On the Parametric Rolling of Container Vessels

Article *in* Brodogradnja · December 2010

Source: OAI

Sergio Ribeiro e Silva
Technical University of Lisbon

Carlos Guedes Soares
University of Lisbon

UDC 629.544:629.5.017.22

Sergio RIBEIRO e SILVA[1]
Anton TURK[2]
C. GUEDES SOARES[1]
Jasna PRPIĆ-ORŠIĆ[2]

On the Parametric Rolling of Container Vessels

Original scientific paper

The main objective of this article is to outline a simple method to identify whether a ship is susceptible to extreme rolling, in order to avoid or to reduce the effects of that phenomenon. A short review of the current classification societies' recommendations is given, as well as a brief description of the physical phenomenon of parametric roll resonance. Analysis criteria to determine the vulnerability of a particular vessel to parametric roll (susceptibility criteria) are applied following the *American Bureau of Shipping* operational guidelines for the assessment of parametric roll resonance in the estimation of container carriers attainable ship speed and heading. The calculations and checks for particular vessels are done for the design wave and forward speed which will most likely lead to the development of parametric roll. Using a time-domain, non-linear numerical model of ship's motions in 6 degrees-of-freedom analysis for regular head waves is conducted for a C11 class containership (severity criteria).

Keywords: *seakeeping, parametric rolling, extreme oscillatory motion*

O parametrijskom ljuljanju kontejnerskih brodova

Izvorni znanstveni rad

Glavni je cilj ovoga rada prikazati jednostavnu metodu za utvrđivanje podložnosti broda ekstremnom ljuljanju radi izbjegavanja ili smanjivanja učinaka te pojave. Prikazan je kratak pregled vrijedećih preporuka klasifikacijskih društava, kao i kratak opis fizikalne pojave rezonancije kod parametrijskog ljuljanja. Izračun kriterija podložnosti pojedinoga plovila parametrijskom ljuljanju provest će se pomoću smjernica za procjenu rezonantnog učinka parametrijskog ljuljanja kod određivanja sigurne brzine i kursa plovidbe kontejnerskih brodova koje daje *American Bureau of Shipping*. Izračuni i provjere za određene brodove izvedeni su za projektni val i brzinu napredovanja kod koje postoji najveća vjerojatnost za razvoj parametrijskog ljuljanja. Za kontejnerski *post- Panamax* brod klase C11 obavljena je analiza tzv. kriterija siline, koristeći nelinearni numerički model gibanja broda sa 6 stupnjeva slobode u vremenskoj domeni.

Ključne riječi: *pomorstvenost, parametrijsko ljuljanje, ekstremna njihanja*

Authors' Addresses (Adrese autora):
[1 -] *Centre for Marine Technology and Engineering (CENTEC), Technical University of Lisbon, Instituto Superior Técnico,* Portugal;
[2] *University of Rijeka, Faculty of Engineering, Department of Naval Architecture and Marine Technology Engineering,* Croatia
Received (Primljeno): 2010-03-09
Accepted (Prihvaćeno): 2010-04-10
Open for discussion (Otvoreno za raspravu): 2011-12-31

1 Introduction

The problem of parametric rolling of ships has been recognized for more than half a century. The fundamental dynamics that creates this kind of behaviour is considered nowadays as reasonably clarified, namely that the frequency of encounter with waves of length similar or larger than the ship length is comparable to twice the ship's roll natural frequency. Also, hull forms with a pronounced bow flare, flat transom stern and non vertical ship sides near the waterline are most vulnerable to parametric roll. Such ships have a wide deck beam to store a large number of containers and at the same time, the underwater hull is streamlined to minimize the resistance. Such features contribute to the variation of ship's stability characteristics due to the constant change of the underwater hull geometry as waves travel along the ship.

As the head sea waves travel along the hull, the righting arm stability varies as the wave crests pass. Two crests in one roll period is an interesting situation to many ship designers. The bow is down due to pitching coupled with roll and the large flare is suddenly immersed in the wave crest. The restoring buoyancy along with the wave force pushes the ship to other side. Coupling and resonance push the ship to roll in larger and larger angles. It has been shown that parametric rolling can occur not only in long-crested head and following seas, but also at slightly oblique heading angles with and without directional wave energy spreading. This fact can simply be explained by the energy balance between damping and change of stability.

The parametric excitation was introduced into the analysis of ship motions by Paulling and Rosenberg [1]. In its service life, a ship will involuntarily experience many occasions of storm weather and rough seas, during which some dynamic problems can happen to the ship. For example, roll motion in resonance with the wave excitation, and as such the GM-variation of a ship in waves is an important evaluation factor in that problem [2].

Initially, it was thought to be a phenomenon mainly limited to the following seas condition and to be of significance for smaller, high-speed displacement vessels such as some fishing boats and seagoing tugs [3] and Ribeiro e Silva and Guedes Soares [4] demonstrated that both linearised and nonlinear theories could be used to predict parametric rolling in regular head waves.

However, in 1998 a post-Panamax, C11 class containership lost 1/3 of her deck containers and damaged another 1/3 in a severe storm. Evidence of parametric rolling in head seas on a post-Panamax C11 class container ship received wide and renewed attention and this incident was analysed by means of numerical simulations and model tests [5].

As the parametric roll phenomenon is caused by time variation of transverse stability, the numerical simulation method must be able to model adequately the changes of geometry of the immersed part of the hull due to large waves and ship motions [6]. The results confirmed that the vessel suffered from a severe case of parametric roll during the storm [7].

Since the publication of these results, ship operators and ship designers have become more aware of the fact that this phenomenon can occur for larger vessels in confused seas [8]. A full numerical procedure that was conducted for the post-Panamax container vessel for predicting the onset of parametric rolling based on the Duffing method with excitation taken from the difference in restoring force from trough to crest condition when the ship is encountering head seas proved that it can push the ship into a parametric mode of rolling [9].

