Lecture 7

Seakeeping methods

The birth of modern seakeeping analysis was in the mid 20th Century as demonstrated by the landmark papers of (Ursell 1949a, Ursell 1949b), and (St Dinis and Pierson Jr 1953). Continuous refinements of analysis methods and mathematical techniques combined with the availability of high-performance desktop computers in the late 20th Century has made routine seakeeping analysis possible in design offices. Today designers have several seakeeping tools to choose from and apply at preliminary design stage. This lecture primarily discusses the basic hydrodynamic modelling methods for the evaluation of seakeeping responses using two- and three-dimensional potential flow models. A brief overview to nonlinear hydrodynamic methods is also made.

7.1 Evaluation of hydrodynamic forces

In traditional seakeeping the problem of linear ship motions in waves is usually numerically tackled by examining three different types of forces, in addition to the restoring forces of hydrostatic origin. Those are:

- Radiation forces (or moments) where the ship is assumed to oscillate in calm seas and accordingly the hydrodynamic added inertia and damping coefficients are determined in still water conditions
- Incident wave or Froude Krylov forces (or moments) where the wave is considered in the absence of the ship and the corresponding wave forces (or moments) acting on the ship are determined. NB: In linear hydrodynamics we assume small displacements, i.e. "true", wetted surface is not considered.
- Diffraction forces (or moments) where the effects of the presence of the ship on the waves are considered and the corresponding diffracted wave forces (or moments) are determined.

The evaluation of these force components within the context of linearity implies that a ship is subject to an *incident wave that is progressive, regular and harmonic* (see Lectures 3,4). Progressive means that it has a translation speed known as celerity. Regular means that the spatial variation of the wave component is repetitive and is expressed by the wavelength λ . Accordingly the spatial frequency is the wave number $k = \frac{2\pi}{\lambda}$. Harmonic means that the variation of the waveform repeats itself after a time interval *T* known as the wave period. The associated circular frequency to this wave period is defined as $\omega = \frac{2\pi}{T}$. The velocity potential associated with the incident wave is determined using linearized description of fluid structure interaction. This is achieved by utilizing the velocity potential function $\Phi(x, y, z, t)$ which describes the fluid flow arising from the existence of the incident wave system. The fluid is assumed to be inviscid and incompressible and fluid flow is assumed to be irrotational and accordingly:

• $\nabla^2 \Phi = 0$ everywhere in the fluid due to incompressibility and irrotationality conditions

- $\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial \Phi}{\partial y} = 0$ on the undisturbed free surface (y = 0) due to requirements for continuity of pressure and velocity across the surface
- $\frac{\partial \Phi}{\partial n} = 0$ on the impermeable seabed





If we assume that the ship is held rigid then the incident wave will strike different parts of the hull at different times. It takes an initial period before the ship structure becomes aware of the existence of a steady state (i.e. the situation for which the loading and responses of the structure are harmonic). The period of time that lapses prior to the persistence of the steady state is known transient. This transient period gives rise to a phase shift between the harmonic incident wave and the diffraction. The fluid flow is described by the sum of incident and diffraction potentials :

$$\Phi = \Phi_{incident} + \Phi_{diffraction} \tag{7-1}$$

The linearized dynamic wave excitation pressure over a dS elemental area is defined as :

$$F_k^{excitation} = -\int_{S_w} p^{excitation} n_k dS = -\rho \frac{\partial}{\partial t} \left[\Phi_{incident} + \Phi_{diffraction} \right]$$
(7-2)

where :

k (=1,2...,6) are the six scalars corresponding to excitations in 6-DOF namely (surge, sway, heave, roll, pitch, yaw)

 n_k is the unit vector in way of the excitation in 6-DOF

 S_w is the surface of the ship structure in way of which n_k applies

The total excitation force is expressed as

$$F_k^{excitation} = F_k^{Froude-Krylov} + F_k^{Diffraction}$$
(7-3)

where:

$$F_k^{Froude-Krylov} = -j\omega\rho \int_{S_w} \varphi_{incident} n_k dS e^{-j\omega t}$$
(7-4)

$$F_{k}^{Froude-Krylov} = -j\omega\rho \int_{S_{w}} \varphi_{diffraction} n_{k} dS e^{-j\omega t}$$
(7-5)

