Lecture 6

Ship motions in regular waves

Moving through water is different than moving through air. This is partly because water is denser and more viscous than air (i.e. damping is increased). This motion requires the water surrounding the floating body to be accelerated. This extra required force shows up in the equation of motion as an addition to the mass of the object known as the added mass. It therefore represents the amount of fluid accelerated by the object. However, something to keep in mind is that the particles of fluid adjacent to the body will also accelerate to varying degrees and the added mass value is a weighted integration of the entire fluid mass effected by the accelerating object.

In this sense the equation of motion of a ship in water can be simplified to read:

$$(a + m)\ddot{x} + b\dot{x} + cx = F_0 sin(\omega t) \qquad (6\text{-}1)$$

where *a* stands for added mass, *b* is the hydrodynamic damping, *c* is the stiffness, F is the excitation due to external environment (assumed herby sinusoidal) and the x - variables represent the response (acceleration $\ddot{x}$, velocity $\dot{x}$ and displacement x). Both the added mass and hydrodynamic damping coefficients are a function of the frequency of oscillation. However, added mass depends primarily on the shape of the object, the type of motion (linear or rotational), and the direction of the motion. In this way, added mass differs from just mass since mass is a quantity independent of motion. Hydrodynamic damping is related to the viscosity of the fluid (and hence the frictional drag). However, when a free surface is involved the damping is dominated by the generation of waves. The larger the waves generated, the larger the hydrodynamic damping. Each degree of freedom that has a restoring force has an associated natural frequency. So, for a ship, there is a natural frequency in heave, roll, and pitch. These natural frequencies depend on the mass and stiffness properties of the system.

For a ship with port-starboard symmetry (e.g. typical ocean going or naval vessel) the coupled motions of heave – pitch and sway – roll – yaw can be examined separately during seakeeping analysis. Of these five motions only heave pitch and roll have a restoring force or moment. The forces provided due to the effects of added mass and damping are referred to as hydrodynamic forces. They arise from pressure distribution around the oscillating hull.

In the following sections the equations of motion heave and pitch and coupled heave pitch and roll will be examined as an introduction to understanding the mathematical background to the seakeeping problem.

6.1 Uncoupled heave motion

Let us consider the case of a ship in still water which is subject to a mechanical excitation in the form of an upward force $F_z(t)$ leading to heave displacement $z(t)$. According to the theory explained in Lecture 5 the linear equation of motion for this 1 DOF system will be:

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z(t) \qquad \text{(6-2)}$$

For a sinusoidally varying mechanical excitation $F_z(t) = F_1 e^{j\omega t}$. Assuming F_1 is a force vector of constant amplitude the response will also be sinusoidal namely $z(t) = Ze^{j(\omega t-\varepsilon)}$ where Z is the amplitude of excitation and ε the phase lag of the response. Accordingly:

$$Z = \frac{F_1}{\sqrt{(C_{zz}-\omega^2 M_{zz})^2+(\omega N_{zz})^2}} \quad \text{and} \quad tan\varepsilon = \frac{\omega N_{zz}}{(C_{zz}-\omega^2 M_{zz})} \qquad \text{(6-3)}$$

where :

- $C_{zz}z = \rho g A_w z$ is the hydrostatic heave restoring force (see Figure 6-1) with ρ representing the water density (kg/m^3) ; g the acceleration of gravity (m/s^2) and A_w the still water lane area (m^2).'
- $N_{zz}\dot{z}$ is the heave damping force provided by the surrounding water
- N_{zz} is the heave damping coefficient.
- $M_{zz} = m + m_{zz}$ is the virtual mass of the ship where the mass of the ship is $m = \rho\nabla$ and m_{zz} is the heave added mass.