Despite the progress in the understanding of the phenomenon, a few issues remain open, such as the development of effective criteria for the prevention of parametric rolling by design, the assessment of the effects of coupling with other motions, and the derivation of optimal experimental/numerical procedures for safety assessment in a realistic sea.

In this work the effect of parametric resonance which possesses a great danger especially for container ships sailing in following or head seas is dealt with. Important parameters that are effective in roll resonance are pointed out. For this purpose, the method worked out by the *American Bureau of Shipping (ABS)* is taken as an example for a series of random container ships to analyze its stability in longitudinal waves. Using *ABS* guidelines [10], for these particular ships, unfavourable sailing conditions such as heading and speed have been determined. Numerical details of the procedure have been worked out and provided as well.

2 Assessment models for parametric rolling on ships

2.1 Prediction of occurrence of parametric rolling in regular waves (susceptibility criteria)

Some essential elements of roll motion are reviewed first. At a fundamental level, the equation of linear rolling motion is that of an excited ($M_x(t)$) or non excited rotational oscillator (free rolling). Considering a single degree of freedom equation for roll motion in head seas, and taking into account the changing *GM* due to wave encounter leads to [7],

$$\ddot{\eta}_4 + \frac{B_{44}}{I_{XX}+A_{44}}\dot{\eta}_4 + \frac{\Delta \cdot GM(t)}{I_{XX}+A_{44}}\eta_4 = 0 \quad (1)$$

where, η_4 represents the roll amplitude, B_{44} is the damping coefficient, Δ is the displacement of a ship, I_{XX} is the transverse moment of inertia, and A_{44} is the added mass moment of inertia in roll.

Parametric rolling typically occurs in various combinations of ship speed and wave frequency, provided that the resulting frequency of encounter is near to ($2/n$) times the natural frequency, where n is any integer. Theoretically it can occur at approximately 2:n ratios between these two frequencies, where the integer n = 1, 2, 3.. . In practice the cases for n = 1, when the wave encounter frequency is approximately twice the roll natural frequency, and n=2, when the wave encounter and roll natural frequencies are nearly equal, are of importance. Typically the former case (n >1) occurs in head and bow quartering seas (e.g. [5], [6], [11]), but it can also occur in following seas [8]. The latter case (n > 2) is typically associated with following and stern quartering seas [12].

Apart from the above condition of frequencies to build-up a parametric rolling, a threshold wave height must be determined, in principle by two factors: the degree of fluctuation of roll restoring due to wave passage, and the ship's roll damping which is speed dependent must be obtained. The damping is a key design parameter for the avoidance of parametric rolling [13].

The following simple rule may be applied, by varying *GM* values on the wave between GM_{min} and GM_{max} (ABS), and the scaled amplitude of variation of metacentric height, defined as [2],

$$h = \frac{GM_{max} - GM_{min}}{2 \cdot \overline{GM}} \quad (2)$$

where *GM*, which is the mean metacentric height of the ship for the considered regular wave, exceeds 4 times roll damping ratio; then the occurrence of parametric rolling is possible. To do this, assuming that *GM* changes sinusoidally with time in waves,

$$GM(t) = GM_m + GM_a \cos(\omega t) \quad (3)$$

where, GM_m is a mean value of *GM* and GM_a is the amplitude of the *GM* changes in waves. By obtaining the so called susceptibility criteria, this phenomenon described as the parametric resonance, can simply be simulated by the Mathieu equation seen to be a linear differential equation with a time varying restoring coefficient of the form,

$$\frac{d^2\eta_4}{d\tau^2} + (p + q\cos\tau)\eta_4 = 0 \quad (4)$$

where p is a function of the ratio of forcing and natural frequency, q the parameter that dictates the amplitude of parametric excitation, and τ represents damping. After the mean roll frequency, its variation amplitude and the linearized damping coefficient δ:

$$\omega_m = \sqrt{\frac{\Delta \cdot GM_m}{I_{XX}+A_{44}}}; \quad \omega_a = \sqrt{\frac{\Delta \cdot GM_a}{I_{XX}+A_{44}}}; \quad 2\delta = \frac{B_{44}}{I_{XX}+A_{44}} \quad (5)$$

being included in (1), the following equation is derived:

$$\ddot{\eta}_4 + 2\delta\dot{\eta}_4 + \left(\omega_m^2 + \omega_a^2\cos(\omega t)\right)\eta_4 = 0 \quad (6)$$

To obtain the standard form of the Mathieu equation it is necessary to transform equation (4) by introducing a dimensionless time: $\tau = \omega t \Rightarrow t = \tau/\omega$. By doing that and dividing both parts by the square of the wave frequency ω^2, the dimensionless quantities are obtained:

$$\mu = \frac{\delta}{\omega}; \quad \bar{\omega}_m = \frac{\omega_m}{\omega}; \quad \bar{\omega}_a = \frac{\omega_a}{\omega} \quad (7)$$

where ω_a is the roll variation frequency and ω_m is the mean value of roll frequency.

With the next substitution the damping part is introduced in the following way:

$$\eta_4(\tau) = x(\tau)\exp(-\mu\tau) \tag{8}$$

This finally expresses roll in the form of a Mathieu equation (4) with coefficients,

$$p = \left(\bar{\omega}_m^2 - \mu^2\right); \quad q = \bar{\omega}_a^2. \tag{9}$$

The solution of the Mathieu equation may be periodic, increasing or decreasing in nature, but strictly depends on the values of p and q. The Mathieu equation may have two types of solutions: bounded and unbounded, commonly referred as "unstable".

Figure 1 is the stability diagram for this equation [3]. The shaded regions are stable corresponding to (p,q) pairs where the motion cannot exist and the unshaded regions which are unstable, i.e. the motion can exist. If (p,q) lie in an unstable region, an arbitrarily small initial disturbance will trigger an oscillatory motion that tends to increase indefinitely with time. In a stable region, the initial disturbance will die out with time. It also means that there is a threshold value for roll damping for each pair of Mathieu parameters p and q. To calculate this threshold, it is necessary to find a way to express the increment of the unbounded solution of the Mathieu equation (e.g. ABS).

