Aalto University
School of Engineering

MEC-E2004 Ship Dynamics (L)

Lecture 7 –Seakeeping methods

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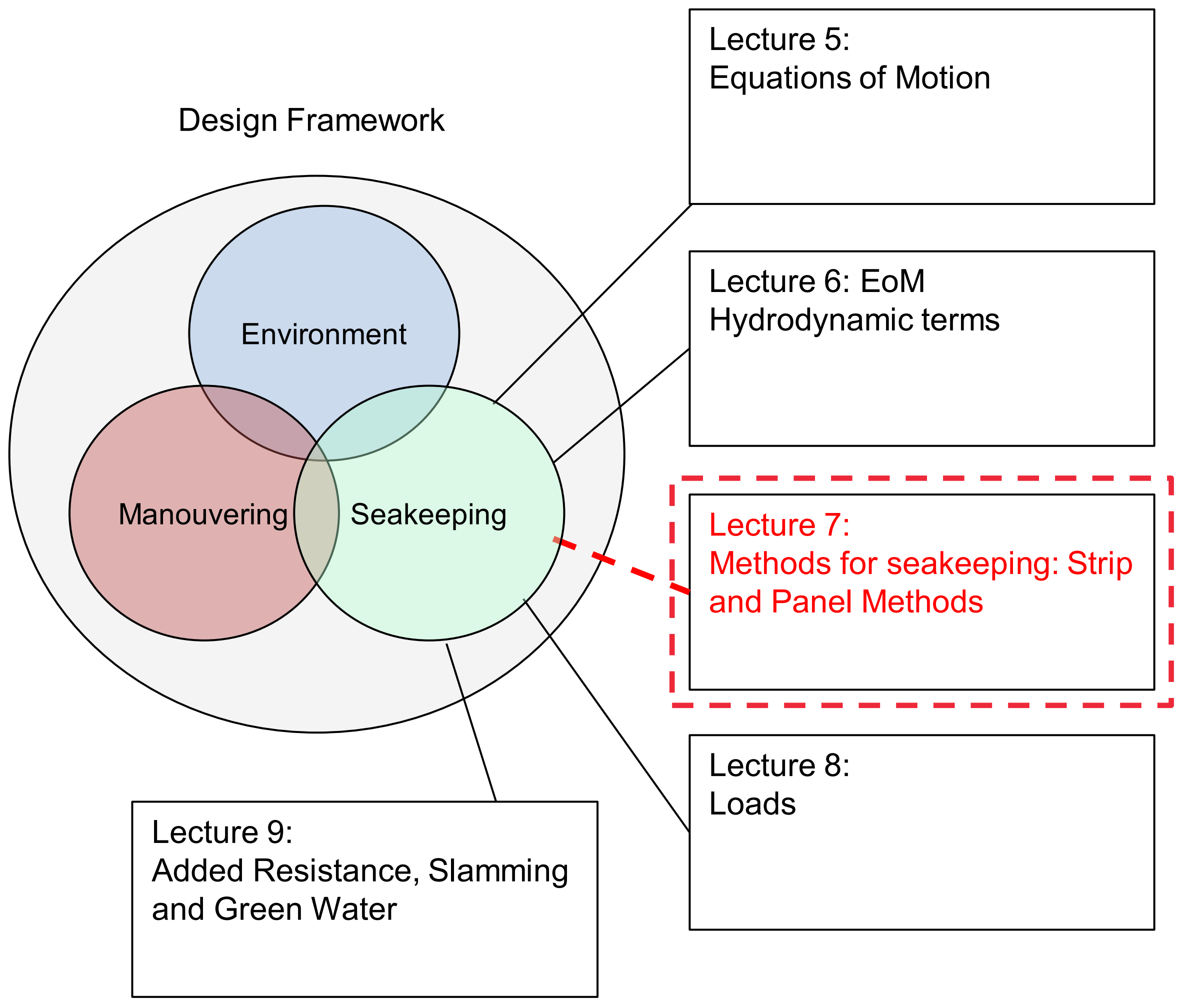
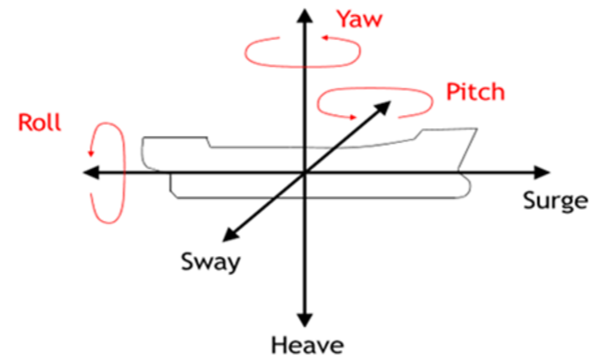
Where is this lecture on the course?



Contents

- **Aim :** Solution of the equations of motion by potential flow methods
  - Overview of existing methods
  - Focus on Strip theory
  - Simple reference to 3D panel methods
  - Overview on the importance of non linearities



- Literature
  - Journee, J.M.J., ”Introduction to Ship Hydromechanics”
  - Lloyd, A.R.J.M, ”Seakeeping – Ship Behavior in Rough Weather”, John Wiley & Sons
  - Bertram, V., ”Practical Ship Hydrodynamics”, Butterworth-Heinemann, Ch. 4.
  - Matusiak, J., ”Ship Dynamics”, Aalto University
  - Lewis, E. V. Principles of Naval Architecture. Vol. 3, ”Motions in waves and controllability”
  - Rawson, K. J., ”Basic Ship Theory. Volume 2, Ship dynamics and design - ch.12 Seakeeping”

Motivation

- The analysis of ship motions is complex engineering task. We need to have appreciation of the key characteristics and limitations of alternative methods and their use in design.

- 2D and 3D linear approaches are useful at the preliminary ship design stage to help us reduce risks associated with seakeeping performance.

- Today seakeeping theory is implemented in various computer programmes and allows for the computation of various sea states, motion components etc. in 2D or 3D.

- There are several codes available with extensions to include non-linear corrections (e.g. strip theory, panel methods). Appreciation of their advantages and limitations from a hydrodynamic modelling perspective is useful.

- The validation of computational methods is imperative especially in those cases that design innovation impacts upon rule and regulations (i.e. cases where credible and validated calculations are important).

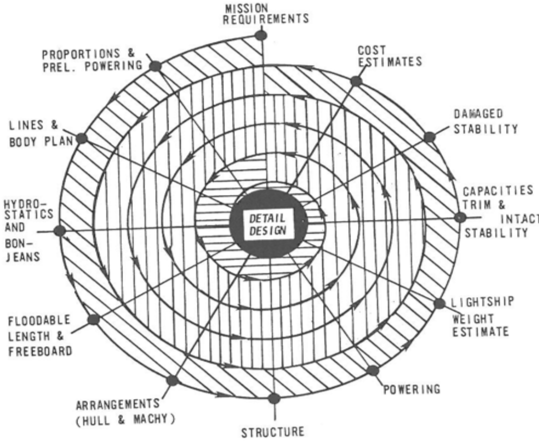
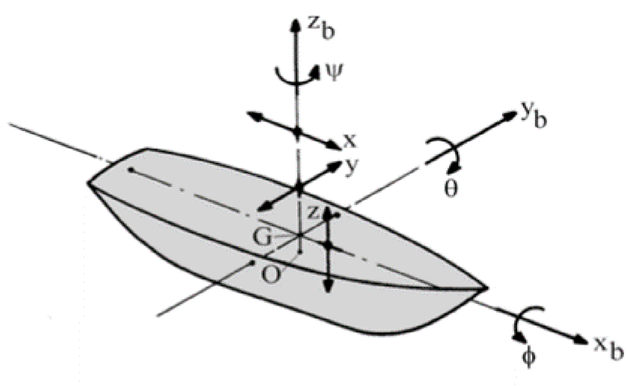


Fig. 1 Basic design spiral

Assignment 4

- Grades 1-3:
  - Select a book-chapter related to determination of ship motions and loads and get acquainted with a tool to predict these
  - Form a seakeeping analysis model from your ship, discuss the simplifications made
  - Perform the computations for Response Amplitude Operators

- Grades 4-5:
  - Compute all motions (6) and global loads (bending moments and shear forces) for your ship for selected sea spectra (e.g. worst case spectra in North Atlantic). You can predict 3 hour maximums
  - Based on scientific literature, discuss the accuracy of your results



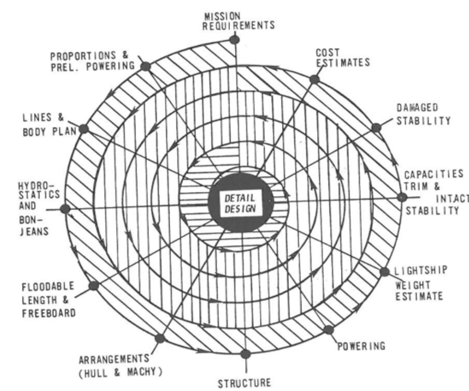


Fig. 1 Basic design spiral

Strip theory

- Three different types of forces, in addition to the restoring forces of hydrostatic origin:
  - **Radiation forces** (or moments),
  - **Incident wave or Froude-Krylov forces** (or moments) and
  - **Diffraction forces** (or moments).

- Two basic types of linear methods (potential flow analysis) are used :
  - **Strip theory**
  - **Panel methods**.