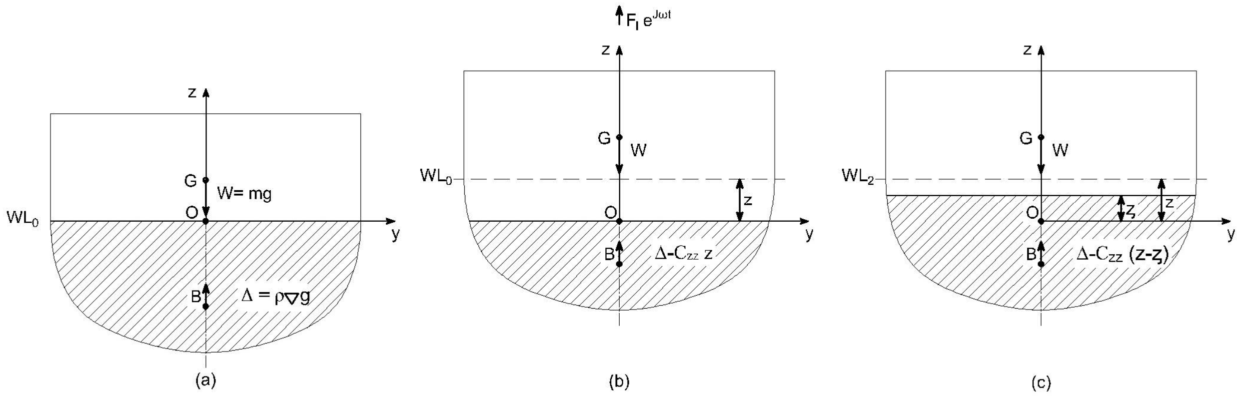

Figure 6-1 Demonstration of uncoupled heave motion. Underwater shaded areas indicate portion of hull underwater section.

The forces provided due to the effect of added mass and damping are the hydrodynamic forces. They arise from pressure distribution around the oscillating hull. In linear hydrodynamic theory the force has a component proportional to the acceleration (i.e. added mass) and a component proportional to the velocity (i.e. damping coefficient). To understand the effect of waves we have to consider the effect of the relative position of the ship with respect to waves. If we ignore the hydrodynamic effects and apply Newton's second law of motion (see Lecture 5) then for the uncoupled heave case :

$$m\ddot{z} = -W + \Delta - C_{zz}(z - \varsigma) \; or \; m\ddot{z} + C_{zz}z = C_{zz}\varsigma \qquad (6\text{-}4)$$

where ς is the wave profile defined with respect to the still water line and z-ς is called the relative displacement. Assuming that the hydrodynamic effects are proportional to the relative velocity and acceleration the equation of motion in waves becomes:

$$m\ddot{z} = -m_{zz}(\ddot{z} - \ddot{\varsigma}) - N_{zz}(\dot{z} - \dot{\varsigma}) - C_{zz}(z - \varsigma) \qquad (6\text{-}5)$$

Or

$$(m + m_{zz})\ddot{z} + N_{zz}\dot{z} + C_{zz}z = m_{zz}\ddot{\varsigma} + N_{zz}\dot{\varsigma} + C_{zz}\varsigma = F_z(t) \qquad (6\text{-}6)$$

Although the above rather simplistic Eq.(6-6) shows that for a ship in waves the surrounding fluid not only provides the hydrostatic and hydrodynamic terms but also the wave excitation which is a function of the wave acceleration, velocity and displacement.

For a stationary ship in a regular wave train of frequency ω the excitation term becomes:

$$F_z(t) = F_{zs}sin(\omega t) + F_{zc}cos(\omega t) = \bar{F}_z sin(\omega t + \psi) \qquad (6\text{-}7)$$

The amplitude of the wave excitation are defined as :

$$\bar{F}_z = \sqrt{F_{zs}^2 + F_{zc}^2} \text{ for } F_{zs} = \bar{F}_z cos\psi \quad \text{and} \quad F_{zc} = \bar{F}_z sin\psi \qquad (6\text{-}8)$$

Thus Eq. (6-6) can be re-written as

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z sin(\omega t + \psi) = \bar{F}_z e^{j(\omega t + \psi)} \qquad (6\text{-}9)$$

The response to this excitation is $z(t) = Zsin(\omega t + \psi - \varepsilon)$ or in complex notation $z(t) = Ze^{j(\omega t + \psi - \varepsilon)}$.

Thus, by substitution to Eq. (6.3) we obtain :

$$Z[(C_{zz} - \omega^2 M_{zz})sin(\omega t + \psi - \varepsilon) + \omega N_{zz}cos(\omega t + \psi - \varepsilon)] = F_z sin(\omega t + \psi) \qquad (6\text{-}10)$$

An easy way to obtain the heave amplitude (Z) and phase lag (**ε**) from Eq.(6-10) is to consider the following two cases:

$$\omega t + \psi - \varepsilon = 0 leading to Z\omega N_{zz} = \bar{F}_z \sin(\varepsilon) \qquad (6\text{-}11)$$

$$\omega t + \psi - \varepsilon = \pi/2 \quad \text{leading to } Z(C_{zz} - \omega^2 M_{zz}) = \bar{F}_z \cos(\varepsilon) \qquad (6\text{-}12)$$