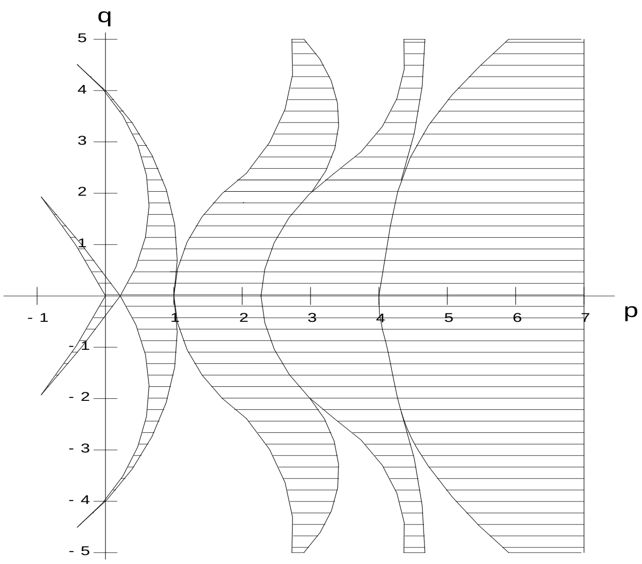


Figure 1 **Stability diagram for the Mathieu equation**
Slika 1 **Dijagram zona stabiliteta Mathieuove jednadžbe**

Depending on the critical values of roll damping, parametric resonance may or may not take place. The physical phenomenon is based on successive alterations of the restoring rolling moment lever between crests and troughs, exhibited by many ships in steep longitudinal waves with a clear analogy with a simple oscillator governed by the Mathieu equation with damping.

There is another way to express this variation of *GM* that can be reflected in the equation of roll motion as follows [8]:

$$\ddot{\eta}_4 + 2\zeta\omega_{\eta_4}\dot{\eta}_4 + \omega_{\eta_4}^2\left(1 - h\cos\left(\omega_e t - \varepsilon_e\right)\right)\eta_4 = M_x(t) \tag{10}$$

In equation (10) ζ is the roll damping ratio, ω_e is the encounter frequency and ε_e is an appropriate phase shift. The variation of the roll natural frequency $\omega_{\eta 4}$

$$\omega_{\eta_4} = \sqrt{mg \cdot \overline{GM} / (I_{XX} + A_{44})} \tag{11}$$

due to longitudinal wave is introduced into equation (10) by the following function:

$$h = \sqrt{\left(2 - \frac{\omega_e^2}{2\omega_{\eta_4}^2}\right)^2 + 4\zeta^2 \frac{\omega_e^2}{\omega_{\eta_4}^2}} \tag{12}$$

For the threshold level of the scaled *GM* fluctuation, in the vicinity of exact principal resonance, value of h is moving upwards from the above presented boundary curve and quickly triggers the build-up of parametric rolling.

The most comprehensive mathematical method to assess susceptibility to parametric rolling is the solution of the Mathieu equation. Obviously, some computer codes of different sophistication levels can be also used in order to check if parametric rolling occurred in their simulation results. For each time step the roll angle can be compared to a threshold level. If two subsequent roll amplitudes (negative or positive) are higher than that threshold, the group of roll angles is marked as parametric rolling.

To conclude the fundamental part from theory and as validated in tests, it can be stated that parametric roll occurs when the following requirements are satisfied:

1. natural period of roll is equal to approximately twice the wave encounter period
2. wave length is on the order of the ship length (between 0.8 and 2 times LBP)
3. wave height exceeds a critical level
4. roll damping is low.

2.2 Prediction of amplitude of parametric rolling in regular waves (severity criteria)

If the susceptibility to parametric roll has been determined, the *ABS* requires a full numerical procedure with different possible analytical methods to be applied (see [8], [14], [15], [16]) as long as the numerical simulation system is based on formulations capable of solving incident wave forces (or Froude-Krylov forces) over an instantaneous submerged body.

Forces due to wave excitation (incident wave forces and diffraction forces) and reaction (restoring and radiation) forces due to wave-induced ship motions have to be taken into account. With the model adopted for this case study, radiation and wave excitation forces are calculated at the equilibrium waterline using a standard strip theory, where the two-dimensional frequency-dependent coefficients of added mass and damping are computed by Frank close fit method and the sectional diffraction forces are evaluated using the Haskind-Newman relations [17].

The non-linear restoring coefficients in heave, roll, and pitch motions in waves are calculated using a quasi-static approach. The time domain simulations, meaning that at each time step the underwater part of the hull is calculated, together with its geometric, hydrostatic and hydrodynamic properties can be represented as follows maintaining the hypothesis of the linear hydrodynamic model [10],

$$\Phi = \Phi_U + \Phi_I + \Phi_D + \Phi_R \qquad (13)$$

where Φ_U denotes the potential of the steady motion in still water, Φ_I is the incident wave potential, Φ_D is the diffraction potential and Φ_R is the radiation potential.

Owing to complex interactions between the hull and ship generated waves, the governing equations can be written in the form of integro-differential equations. Combining all hydrodynamic forces with the mass forces, using subscript notations, one can obtain six linear coupled differential equations of motion, in abbreviated form, such as:

$$\sum_{J=1}^{6}\left\{\left(M_{kj}+A_{kj}\right)\ddot{\xi}_j + B_{kj}\dot{\xi}_j + C_{kj}(t)\xi_j\right\} = F_k e^{i\omega_e t} \quad k,j=1,.....6 \qquad (14)$$

where the subscripts k, j are associated with forces in the k-direction due to motions in the j-mode (k =1, 2, 3 represent the surge, sway and heave directions, and 4, 5, 6 represent roll, pitch and yaw directions). M_{kj} are the components of the mass matrix for the ship, A_{kj} and B_{kj} are the added mass and damping coefficients, $C_{kj}(t)$ are the hydrostatic (time dependent) restoring coefficients, and F_k are the amplitudes of the exciting forces, where the forces are given by the real part of $F_k e^{i\omega_e t}$.