The first term on the right hand side of Eq.(7-3) is equivalent to summing the pressure due to the progression of a regular harmonic wave acting on a virtual structure of the same shape and position as the actual structure. The second term represents the extent of the interaction of the incident wave with the ship. Having investigated the interaction of the incident wave with the fixed structure (namely diffraction) we can next consider the forced oscillation of the structure in calm waters. The reactive or radiation forces and moments are expressed as :

$$F_{kj} = -\int_{S_w} p^{radiation} n_k dS = \int_{S_w} \rho \frac{\partial}{\partial t} (\varphi_{radiation}^j) n_k dS = -j\omega\rho \int_{S_w} \varphi^j n_k dS e^{-j\omega t}$$
(7-6)

Given that we have 6 force components (k) in 6 different directions of motion (j) there would be 36 values of F_{kj} at each incident wave frequency ω . If we resolve the radiation forces into added mass and fluid damping then

$$F_{kj} = -A_{kj}\dot{s}_j - B_{kj}\dot{s}_j \tag{7-7}$$

leading to

$$A_{kj} = \frac{\rho}{a_j \omega} \int_{S_w} \varphi^j n_k dS \tag{7-8}$$

$$B_{kj} = -\frac{\rho}{a_j} \int_{S_w} \varphi^j n_k dS \tag{7-9}$$

7.2 Equations of motion in 6-DOF

According to Newton's 2nd law of motion the rate of change of *linear or angular momentum* is equal to the sum of the *external forces and moments* acting on the ship structure. Thus, for translation and rotation in the *j*th degree of freedom :

$$\frac{dMS_j}{dt} = F_{excitation} + F_{radiation} + F_{hydrostatic} + F_{other}$$
(7-10)

$$\frac{dI_{jj}S_j}{dt} = M_{excitation} + M_{radiation} + M_{hydrostatic} + M_{other}$$
(7-11)

The arbitrary shape of the structure means that all motions are coupled. Consequently, the radiation forces in the *j*th direction will have contributions from motions in all 6-DOF. The hydrostatic restoring forces based on Archimedes principle involve only the vertical plane motions for heave roll and pitch. Those arguments allow us to write down the six equations of motion as :

$$-\omega^2 M s_k = F_{kexcite} - \sum_{j=1}^6 \left(-\omega^2 A_{kj} - i\omega B_{kj} \right) s_j - \sum_{j=3,4,5} C_{\nu j} s_j$$
(7-12)

where k = 1,2,...,6 correspond to 6 – DOF (surge, sway, heave, roll, pitch, yaw) and v notations next to restoring term *C* correspond to roll, pitch and yaw. If we transfer the non-wave excitation terms to the left hand side of each equation and we re-arrange terms so that motion dependence terms are arranged in a strict order we get a system of 6 coupled algebraic equations expressed in matrix format. These equations can be solved in matrix format for each wave frequency and heading and they are used to evaluate the Response Amplitude Operators (RAOs).

7.3 Linear seakeeping analysis methods

Solving the linearized equations of motion presented in section 7.1 requires evaluation of the coefficients and the excitation amplitudes and phases. Considerable effort has therefore been devoted to developing theoretical methods of determining the coefficients and excitations to allow ship motions to be calculated without recourse to experiment. These methods have been developed on the basis that seakeeping analysis is essentially a three - part problem:

- Estimation of the likely environmental conditions to be encountered by the vessel
- Prediction of the response characteristics of the vessel
- Specification of the criteria used to assess the vessel's seakeeping behavior

This logic also defines the way in which the performance of different vessels is compared in ship design development. Comparison of different designs or assessment of a single design against specified criteria is dependent on accurate information for all three items listed above. Evaluation of seakeeping performance depends heavily on the environment (wave spectra) that the vessels are being subjected to and the criteria which are being used to compare the designs.