- **Strip theory is a 2D analysis method** where the hull is divided into a uniform number of strips.
  - Hydrodynamic properties are obtained for each strip considering the flow around an infinitely long uniform cylinder with the cross section of a slice.
  - Each strip is independent and interactions in flows are neglected
  - The sectional added inertia and damping coefficients are obtained for **heaving and coupled swaying - rolling slices**.
  - To obtain the added inertia and damping coefficients for the entire hull the sectional properties are integrated along the hull using the moments for the pitch and yaw coefficients.
  - Diffraction effects are formulated w.r.t. added inertia and damping coefficients as

$$(m+m_{zz})\ddot{z}+N_{zz}\dot{z}+C_{zz}\,z=m_{zz}\ddot{\zeta}+N_{zz}\dot{\zeta}+C_{zz}\,\zeta=F_z(t)$$

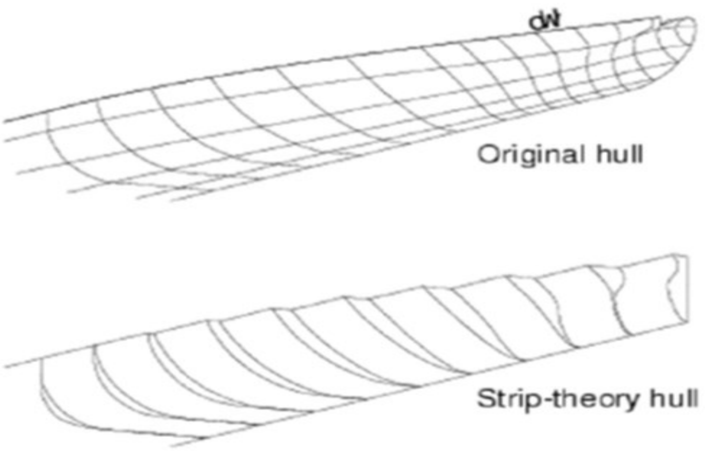
Reference : Tuck & Faltisen, Ship Motions and Sea Loads, Trans SNAME, 1970, 78:250-287

Strip theory

- Strip theory ignores the influence of longitudinal effects on the flow around the ship. This influence is important when forward speeds are high. So **strip theory is limited to small or moderate Froude numbers**.

- **In strip theory the 2D velocity potential can be formulated by** :
  1. Source or dipole distributions on the section contour. This method is preferred due to irregular frequencies leading to added mass and damping coefficients tending to infinite values at these frequencies
  2. A source + a dipole (in the case of roll) representing the oscillations of a **semi circular cross section + conformal mapping** to transform semi circle into a contour shape form.

- **In conformal mapping the so called Lewis sections are broadly used**. They are accurate in terms of mapping beam, draft and area. Another more accurate technique is the multi-parameter conformal mapping. It maps better the section contour.



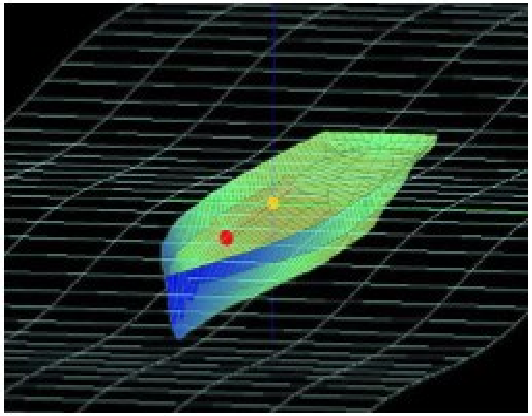
Reference : Tuck & Faltisen, Ship Motions and Sea Loads, Trans SNAME, 1970, 78:250-287

Panel Methods

- To overcome the restrictions imposed by strip theory the **3D potential flow analysis** was developed.

- The method discretises the mean or still water wet surface of the hull by panels and places a pulsating (or translating and pulsating source) in each panel.

- After the evaluation of the strength of all sources the radiation and diffraction forces can be obtained by integration over the mean water surface of the hull. This is also known as **boundary element method**



- The **velocity potentials** associated with the singularities are referred to as **green functions**. This is also known as a ***near field method***

- Another approach is the Rankine singularity method. In this case the domain of idealisation is extended to include also the free surface boundary conditions to infinity **–** ***far field method***

- Green Function and Rankine Panel methods can be combined to include both near- and far-field effects simultaneously

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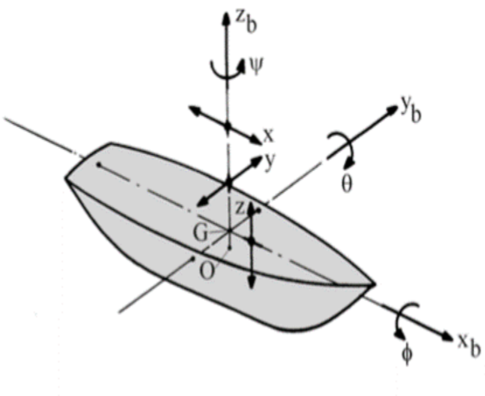
Hirdaris, S.E et al. (2016) The influence of nonlinearities on the symmetric hydrodynamic response of a 10,000 TEU Container ship. Ocean Engineering, 111:166-178.

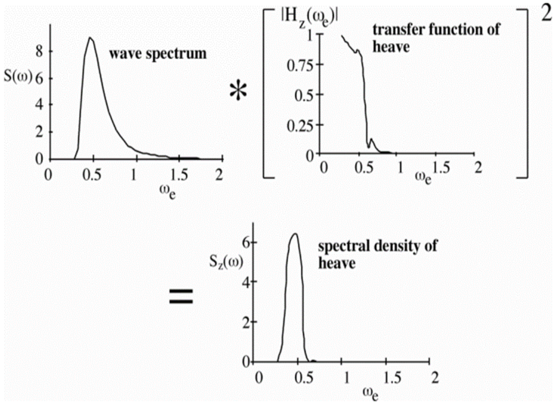
The perfectly linear seakeeping problem

Assumptions :

- Arbitrary shaped ship with port/starboard symmetry moves in waves in 6 dof.
- The ship is slender (i.e. length is much larger than the beam or draught)
- The hull is rigid (i.e. it does not deform due to waves)
- Speeds are low to moderate, there is no planning lift, the ship sections are wall sided (no wave elevation), motions are small
- The water depth is much greater than wave length (deep water aprox. is valid)
- The presense of the hull has no effect on waves ; the waves are linear
- There are no moving masses on the ship (e.g. free surface effects) that interfere with motions

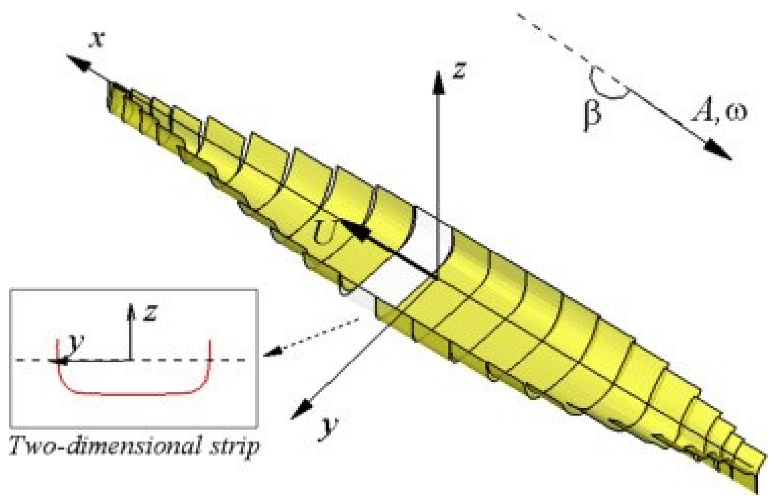
Aim : To evaluate the RAOs and use relevant sea spectra to assess motions by **Strip Theory aprox.**

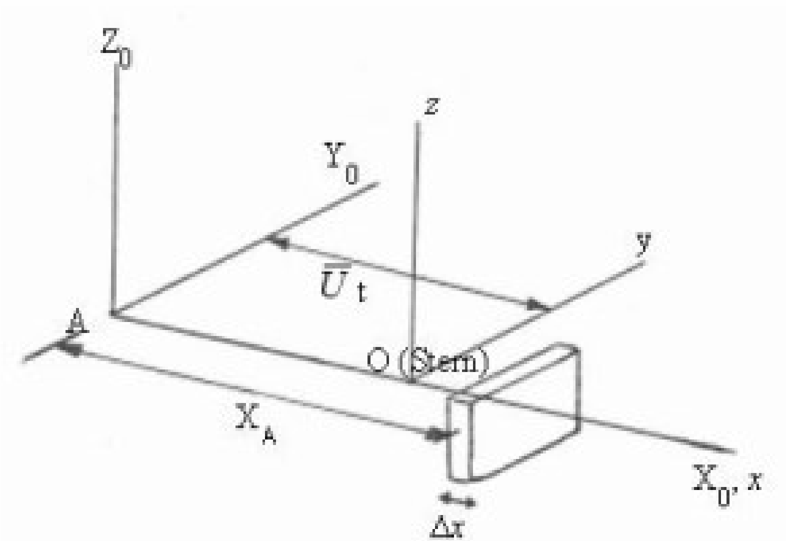




Strip Theory hydro-modelling

- Potential flow analysis (irrotational, inviscid assumptions – velocity potential idealisation)
- Ship modelled as an infinitely long uniform rigid cylinder of arbitrary cross section.
- Variation of the flow in the cross-directional plane >> Variation ship longitudinal direction
- No 3D effects and no 2D or 3D flow in hull proximities
- No flow interactions between strips
- 2D coefficients for added mass are computed for each strip and then summed over the length