Another important aspect of parametric resonance is the amplitude of the roll motion which depends on the character of the stability curve (*GZ* curve). In longitudinal waves, except for small values of roll (8–10 deg), *GM* depends on the roll angles. On the other hand, roll period (frequency) is also a function of *GM*. Therefore, after certain roll angles, conditions for parametric resonance cannot be established and in short the growth stops.

Applicability of any solution based on the Mathieu equation is limited because it is linear: it can only indicate the conditions when parametric roll can be generated, but it is unable to predict the roll amplitude. Such an answer is not enough for engineering practice: the solution must determine how large the parametric roll might develop if conditions satisfy the susceptibility criteria.

So far it can be said that the probability of parametric rolling has not been yet "controlled" completely and eliminated in design. It is essential to ensure that the amplitude of parametric roll oscillation *A*, which might be generated in an extreme seaway, is kept small. The amplitude A may be expressed by the following solution of (10) and (12), [13]:

$$A^2 = \frac{4}{3c_3}\left[\left(1-\frac{1}{a}\right) \pm \sqrt{\frac{h^2}{4}-\frac{4\zeta^2}{a}}\right] \qquad (15)$$

where $a = 4\omega_{\eta_4}^2 / \omega_e^2$. If the amplitude of parametric roll is moderate to large, a fifth order polynomial is likely to be required for modelling the restoring moment. The solution of the Mathieu equation in that case is unbounded and grows indefinitely, however, in the real world, parametric roll is limited to finite, though sometimes large, amplitude. It is known that nonlinear terms in the rolling equation stabilize parametric rolling. The nonlinear equation of roll in a longitudinal sea that could be used for predicting the steady roll amplitude in the vicinity of principal resonance would be like:

$$\ddot{\eta}_4 + 2\zeta\omega_{\eta_4}\dot{\eta}_4 + \omega_{\eta_4}^2\left(1-h\cos\left(\omega_e t-\varepsilon\right)\right)\eta_4 - c_3\omega_0^2\eta_4^{\,3} = 0 \qquad (16)$$

which yields

$$\ddot{\eta}_4 + 2\zeta\omega_{\eta_4}\dot{\eta}_4 + \omega_{\eta_4}^2\left(1-h\cos\left(\omega_e t-\varepsilon\right)\right)\eta_4 - c_3\omega_{\eta_4}^2\eta_4^{\,3} - c_5\omega_{\eta_4}^2\eta_4^{\,5} = 0 \qquad (17)$$

where c_3 and c_5 are nonlinear stiffness coefficients, corresponding respectively to the third and fifth order restoring terms. Nonlinear damping has a tendency to increase with roll velocity, so sooner or later it will grow above the damping threshold.

The *ABS* [10] treats the following severity check problem with the second order differential equation for nonlinear damping, which is the modification of (16),

$$\ddot{\eta}_4 + 2\zeta\dot{\eta}_4 + \omega_m^2 f(\eta_4,t) = 0 \qquad (18)$$

The equation (18) may be converted into a set of first-order equations by a suitable change of variables. The set of equations is given as follows:

$$\begin{aligned}\dot{\eta}_4 + 2\zeta\eta_4 + f(\eta_4,t) &= 0\\ \dot{\eta}_4 - \eta_4 &= 0\end{aligned} \qquad (19)$$

Time-dependent restoring term of the roll equation depends on the characteristics of the *GZ* curve:

$$f(\eta_4,t) = \frac{\pm(\eta_4)}{GM_0} GZ\left(|\eta_4|,t\right) \qquad (20)$$

Thus, it can be manipulated by a step-by-step solution covering all values on the 3-D *GZ* curves approximated by a third or higher order polynomial to simplify the case. The above-mentioned equation of roll motion can be solved by a fourth-order Runge–Kutta algorithm for damping values of μ = 0.03, 0.05, 0.075 and 0.10. Another way to get it done, is to re-write rolling equation with only the restoring nonlinearity, meanwhile retaining only the linearized damping coefficient δ,

$$\ddot{\eta}_4 + 2\delta\dot{\eta}_4 + \omega_m^2\left(1+a_p\cos\left(\omega_e t\right)\right)\eta_4 - a_3\eta_4^{\,3} = 0; \quad a_p = \frac{\omega_a^2}{\omega_m^2} \qquad (21)$$

where a_3 is a third power coefficient used to approximate the nonlinear *GZ* curve, and thus by applying the method of multiple scales the steady-state amplitude can be obtained and expressed as [7]:

$$A = \frac{\sqrt{\omega_e - 4\omega_m^2 - 0.5\omega_e\sqrt{\omega_e^2 a_p^2 - 64\delta^2}}}{\sqrt{3a_3}} \qquad (22)$$

3 Assessment models for parametric rolling on ships

3.1 Classification guides regarding parametric rolling

The problem of parametric roll returned to prominence quite recently as a result of significant cargo loss and damage sustained by post-Panamax container carriers which led classification societies to take interest in such phenomenon.

The *American Bureau of Shipping* (*ABS*) is the leading society concerning the documentation on parametric roll, as they issued the first guide for the assessment of parametric roll resonance in the design of container carriers [10] in September 2004, with the main purpose to supplement the Rules and the other design and analysis criteria that the *ABS* issues for the classification of container carriers. The Guide contains a brief description of the physical phenomenon of parametric roll resonance, which may cause an excessive roll of a containership in longitudinal (head and following) waves. The Guide also contains a description of the criteria used to determine if a particular vessel is vulnerable to parametric roll (susceptibility criteria) and how large these roll motions might be (severity criteria). The procedure based on work conducted by [7] which gave the technical background of the *ABS* Guide is explained in detail in the following chapter.