Two classic types of analyses are in use to obtain these forces using potential flow analysis. The first is known as strip theory and the other as panel method. Details related with the basic assumptions associated with each of these methods follow.

7.3.1 Strip theory

Strip theory is a two-dimensional analysis whereby the hull is divided into a number of uniform slices. The hydrodynamic properties (that is added mass, damping, and stiffness) obtained for each slice considering the flow around the an infinitely long uniform cylinder with the cross-section of the slice. The sectional added inertia and damping coefficients are obtained for heaving and coupled sway-roll slices. There are limitations concerning what assumptions may be made to use strip theory depending on the problem specifics and over the years various methods have been developed. The global hydrodynamic values for the complete hull are then computed by integrating the two - dimensional values of the strips over the length of the ship. Linear strip theory assumes the vessel's motions are linear and harmonic, in which case the response of the vessel in both pitch and heave, for a given wave frequency and speed, will be proportional to the wave amplitude and slope, respectively.

The basic assumptions required for linear strip theory are:

- The fluid is inviscid, that is, viscous damping is in principle ignored or implemented independently via an empirical coefficient usually associated with roll damping.
- The ship is slender (i.e. the length is much greater than the beam or the draft, and the beam is much less than the wave length).
- The hull is rigid so that no flexure of the structure occurs.
- The speed is moderate so there is no appreciable planing lift.
- The motions are small (or at least linear with wave amplitude).
- The ship hull sections are wall—sided.
- The water depth is much greater than the wave length so that deep water wave approximations may be applied.
- The presence of the hull has no effect on the waves (Froude -Krilov hypothesis).



Figure 7-2 Strip theory idealisation

The theory presented below is based on the frequency domain strip theory introduced by (Salvesen, TUCK, and Faltinsen 1970) The surge motion is not considered in our analysis since the hydrodynamic forces associated with surge motion are much smaller than those associated with the other five modes of motion (assuming a slender body). Along the lines of Eq. (7-12) by summing the inertia force, the hydrodynamic force, and the hydrostatic resorting force, the equation of motion in the frequency domain can be re-written as:

$$\sum_{k=1}^{6} \left[-\omega_{e}^{2} (M_{jk} + A_{jk}) + i\omega_{e}B_{jk} + C_{jk} \right] \zeta_{k} = F_{j}$$
or
$$\sum_{k=1}^{6} (M_{jk} + A_{jk}) \ddot{\eta}_{k} + B_{jk} \dot{\eta}_{k} + C_{jk} \eta_{k} = F_{j} e^{i\omega_{e}t} \quad \text{for} \quad j = 2, 3, \dots 6$$
(7-13)

where M_{jk} is the generalized mass matrix. for free motions the only non-zero hydrostatic coefficient C_{jk} are the C_{33} , C_{44} , C_{55} , and $C_{35} = C_{53}$. If the ship is assumed to be with lateral symmetry(symmetric about the xz plane), and the center of gravity is located at (0,0, z_{jk}) then the generalized mass matrix is given by:

$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_g & 0 \\ 0 & M & 0 & -Mz_g & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_g & 0 & I_4 & 0 & -I_{46} \\ Mz_g & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix}$$
(7-14)

The added mass and damping matrix are given by:

$$A_{jk}(orB_{jk}) = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0\\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0\\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0\\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}$$
(7-15)

By substituting the damping matrix, the added mass matrix, and the restoring force matrix in the equation of motion (7-13) with applying the lateral symmetry assumption, the six coupled equations of motion reduce to

- three coupled equations for surge, heave, and pitch and
- three coupled equations for sway, roll, and yaw.