Squaring equations (6-11) and (6-12) and adding them produces the heave amplitude as follows :

$$Z = \bar{F}_z / \sqrt{(C_{zz} - \omega^2 M_{zz})^2 + (\omega N_{zz})^2} \qquad (6\text{-}13)$$

Dividing Eq. (6-11) by Eq. (6-12) defines the phase lag as

$$tan(\varepsilon) = \omega N_{zz}/(C_{zz} - \omega^2 M_{zz}) \qquad (6\text{-}14)$$

For a ship progressing in regular waves with forward speed *U* and heading χ the variation of the wave elevation (and wave velocity and acceleration) with time is sinusoidal with the wave encounter frequency (ω_e). Thus Eq. (6-9) becomes:

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z sin(\omega_e t + \psi) = \bar{F}_z e^{j(\omega_e t+\psi)} \quad (6\text{-}15)$$

In the above equation the amplitude and phase of the wave sinusoidal wave excitation is proportional to the wave amplitude and is a function of the wave frequency and the ship's forward speed and heading.

The hydrodynamic damping coefficient and added mass are not constant values but they vary with the frequency of the ship's oscillation, i.e. $m_{zz} = m_{zz}(\omega_e)$; $N_{zz} = N_{zz}(\omega_e)$ *and* $\bar{F}_z = aF_z(\omega, \omega_e)$. Thus in more explicit form Eq. (6-15) becomes:

$$[m + m_{zz}(\omega_e)]\ddot{z} + N_{zz}(\omega_e)\dot{z} + C_{zz}z = aF(\omega,\omega_e) sin(\omega_e t + \psi) = F_z(\omega,\omega_e)e^{j(\omega_e t+\psi)} \quad (6\text{-}16)$$

and the heave amplitude and phase lag are:

$$Z = \frac{aF_z(\omega,\omega_e)}{\sqrt{[C_{zz}-\omega_e^2(m+m_{zz}(\omega_e))]^2+[\omega_e N_{zz}(\omega_e)]^2}} \text{ and } tan\varepsilon = \frac{\omega_e N_{zz}(\omega_e)}{C_{zz}-\omega_e^2(m+m_{zz}(\omega_e))} \quad (6\text{-}17)$$

If we assume free motions in waves then Eq. (6-16) becomes:

$$[m + m_{zz}(\omega_e)]\ddot{z} + N_{zz}(\omega_e)\dot{z} + C_{zz}z = 0 \quad (6\text{-}18)$$

Analytical solution of this equation is not possible due to the presence of coefficients which are not constants but functions of the encounter frequency. Nevertheless, the free heave displacement will be exponentially decaying oscillatory function of time. For undamped motion

$$[m + m_{zz}(\omega_e)]\ddot{z} + C_{zz}z = 0 \quad (6\text{-}19)$$

leading to the characteristic equation

$$C_{zz}z - \omega_e^2[m + m_{zz}(\omega_e)] = 0 \quad (6\text{-}20)$$

that cannot be solved analytically. However, as it can be seen from Fig. 6.2a it is possible to assume a constant value of added mass namely $\bar{m}_{zz} = m_{zz}(\omega_e \to \infty)$ and therefore the heave natural frequency in water can be approximated as :

$$\omega_{3n} = \sqrt{\frac{c_{zz}}{m+m_{zz}}} \quad (6\text{-}21)$$

6.2 Uncoupled pitch motion

If we consider that the ship is an 1DOF system subject to pitch excitation namely $\vartheta(t)$ then the corresponding mathematical expression to Eq. (6-16) is :

$$[I_{yy} + I_{\vartheta\vartheta}(\omega_e)]\ddot{\vartheta} + N_{\vartheta\vartheta}(\omega_e)\dot{\vartheta} + C_{\vartheta\vartheta}\vartheta = \bar{M}_\vartheta(\omega,\omega_e)e^{j(\omega_e t+u)} \quad (6\text{-}22)$$

where :

I_{yy} is the mass moment of inertia about axis Oy

$I_{\vartheta\vartheta}$ is the pitch added mass moment of inertia

$N_{\vartheta\vartheta}$ is the pitch damping coefficient

$C_{\vartheta\vartheta} = \rho g I_{long}$ for $I_{long} = longitudinal\ 2^{nd}\ moment\ of\ water\ plane\ area$

$\bar{M}_{\vartheta}$ is the amplitude of the wave excitation vector

The solution of this equation is similar to the one presented in Eq. (6-17). The pitch natural frequency in water can be approximated as

$$\omega_p = \sqrt{\frac{c_{\theta\theta}}{I_{yy}+\bar{I}_{\theta\theta}}} \quad (6\text{-}23)$$

where $\bar{I}_{\theta\theta} = I_{\theta\theta} \rightarrow \infty$.