In March 2008 the *ABS* awarded the first class notation specific to parametric roll to three ships in the Hyundai Merchant Marine Fleet. The optional class notation was issued against criteria contained in the *ABS* Guide for the Assessment of Parametric Roll Resonance in the Design of Container Carriers, which provides design and analysis measures to determine if a particular vessel is vulnerable to parametric roll and the potential magnitude of the roll motions. The "PARR C1" notation was granted to the 4,700 TEU *Hyundai Forward,* and 8,600 TEU vessels *Hyundai Faith and Hyundai Force* [18].

Lloyd's Register of Shipping supports initiatives made to introduce guidance on avoidance of parametric roll. At present the *IMO* sub-committee on stability and load lines and on fishing vessel safety (*IMO SLF*) is tasked with addressing this issue. In response to concerns voiced in the industry, *Lloyd's Register* has investigated its container securing requirements. *LR* suggests simplified Susceptibility Assessment Method which considers the roll motions of the vessel, treating them as a simple mass-spring damper system with a single degree of freedom. Any seakeeping software can be used to determine the ship motions in chosen wave conditions, for a range of speeds and headings. From the resulting data, the variation in vessel stability through the waves and the functional relationships between the wave encounter frequency and the roll natural frequency was used to help avoid the possibility of parametric rolling [19].

Bureau Veritas is recommending appropriate solutions of the simplest mathematical model of parametric roll considering the one degree of freedom roll motion equation, in which the restoring coefficient is made time dependent. Calculated hydrostatic variations are approximated by the sinusoidal function which leads to the Mathieu type equation for roll, from which the regions where the roll instability takes place can be identified. In preliminary stages the *BV* suggest simplified analytical models, as presented above, which allows for quick identification of the dangerous zones, however in the final phase of parametric roll evaluation, it is preferred to use the nonlinear numerical models which are able to include all sort of nonlinearities in a natural way. To summarize, the *BV* approach defines a two step procedure,

- preliminary checks using the simplified semi-analytical model
- fully non-linear simulations for critical cases,

after which it is possible to produce the polar plots which represent the maximum expected roll motion for a given sea state with respect to ship speed and heading [20].

Det Norske Veritas issued a containership update, in addition to traditional class services during the design, construction and operational phases. They provide owners and operators with increasingly 'popular' services, such as Active Operator Guidance, advice on extreme roll motions (parametric rolling) and how to avoid these for a sea state in which the vessel's hull, given the wave length and height as well as the distance between waves, may be subject to extreme roll motions.

IMO has published revised guidance to the master for avoiding dangerous situations in adverse weather and sea conditions where the physical phenomena of parametric roll motions are explained. Operational guidance on how to avoid dangerous conditions with risk of successive high wave attack in following and head seas is given for the encounter period that is close to one half of the ship roll period. Therefore, apart from special checks on hull forms and the need to fit roll suppression devices, a revision of the "Guidance to the Master for Avoiding Dangerous Situations in Following and Quartering Seas" is seen as appropriate. More specifically, as statistically demonstrated in section 6.3 a special attention to the referred bow seas scenario should be given, where the best first course of action should be rather to increase speed than reduce the ship's speed, as mentioned in the document [21].

In addition, the various classification society criteria may not predict equivalent lifetime roll responses and accelerations. Accordingly, current classification society criteria do not provide uniform design standards for container ships experiencing head-sea parametric rolling.

3.2 Susceptibility criteria examples from the *ABS* assessment model

The procedure for susceptibility criteria check is described in detail in [10]. Stability in longitudinal seas and parametric roll susceptibility criteria is to be calculated for the design wave, according to the *ABS*. Its length, λ, is equal to the length between perpendiculars, *L*.

The coordinate system is to be assumed as shown in Figure 2.

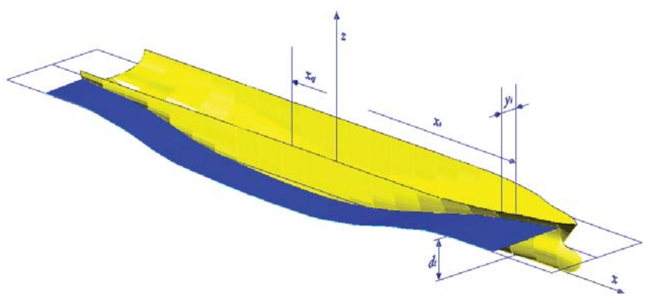


Figure 2 **Coordinate system**
Slika 2 **Koordinatni sustav**

Following the above *ABS* recommendations, a series of containerships have been tested. It is recognized that parametric rolling reappeared with the advent of latest generation of post-Panamax vessels. Since such events are quite rare phenomena occurring in head or following seas and only now being studied in a rigorous way, it was difficult to find a vessel sufficiently fast with a heavily flared bow and flat transom stern that might be prone to rapidly develop large, unstable and violent rolling motions. The results are reported for the 6,586-TEU container carrier *NYK PEGASUS* (*LBP* = 287.0 m), which among other similar vessels did not exhibit susceptibility to parametric rolling (Figure 3).

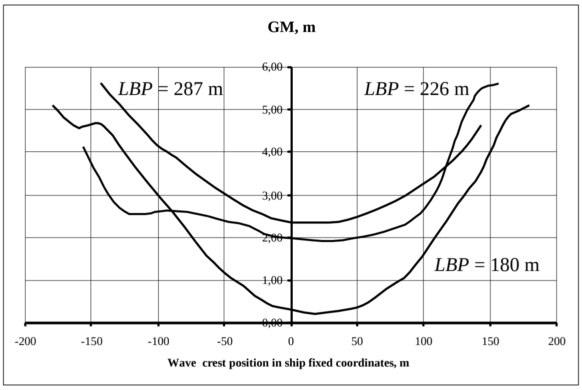
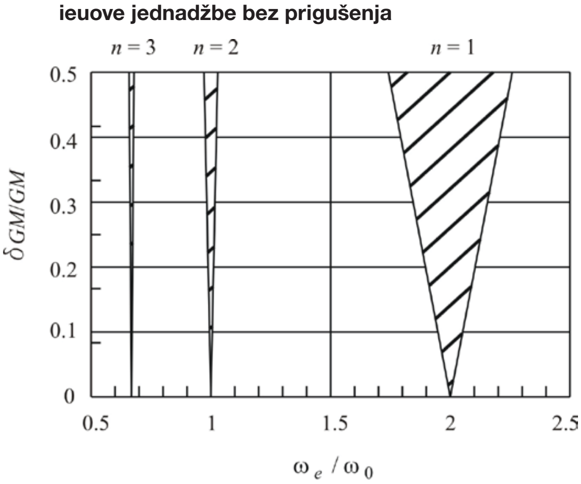