If we consider as an example the two coupled equations of heave and pitch:

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{i\omega_e t}$$
(7-16)

$$A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (A_{55} + I_5)\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5 e^{i\omega_e t}$$
(7-17)

The relation between the different added mass and damping coefficients are given in detail by Salvesen et al. (1970). For example, added mass, damping and hydrostatic restoring coefficients are defined as :

$$A_{33} = \int a_{33} d\xi - \frac{U}{\omega_e^2} b_{33}{}^A \tag{7-18}$$

$$B_{33} = \int b_{33} d\xi - U a_{33}{}^A \tag{7-19}$$

$$C_{33} = \rho g \int_{L} B d\xi = \rho g A_{wp} \tag{7-20}$$

$$C_{35} = C_{53} = -\rho g \int_{L} \xi B d\xi = -\rho g M_{wp}$$
(7-21)

$$C_{55} = \rho g \int_{L} \xi^{2} b d\xi = -\rho g I_{wp}$$
(7-22)

In Eq.(7-20) – (7-22) A_{wp} , M_{wp} , and I_{wp} represent the area, first moment, and moment of inertia of the waterplane.

The solutions of local hydrodynamic coefficients is mathematically challenging and typically computers are used. Once we know the added mass etc. we can use conformal mapping in form of Lewis sections to estimate the properties of 2D ship-like sections. The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrarily shape of a cross-section in a complex plane can be derived from the more convenient circular cross-section in another complex plane. In this manner hydrodynamic problems can be solved directly with the coefficients of the mapping function. The advantage of making use of the two-parameter Lewis conformal mapping is that the dimensionless frequency-depending potential coefficients are a function of two parameters only namely the half the breadth to draught ratio and the area coefficient of the cross-section.

The conformal mapping technique is a transformation operation of a known potential around a uniform geometry (infinitely long cylinder in our case) into flow around a contour (the ship section) by use of transformation series. Truncating the transformation series to only three parameters, the mapped cross-sections will become what is known by Lewis forms. The main requirements to use Lewis forms that the cross-section must be symmetric, semi-submerged, and the hull needs to intersect the water surface perpendicularly. For instance, based on this method the sectional added mass coefficient a_{33} can be evaluated according to the equation:

$$a_{33} = K_2 K_4 A(x) \tag{7-23}$$

where:

- *K*₂ is a non-dimensional coefficient that helps us determine the mapping of the geometry into the flow around the cylinder ignoring the free surface
- K_4 is a non-dimensional frequency correction coefficient for the free surface.
- *A*(*x*) is the immersed cross sectional area.

The non-dimensional coefficient K_2 is defined as :

$$K_2 = \frac{(1+C_1)^2 + 3C_2^2}{1 - C_1^2 - 3C_2^2} \tag{7-24}$$

where C_1 and C_2 are the Lewis' mapping coefficients given by:

$$C_1 = \frac{B}{B_0}(1 - \Lambda), \ C_2 = \frac{B}{B_0}(1 + \Lambda) - 1 \text{ for } \frac{B}{B_0} = \frac{1}{2}, \ \Lambda = \frac{2T}{B}, \ C_M = \frac{A}{BT}$$
 (7-25)

In Eq.(7-22) B and T present the sectional waterline breadth and draught respectively.

The non-dimensional coefficient K_4 reflects the effect of the free surface. It depends on the nondimensional frequency $\xi_0 = \frac{\omega_e^2 B}{2g}$ and is expressed as :

$$K_{4} = \begin{cases} \frac{-8}{\pi^{2}} ln(0.795(1+\frac{2T}{B})\xi_{0}) & \text{for } \xi_{0} < -\frac{1.3503}{\frac{T^{-0.9846}}{B}+2.3567} + 0.5497 \\ 0.2367\xi_{0}^{2} - 0.4944\xi_{0} + 0.8547 + \frac{0.01}{\xi_{0}+0.0001} & \text{for } -\frac{1.3503}{\frac{T^{-0.9846}}{B}+2.3567} + 0.5497 < \xi_{0} < 1.388 \\ 0.4835 + \sqrt{-0.0484} + 0.0504\xi_{0} - 0.001\xi_{0}^{2} & \text{for } 1.388 < \xi_{0} \le 7.31 \end{cases}$$
(7-26)