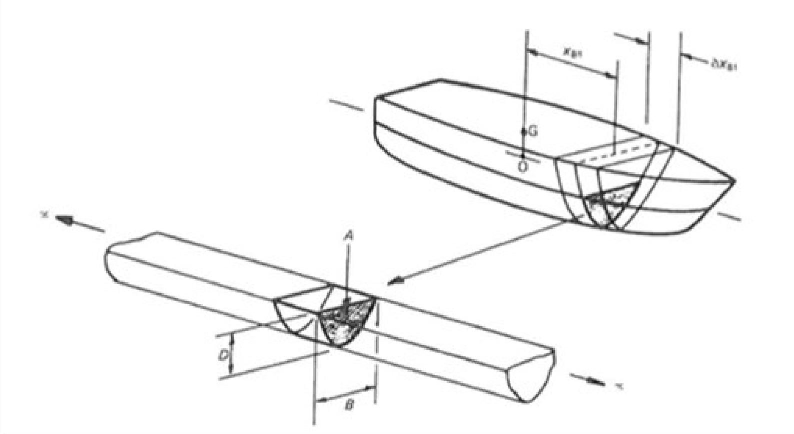


Strip Theory - Fundamentals

- Each strip operating at a certain idealised wave elevation is subject to hydrodynamic actions

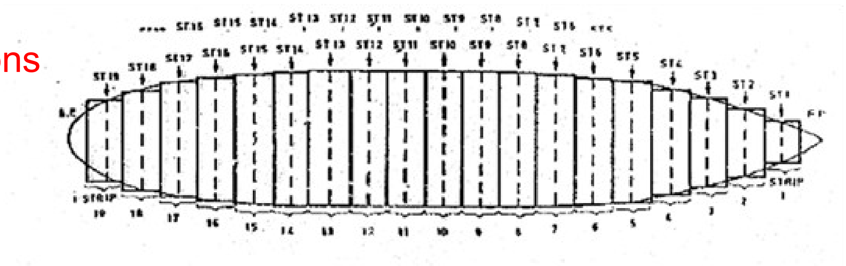
- Accordingly each strip has local hydrodynamic properties, i.e.

  1. Added mass related to accelerations
  2. Damping related to velocities
  3. Stiffness related to the motion components

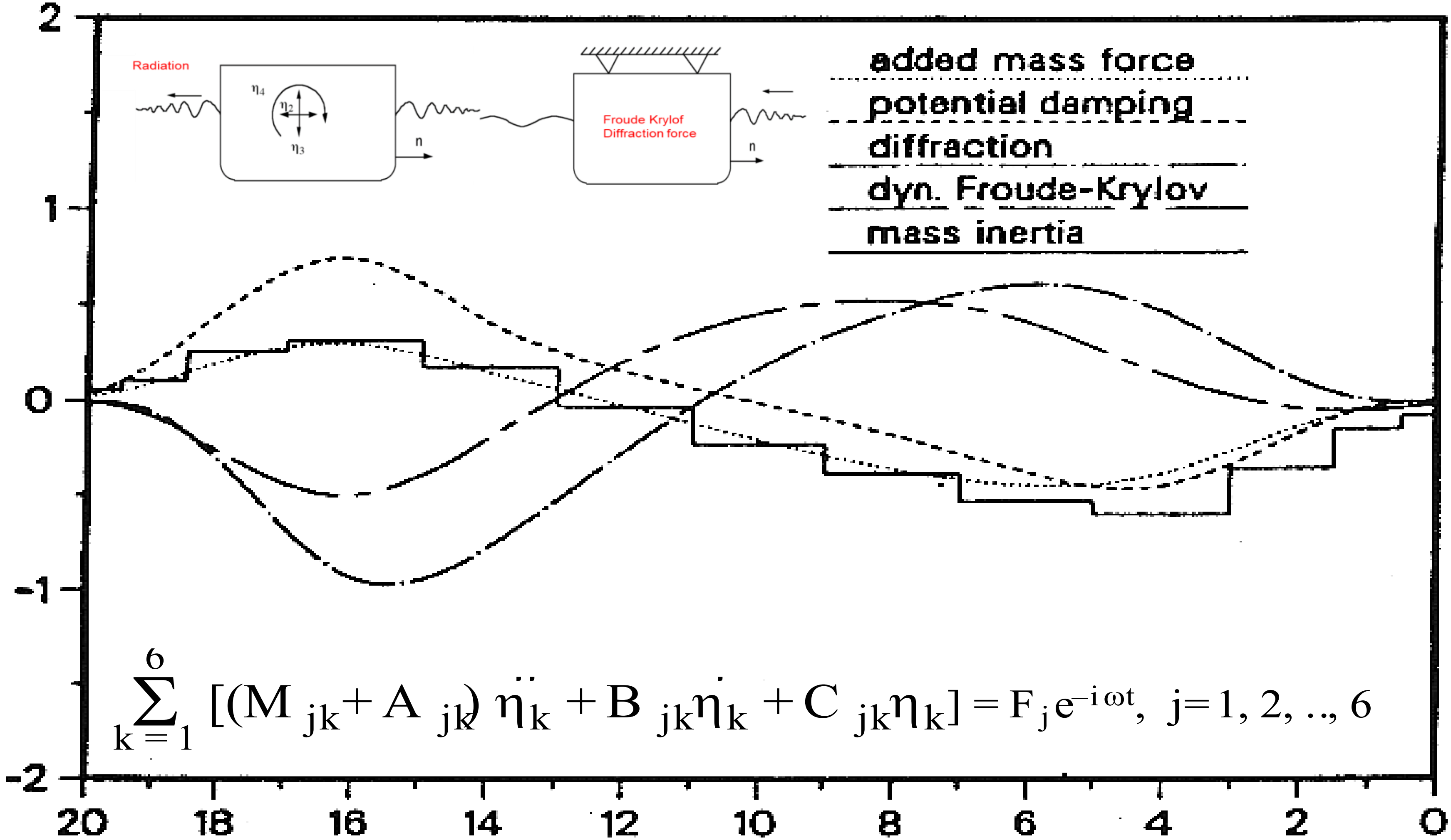


- Key topics

  1. Basic EoM
  2. Radiation & Relative Displacement
  3. Fluid force on strips
  4. Added mass
  5. Conformal mapping - Lewis hull form
  6. Free surface effects & boundary conditions
  7. Fluid Damping
  8. Hydrodynamic coefficients

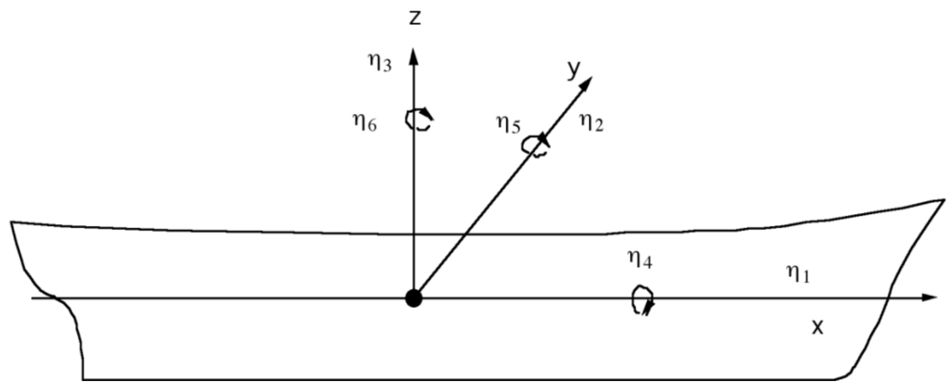


Strip Theory – Basic EoM

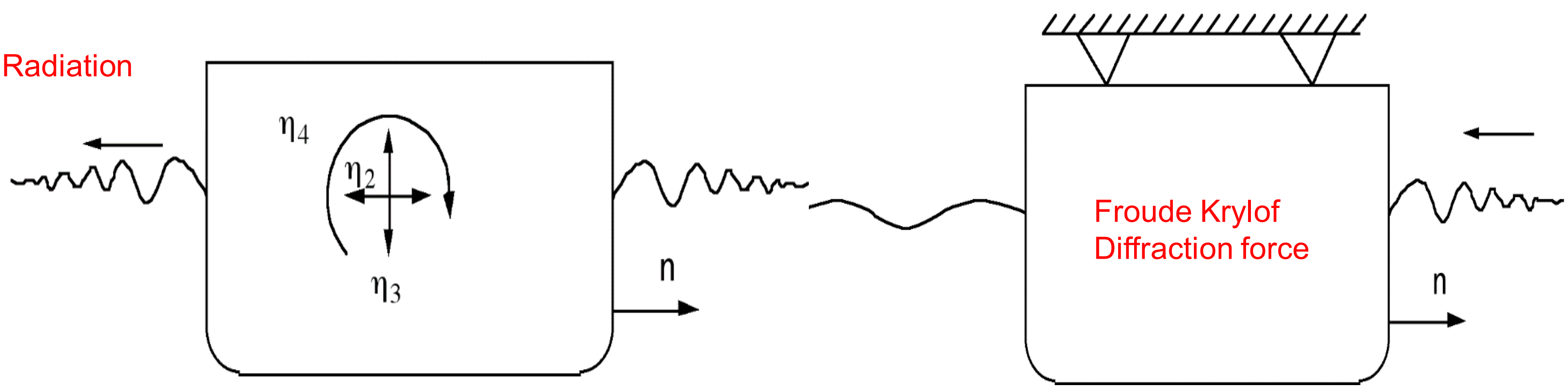


$$\sum_{k=1}^{6}\left[(M_{jk}+A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k\right] = F_j e^{-i\omega t},\quad j = 1, 2, .., 6$$