Figure 3 ***GM* value for different wave crest positions along the ship hull**
Slika 3 **Vrijednosti *GM* za različite položaje valnog brijega duž trupa**

However, two of the analyzed examples have been chosen and presented, as they show to be somewhat surprising in comparison to the results of numerical calculation of severity criteria which were known from previous investigations [9, 22]. It is commonly perceived that mostly larger vessels such as Post-Panamax container series can be prone to parametric rolling but there have been some occasions where that presumption does not apply. Namely, a relatively small containership [18] with body lines (Figure 5) and main particulars is taken as an example to analyze its stability in longitudinal waves while sailing in unfavourable environmental conditions such as heading and speed. It has shown to be vulnerable to parametric roll resonance as the susceptibility criterion is not satisfactory.

One of the possibilities for that outcome is that despite the fact that the instability zones for smaller vessels derived from linear undamped Mathieu equation are small (Figure 4), it is still possible that by a combination of selected wave length, ship speed V_S and heading, an effect of parametric rolling can occur. Further numerical procedure for investigation of heading, speed and damping combinations that lead to the rise of roll amplitudes (severity check) must be defined and governed, being thus able to avoid them. This will be the part of the future work relating to the subject.

Figure 4 **Threshold boundaries of the first three instability zones for the linear Mathieu equation without damping**
Slika 4 **Granice za prve tri zone nestabilnosti linearne Mathieuove jednadžbe bez prigušenja**



Main particulars of midsize container ship (A):
Length *LBP* = 168.0 m
Breadth *B* = 28.0 m
Depth *D* = 16.0 m
Design draught *d* = 10.0 m
Vertical centre of gravity *VCG* = 12.0 m
Metacentric height *GM* = 0.596 m
Speed V_S = 20 kn

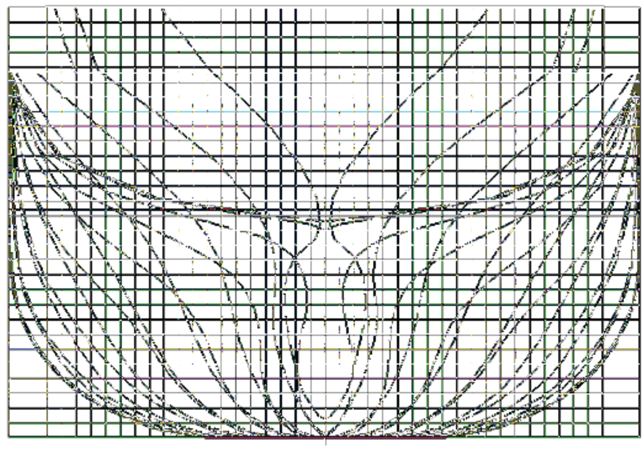


Figure 5 **Body plan of midsize container ship (*A*)**
Slika 5 **Nacrt linija kontejnerskog broda (*A*)**

The following example is taken from [9] for predicting the onset of parametric rolling. The full numerical procedure for a Post-Panamax container vessel (Figure 6) based on the Duffing method with excitation taken from the difference in restoring force from trough to crest condition when the ship is encountering head seas proved it can push the ship into a parametric mode of rolling. However, after applying the *ABS* procedure to the above mentioned example, the outcome has not shown susceptibility to parametric rolling. This led to the conclusion that, while there is a need for standardized procedure such as this one, it is not by any means full proof. Certainly for the vessels of such dimension it is advisable to conduct full numerical procedure for the prediction of amplitude of parametric rolling in irregular waves. It is possible that, because of the simplified analytical model presented by the *ABS*, such a vessel may not seem susceptible to parametric roll since it was found to be a problem for the slender Panamax ships. Examples exist where large numbers of containers were lost overboard from Panamax and relatively smaller Post-Panamax vessels due to sudden excessive rolling. As the Post-Panamax ships became ever bigger, concern was expressed that parametric roll may become even more threatening for these ships. This might not be the case. Because of their large stability, parametric roll generally seems to be less of a problem for very large Post-Panamax ships. GM value for different wave crest positions along the ship hull are given in Figure 7.

Main particulars of Post-Panamax containership (B):
Length *LBP* = 307.0 m
Breadth *B* = 42.8 m
Depth *D* = 24.5 m
Draught *T* = 13.0 m
Design draught *d* = 10.0 m
Vertical centre of gravity *VCG* = 12.0 m

Metacentric height GM = 0.596 m
Speed V_S = 20 kn

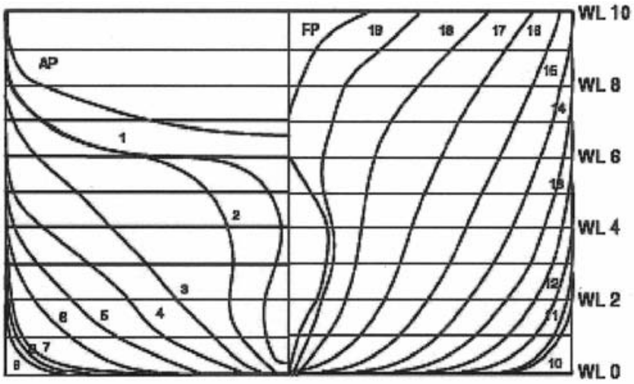


Figure 6 **Body plan of the Post-Panamax container ship (*B*)**
Slika 6 **Nacrt linija kontejnerskog broda Post-Panamax (*B*)**