7.3.2 Pulsating source Green Function method

In the green function panel method, the three-dimensional flow around the ship is calculated in order to obtain the pressure, forces and moments acting on the wetted hull surface. As the method is three dimensional the approximations inherent in strip theory are avoided. In the pulsating source method the source density is computed on the center of each panel in a way that assumes that there is no flow through. A distribution of sources is applied, either on the panel (hull surface) or at some distance from it within the body, to smooth flow irregularities occurring at the boundaries of the panels. All these potentials fulfill the Laplace equation and the radiation, the bottom or infinite depth and linearized free-surface conditions. The ship response can be determined in the frequency domain, where the motions of the ship are defined for various regular waves frequencies.



Figure 7-3 Green function idealization of a 10,000 TEU container ship (Hirdaris et al. 2016)

The response is usually calculated for different wave directions and zero forward speed and then combined to present the behavior of the ship in actual sea states. Accurate solutions using the zero-speed free-surface Green function method are obtained for problems with linearized free-surface boundary conditions at zero forward speed, but good or reasonable approximations are possible with moderate steady forward speed.

The most important effect of forward speed can easily be taken into account by accounting for the influence of the encounter frequency ω_e which as explained in Lecture 3 can be expressed as :

$$\omega_e = \omega - kU\cos\mu \tag{7-27}$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, U is the ship speed, and μ is the wave direction.

The amplitude of the incoming linear wave of a unit amplitude in deep water (for simplification only) is:

$$\hat{\phi}_0 = -i \frac{\omega}{k} e^{-ikx \cos \mu + iky \sin \mu}$$
(7-28)

The remaining potentials $\hat{\phi}_{J=1,2,3,..,7}$ can be determined numerically by the panel method. To obtain these potentials the model should satisfy the Laplace equation in the fluid domain and the zerospeed linearized free surface boundary condition at the non-oscillating water surface z = 0. The diffraction potential (j = 7) and radiation potentials (j = 1:6) can be determined by superimposing the potentials of all panels as

$$\hat{\phi}_{j} = \sum_{p=1}^{P} q_{j,p} \hat{\phi}_{p}$$
 (7-29)

where *P* is the number of panels and $q_{i,p}$ is the source density of each panel.

The source densities can be obtained by solving a linear equation system with seven different righthand sides (j = 1:7); and then satisfy the body boundary conditions at the center of all panels. The radiation potentials are divided into two parts:

$$\hat{\phi}_j = \hat{\phi}_j^0 + \frac{U}{i\,\omega_e}\,\hat{\phi}_j^U \tag{7-30}$$

where $\hat{\phi}_j^0$ and $\hat{\phi}_j^U$ are the speed-independent conditions that satisfy the body boundary condition.

The fluid pressure (P) can be obtained using the Bernoulli equation for unsteady flow:

$$\frac{p}{\rho} = -\dot{\phi} - \frac{1}{2} |\nabla\phi|^2 + gz + \frac{1}{2}U^2$$
(7-31)

The pressure of a specific motion on each panel can be evaluated by solving the above equation separately for the wave potential ϕ_0 , diffraction potential ϕ_7 and the six radiation potentials from ϕ_1 to ϕ_6 . Consequently, the forces and moments can be evaluated by summing up the forces and moments on each panel namely :

$$\hat{\mathbf{F}}_{j} = \sum_{p=1}^{P} \hat{p}_{j} \mathbf{n}_{p}$$
 and $\hat{\mathbf{M}}_{j} = \sum_{p=1}^{P} \hat{p}_{j} \mathbf{x} \times \mathbf{n}_{p}$ (7-32)

where \mathbf{n}_p is a normal vector directed into the hull and its absolute equals the panel area; while the pressure \hat{p}_j at the panel center equals the average pressure on each panel for J = 0:7.

