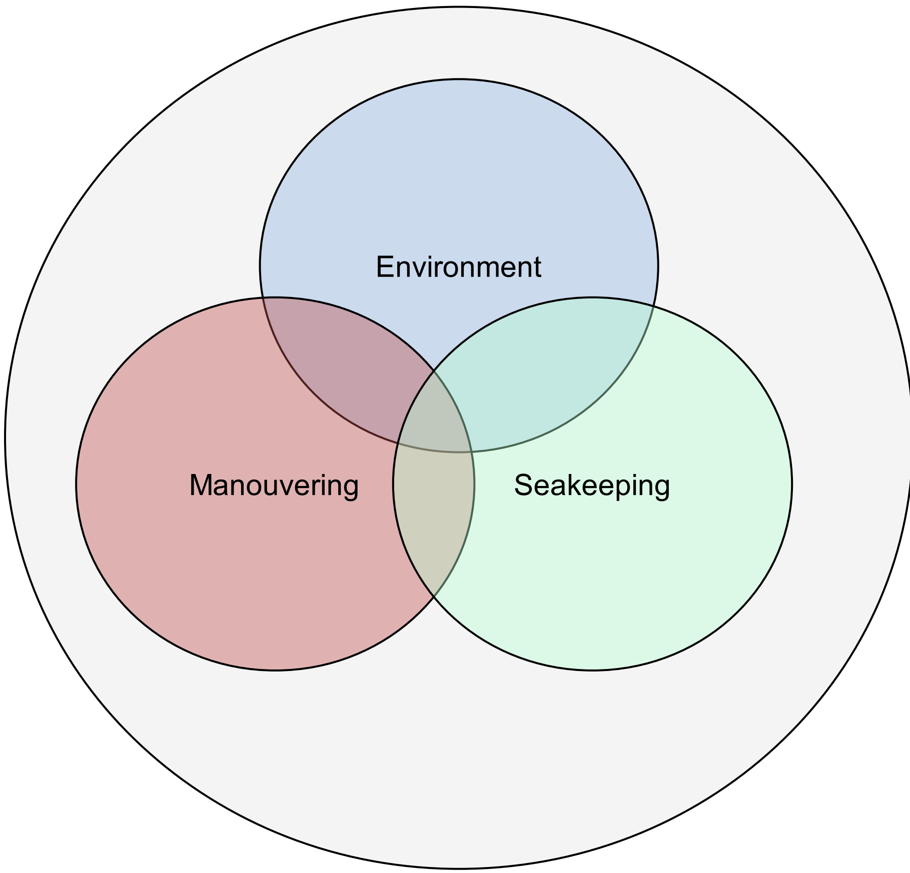
Exam Set up and Rules

- Open book exam from 13:00 - 16 :00 hrs on 12.04.2021
- **The exam starts at 13:00 hrs AND NOT 13:15 hrs ! Please** make yourselves avaliable from 12:50 on the day so that I can answer any questions you may have before hand
- To access the exam we will use MyCourses System. Access through special tag on the system will be given on 13:00hrs on the day.
- In case you need to take a comfort break ask the invigilators !

Exam Set up and Rules

- You may use calculators, the internet, your books, papers and course notes.
- You cannot talk to each other or text each other. Your cameras will have to be switched on for continuous observation.
- You will be given the option to answer up to 6 questions including one bonus question.
- Out of these 6 questions you will answer your best scores in 5 questions are the ones that will count toward your exam mark.

Course Overview



Design Framework

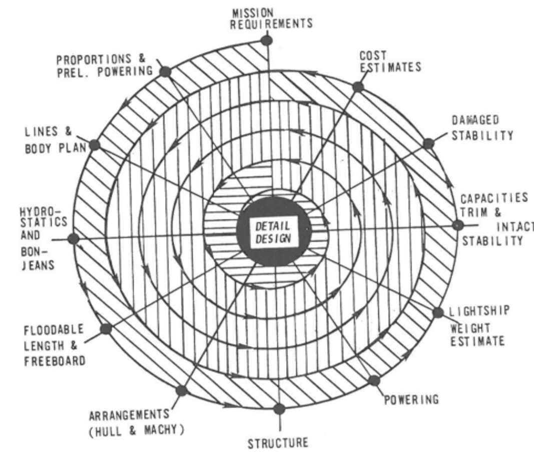


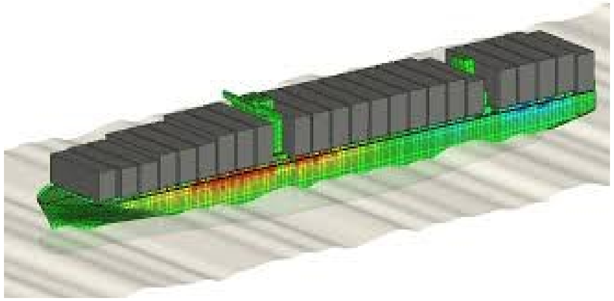
Fig. 1 Basic design spiral

Ship Dynamics – A very broad subject

- The term implies that all operational conditions of a vessel where *inertia forces* play role are important.

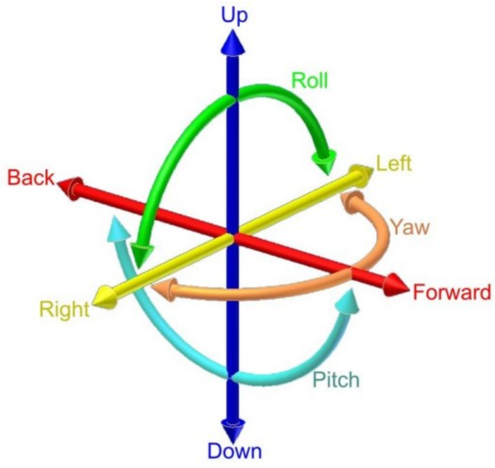
- Thus, all situations that differ from the ideal still water condition, constant heading and constant forward speed should be considered.

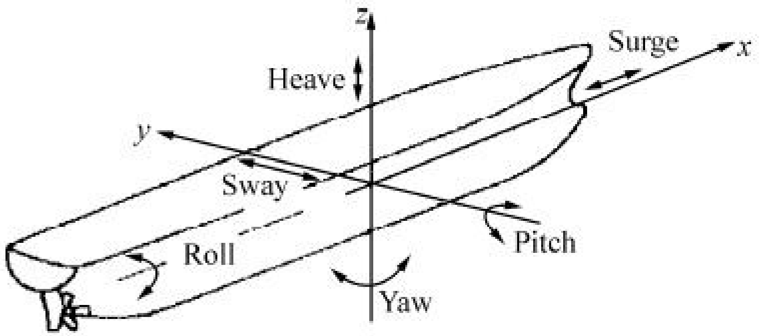
- Traditionally different simplified models are used within the context of
  - ✓ *seakeeping*
  - ✓ *manoeuvring*
  - ✓ *structural vibrations*
  - ✓ *hydroelasticity*
  - ✓ *Stability (intact, damage, static/dynamic)*



Motivation ... (cont.)

- A ship should be able to operate according to her mission in various environmental conditions
- The dynamics are affected by the hull form and the appedanges (propulsive equipment)
- A ship is a body moving in 6 rigid dof + N distortions
- The hull form interaction with water waves affects the seakeeping performance and loading / dynamics of the ship
- Appendages (propeller, rudder, pods, thrusters, etc.) imply forces and moments on the hull and they interact with the wave environment
- Moving cargo loads and onboard equipment (e.g. Heavy lift Cranes) may also affect motions and should be considered if/as applicable





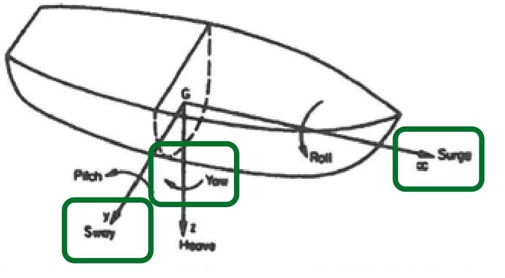
Manouevring – focus and basics

- In manoeuvring the design aspects are
  - Course-keeping and changing
  - Track keeping
  - Speed-changing

- The terms directional stability and control are also used

- Manoeuvring concerns shipyard / owner
  - IMO sets minimum requirements for all ships (IMO A751)
  - Ship-owners may be much more strict (e.g. port of Miami)
  - *Practical Questions: Does the ship keep straight course? Is tug assistance needed to berth and under which wind speeds? Could the vessel initiate/sustain/stop turning? Could the vessel stop and accelerate safely?*

- Manoeuvring requirements affect the equipment to be selected (e.g. rudders and pods, waterjets, fixed fins, jet thrusters, propellers, ducts and steering nozzles etc.)



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J. E. Matusiak, *Dynamics of a Rigid Ship*. Aalto University publication series on Science and Technology, e-ISBN: 978-952-60-7262-3; e-ISSN: 1799-490X, 2017.

Rudder dynamics

- Rudder set at angle δ develops a +ve force in Y–dir defined as *$Y_\delta \delta$ (in simplified format)*

- As this force acts on the ship's stern aprox. half way aster from the origin [0] a – ve turning moment $N_\delta \delta$ develops

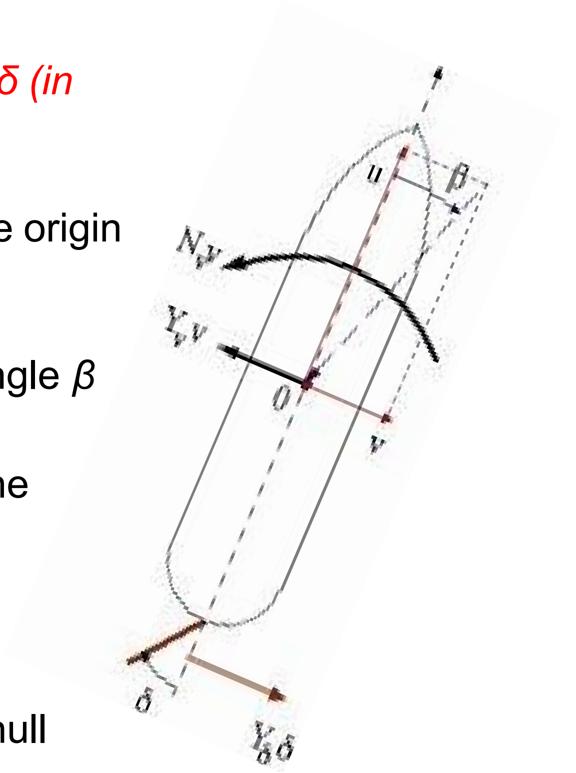
- This moment makes the ship to turn and sets it at a certain drift angle β