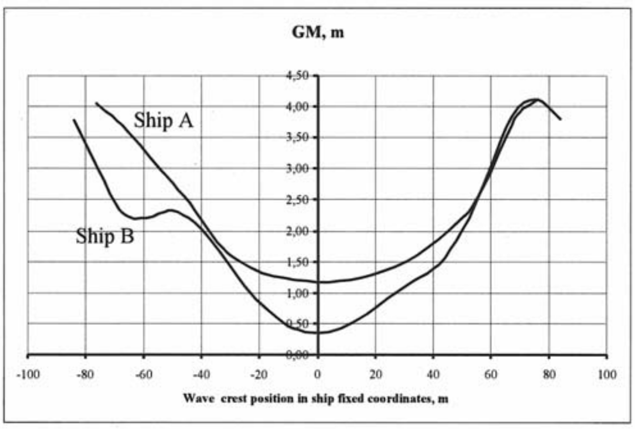


Figure 7 ***GM* value for different wave crest positions along the ship hull, (*A*)**
Slika 7 **Vrijednosti *GM* za različite položaje valnog brijega duž trupa, (*A*)**

3.3 Application of usceptibility and severity criteria for a Post-Panamax, C11 class containership

A full numerical procedure [16] is applied to compare with the susceptibility guidelines for the assessment of parametric rolling resonance in the case of a container carrier. Parametric rolling as an unstable phenomenon with rolling angles coupled with significant pitching introduces high loads into the containers and their securing systems. As already mentioned, it is well documented that a Post-Panamax, C11 class containership, (see Table 2), encountered extreme weather (storm encounter that lasted for some 11 hours) and sustained extensive loss and damage to the deck stowed containers [5].

This event by itself was the main reason to choose that vessel for the experimental study whose partial results are presented here. In this case, the energy in heave and pitch motions was transferred to roll motion mode through non-linear coupling, which leads to excessive resonant rolling and stability problems.

This is in a way a backward problem since the application of both of these criteria is a direct consequence of the motions that the vessel exhibited during this storm event in which port and starboard rolls of 35 deg to 40 deg were reported.

Table 2 **Main particulars of the container vessel**
Tablica 2 **Glavne izmjere kontejnerskog broda**

Length between perpendiculars (L_{PP})	262.00 m
Depth at main deck (D)	24.40 m
Breadth, design waterline (B_{DWL})	40.00 m
Displacement, design waterline (Δ_{DWL})	76056 t
Block coefficient (C_B)	0.66
Draught at amidships (T_{DWL})	12.34 m
Transverse metacentric height, still water (GM_T)	1.973 m
Natural roll period, linearized in waves (T_{44})	22.78 s
Cruise speed (U)	20 knots

Roll Damping Curves

Normally, the damping coefficients can be obtained from free decay experiments, in which the model is released for a free roll from a given inclination. It has to be noted that there are still differences between the measured values and those predicted from existing methods.

Usage of such an empirical roll damping assessment, established from the free-decay model tests led to a few applicable analytical methods [23], [24]. Roll damping has to be considered as one of the most difficult properties to predict due to the non-linear characteristics (effects of fluid viscosity) as well as due to its strong dependence on the ship's forward speed. For this study Miller's method was adopted since it was found to present a better fit especially for a slender vessel such as the C11 Post-Panamax container ship, taken as a case study example (Figure 8).

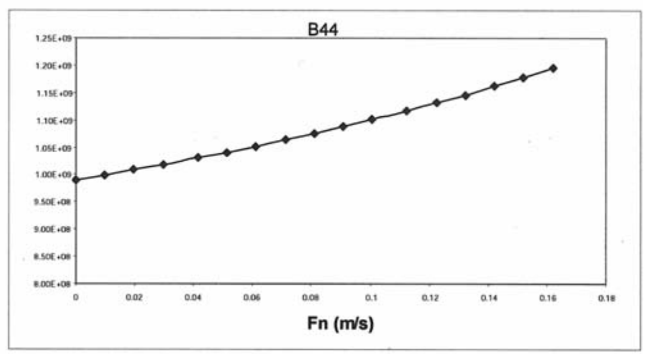


Figure 8 **Prediction of roll damping by Miller's original method**
Slika 8 **Predviđanje prigušenja ljuljanja izvornom Millerovom metodom**

Bounded and Unbounded Solutions of the Mathieu Equation (Ince-Strutt Diagram)

The solution of the Mathieu equation depending on the values of p and q, with two types of solutions - bounded and unbounded, can establish certain boundaries, subsequently leading towards susceptibility criteria.

The loading condition adopted corresponds to a displacement of 76056 [ton] and a position of the centre of gravity of the vessel and its cargo which leads to a transversal metacentric height (GM_T) of 1.973 m (in still water). As shown in Figure 9, operational conditions fall into a Mathieu type zone of instability.

The ship may be susceptible to parametric roll and the severity criterion has to be checked. Parametric rolling is a result of the periodic variation of the vessel's stability characteristics and does not come from direct excitation by external wave forces. For a ship in head or stern seas, the uneven wave surface together with pitch and heave motions results in significant time-varying changes to the GM. When that time variation of stability matches a wave encounter period of approximately half the vessel's natural rolling period, extreme rolling motions can occur, therefore a full numerical procedure is needed.