Strip Theory – Basic EoM

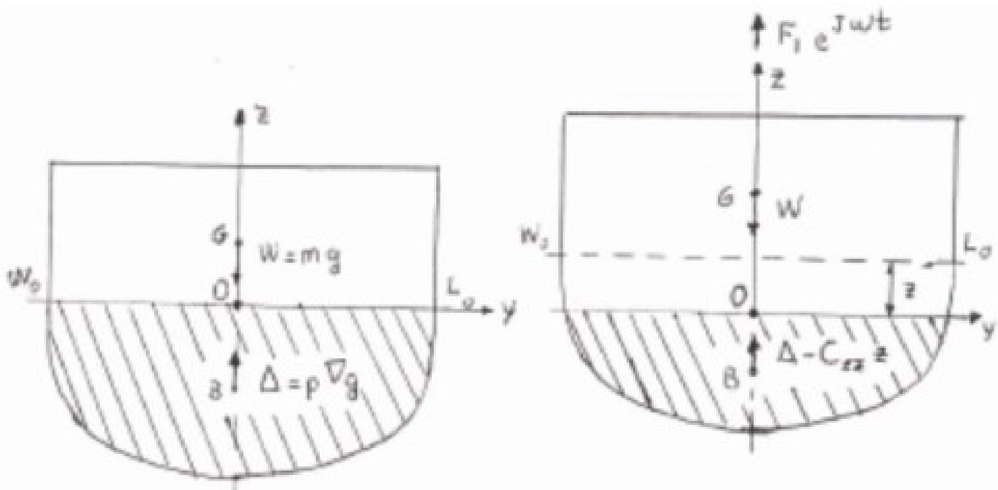



$$\sum_{k=1}^{6}\left[(M_{jk}+A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k\right] = F_j e^{-i\omega t}, \quad j=1,2,..,6$$



Strip Theory – strip model (the radiation problem)

Assume still water conditions and hull in perfect equilibrium (for more details refer to SD6)



F_1 : Mechanical excitation (SHM)

C_{zz} : Restoring coefficient

$C_{zz} z = \rho V g$

V : Volume between $W_0 L_0$ and Oy axis

So C_{zz} : ρ g Water Plane Area assuming hull wall - sided

Hull in equilibrium in still water

$\Delta = W$

$\Delta = \rho \nabla$

$W = m g$

$$m\ddot{z} = -W + \Delta - C_{zz} z + F_1(t) \quad \text{or} \quad m\ddot{z} = -C_{zz} z + F_1(t)$$

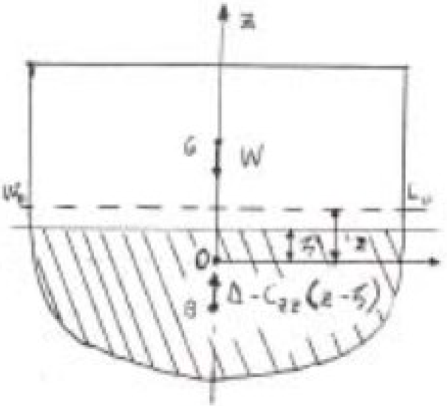
Mechanical excitation in still water (hydro effects are ignored)

$$m\ddot{z} = -m_{zz}\ddot{z} - N_{zz}\dot{z} - C_{zz} z + F_1(t)$$

Assuming hydro-pressure proportional to velocity (N_{zz} fluid damping) and acceleration (m_{zz} added mass)

Strip Theory – Relative displacement & Total Derivative

For the hull operating in regular waves, elevated in perfect equilibrium in way of centre line (CL)

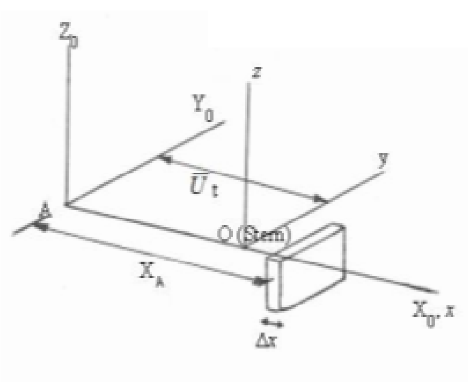


$$m\ddot{z} = -W + \Delta - C_{zz}(z-\zeta) = -C_{zz}(z-\zeta)$$ - *ignoring hydro effects*

$$m\ddot{z} = -m_{zz}(\ddot{z}-\ddot{\zeta}) - N_{zz}(\dot{z}-\dot{\zeta}) - C_{zz}(z-\zeta)$$

Assuming hydro pressure with components proportional to relative velocity (N_{zz} fluid damping) and relative acceleration (M_{zz} added mass)

The concept of total derivative



Distance in the space - fixed $AX_0Y_0Z_0$ axes

$X_A = \overline{U}\,t + x$

with

$$\frac{d}{dt}(X_A) = \overline{U} + \frac{dx}{dt}.$$

At time $t = 0$, O and A coincide; hence

$$\frac{dx}{dt} = -\overline{U}.$$

$$\Delta f = \frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial x}\Delta x$$

or

$$\frac{\Delta f}{\Delta t} = \frac{\partial f}{\partial t}\frac{\Delta t}{\Delta t} + \frac{\partial f}{\partial x}\frac{\Delta x}{\Delta t}$$

$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt}$$

$$= \frac{\partial f}{\partial t} - \overline{U}\frac{\partial f}{\partial x}$$

TOTAL DERIVATIVE

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - \overline{U}\frac{\partial}{\partial x}$$

Strip Theory – Fluid Force on each strip

Total derivative

NB: Just a function of position along length

$$F(x,t) = -\left\{ \frac{D}{Dt}\left[m(x) \frac{D\bar{z}(x,t)}{Dt} \right] + N(x)\frac{D\bar{z}(x,t)}{Dt} + \rho g B(x)\bar{z}(x,t) \right\}$$

$m(x)$: Added mass (heave) per unit length

$N(x)$: Fluid damping (heave) per unit length

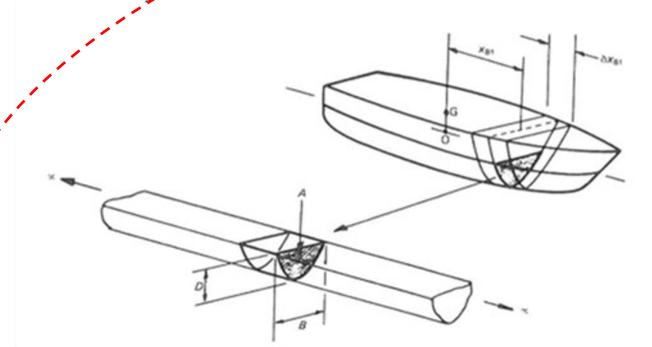
$B(x)$: Breadth along calm water line

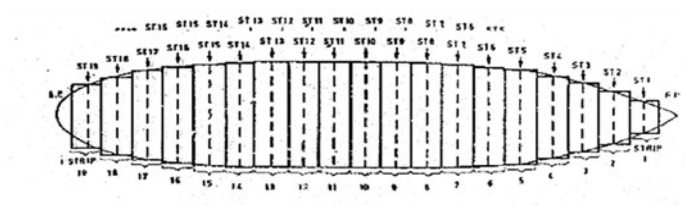
$\bar{z}(x,t) = w(x,t) - \zeta(x,t)$

$w(x,t)$: Vertical displacement

$\zeta(x,t)$: Regular wave profile at hull's centre line

Note that $\left[m(x) \frac{D\bar{z}(x,t)}{Dt} \right]$ represents fluid momentum





Leading to

$$F(x,t) = -m(x)\frac{D^2\bar{z}(x,t)}{Dt^2} - \left[N(x) - \bar{U}\frac{\mathrm{d}\,m(x)}{\mathrm{d}\,x} \right]\frac{D\bar{z}(x,t)}{Dt} - \rho g B(x)\bar{z}(x,t)$$

Strip Theory – Incident wave force Z(x,t)

We can break down the fluid action into components of $w(x,t)$ and $\zeta(x,t)$ so that $F(x,t) = -H(x,t) + Z(x,t)$

$$H(x,t) = m(x)\frac{D^2 w(x,t)}{Dt^2} + \left[N(x) - \bar{U}\frac{\mathrm{d}\,m(x)}{\mathrm{d}\,x}\right]\frac{D\,w(x,t)}{D\,t} + \rho\, g\, B(x)\, w(x,t)$$

$$Z(x,t) = m(x)\frac{D^2 \zeta(x,t)}{Dt^2} + \left[N(x) - \bar{U}\frac{\mathrm{d}\,m(x)}{\mathrm{d}\,x}\right]\frac{D\,\zeta(x,t)}{D\,t} + \rho\, g\, B(x)\, \zeta(x,t)$$

Wave elevation : $\zeta(x,t) = a\exp(-k\bar{T})\exp[i(kx\cos\chi - \omega_e t)]$ → $\mathrm{Real}[\zeta(x,t)] = a\exp(-k\bar{T})\cos(kx\cos\chi - \omega_e t)$

$$\frac{D^2}{Dt^2}\zeta(x,t) = \frac{D}{Dt}[-i\,\omega\,\zeta(x,t)] = (-i\,\omega)(-i\,\omega)\zeta(x,t) = -\omega^2\zeta(x,t).$$

$$\frac{D}{Dt}\zeta(x,t) = \left(\frac{\partial}{\partial t} - \bar{U}\frac{\partial}{\partial x}\right)\zeta(x,t) = -i\,\omega_e a\exp(-k\bar{T})\exp[i(kx\cos\chi - \omega_e t)]$$
$$-\,i\,\bar{U}k\cos\chi\, a\exp(-k\bar{T})\exp[i(kx\cos\chi - \omega_e t)]$$
$$= -i\,(\omega_e + \bar{U}k\cos\chi)\, a\exp(-k\bar{T})\exp[i(kx\cos\chi - \omega_e t)]$$
$$= -i\,\omega\,\zeta(x,t)$$