- The turning motion initiated by the rudder is greatly amplified by the turning moment $N_v v$ developed by a hull set in inclined flow

- Thge rudder action can be modelled by :

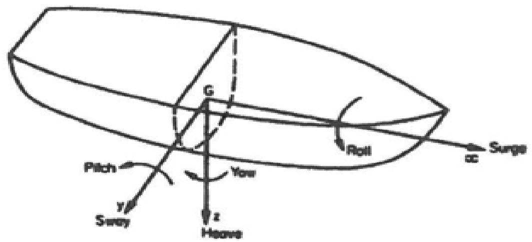
  ✓ **Method 1 :** Stability derivatives (direct representation of the hull forces asdependent to the rudder angle)

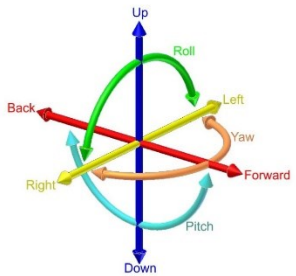
  ✓ **Method 2 :** Modular model (kinematic of inflow into the rudder & modelling of effect of propeller flow on rudder action)

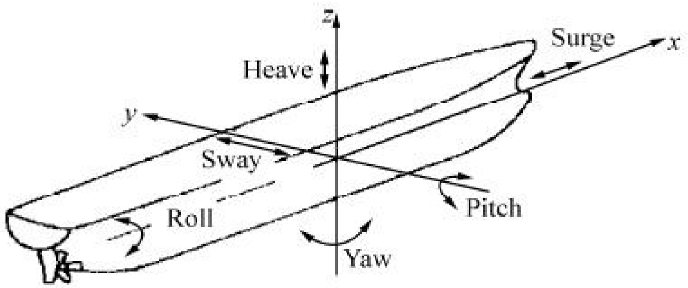


Manouevring vs Seakeeping models

- Time dependent investigations are a norm in manoeuvring but an option in seakeeping
- Manoeuvring is often studied in shallow waters but seakeeping in open seas
- Seakeeping is studied by an inertial coordinate system while manoeuvring by a ship fixed system
- Viscosity is not always neglected in manoeuvring but can be neglected or simply superimposed in seakeeping. This is mostly because of mathematical difficulties and computational cost





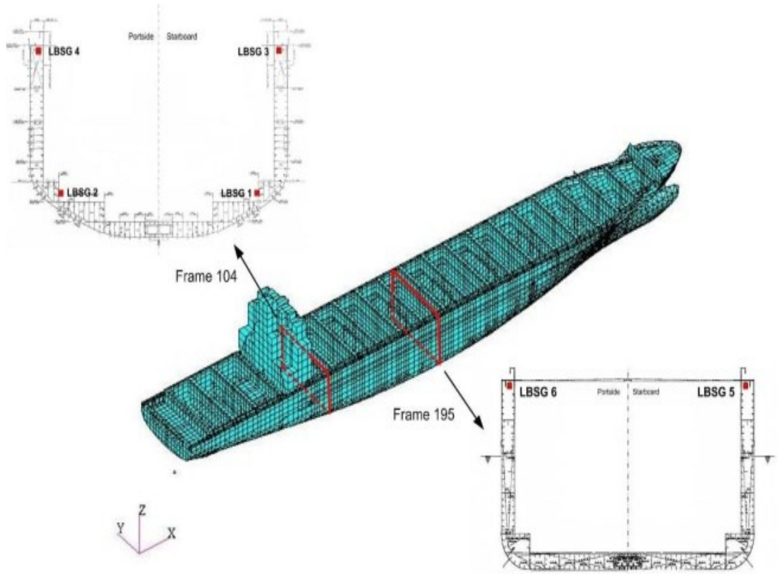


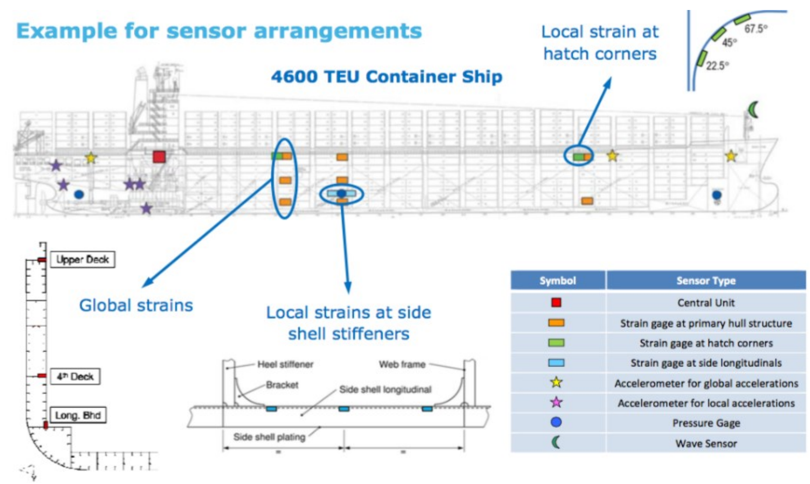
G. Taimuri, T. Mikkola, J. Matusiak, P. Kujala, and S. Hirdaris (2019), The Influence of Hydrodynamic Assumptions on Ship Maneuvering, in *22nd Toowing Tank Symposium - NuTTS, Tomar,* pp. 6.

Engineering Tools – Full Scale Measurements

Ships can be assembled with gyros, strain gauges etc. to measure the responses

- Accelerometers for motions
- Strain gauges to extract wave bending moments
- Uncertainties relate with accurate seaway measurements and measurement equipment failures





S. E. Hirdaris, N. J. White, N. Angoshtari, M. C. Johnson, Y. Lee & N. Bakkers (2010) Wave loads and flexible fluid-structure interactions: current developments and future directions, Ships and Offshore Structures, 5:4, 307-325, DOI: 10.1080/17445301003626263

Hull form dynamics - Motion Reduction

- The key idea of motion reduction is to reduce the levels of kinetic energy lost in the system. In terms of dynamics this means that we should attempt by re-design the ship along the lines of ‘conservative system’ dynamics.

- Loss of kinetic energy is implied by wave making, friction, production of eddies and additional forces

- Motion reduction systems may also produce unwanted side effects, e.g. added resistance

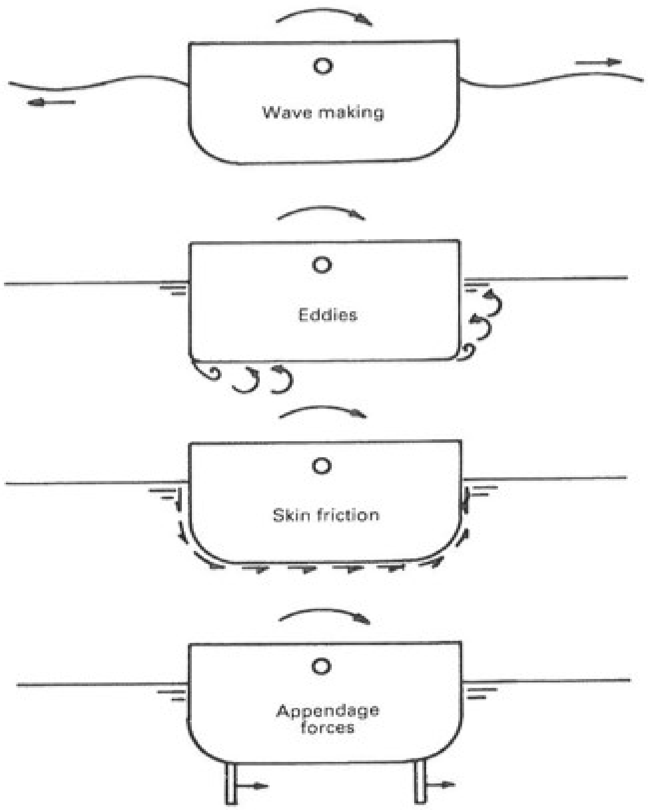
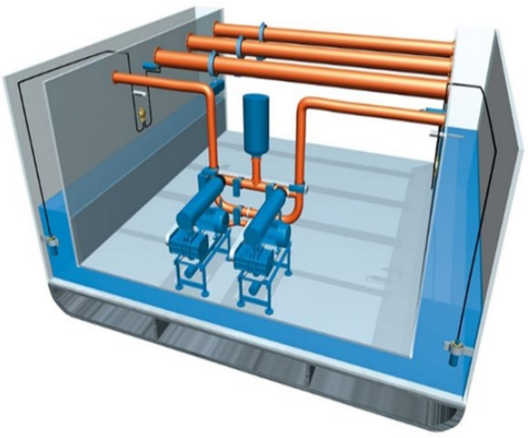
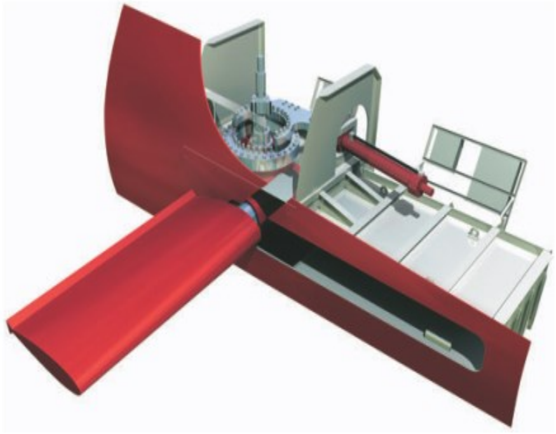
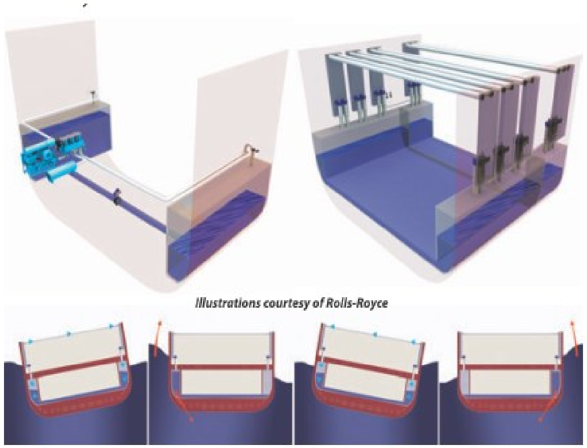


Fig. 12.1 — Sources of roll damping.

Ship Stabilisation systems







Wave Formation

- Waves are typically generated by
  - wind
  - earthquakes etc.