6.3 Coupled heave and pitch motions

The coupled equations of motion for heave and pitch can be expressed in matrix format as:

$$\begin{bmatrix} m + m_{zz} & m_{z\theta} \\ m_{\theta z} & I_{yy} + I_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} N_{zz} & N_{z\theta} \\ N_{\theta z} & N_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_{zz} & C_{z\theta} \\ C_{\theta z} & C_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} z \\ \theta \end{bmatrix} = \begin{bmatrix} F_z(\omega, \omega_e) e^{j(\omega_e t + \psi)} \\ \bar{M}_{\vartheta}(\omega, \omega_e) e^{j(\omega_e t + u)} \end{bmatrix} \quad (6\text{-}24)$$

In this equation in addition to heave added mass m_{zz} and pitch added inertia $I_{\theta\theta}$ we have two additional heave into pitch (and *pitch into heave*) added mass terms namely $m_{z\theta}$ and $m_{\theta z}$. Similarly, in addition to heave damping N_{zz} and pitch damping $N_{\theta\theta}$ coefficients we have the heave into pitch (and pitch into heave) terms defined as $N_{z\theta}$ and $N_{\theta z}$ respectively. There are no terms in the form of first moments of mass in the inertia matrix. The heave into pitch restoring terms are defined as :

$$C_{z\theta} = C_{\theta z} = \rho g M_l \quad (6\text{-}25)$$

where $M_l = \int_L xB(x)dx$ represents the longitudinal first moment of water plane area and $B(x)$ is the beam in way of the water line.

The above equations indicate that coupling takes place through hydrodynamic and hydrostatic actions. All added masses and damping coefficients are dependent on the frequency of oscillation.

6.4 6.4 Roll in small amplitudes

There are three rotational degrees of freedom namely roll, pitch and yaw and each have a subscript number associated with the direction. For typical ship shapes, the radii of gyration have a relationship to the ship's geometry. So, in general, $k_4 = 0.3 \times B_{WL}$ (for roll); $k_5 = 0.25 \times L_{pp}$ (for pitch); $k_6 = 0.25 \times L_{pp}$ (for yaw). If we assume that a ship rolls in small oscillation format about her center of mass which is usually close to the undisturbed water line then dynamics are described by the equation :

$$[J_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(t) \qquad (6\text{-}26)$$

where :

$K_{\varphi}(t)$ is a sinusoidal mechanical excitation producing a rolling moment $K_{\varphi}(t) = K_1 e^{j\omega t}$

ϕ is the angle of roll

I_{xx} is the mass moment of inertia about the longitudinal axis through the centre of mass

$C_{\phi\phi} = \Delta GM_T = \rho g \nabla GM_T$ is the hydrostatic roll restoring coefficient

$I_{\phi\phi}$ = roll added inertia (frequency of oscilation dependent)

$N_{\phi\phi}$ = roll damping coefficient due to hydrodynamic effects associated with fin and tank stabilizaers

It is noted that roll damping increases with forward speed. The increase in damping results in a smaller maximum resonant peak, but also a slight reduction in the frequency at which the peak response will occur. The mass of a ship is determined by its total weight or displacement. Thus rotational inertia associated with roll are determined by the distance of each weight from center of gravity. The further the heaviest weights are from the CG, the larger the rotational moment of inertia. If all of the masses were located equidistant from the center of gravity the moment of inertia would be easy to calculate and would be equal to the total mass times the distance from the CG squared. If the roll added inertia and damping coefficients are constant the free damped equation of motion becomes

$$[J_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = 0 \qquad (6\text{-}27)$$

and if we ignore damping

$$[J_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + C_{\varphi\varphi}\varphi = 0 \qquad (6\text{-}28)$$

Although the mass in a ship is never located equidistant from the center of gravity, we can find the representative distance the mass would need to be were the ship a sphere. This representative distance is the radius of gyration, *k*. If we have the radius of gyration, we can find the ship's moment of inertia. Accordingly. Eq.(6-28) leads to

$$\rho\nabla(k_{xx}^2\ddot{\varphi} + gGM_T\varphi)=0 \qquad (6\text{-}29)$$

where the roll radius of gyration $k_{xx}$is defined as $I = I_{xx} + I_{\varphi\varphi} = mk_{xx} = \rho\nabla k_{xx}^2$. and the roll natural frequency including the effects of added inertia becomes

$$\omega_{\varphi} = \sqrt{\frac{C_{\varphi\varphi}}{I_{xx}+I_{\varphi\varphi}}}\sqrt{\frac{gGM_T}{k_{xx}^2}} \qquad (6\text{-}30)$$