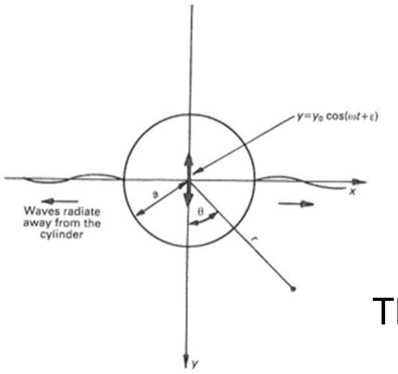
$$\mathrm{Z(x,t)} = \left\{-\omega^2 m(x) - i\,\omega\left[N(x) - \bar{U}m'(x)\right] + \rho\, g\, B(x)\right\}\zeta(x,t)$$

Strip Theory – example on added mass

- To set a body in motion it requires Kinetic Energy.
- The fluid displaced by the body's motion also has kinetic energy.
- The work done in accelerating the body in fluid is therefore greater than that for the body only.
- It is as if work is done on the body mass + added mass OR the mass of the body appears to be greater by an amount of added mass.
- Physically speaking the result of a body moving in infinite ideal fluid is hydrodynamic pressure proportional to body's acceleration

Example

Consider a circular cylinder of radius a = B/2 The cylinder is infinitely long, moving with velocity U(t) perpendicular to its longitudinal axis (i.e. in x – dir). Assume the fluid is stationary.



The velocity potential in polar coordinates (r,θ) is : $\Phi = U(t)\dfrac{a^2}{r}\cos\theta$

The velocity components in radial coordinates are defined as :

$$q_r = -\frac{\partial \Phi}{\partial r} = -U\frac{a^2}{(-r^2)}\cos\theta; \qquad q_\theta = -\frac{\partial \Phi}{r\,\partial \theta} = -U\frac{a^2}{r^2}(-\sin\theta)$$

The KE / unit length is defined as : $0.5\rho\int\int (u^2 + w^2)\,dx\,dz = 0.5\rho\int_a^\infty \int_0^{2\pi} (q_r^2 + q_\theta^2)\,r\,dr\,d\theta$

Strip Theory – Example on added mass

The KE / unit length is defined as : $0.5\rho\int\int(u^2+w^2)\,dx\,dz = 0.5\rho\int_a^{\infty}\int_0^{2\pi}(q_r^2+q_\theta^2)\,r\,dr\,d\theta$

$$(u^2+w^2) = (q_r^2+q_\theta^2) = U^2\frac{a^4}{r^4}$$

Added mass pull

$$\overline{\mathrm{m}} = \rho\pi a^2$$

Thus the KE of fluid pull is defined as $\frac{1}{2}\rho U^2 a^4\int_a^{\infty}\frac{r}{r^4}dr\int_0^{2\pi}d\theta = \frac{1}{2}\rho U^2 a^4\left(-\frac{1}{2r^2}\right)\Bigg|_a^{\infty}2\pi = \frac{1}{2}\rho U^2 a^2\pi = \frac{1}{2}\overline{m}U^2$

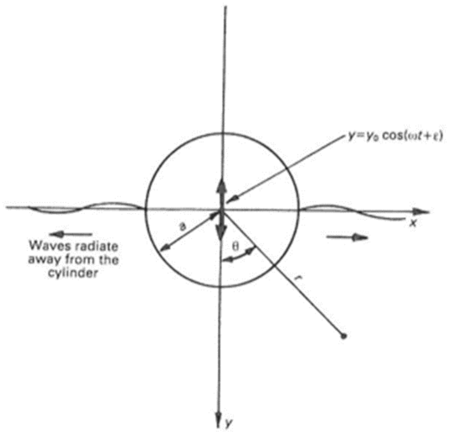
The fluid force acting on the cylinder is : $\mathrm{F\,U(t)} = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}(\mathrm{m}+\overline{\mathrm{m}})\mathrm{U}^2\right] = \dot{\mathrm{U}}(\mathrm{t})\mathrm{U(t)}\,(\mathrm{m}+\overline{\mathrm{m}})$

Virtual mass pull

Added mass coefficient is : $C = \frac{\text{Added mas pul of ship shaped section}}{\text{Added mass pul of } \textit{comparable} \text{ infinite cylinder}}$

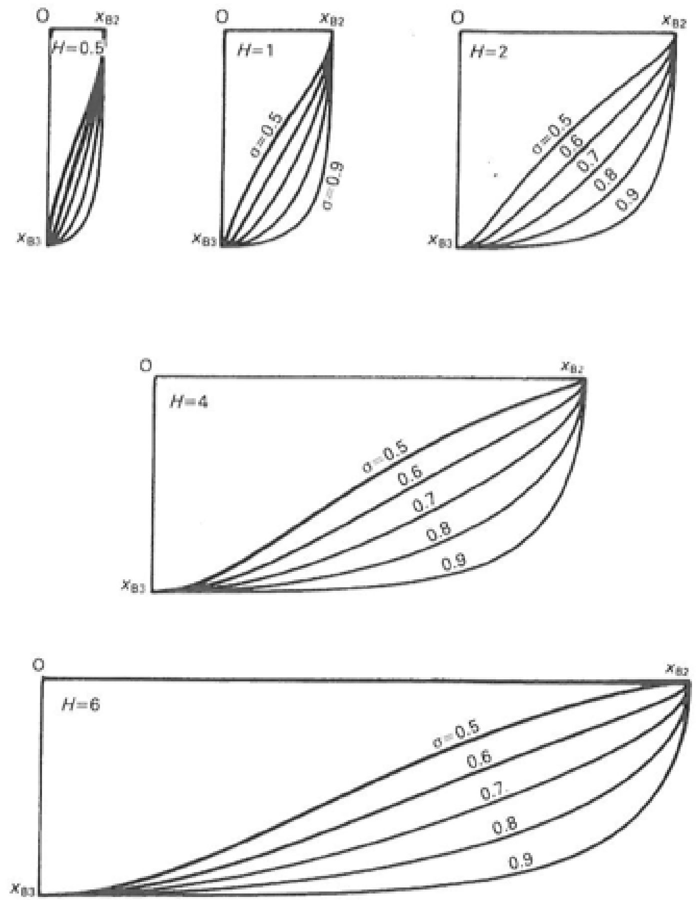
$$C_V = \frac{m_V}{\rho\pi B^2/8}; \qquad C_H = \frac{m_H}{\rho\pi T^2/2} \qquad C_T = \frac{\bar{I}}{\rho\pi T^4}$$

Heave (vertical) | Sway (horizontal) | Roll (rotational)



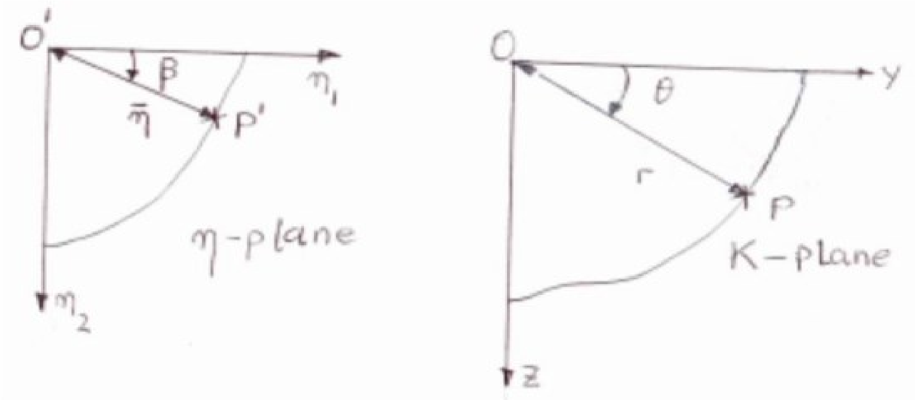
Strip Theory – Conformal mapping & Hull forms

- The derivation of hydrodynamic coefficients is mathematically challenging and typically computers are used
- We often utilize oscillating cylinders
- Once we know the added mass etc we can use with conformal mapping in form of Lewis forms to estimate the properties of our ship sections
- This deals mainly with the areas of sections and there are requirements for these
- This is why in this section we try to explain the background to conformal mapping techniques



Strip Theory – Conformal mapping basics

A shape in η-plane mapped into a shape in K-plane $K=f(\eta)$, by coordinate transformation



$$K = \sum_{n=-1}^{\infty} c_n \, \eta^{-n}$$

$$\eta = \bar{\eta}\, e^{i\beta} = \bar{\eta}(\cos\beta + i\sin\beta)$$

$$K = Y + iZ = r e^{i\theta} = r(\cos\theta + i\sin\theta)$$

- **<u>Advantage :</u>** We can carry out calculations on a simple shape (e.g. semi circular section in way of free surface) and find results for a complex shape (ship section at free surface) by mapping the semi circle onto this section

- For a port/starboard symmetric section the transformation is

$$K = Y + iZ = a_0\left(\eta + a_1\frac{1}{\eta} + a_3\frac{1}{\eta^3} + a_5\frac{1}{\eta^5} + a_7\frac{1}{\eta^7} + \ldots\ldots\right)$$