- Two mechanisms for wind-generated waves
  - Pressure fluctuations in the sea surface [Phillips, Phillips, O. M.: 1957, ‘‘On the Generation of Waves by Turbulent Wind’’, J. Fluid Mech. 2, 417–445] https://doi.org/10.1017/S0022112057000233
  - Shear force in the interface of water and air [Miles, J. W.: 1957, ‘‘On the Generation of Surface Waves by Shear Flows’’, J. Fluid Mech. 3, 185–204]https://doi.org/10.1017/S0022112057000567

- Today we accept that usually the formation of waves starts from pressure fluctuations
  - waves are enlarged by shear forces and then
  - waves interact forming longer waves

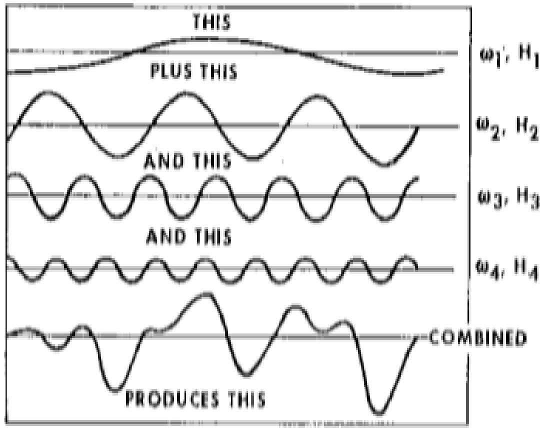
Key wave definitions

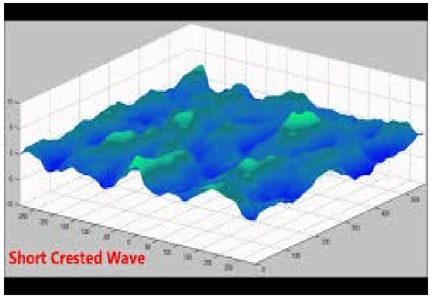
- A **regular wave** (also known as single wave component) has a single frequency, wavelength and amplitude (height)

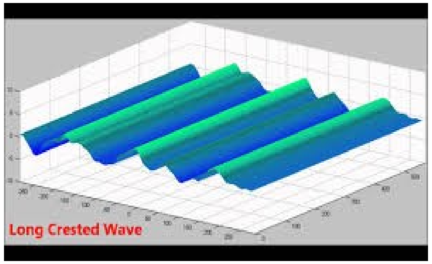
- **Irregular waves** can be viewed as the superposition of a number of regular waves with different frequencies and amplitudes

- **Long-crested waves** are waves formed toward the same direction; **Short-crested waves** are waves formed toward different directions

- **Short term (ST) wave loads** generally relate with ocean or coastal wave formations over 0.5h-3h. **Long term (LT) wave loads** are assessed over life time (e.g. 20 years for ships) and comprise of a sequence of short term events. LT predictions consider multiple sea areas, routes, weather (see IACS URS11, Rec. 14, BSRA Stats etc.)

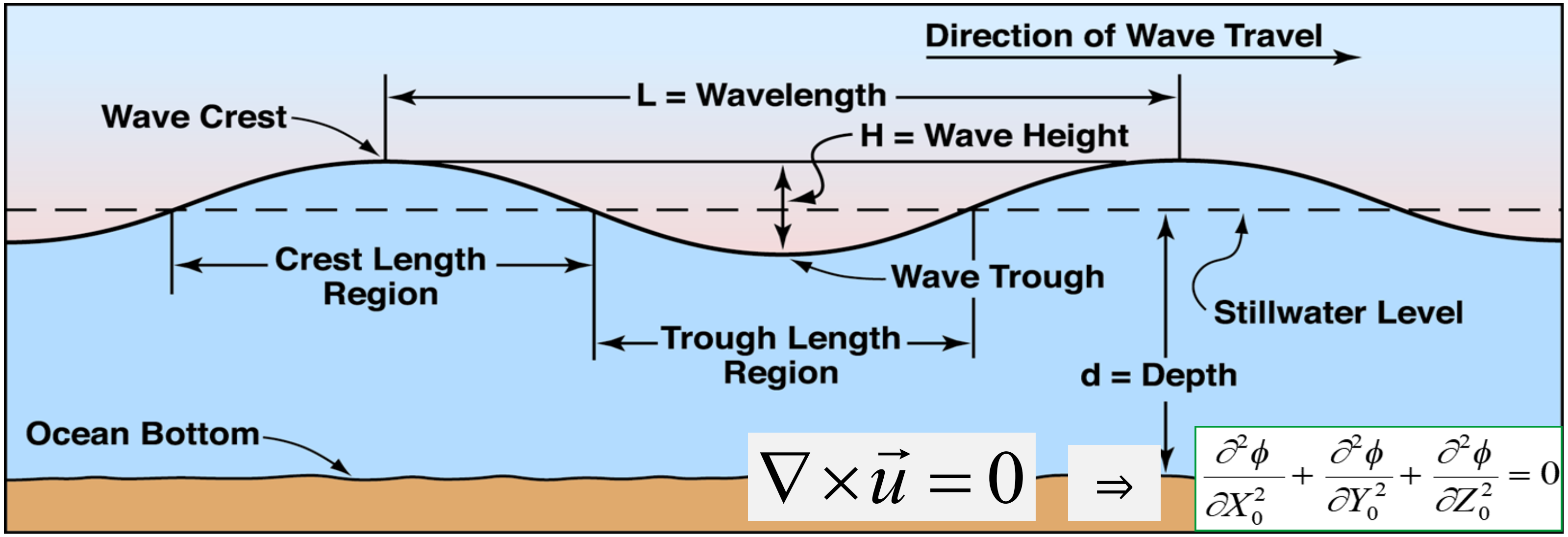






Linear wave - Potential Flow (Airy 1845)

- Single component small amplitude wave
- 2D wave motions do not change with time
- Ideal (i.e. inviscid and irrotational) flow. The water is incompressible and the effects of viscosity, turbulence and surface tension are neglected.
- Laplace velocity potential equation with boundary conditions is used to idealise (1) the sea bed and (2) the deforming sea surface



$$\nabla \times \vec{u} = 0 \quad \Rightarrow \quad \frac{\partial^2 \phi}{\partial X_0^2} + \frac{\partial^2 \phi}{\partial Y_0^2} + \frac{\partial^2 \phi}{\partial Z_0^2} = 0$$

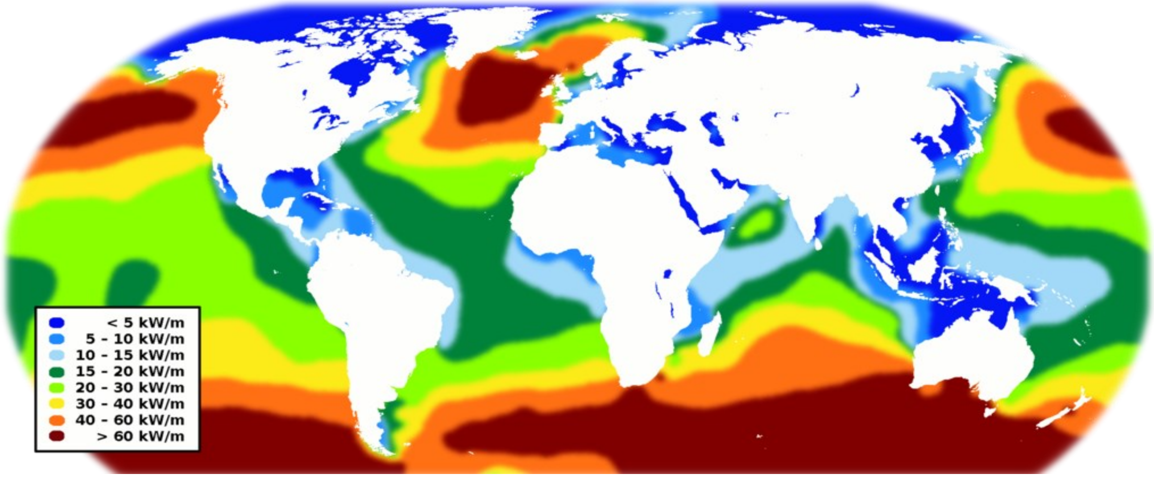
$$\zeta(X,t) = a\cos(kX - \omega t + \alpha) \quad \textbf{(1)}$$

WAVE ENERGY AND POWER

Kinetic + Potential = Total Energy of Wave System

Kinetic: due to H_2O particle velocity

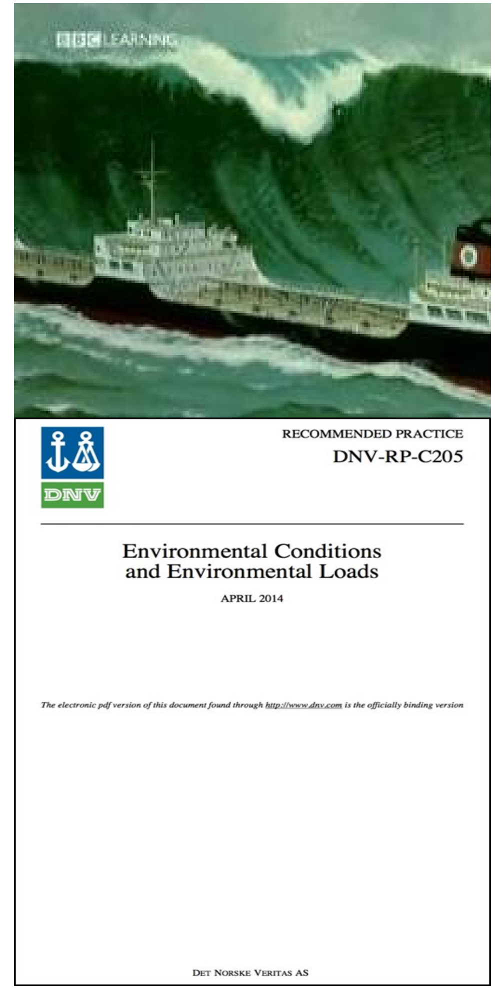
Potential: due to part of fluid mass being above trough. (*i.e.* wave crest)