If we define the constant roll damping coefficient as $N_{\varphi\varphi} = 2\varsigma\left(I_{xx} + I_{\varphi\varphi}\right)\omega_{\varphi}$ the damped equation of motion in still water after dividing terms by the inrtia term becomes :

$$\varphi + \ddot{2}\varsigma\omega_{\varphi}\dot{\varphi} + \omega_{\varphi}^2\varphi = 0 \qquad (6\text{-}31)$$

where ς is the damping ratio.

For a ship rolling in long beam waves (i.e. waves that are long compared to her beam) the instantaneous wave surface can be represented by the wave slope shown in Figure 6-2.

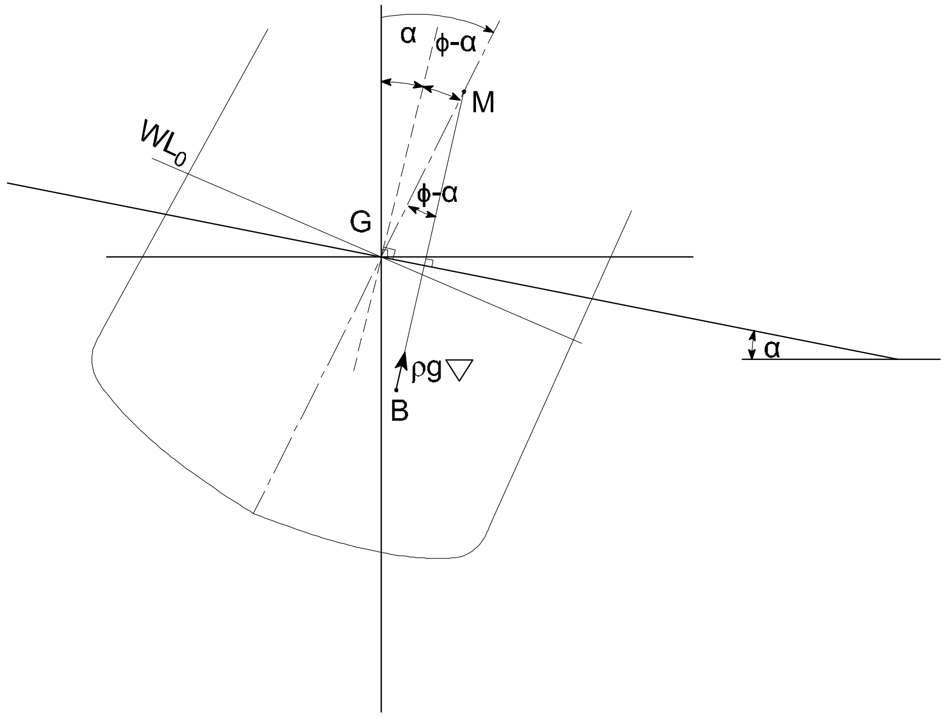

Figure 6-2 *Illustration of Roll motion*

If we ignore the effects of roll damping associated hydrodynamic forces (and any other environmental forces such as current, wind etc.) and we only consider the hydrostatic buoyancy force acting perpendicular to the wave surface (i.e. the wave slope) then taking moments about G (see Figure 6-2) leads to the equation of motion for small angles becomes:

$$I_{xx}\ddot{\varphi} = -\rho g \nabla GM_T sin(\varphi - \alpha) \to \rho g \nabla GM_T(\varphi - \alpha) \quad (6\text{-}32)$$

William Froude experimental observations indicate that for such representation the use of maximum surface wave slope is recommended. This is because the wave slope has the same representation as the wave profile. Having obtained a simplified form of wave excitation (valid essentially for long waves) we can generalize including the effects of damping and added inertia namely :

$$(J_{xx} + I_{\varphi\varphi})\ddot{\varphi} + N_{\varphi\varphi}\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}cos(\omega t) \quad (6\text{-}33)$$

The response to this sinusoidal excitation is $\varphi(t) = \Phi cos(\omega t - \varepsilon)$; thus, by substitution to Eq (6-33) we obtain:

$$[C_{\varphi\varphi} - \omega^2(I_{xx} + I_{\varphi\varphi})]\Phi cos(\omega t - \varepsilon) - \omega N_{\varphi\varphi}\Phi sin(\omega t - \varepsilon) = K_{\varphi}cos(\omega t) \quad (6\text{-}34)$$