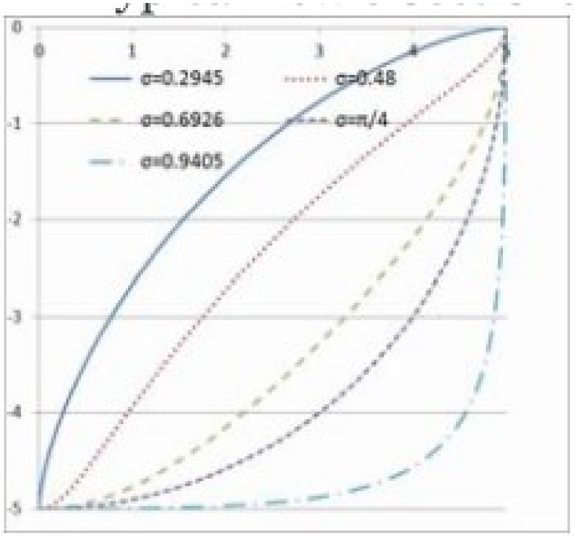
for $\bar{\eta} = 1$ and a_0 : Scale factor

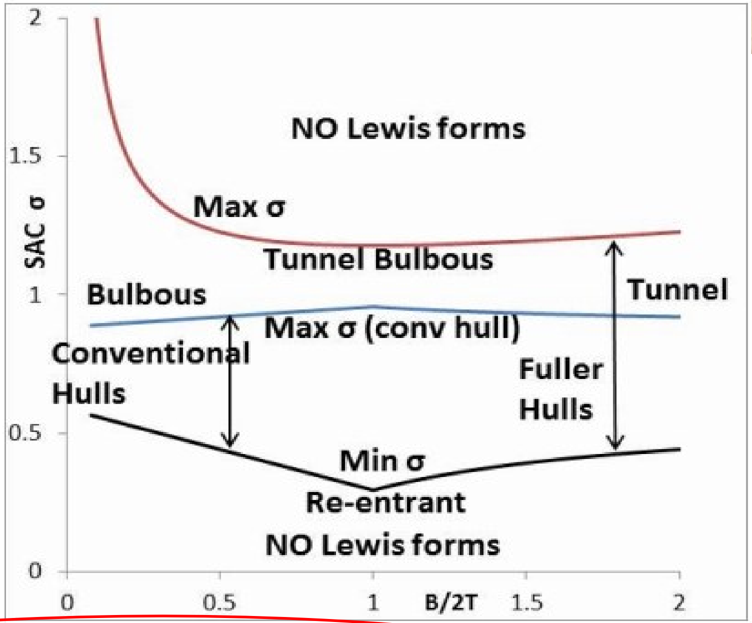
Strip Theory – Conformal mapping & Lewis form

- Lewis developed a 2D parameter transformation using sectional area, beam and draught to obtain the parameters

- Another way is to use a multi parameter transformation that considers 2nd moment of area based on defining points along section contours and obtaining parameters from least squares.

- Independently to the case we always assume that the resultant section is perpendicular at free surface and centre line



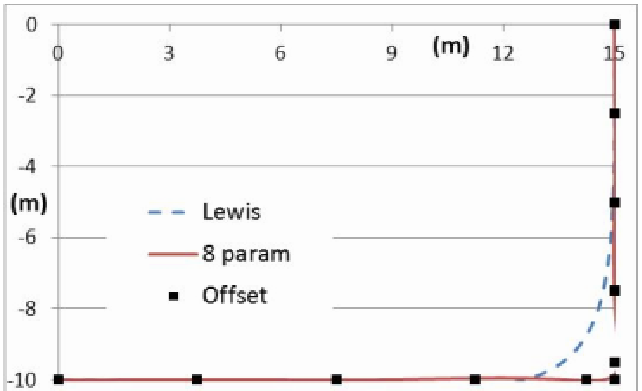


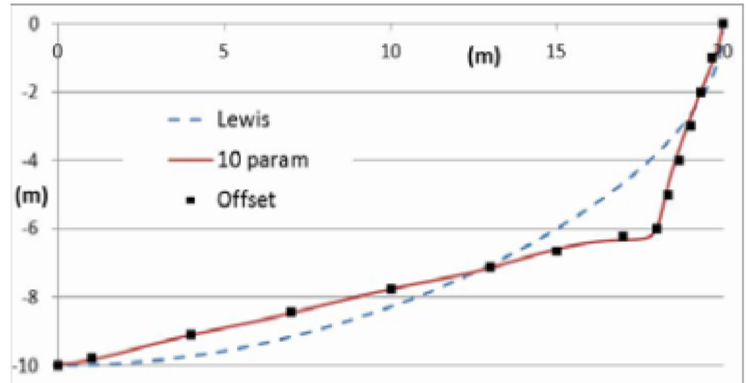
Wetted are for half section

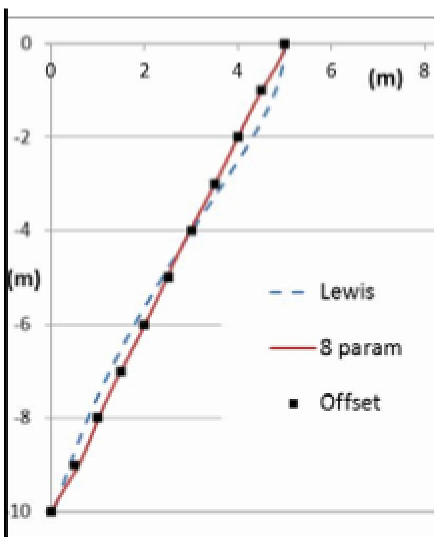
Sectional Area Coefficient $\sigma = S / BT$

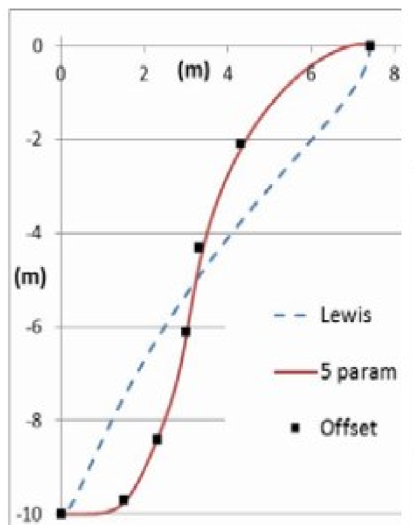
Half - breadth/Draught ratio $\Lambda = B / 2T$

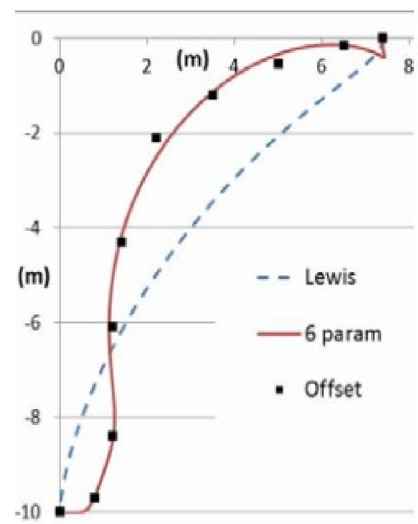
Strip Theory – Lewis form vs multiparamter mapping











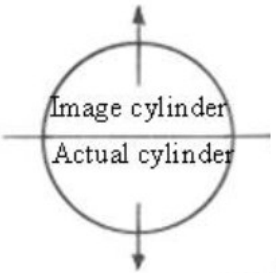
Strip Theory – free surface effects

- Velocity potential : Superposition of source at origin (for heave; dipole in the case of sway and roll) and a sufficient number of multipole potentials that satisfy the linearised free surface condition. For a section harmonically oscillating on free surface:

$$-\omega^2 \Phi + g \frac{\partial \Phi}{\partial Z} = 0 \quad \text{on} \quad Z = 0$$

Frequency of oscillation = encounter frequency

- Added mass coefficient $\text{AMR} = \dfrac{\text{Added Mass at free surface}}{\text{Added Mass in infinite fluid}}$



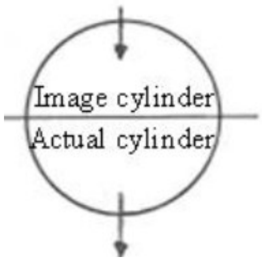
- Free surface boundary condition

(a) Limit case ω → 0 ; rigid boundary at free surface (zero normal velocity)

$$\frac{\partial \Phi}{\partial Z} = 0 \quad \text{on} \quad Z = 0$$ Added mass tends to infinite value

(b) Limit case ω → infinity ; free surface acts as a mirror – double body in infinite fluid

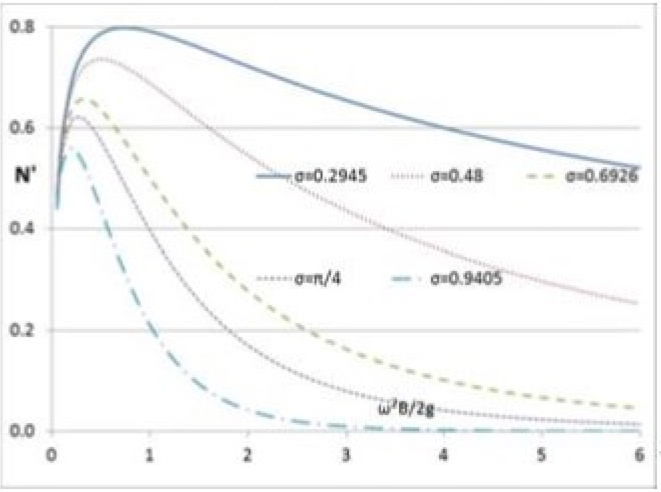
$$\Phi = 0 \quad \text{on} \quad Z = 0$$ Added mass tends to value in infinite fluid