World map showing wave energy flux in kW per meter wave front

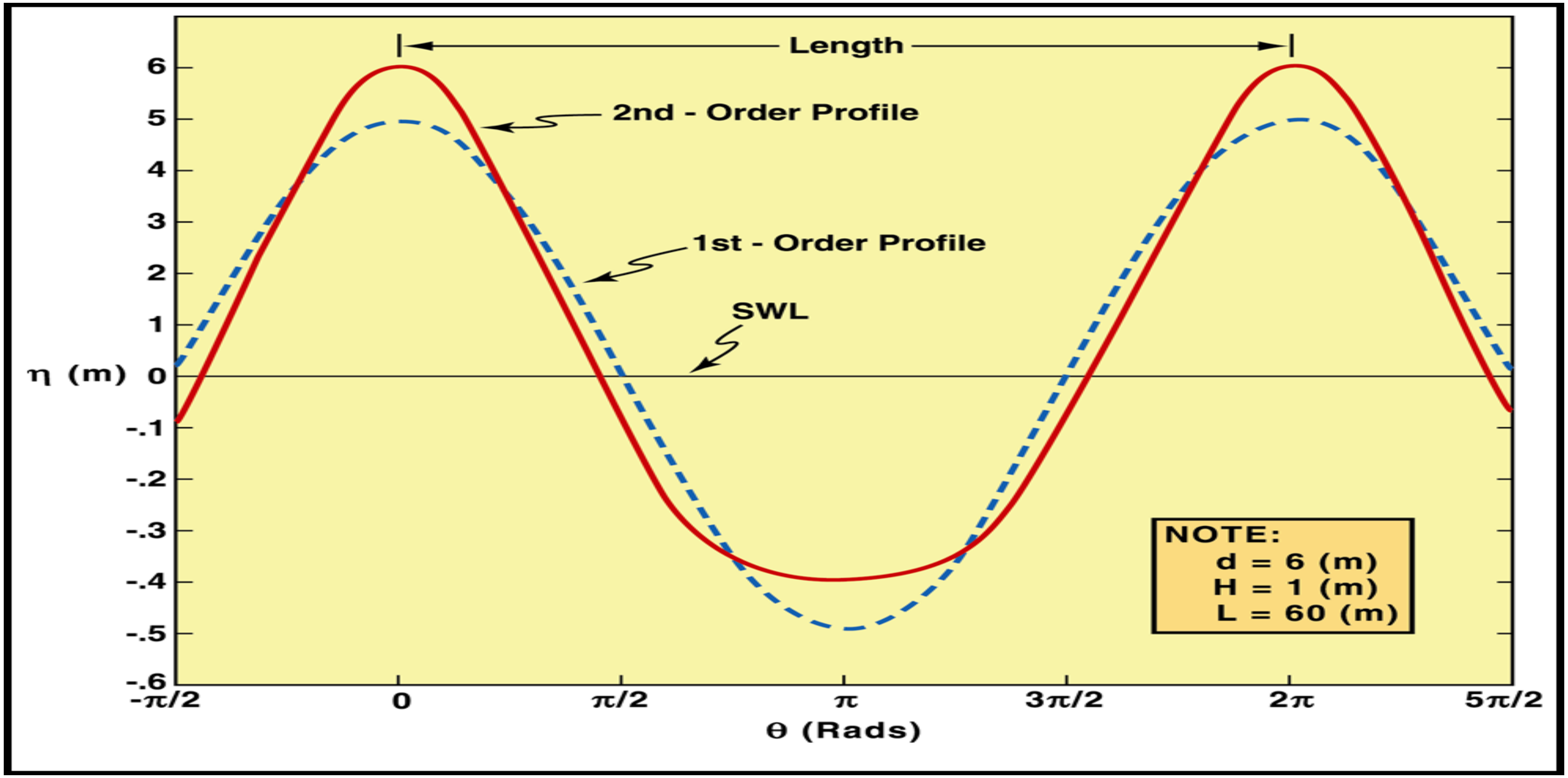
Deviations from Linear Wave Theory

- A linear wave model is very useful in practical engineering work. This is because :
  - It is easy to use,
  - it complies well with the linear modelling of ship responses
  - it enables modelling of the sea by superimposing waves of different lengths and heights
- In some cases, certain non-linear effects have to be considered
  - The information provided by the linear wave model up to the still water level is not sufficient, e.g. we are dealing with local wave pressure loads on ship's side shell
  - An increase of wave steepness results in wave profiles that differ from the ideal cosine form
  - In some cases, certain non-linear effects have to be considered
- Suitably validated NL wave theories and hence NL wave idealisations can provide
  - Better agreement between theoretical and observed wave behavior.
  - Useful in calculating mass transport.



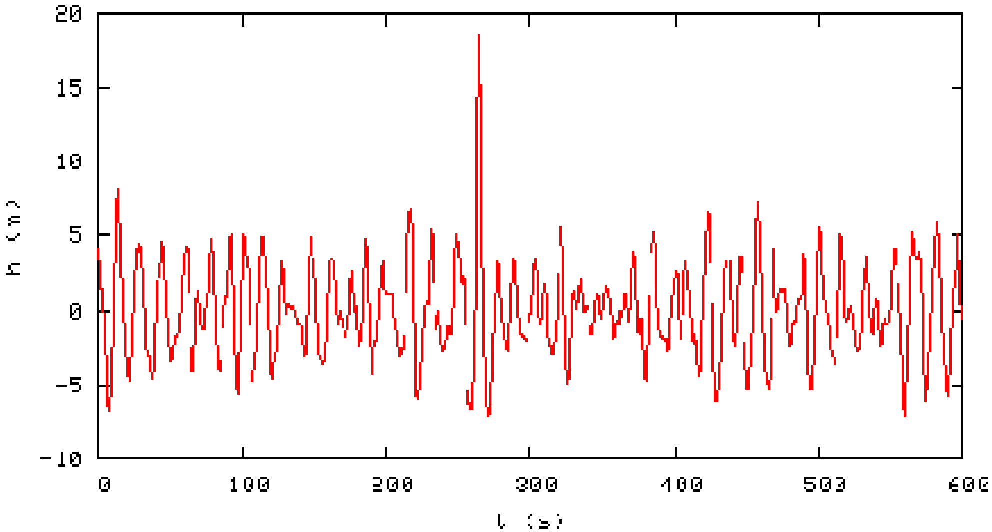
Deviations from Linear Theory – Airy vs Stokes waves

<u>Comparison of second-order Stokes' NL wave profile with linear profile :</u>
Higher order waves are more peaked at the crest, flatter at the trough and with distribution slightly skewed above SWL



Freak Waves

Rogue waves (also known as freak waves, monster waves, episodic waves, killer waves, extreme waves and abnormal waves) are large, unexpected and suddenly appearing surface waves.

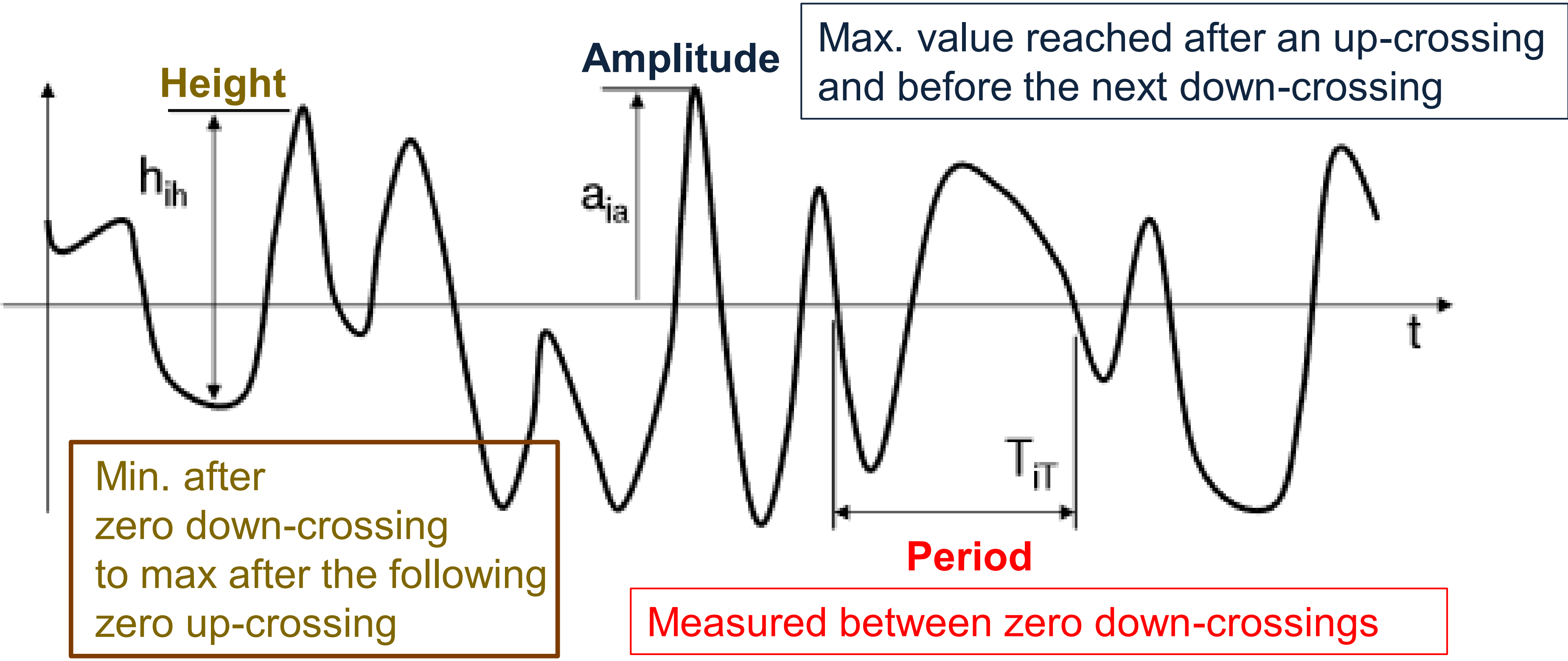


The Draupner wave, a single giant wave measured on New Year's Day 1995, finally confirmed the existence of freak waves, which had previously been considered near-mythical.

https://www.youtube.com/watch?v=eMBU1eXDYDc

https://www.youtube.com/watch?v=qvjJFUTliEM

The Irregular Wave



Statistics on Sea States

- Often idealized wave spectrum that neglect time and place are used, e.g.
  - Pierson-Moskowitz for fully developed sea
  - JONSWAP for developing sea

- Often the sea is considered as *long-crested* (conservative assumption) meaning that the waves are assumed to come from the same direction. Realistic seas are however often *short-crested* meaning that the waves come from different directions

- The sea states are described in global wave statistics

Table 5—Observed Percentage Frequency of Occurrence of Wave Heights and Periods (Hogben and Lumb data)
Northern North Atlantic