Finally, the generalized motion vectors are evaluated by the equation :

$$\begin{bmatrix} -\omega_e^2 \mathbf{M} - \begin{pmatrix} \hat{\mathbf{F}}_1 & \dots & \hat{\mathbf{F}}_6 \\ \hat{\mathbf{M}}_1 & \dots & \hat{\mathbf{M}}_6 \end{pmatrix} \end{bmatrix} \begin{pmatrix} \hat{\mathbf{u}} \\ \hat{\alpha} \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{F}}_0 + \hat{\mathbf{F}}_7 \\ \hat{\mathbf{M}}_0 + \hat{\mathbf{M}}_7 \end{pmatrix}$$
(7-33)

This equation may be further complemented by adding corrections to account for the surge and roll damping, forces on fins, etc., however, this is not covered in this section. The complex amplitude of the translation $\hat{\mathbf{u}}$ and rotation $\hat{\alpha}$ motions can be obtained by solving the above system of the six

complex scalar linear equations. After obtaining the amplitude of the motion we can get the hull pressure, forces and moments in virtual cross-section, drift forces, etc.

7.4 Non-linear seakeeping analysis methods – a brief reference

Technical difficulties in the computations of modern hull ship motions are mainly related with understanding, simulating and validating the effects of nonlinearities. There are nonlinear phenomena associated with the fluid in the form of viscosity and the velocity squared terms in the pressure equation. The so-called free surface effect also causes nonlinear behavior due to the nature of corresponding boundary conditions (e.g. (Bailey 1997)) and the nonlinear behavior of large amplitude incident waves (e.g. (Mortola et al. 2011)). Forward speed effects and the body geometry often cause nonlinear restoring forces and nonlinear behavior in way of the intersection between the body and the free surface (e.g. (Chapchap et al. 2011)). A large variety of different nonlinear methods have been presented in the past three decades (Hirdaris et al. 2016). Clearly, as techniques become more sophisticated assumptions become more complex (see Figure 7-4). Computational time and complexity may be an issue in the process of understanding, simplifying or validating the modelling assumptions. In this sense the accuracy of the solution must be balanced against the computational effort. Figure 7-4 and Table 7-1 summarise the taxonomy and some key qualitative features of the methods available. From an overall perspective one may distinguish between methods based on linear potential theory (Level 1 methods) and those solving the Reynolds-Averaged Navier-Stokes (RANS) equations (Level 6 methods). The majority of methods currently used in practise is based on linear potential flow theory assumptions and account for some empirical forward speed corrections (Chapchap et al. 2011).

Within the group of weakly nonlinear potential flow methods (Levels 2–5) there is a large variety of partially nonlinear, or blended, methods, which attempt to include some of the most important nonlinear effects. For example, Level 2 methods present the simplest nonlinear approach where hydrodynamic forces are linear and all nonlinear effects are associated with the restoring and the Froude–Krylov forces. On the other hand, Level 3 and 4 methods refer to the so called "body nonlinear" and "body exact" methods. In these methods the radiation problem is treated as nonlinear and is solved partially in the time and frequency domains using a retardation function and a convolution integral.

The difference between these two levels is that the "body nonlinear" approach (Level 3) solves the radiation problem using the calm water surface and the "body exact method" (Level 4) uses the incoming wave pattern as in way of the free-surface for the solution of the radiation problem. Level5 methods are highly complex and computationally intensive. They have no linear simplifications and the solution of the equations of motion is carried out directly in the time domain. The hydro-dynamic problem is solved using an MEL (Mixed Euler–Lagrange) approach. They are usually based on the assumption of "smooth waves". Therefore, wave breaking phenomena that may, for exam-ple, be associated with large amplitude motions in irregular seaways cannot be modelled. Large advances in reducing computer processing times resulted in making basic RANS methods, excluding DES (Detached Eddy Simulations), URANS (Unsteady RANS) and DNS (Detached Navier Stokes), attractive

for 3D fluid-structure interaction problems and hence for the prediction of wave-induced motions and loads.