An easy way to obtain the roll amplitude (**Φ**) and phase lag (**ε**) from Eq.(6-16) is to consider the following two cases:

$$\omega t - \varepsilon = \frac{\pi}{2} leadingto - \Phi\omega N_{\varphi\varphi} = -K_{\varphi}\sin(\varepsilon) \quad (6\text{-}35)$$

$$\omega t - \varepsilon = 0 \quad \text{leading to} \quad \Phi\left[C_{\varphi\varphi} - \omega^2\left(I_{xx} + I_{\varphi\varphi}\right)\right] = K_{\varphi}\cos(\varepsilon) \quad (6\text{-}36)$$

Squaring equations (6-35) and (6-36) and adding them produces the roll amplitude as follows :

$$\Phi = \frac{K_{\varphi}}{\sqrt{\left[C_{\varphi\varphi} - \omega^2\left(I_{xx} + I_{\varphi\varphi}\right)\right]^2 + \left(\omega N_{\varphi\varphi}\right)^2}} \quad (6\text{-}37)$$

Dividing Eq. (6-35) by Eq. (6-36) defines the phase lag as

$$tan\varepsilon = \frac{\omega N_{\varphi\varphi}}{C_{\varphi\varphi} - \omega^2\left(I_{xx} + I_{\varphi\varphi}\right)} \quad (6\text{-}38)$$

The amplitude and phase lag of the roll oscillation can be put into the following form

$$\Phi = \frac{a_m}{\sqrt{[1-\Lambda^2]^2 + (2\varsigma\Lambda)^2}} = \mu a_m \quad \text{and} \quad tan\varepsilon = \frac{2\varsigma\Lambda}{(1-\Lambda^2)^2} \quad (6\text{-}39)$$

where μ is referred to as the magnification factor and $\Lambda = \frac{\omega}{\omega_{\varphi}}$ is the tunning factor. Both μ and amplitude a_m vary with the frequency of oscillation. At resonance $\Lambda = 1$ and $\Phi_{res} = \frac{a_m}{2\varsigma}$.

For a ship moving in regular waves with forward speed U and heading angle χ and equation of motion similar to Eqs. (6-16) and (6-22) can be written as:

$$\left[I_{xx} + I_{\varphi\varphi}(\omega)\right]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(\omega, \omega_e)sin(\omega_e t + \delta) = \overline{K}_{\varphi}(\omega, \omega_e)e^{j(\omega_e t + \delta)} \quad (6\text{-}40)$$

where the hydrodynamic coefficients and the wave excitation are evaluated from potential flow hydrodynamic theory.

6.5 6.5 Roll in large amplitudes

In Lecture 2.3 we briefly reviewed different stabilisation systems that may be used for controlling large amplitude motions. Here we briefly address some of the mathematical basis of the problem.

The main problem with roll motion is that large amplitudes may cause discomfort compared to other motions. The amount of damping which is provided by the fluid is not always sufficient to reduce the roll amplitude to acceptable levels. Therefore, additional mechanisms are commonly in use to increase the amount of roll damping. These can be grouped as (i) passive systems which make use of the roll motion and do not require any power source and control system (ii) active systems which use power to move masses or control surfaces and a control system. Typical passive systems are bilge keels, fixed fins, passive tanks and passive moving weights. Bilge keels are longer than fins (approximately 2/3 of a sip’s length), whilst fins have longer chord length. Typical active systems are moving fins (retractable or not), active tans and moving weights. For roll stabilisation with active fins the equation of motion has some additional terms on the right hand side which are referred to as three term controller:

$$[I_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + N_{\varphi\varphi}\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(t) + C(C_1\ddot{\varphi} - C_2\dot{\varphi} + C_3\varphi) \quad (6\text{-}41)$$

Moving the terms relating to φ to the left hand side of the equation leads to

$$[I_{xx} + I_{\varphi\varphi}(\omega) - CC_1]\ddot{\varphi} + (N_{\varphi\varphi} + CC_2)\dot{\varphi} + (C_{\varphi\varphi} - CC_3)\varphi = K_{\varphi}(t) \quad (6\text{-}42)$$

as it can be seen C_1 and C_3decrease with virtual mass moment of inertia and restoring moment whilst C_2 increases the damping. C is associated with the lift generated by the fin stabilizers and can be evaluated from the flow around airfoils as :

$$C = \rho a A V^2 \frac{\partial C_L}{\partial a_L} \quad (6\text{-}43)$$

where

a is the distance from roll axis to center of pressure fin

A is the fin area (i.e. the product of fin chord and span)

V is the velocity into the fin usually assumed to be equivalent to U, i.e. the forward speed of the ship

a_L is the angle of attack

$\frac{\partial C_L}{\partial a_L}$ is the slope of the lift coefficient surve

Typical damping ratios without any active or passive measures are 0.05 – 0.1. With the use of active stabilizers the damping ratio can be increased to 0.5 – 0.8.