Wave height, m	Wave Period T_1, sec: 2.5	6.5	8.5	10.5	12.5	14.5	16.5	18.5	20.5	Over 21	Total
0-1	13.7204	3.4934	0.8559	0.3301	0.1127	0.0438	0.0249	0.0172	0.0723	0.3584	19.0291
1-2	11.4889	15.5036	6.4817	1.8618	0.5807	0.1883	0.0671	0.0254	0.0203	0.0763	36.2941
2-3	1.5944	7.8562	8.0854	3.7270	1.1790	0.3713	0.1002	0.0321	0.0091	0.0082	22.9629
3-4	0.3244	2.2487	4.0393	2.9762	1.3536	0.4477	0.1307	0.0428	0.0050	0.0040	11.5724
4-5	0.1027	0.7838	1.6998	1.5882	0.9084	0.3574	0.1443	0.0433	0.0072	0.0049	5.6400
5-6	0.0263	0.1456	0.3749	0.4038	0.2493	0.1200	0.0382	0.0067	0.0027	0.0027	1.3702
6-7	0.0277	0.1477	0.3614	0.4472	0.2804	0.1301	0.0504	0.0113	0.0011	0.0032	1.4605
7-8	0.0084	0.0714	0.1882	0.2199	0.1634	0.0785	0.0353	0.0069	0.0018	0.0034	0.7772
8-9	0.0037	0.0325	0.0856	0.1252	0.1119	0.0558	0.0303	0.0045	0.0027	0.0033	0.4555
9-10	0.0034	0.0204	0.0674	0.1173	0.0983	0.0550	0.0303	0.0173	0.0079	0.0047	0.4220
10-11		0.0005	0.0012	0.0023	0.0031	0.0012		0.0005			0.0088
11+		0.0005	0.0007	0.0019	0.0035	0.0002			0.0005		0.0073
Totals	27.3003	30.3043	22.2415	11.8009	5.0143	1.8493	0.6517	0.2080	0.1306	0.4691	100.000

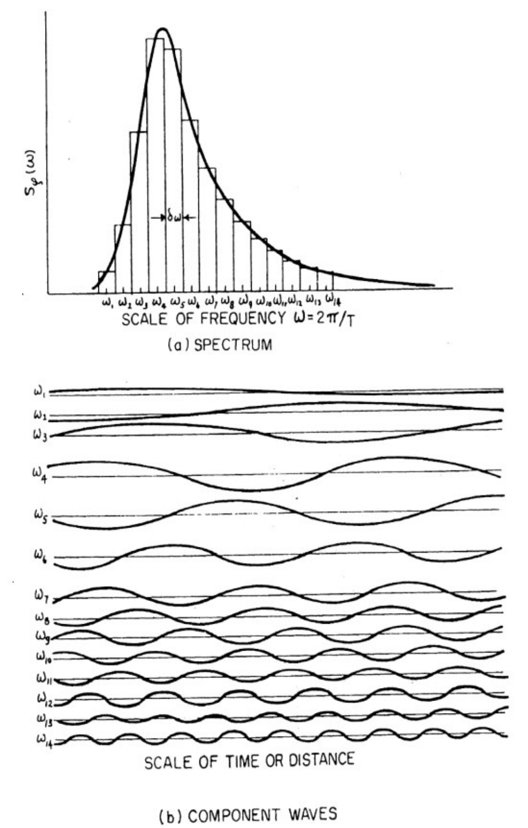


Fig. 8 Typical variance spectrum of waves, showing approximation by a finite sum of components

Sea States for ship structures (Long Term)

- For unlimited operation the North-Atlantic (Area 25 of BSRA statistics)
- For restricted service at the discretion of the Class Society Service Factor Analysis can be employed
- Some Key References :
  - ➢ IACS URS 11A, Rec. 34 ;
  - ➢ Lloyd's Register Rules (Part 4 Ship Structures) and

© Marine Technology, Vol. 46, No. 2, April 2009, pp. 116–121

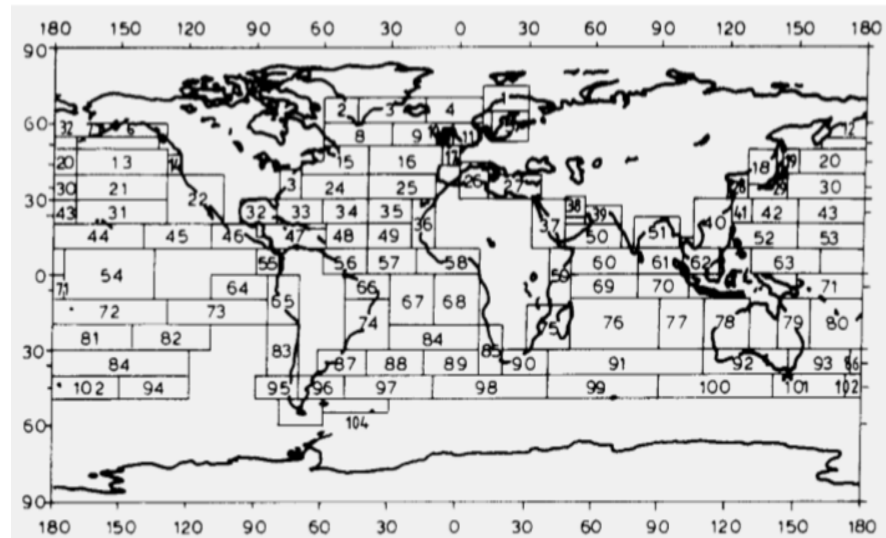
Service Factor Assessment of a Great Lakes Bulk Carrier Incorporating the Effects of Hydroelasticity

Spyridon E. Hirdaris,[1] **Norbert Bakkers,**[2] **Nigel White,**[2] and **Pandeli Temarel**[3]

This paper presents a summary of an investigation into the effects of hull flexibility when deriving an equivalent service factor for a single passage of a Great Lakes Bulk Carrier from the Canadian Great Lakes to China. The long term wave induced bending moment predicted using traditional three-dimensional rigid body hydrodynamic methods is augmented due to the effects of springing and whipping by including allowances based on two-dimensional hydroelasticity predictions across a range of headings and sea states. The analysis results are correlated with full scale measurements that are available for this ship. By combining the long term "rigid body" wave-bending moment with the effects of hydroelasticity, a suitable service factor is derived for a Great Lakes Bulk Carrier traveling from the Canadian Great Lakes to China via the Suez Canal.

Keywords: Great Lakes; hydrodynamics; longitudinal strength

Hogben, N., Dacunha, N.M. and Olliver, G.F. (1986). Global wave statistics, British Maritime Technology.



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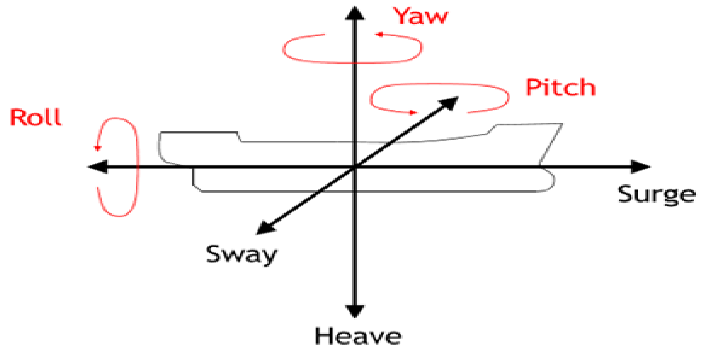
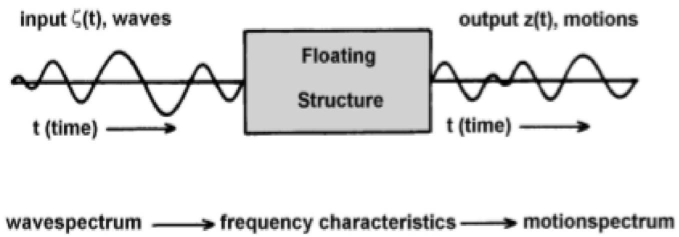
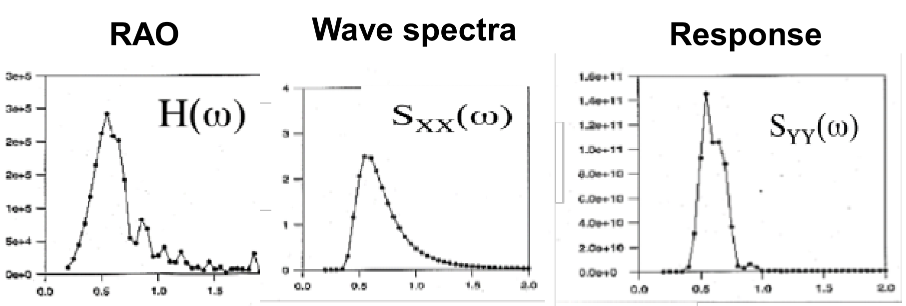
Ship Motions - Introduction

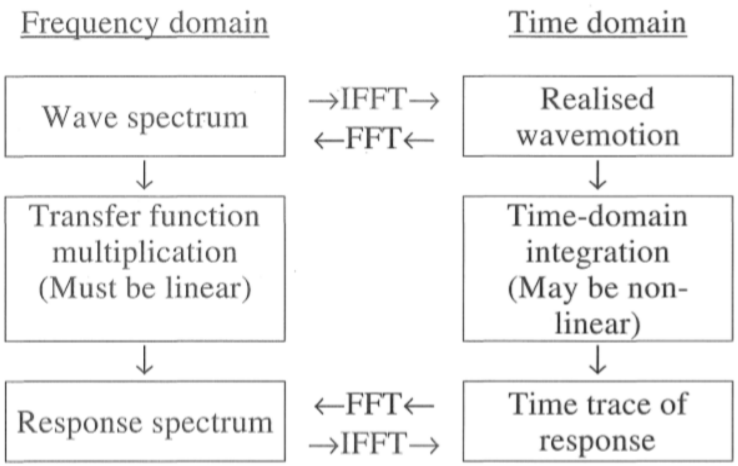
- When ship moves in waves it will have 6 degrees of freedom (DOF)
- This means that for arbitrarily-shaped ship we will have
  - 6 equations of motion
  - 6 unknowns
- These must be solved simultaneously
- For port-starboard-symmetry these equations reduce to two sets of uncoupled EoM containing 3 unknows namely :
  - Vertical: surge, heave, pitch
  - Horizontal: Sway, yaw, roll
- We approximate the response by superposition of elementary waves
  - Different lengths
  - Different directions