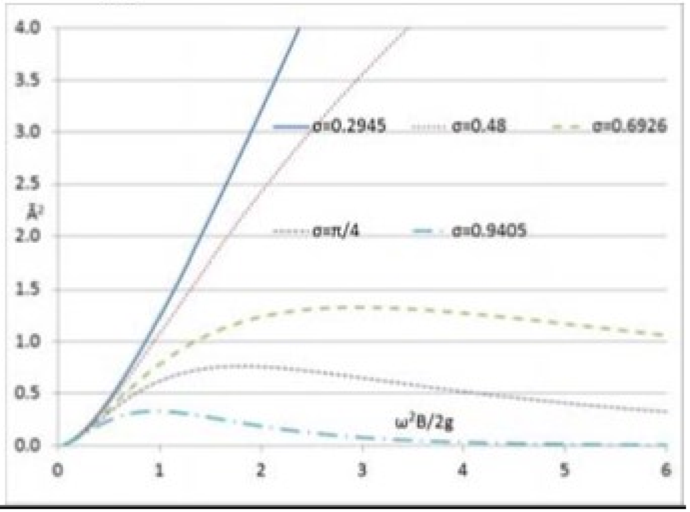


Strip Theory – fluid damping

In presence of free surface effects the oscillation of the hull or section results in the generation of dissipative waves travelling away from the body **(radiation)**

$$N(x) = \frac{\rho g^2}{\omega_e^3} \bar{A}^2 \quad \text{where} \quad \bar{A} = \frac{\text{amplitude of radiated wave at infinity}}{\text{amplitude of section's vertical oscillation}}$$





Different Codes

- There are numerous codes available for the job
- You can find these from different websites
- The codes can handle different assumptions and there are numerous extensions, e.g. to remove
  - Vertical wall assumption
  - No flow between strips etc

Linear seakeeping models

- Example 1: Small amplitude assumption
- Example 2 : Restoring and Froude-Krylov actions evaluated on still water

Advantages (in general)

- Well established numerical codes for linear method
- It is considered the fastest and easiest approach
- In case of irregular sea, it is possible to analyze the response of each wave component separately and then combine them together;
- Frequency domain
- It is possible to obtain the spectrum of the response directly from the sea spectrum

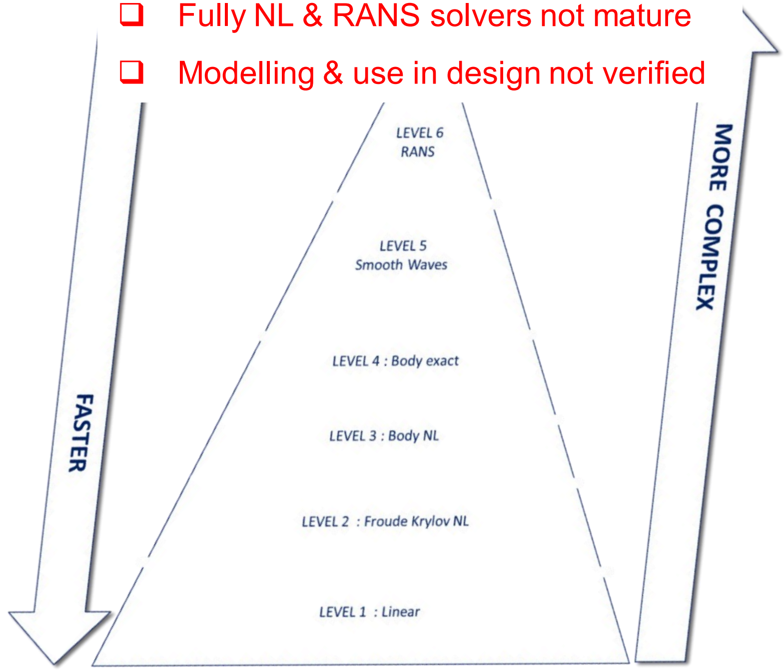
Engineering Tools – From Linear to NL Simulations

- Methods: Levels 1 – 6

- To identify the significance of non-linearity often we need two types of analyses:
  - Screening in frequency domain: what are the worst conditions for our ship?
  - Simulations in time domain: when wave-amplitude dependency is violated, what is the impact?

- Methods contain different assumptions on
  - Stochastics of the problem
  - Fluid mechanics idealisations:
    1. Navier-Stokes, i.e. full CFD
    2. Reynolds-Averaged Navier-Stokes (little turbulence fluctuations omitted in boundary layer, averaged)
    3. Euler equations (viscosity neglected, coarser meshes, faster simulations)
    4. Potential flow solvers (irrotational flow, 1 non-linear Eq. instead of 4, cannot model breaking waves or splashes)

- ❑ Most applications use potential flow
- ❑ Ad-hoc codes use partly NL methods
- ❑ Fully NL & RANS solvers not mature
- ❑ Modelling & use in design not verified



Paper : Hirdaris, S.E et al. (2016) The influence of nonlinearities on the symmetric hydrodynamic response of a 10,000 TEU Container ship. Ocean Engineering, 111:166-178.

Linear vs NL models - example

Linear model disadvantages

- Large amplitude effect neglected;
- Not capable to simulate non-linear phenomena, like parametric rolling.

Non-Linear model

- Restoring and Froude-Krylov actions evaluated on the effective immersed hull in wave at each time step;
- Memory effect on damping and added mass actions;
- Time domain simulation.

Non-Linear model advantages

- More precise on large amplitude responses;
- Suited to simulate Parametric Roll;
- Precise axial forces, usually approximated in the linear model.

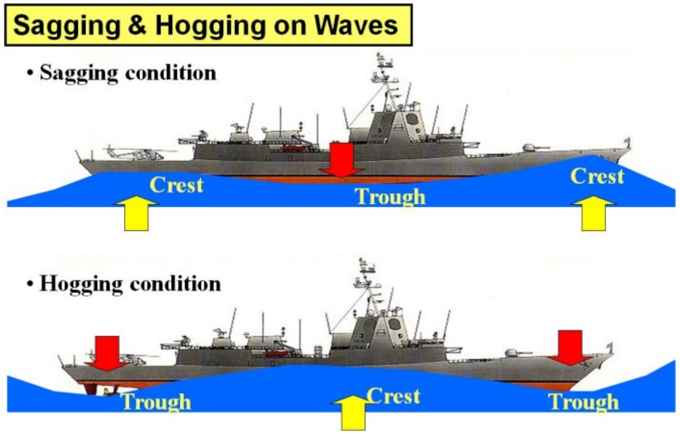
Non-linearities

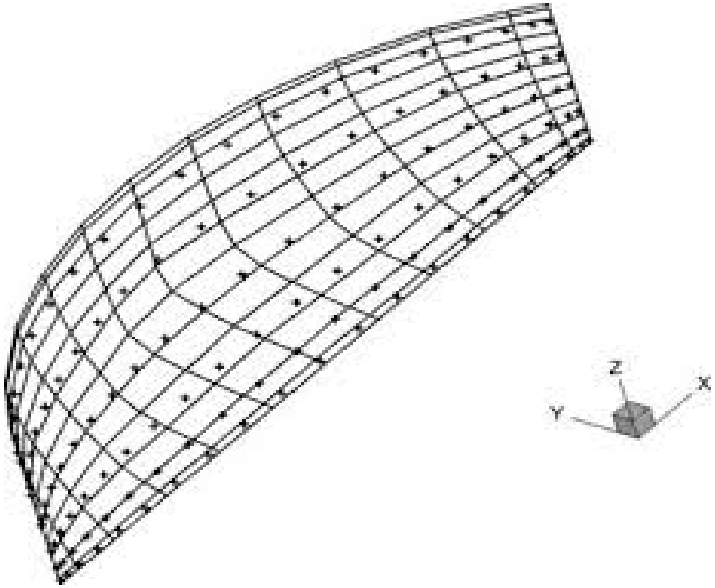
- If the motions are excessive strip method may become invalid due to NL effects

- We therefore need to consider :
  - change of wetted surface
  - true draught and corresponding breadth of the ship sections
  - the flow between sections