Implementation of potential flow hydroelastic methods in the "frequency domain (FD)" or "time domain (TD)" may be possible irrespective to the type of hydrodynamic idealisation (e.g. (Chapchap et al. 2011), and (Mortola 2013)). More recent developments enabling full coupling between RANS with FEA software, may ensure the inclusion of hydroelasticity also within this more advanced CFD framework (Lakshmynarayanana and Hirdaris 2020).Nevertheless, there are quite a few issues to resolve even for the application of RANS methods to the conventional, rigid body, sea-keeping problem. For example, these include issues with the time efficiency for computations, the efficient and convergent meshing of the fluid domain associated with the movement of the body and the deforming free surface, as well as the influence of turbulence modelling.



Figure 7-4 Level of idealisation for forward speed hydrodynamic solutions (Numbers 1–6 refer to Levels 1–6 of idealisation as per (Hirdaris et al. 2016)).

Table 7-1 Taxonom	y of hydrod	ynamic solution	methods as per	r (Hirdaris et al. 2	2016).
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Level	Description	Key features	Additional comments
1	Linear	 The wetted body surface is defined by the mean position of the hull under the free surface The free surface BC are applied in way of the internment wetted body surface Hydrodynamics are solved in FD by strip theory or BEM using a range of GFM 	 Computations are fast Viscous forces are not part of the solution and must be obtained by other methods, if important or required The boundary integral methods cannot handle breaking waves, spray and water flowing onto and off the ship's deck.
2	Froude– Krylov NL	 The disturbance potential is determined as in Level 1 Incident wave forces evaluated by integrating incident wave and hydrostatic pressures over the wetted hull surface The wetted hull surface is defined by the instantaneous position of the hull under the incident wave surface Hydrodynamics are solved in FD or TD by GFM and convolution integrals are used for memory effects. 	 Computations are moderately fast NL modification forces can be included in addition to Froude–Krylov and restoring forces to account for slamming and green water
3	Body NL	 The disturbance potential is calculated for the wetted hull surface defined by the instantaneous position of the hull under the mean position of the free surface. 	• Computations are slow since re-gridding and re-calculation of the disturbance potential for each time step is required.
4	Body exact	 The disturbance potential is calculated for the wetted hull surface defined by the instantaneous position of the hull under the incident wave surface The disturbed, or scattered waves, caused by the ship are disregarded when the hydrodynamic boundary value problem is set up The scattered waves are considered small compared to the incident waves and the steady waves 	 Computations are mathematically complex and slow. This is because common CFM satisfies the free surface condition on the mean free surface and not on the incident wave surface.
5	Smooth waves	 Scattered waves are no longer assumed to be small, and they are included when the boundary value problem is set up. In MEL methods the Eulerian solution of a linear boundary value pro- blem and the Lagrangian time integration of the nonlinear free surface boundary condition is required at each time step. Wave breaking or fragmentation of the fluid demain is ignored 	 Computations are typically forced to stop based on a wave breaking criterion. The stability of the free surface time-stepping can cause numerical problems
6	Fully NL	 The water/air volume is normally discretised, and a finite difference, finite volume or a finite element technique is used to establish the equation system. Particle methods, where no grid is used, can be applied to solve the Navier-Stokes equations. Examples are the Smoothed Particle Hydro- dynamics (SPH), the Moving Particle Semi-implicit (MPS) and the Con- strained Interpolation Profile (CIP) methods, with the latter believed to be more suitable for violent flows. 	 Mathematics and computations are complex There is no unification in the approaches used to solve sea-keeping problems, hence extensive efforts for validation of solution and the benefits of practical implementation are necessary.

7.5 References

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