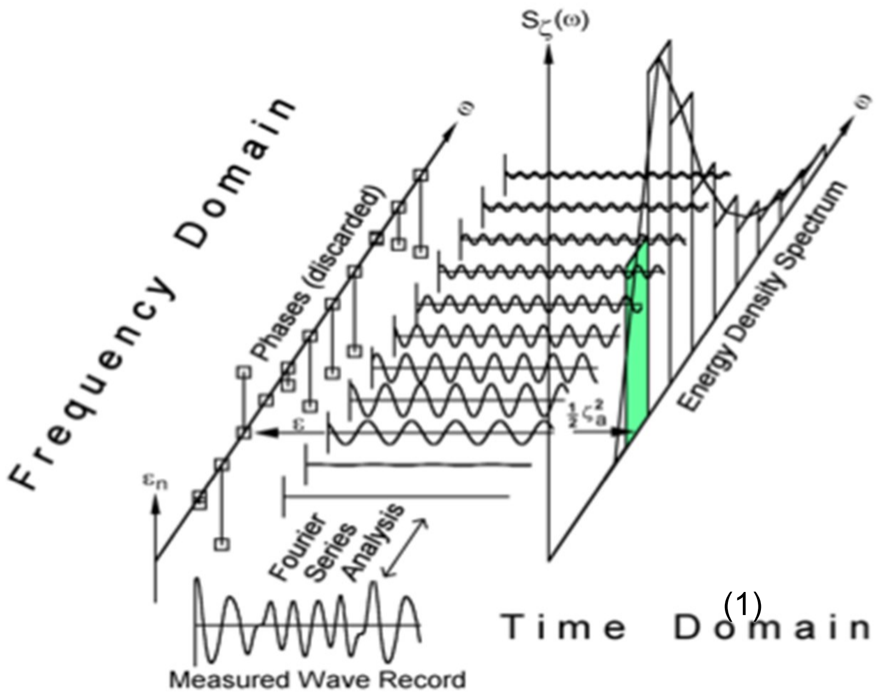






Frequency and Time domains





(1)

$$\mathscr{F}^{-1}\{G(f)\} = \int_{-\infty}^{\infty} G(f)e^{2\pi i f t}\,df = g(t)$$

$$\mathscr{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t}\,dt$$

(2)

Case 1 : Undamped free vibration (1 - DOF)

- Assume the system is conservative and the vibration is free. The equation of motion reduces to:

$$m\ddot{x} + kx = 0$$

- Assume sinusoidal solution $x = e^{\lambda t}$

Natural frequency of the system

$$\lambda^2 m + k = 0,\ \lambda = \pm\sqrt{\frac{k}{m}} = \pm j\omega_n$$

- The response is defined as :

$$x = A_1 e^{j\omega_n t} + B_1 e^{-j\omega_n t} = A\,sin(\quad \omega t) + B\,cos(\quad \omega t) = X\,sin(\quad \omega t + \phi)$$

- The amplitude and phase are defined as :

$$X = \sqrt{A^2 + B^2} \qquad \phi = \tan^{-1}(B/A)$$

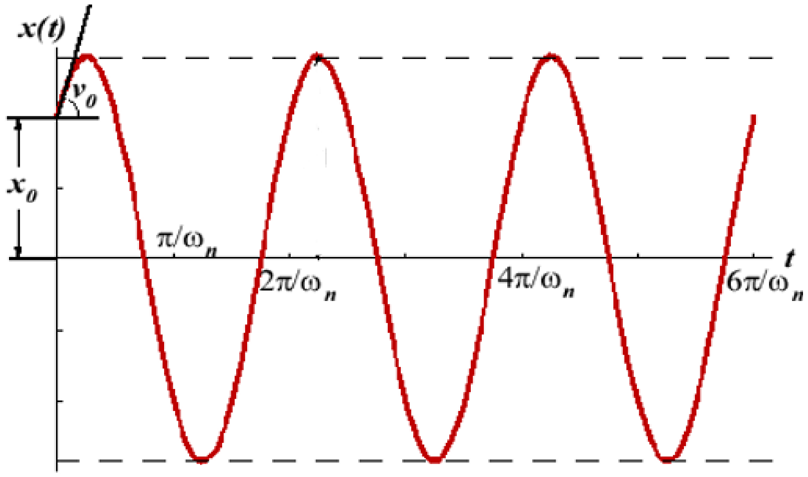
Case 1 : Undamped free vibration (1 dof)

- If we assume the initial conditions: $x(t=0) = x_0, \dot{x}(t=0) = v_0$

$$X = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \qquad \phi = tan^{-1}\left(\frac{\omega_n x_0}{v_0} \right)$$

- Therefore the final solution of this system is defined as:

$$x(t) = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} sin\left(\omega t + \phi \right)$$



Case 2 : Damped free vibration (1- DOF)

- The amplitude of oscillation of the spring, mass, damper system will reduce with time due to damping effects. The damper works by dissipating the energy of the system to zero. For this case Newton's equation becomes :

$$m\ddot{x} + c\dot{x} + kx = 0$$

- Assume sinusoidal solution $x = e^{\lambda t}$

$$m\lambda^2 + c\lambda + k = 0, \qquad \lambda_{1,2} = \frac{-c + \sqrt{c^2 - 4mk}}{2m}$$

- There are three solutions to the above differential equation that link to three different types of motions:

1. If $\lambda_{1,2}$ are real ($c^2 - 4mk > 0$ (corresponding to **overdamped** case.
2. If $\lambda_{1,2}$ are imaginary ($c^2 - 4mk < 0$ (corresponding to **underdamped** case.
3. If $\lambda_1 = \lambda_2$ are real ($c^2 - 4mk = 0$ (leading to $c_{cr} = \sqrt{4mk} = 2m\omega_n$ that corresponds to **critically damped** case (i.ethe system overshoots and comes back to rest).

Case 2 : Damped free vibration (1- DOF)

- Another approach to solve Newton's equation is the damping ratio (ζ). This is the ratio of the damping coefficient of the system to the critical damping coefficient:

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}} \rightarrow c = 2m\omega_n\zeta$$

$$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]$$

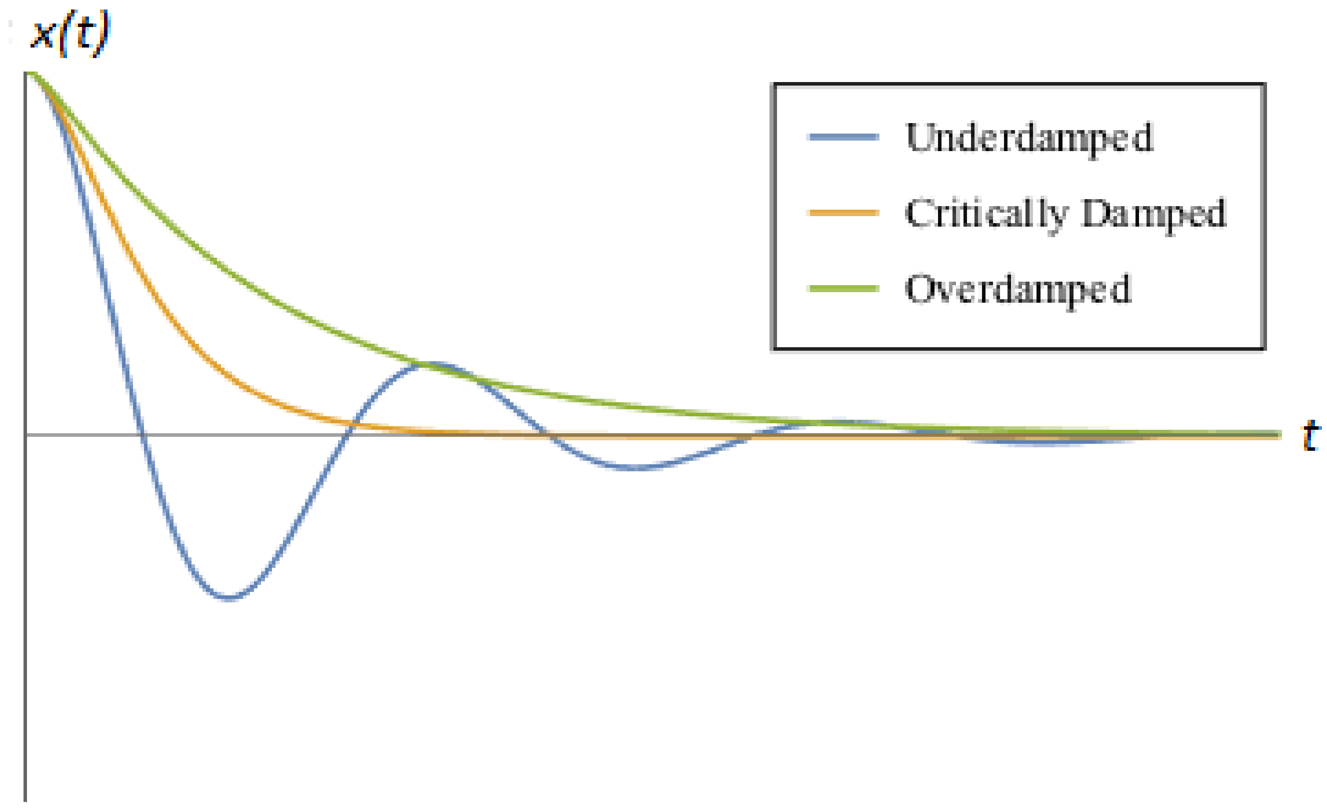
- The three types of motions can then be defined by the damping ratio as:

1. $\zeta > 1$ (for overdamped case);
2. $\zeta < 1$ (for underdamped case) and
3. $\zeta = 1$ for the critically damped case.

- The response of the system in terms of these two roots is defined as:

$$x(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t}$$

Case 2 : Damped free vibration (1 dof)



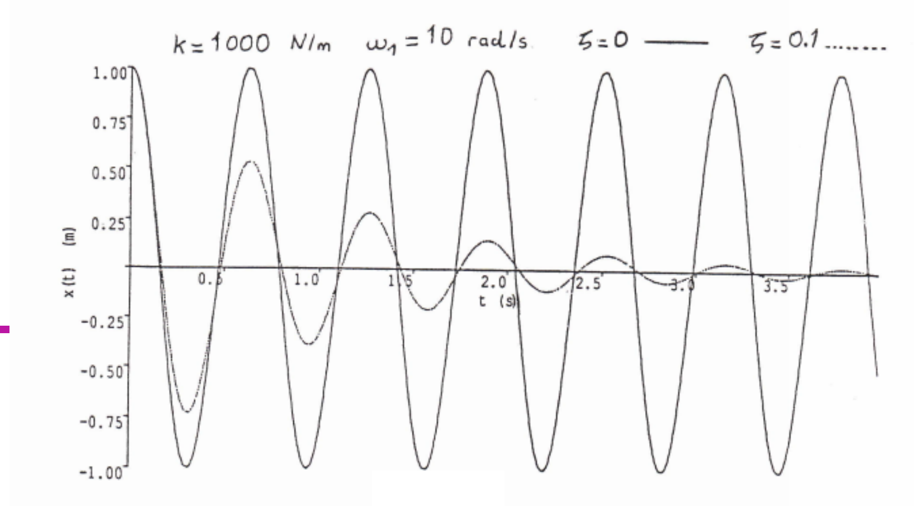
How can we practically assess damping ?