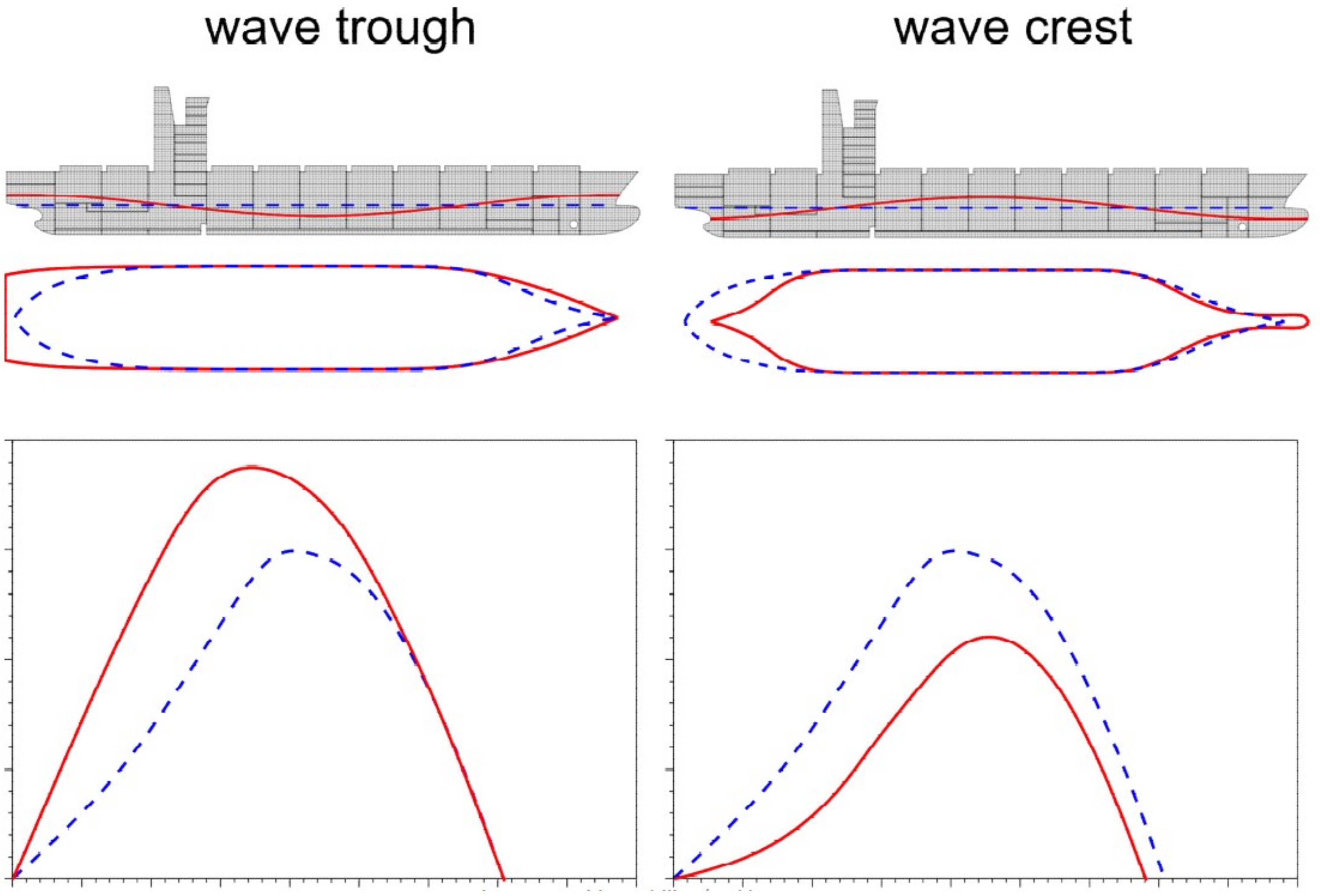
- In such cases we can use – for example - panel methods (e.g. Rankine) to solve the problem

- CFD is an emerging method that may be used to solve the problem as computational capabilities increase. There is need for research work in this area.





Non linear restoring hydro-actions



The impact of NL restoring forces

Restoring and Froude-Krylov actions evaluated on the effective immersed hull in wave at each time step

$$p_t = p - p_a = \rho g \zeta e^{k(Z_0 - \zeta)} - \rho g Z_0 = p_d + p_h \, .$$

p_d: dynamic pressure due to the wave

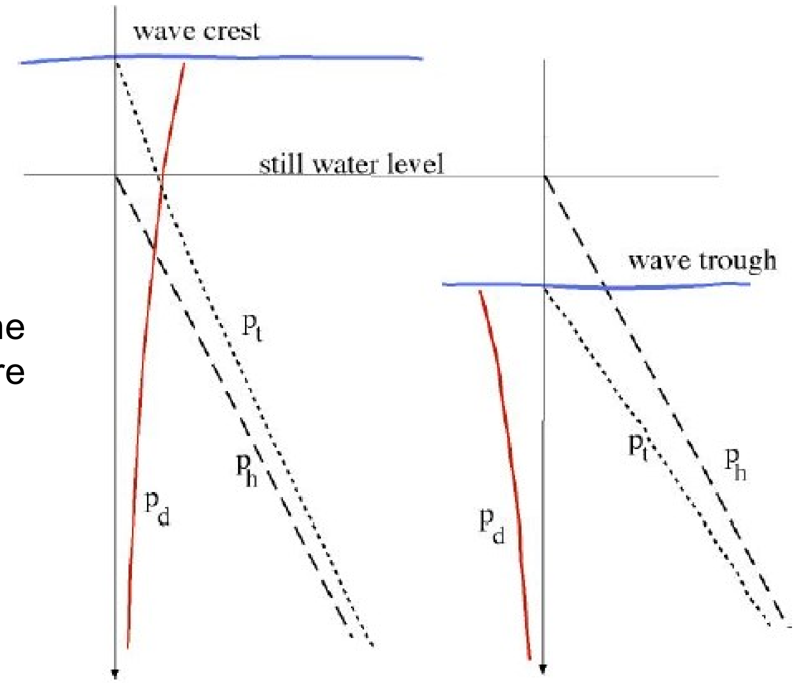
p_h: hydrostatic pressure

p_t: total pressure

The knowledge of the dynamic pressure allows calculating the Froude-Krylov forces and moments; while hydrostatic pressure allows restoring force and moment calculation

$$\mathbf{F}_{\mathrm{F.K}}^{\mathrm{total}} = \sum_{i}^{N} \mathbf{F}_{\mathrm{F.K;i}}^{\mathrm{total}} = \sum_{i}^{N} p_{\mathrm{i}} \Delta S_{\mathrm{i}} \mathbf{n}_{\mathrm{i}}$$

$$\mathbf{M}_{\mathrm{F.K}}^{\mathrm{total}} = \sum_{i}^{N} \mathbf{r}_{\mathrm{i}} \times \mathbf{F}_{\mathrm{F.K;i}}^{\mathrm{total}},$$



Matusiak, J., "Ship Dynamics", Aalto University

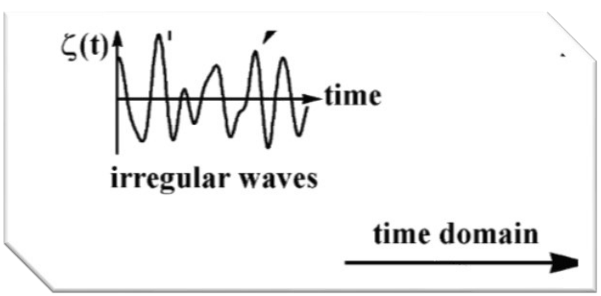
Non linear model – fluid memory effects

- Non-linear assumption does not allow to use spectral analysis: the irregular sea cannot be described anymore as the sum of several regular waves of different frequencies.

- We cannot work in frequency domain, we have to work in time domain!

- We need to rearrange in time domain the added mass and the damping actions (i.e. radiation force and moment) evaluated in frequency domain (evaluated according to the linear method).

- This is done by means of the memory functions

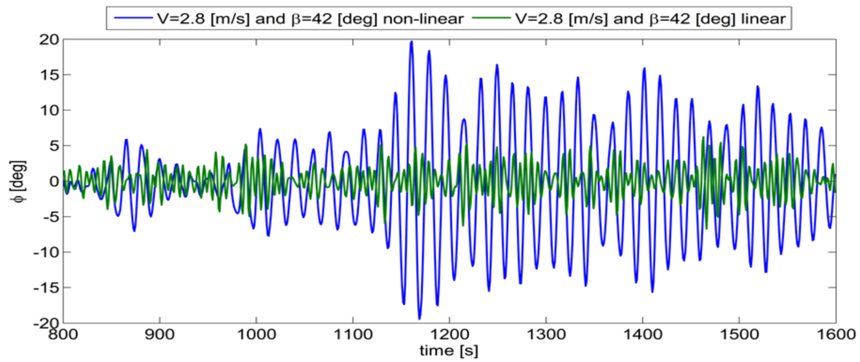


$$\mathbf{X}_{rad}(t) = -\mathbf{a}_{\infty}\ddot{\mathbf{x}}(t) - \int_{-\infty}^{t} \mathbf{k}(t-\tau)\dot{\mathbf{x}}(\tau)d\tau,$$

(Cummins – Johansson)

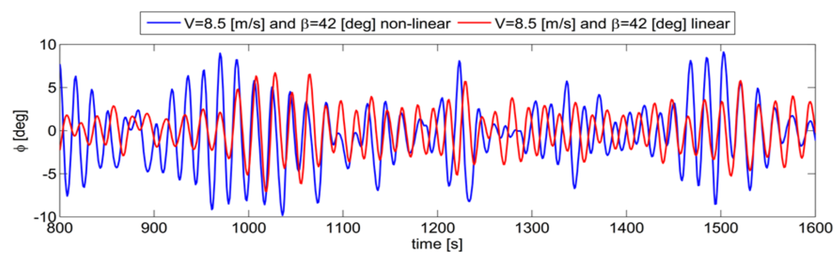
NL codes examples

- **LaiDyn** is a non-linear numerical simulation model in time domain, that is capable of evaluating ship motions in regular and irregular seas. It is meant for research purposes.
- **ShipX Vessel Responces (VERES)** by *Marintek*, also includes the possibility to perform linear and non-linear numerical simulation.



Synchronous Roll Resonance in stern quartering irregular sea

$$H_S = 4.6\ m$$
$$T_1 = 6.5\ s$$



Summary

- The analysis of ship motions is demanding task and for this reason we should select always rational approach that may be suitable for design

- Linear approaches allow us to use spectral techniques and use theory of linear systems

- Linear seakeeping theory is fast to use with modern computers and allows computation of various sea states, motion components etc
  - There are several codes available
  - Some codes have different extensions to include non-linearities and various corrections to made assumptions

- If the motions are excessive strip theories may become invalid due to various non-linearities such as those emerging from
  - Waves
  - Hull form and change of weight distribution due to motions, e.g. free surface etc.
  - Response