6.6 Idealisation of responses in regular waves

To predict the ship motion in a set of regular waves, we need to have a way to predict the ship response as a function of the excitation amplitude and frequency. Names for this relationship include Frequency Response Function (FRF) recognized by naval architects as the Response Amplitude Operator (RAO) or Transfer Function (TF). In all cases, the result can be represented as a plot with the ratio of ship response to excitation amplitude on the vertical axis and the ratio excitation frequency to natural frequency on the horizontal axis. Figure 6-3 shows a typical transfer function for roll motion. The response depends on the ship mass, added mass, hydrodynamic damping, buoyancy and excitation frequency in the direction of motion. For the zero forward ship speed case the excitation frequency would match the wave frequency. However, when the ship has forward speed the excitation frequency depends on the wave frequency and the relative direction of the ship and waves. This resulting excitation frequency is the encounter frequency since it is the frequency at which the ship encounters the waves. Figure 6-4 demonstrates the influence of stabilizers on ship roll motion.

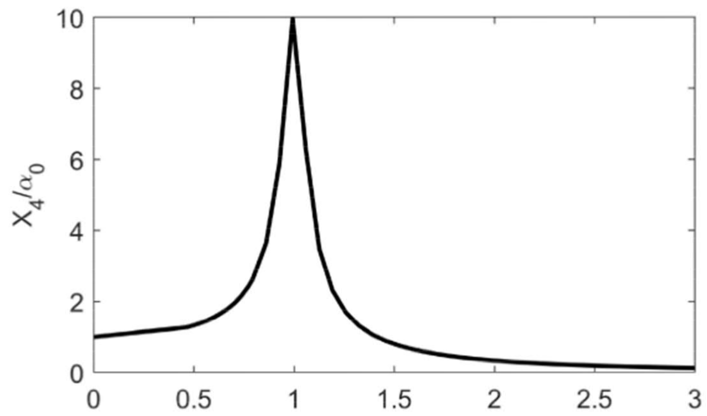

Figure 6-3 Typical ship transfer function for Roll

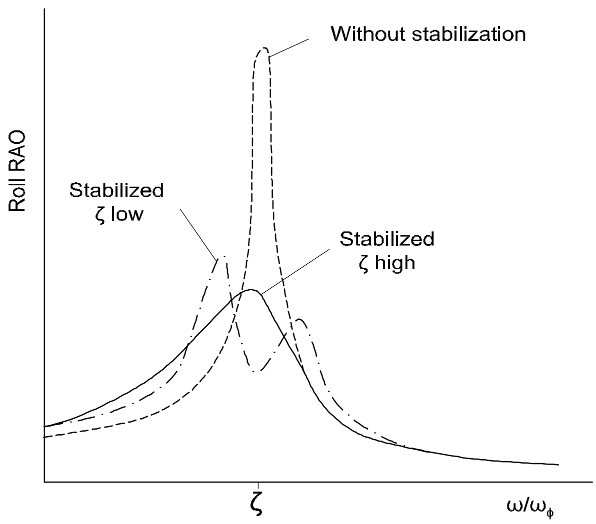

Figure 6-4 Influence of passive tank stabilization on ship roll RAO (roll/wave slope)

Variation of the nondimensionalized heave added mass, damping coefficient and excitation amplitude (for head waves of amplitude 1m) are shown in Figure 6-5 (a,b,c). as a function of the encounter frequency for a naval ship. Note that the variations of added mass and damping coefficients with speed are very small whilst those of the wave excitation are significant. The corresponding heave amplitude (per unit wave amplitude, i.e. the Response amplitude Operator) is sown in Figure 6-5d for head waves. Both amplitudes and phases of the wave excitation terms are functions of wave and wave encounter frequencies. Examples of heave (m/m) and pitch (rad/m) RAOs for a naval ship are shown in Figure 6-6. In these figures three different axes were used to illustrate the variation of the RAOs namely ω_e, L/λ and λ/L; where L : ship length and λ : wave length. It is interesting to note that the peaks of heave and pitch occur in the viscinity of $L = \lambda$. This phenomenon is called ship wave matching.