A practical way to assess damping that is broadly applicable in the area of ship hydrodynamics is the damping decay test. This can be mathematically expressed using the log decrement that is the natural logarithm of the ratio of two successive amplitudes.

$$\delta = ln\frac{X_1}{X_2} = ln\frac{Ae^{-\zeta\omega_n t_1}}{Ae^{-\zeta\omega_n(t_1+T_d)}} = ln\, e^{\zeta\omega_n T_d} = \zeta\omega_n T_d$$

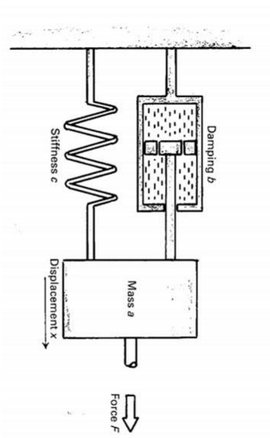
$$\because T_d = 2\pi/\omega_d \rightarrow \therefore \delta = \frac{2\pi\zeta\omega_n}{\omega_d} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Since the damping ratio is very small in that case, the log decrement can be approximated by: $\delta = 2\pi\zeta$



Case 3 : Forced Vibration – 1 DOF

Consider adding harmonic excitation to the vibration system where $F(t)$ varies in sinusoidal manner instead of being arbitrary function in time:



$$m\ddot{x} + c\dot{x} + kx = F(t) = F_0\, cos(\omega t)$$

$$\rightarrow \ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = f_0 \cos(\omega t)$$

$$f_0 = F_0 / m$$

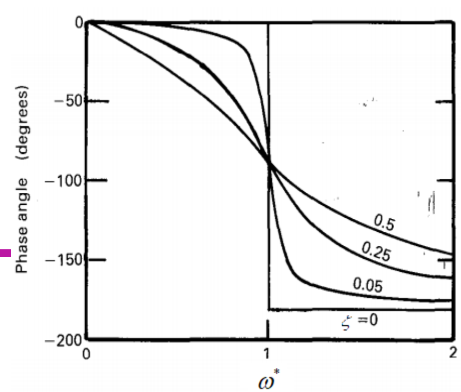
This is a differential equation of the 2nd order. Accordingly, it is prone to a general and particular solution which when combined together they may give the response function of the system.

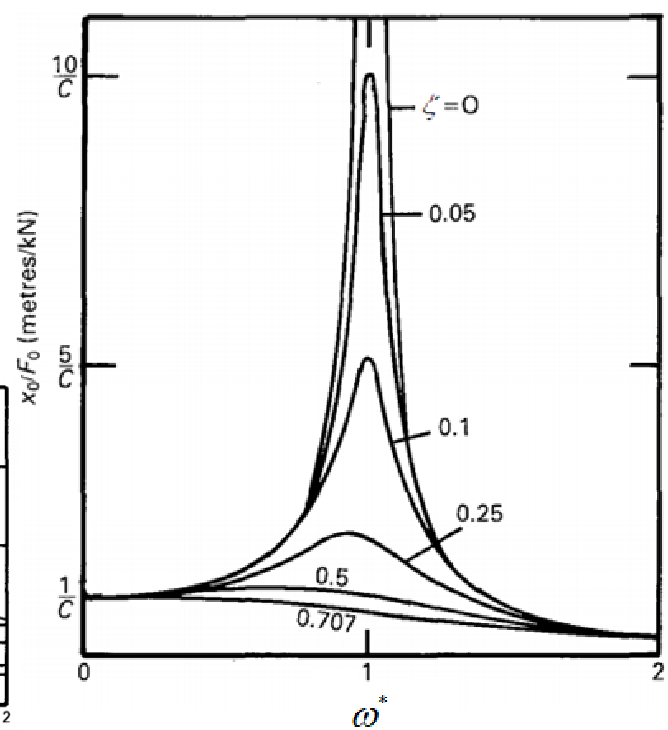
Case 3 : Forced Vibration – 1 DOF

If we rewrite these equations as a function of the frequency ratio $\omega^* = \omega / \omega_n$ we get the expression

$$\frac{Xk}{F_0} = \frac{X\omega_n^2}{f_0} = \frac{1}{\sqrt{\left(1-\omega^{*2}\right)^2 + \left(2\zeta\omega^*\right)^2}} \qquad \theta = tan^{-1}\left(\frac{2\zeta\omega^*}{1-\omega^{*2}}\right)$$

The term in the left-hand side is known as the **amplitude ratio**. When the system is undamped, the amplitude ratio, when the frequency of vibration approaches the natural frequency, gets to extremely significant value, and such case is known by resonance.





Forced Vibrations due to Harmonic Excitation

If we apply a Fourier integral on the excitation force of Newton's equation of motion external loading and response are defined as

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_P(\omega)e^{i\omega t}d\omega \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_x(\omega)e^{i\omega t}d\omega$$

Then Newton's equation of motion becomes :

$$-m\int_{-\infty}^{\infty} A_x(\omega)\omega^2 e^{i\omega t}d\omega + c\int_{-\infty}^{\infty} A_x(\omega)i\omega e^{i\omega t}d\omega + k\int_{-\infty}^{\infty} A_x(\omega)e^{i\omega t}d\omega = \int_{-\infty}^{\infty} A_P(\omega)e^{i\omega t}d\omega$$

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- For $A_x(\omega) = \dfrac{A_P(\omega)}{-m\omega^2 + ci\omega + k}$ ⇨ $A_x(\omega^*) = \dfrac{A_P^n(\omega^*)}{-m\omega^{*2} + \delta i\omega^* + 1}$

$$\begin{cases} A_P^n(\omega^*) = \dfrac{A_P(\omega)}{m\omega_n^2} \\ \delta = \dfrac{c}{m\omega_n} \end{cases}$$

Quasi-Static Response

- **At sub-critical case (also known as quasi-static response)** the system can reach high values of spectral density at small frequencies relative to the natural frequency and the stiffness has the major effect on the system
- Only stiffness affects the system response

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2}} \rightarrow S_x \approx S_P^n$$

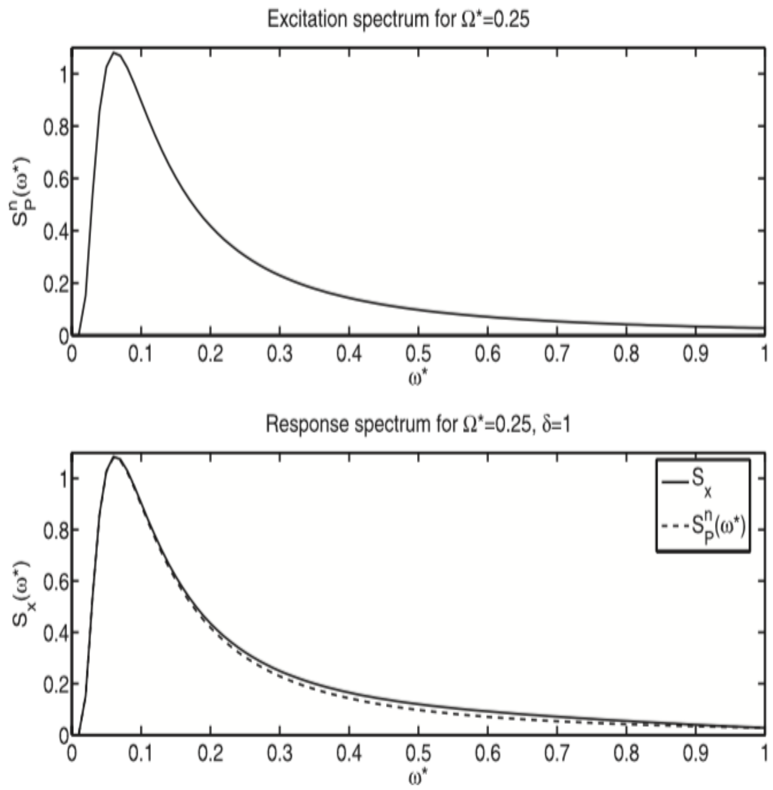


Figure 4.2. Quasi-static or sub-critical response.

Dynamic Response

- At super-critical stage (also known as dynamic response) the highest values of spectral density lie in only high values of frequencies with respect to the natural frequency and damping plays an important role:
- **Only inertia forces affect the system response**

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2}} \rightarrow S_x \approx \frac{S_P^n}{\omega^{*4}}$$

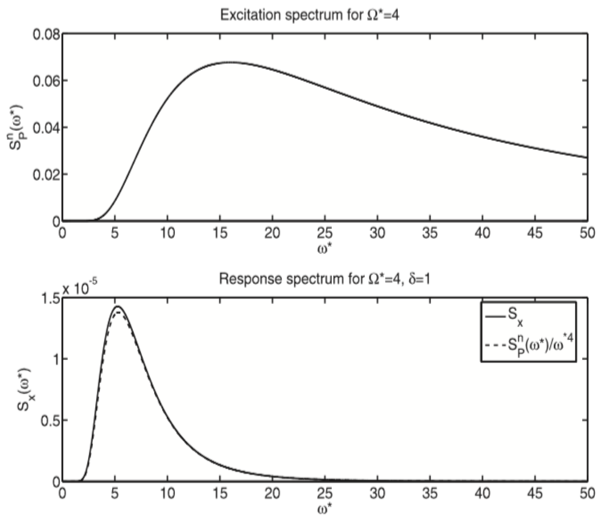


Figure 4.3. Dynamical or super-critical response.

Resonance

- At resonance condition when there is very low damping the frequency ratio ω^* approaches unity. The denominator approaches zero, and the spectral density approaches extremely large value:

- Serious problems which can be controlled only by adjusting damping

$$S_x(\omega^*) = \frac{S_P^n(\omega^*)}{\left(1-\omega^{*2}\right)^2 + \delta^2\omega^{*2} \to \approx 0} \to S_x >>>>$$

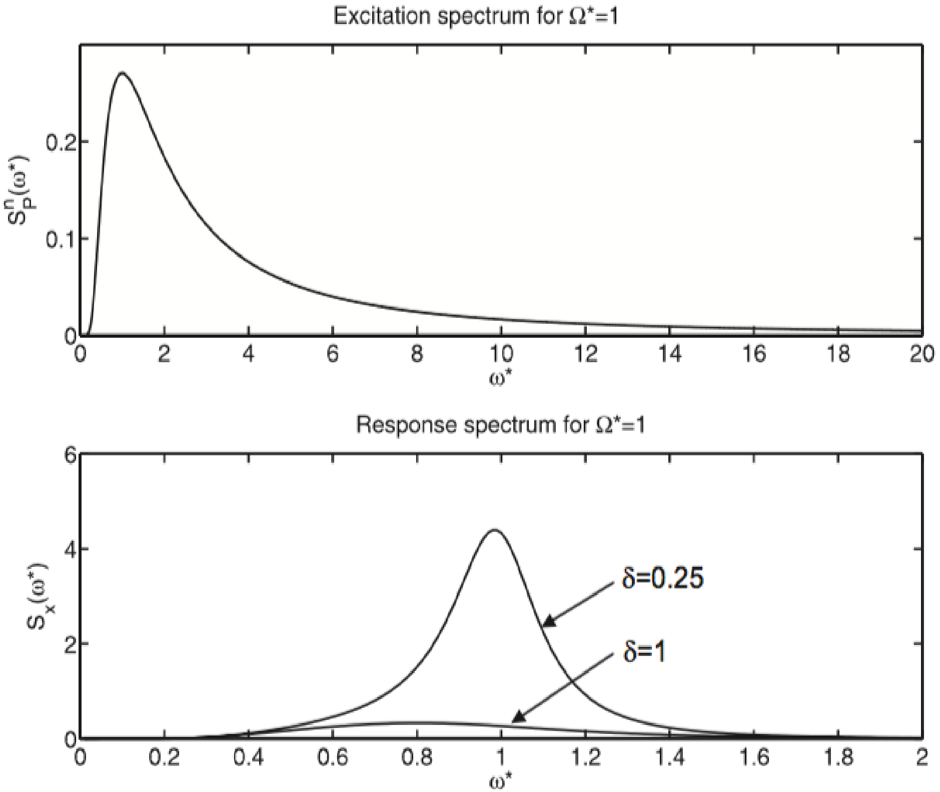
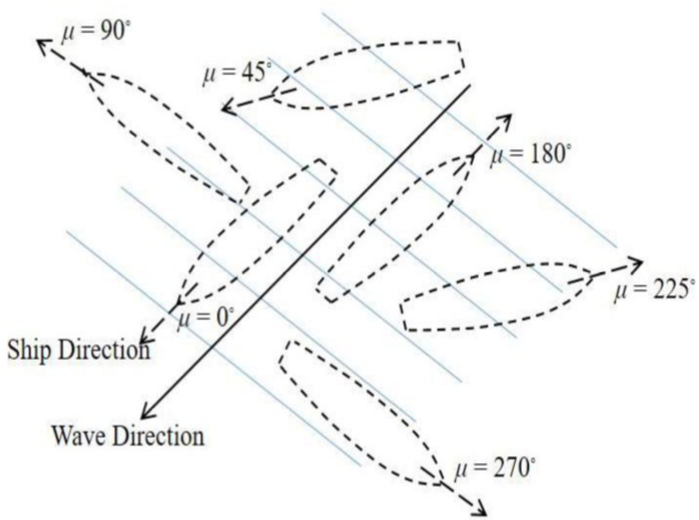
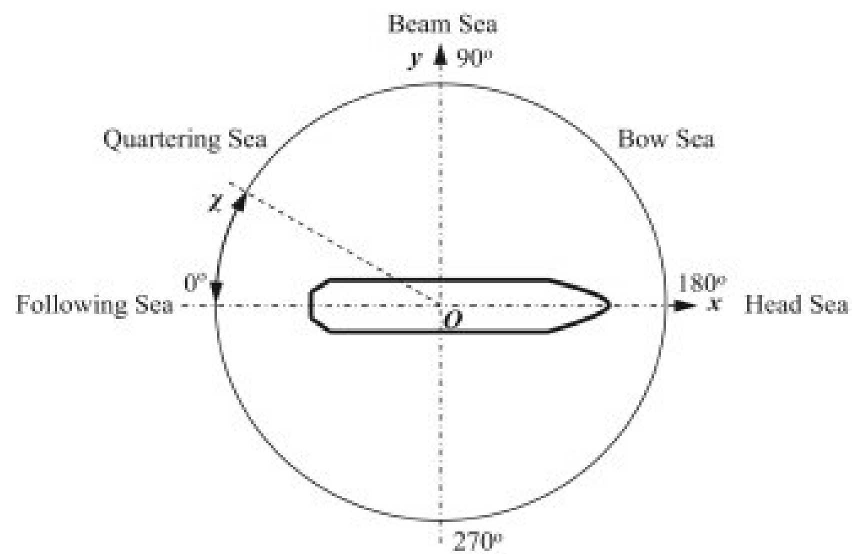


Figure 4.4. Resonant response.

Ship encounter frequency

In ship dynamics the encounter frequency with the waves is used instead of the absolute wave frequency. This is because the ship is moving relative to the waves, and she will meet successive peaks and troughs in a shorter or longer time interval depending upon whether it is advancing into the waves or is travelling in their direction.





Ship encounter frequency

- Assuming the waves and ship are on a straight course, the frequency with which the ship will encounter a wave crest depends on the distance between the waves crests (λ — wavelength), the speed of the waves (c — wave celerity that depends on the wavelength), the speed of the ship (U), and the relative angle between the ship heading and the wave heading (μ)

- The encounter period is thus the distance traveled (λ) divided by the speed the ship encounters the waves (c - U cos(μ))

$$\omega_e = \omega - \frac{\omega^2 U}{g} cos(\mu)$$

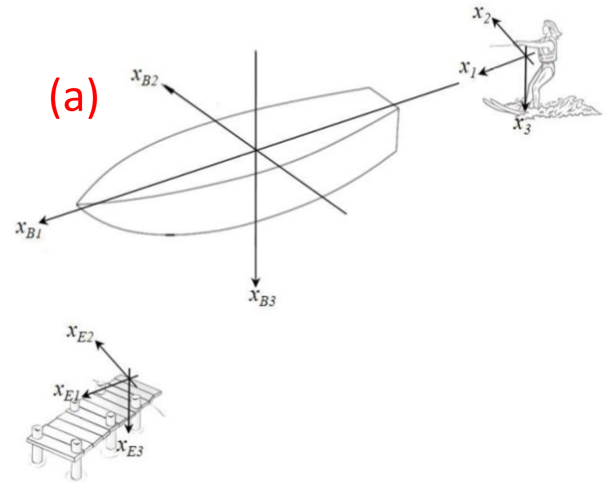
$$T_e = \frac{\lambda}{c - U \times cos\mu}$$

Coordinate Systems

- We have several coordinate systems for different purposes

  - **Ship CoG or body bound system – xB**
  - **Earth bound system – xE**
  - **Steadily translating system xi**

(a)



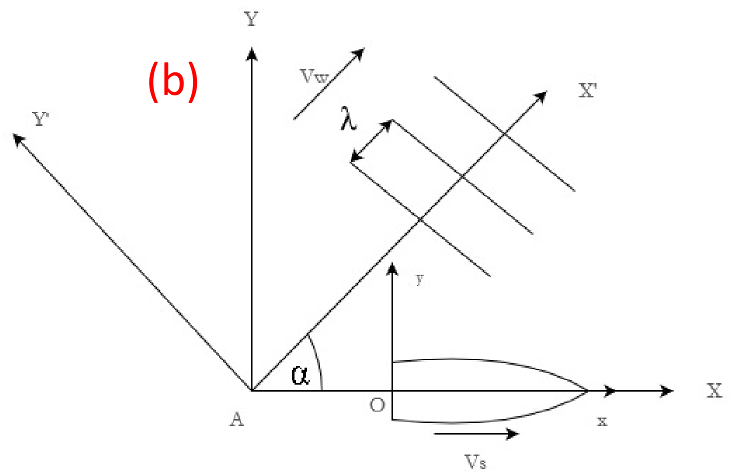
- Considering transformation of coordinates for a regular wave propagating at an angle α (from A X’ Y’ Z’ to A X Y Z) as illustrated in (b)

$$X' = X\cos\alpha + Y\sin\alpha$$

- Then the transformation to the ship’s-fixed coordinate system (oxy) coordinate system:

$$X = x + V_s t,\ \ Y=y$$

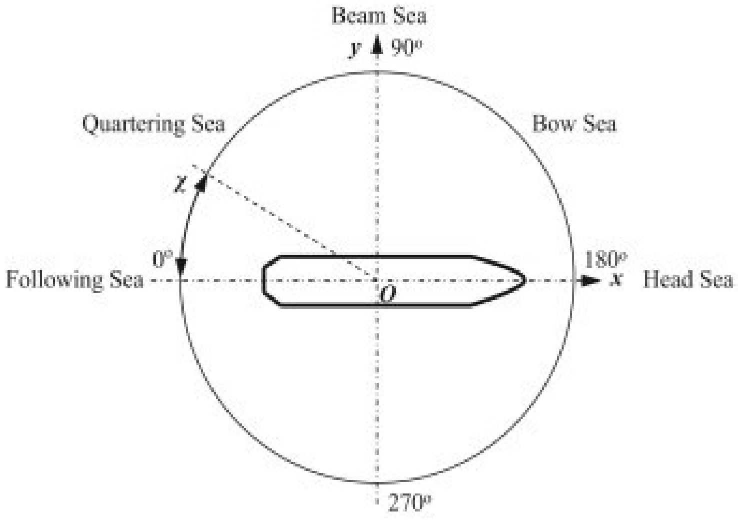
(b)



Definition of headings

The heading angle determines the “type” of seas the ship experiences. Heading angles are defined as follows :

- $\mu = 0^0$ – following seas
- $\mu = 180^0$ – head seas
- $\mu = 90^0$ – starboard beam seas
- $\mu = 270^0$ -port beam seas
- $0 \leq \mu \leq 90^0$ – quartering waves on the ship starboard side
- $270^0 \leq \mu \leq 360^0$ – quartering waves on the ship port side
- $90^0 \leq \mu \leq 180^0$ -bow waves on the starboard side
- $180^0 \leq \mu \leq 270^0$ – bow waves on the port side



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Good Luck !!!