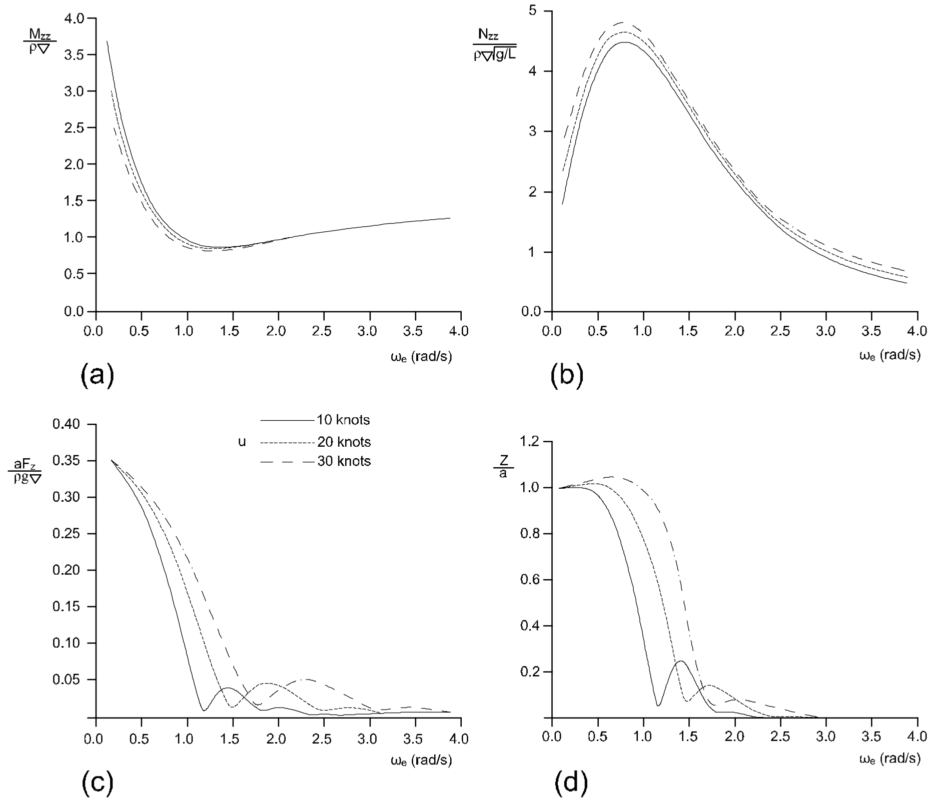

Figure 6-5 Typical nondimensionalized heave added mass, damping coefficients and excitation amplitude for a naval ship.

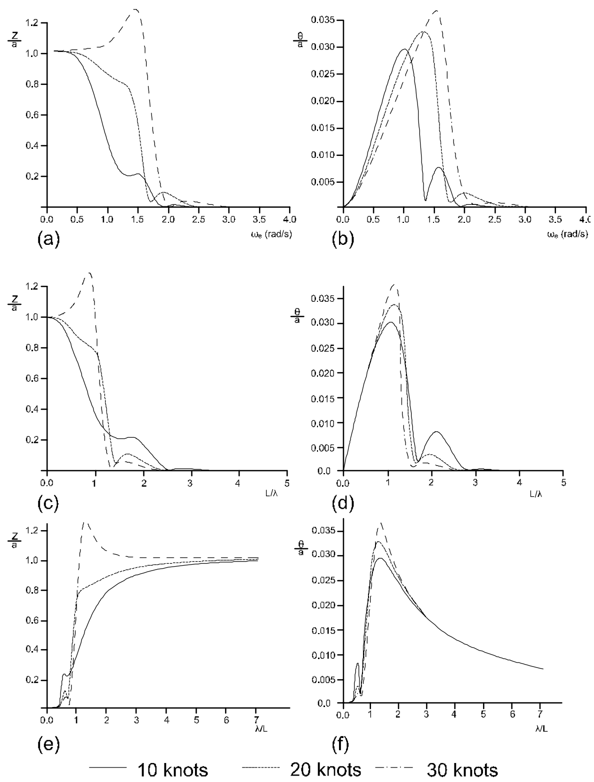

Figure 6-6 Examples of heave (m/m) and pitch (rad/m) RAOs for a naval ship