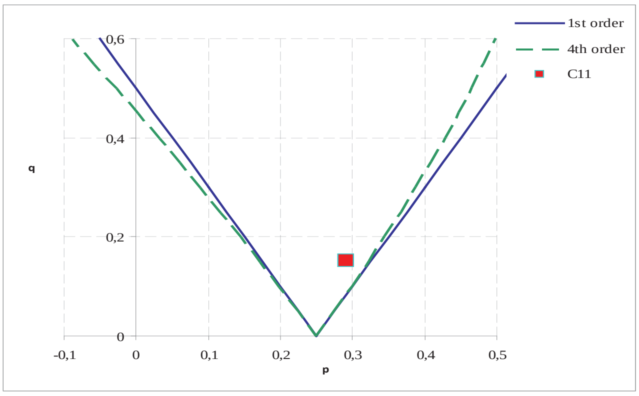


Figure 9 **Basic verification of susceptibility criteria (wave amplitude 4 m, frequency 0.485 s^{-1})**

Slika 9 **Osnovna verifikacija kriterija podložnosti (valna amplituda 4 m, frekvencija 0,485 s^{-1})**

Simulated instability in regular head seas

Following the theory presented in Sec. 2.1, a computer code was developed implementing the features necessary to predict ship motion instabilities, particularly utilized for a design wave and forward speed that could lead to the occurrence of parametric rolling. An investigation was undertaken to determine whether head-sea parametric rolling could have been the cause of the vessel's extreme motions. Based on previous investigations, in which the conditions upon which head wave parametric rolling resonance is likely to occur, such as the effects of the encounter period, the wave height and the encounter angle on parametric rolling amplitude were clarified. In order to run simulations, the program is set up by tuning the parameters so that parametric rolling can be expected, such as that the natural period of roll is equal to approximately twice the wave encounter period and the wave length is in the order of the ship length (between 0.8 and 2 times *LBP*). Wave height needs to exceed a critical level and roll damping is small. The duration of each regular wave simulation record should be long enough for steady state ship motions to be established. The initial conditions for each simulation should be chosen from the steady state regime corresponding to the initial phase of the waves.

For the specific case the wave length to ship ratio varies from 0.8 to 1.4. The investigation was carried out for wave heights ranging from 5 m to 9 m. Finally, all the calculations have to be checked for the forward speed dependence up to the maximum achievable speed (within operational range). More specifically, forward speeds will be considered up to 20 knots with the increment of 1 knot. By varying each of these parameters, significant amplification of roll motion is observed and studied.

The simulations have shown that at wave height of 5 m parametric rolling does not occur. The very first condition (Figure 10) upon which the parametric rolling resonance ($\eta_4 = 18°$) is evident is for the forward speed of 7 knots, wave height of 6 m, and the wave length equal to the ship length (period of wave T_W = 12.95 s).

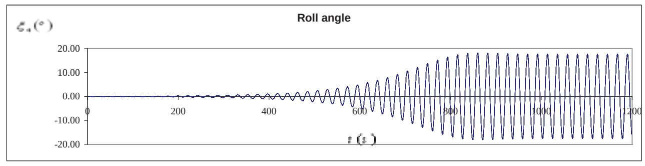


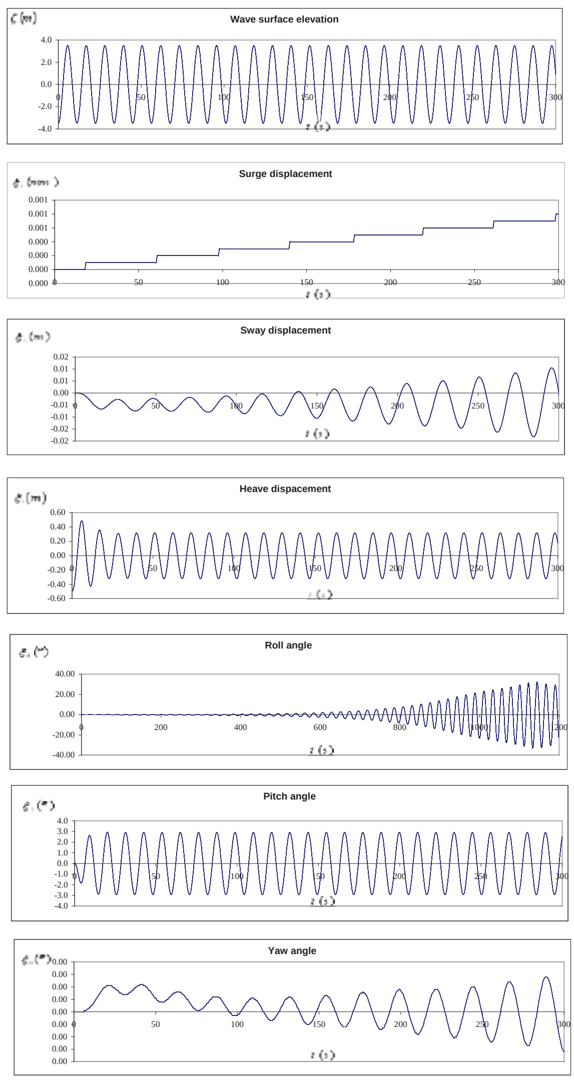
Figure 10 **Development of parametric rolling in regular waves (H_W = 6 m, T_W = 12.95 s, speed 7 knots)**

Slika 10 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 6 m, T_W = 12,95 s, brzina 7 čv)**

This is rather important as a starting point whereupon all the simulations following similar parameters can be closely observed and finely tuned.

Figure 11 **Development of parametric rolling in regular waves (H_W = 7 m, T_W = 12.95 s, speed 5 kn)**

Slika 11 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 7 m, T_W = 12,95 s, brzina 5 čv)**



However, the first prolific effect of parametric resonance is noticeable for a wave height of 7 m with the forward speed ranging from 5 to 11 knots respectively. As seen in Figures 11-14 by varying ship speed from 5 to 11 knots and retaining the other parameters (wave amplitude 3.5 m, period 12.95 s) the significant roll amplification is varying from η_4 from 30° at 5 knots up to the maximum value of η_4 = 34° at 7 knots, and subsequently decreasing η_4 = 6° (11 knots) up to the point where at 12 knots it becomes insignificant.

In Figure 11 numerical results of motions excited by longitudinal regular head waves are displayed. It can be seen that sway and yaw is negligible in comparison to heave, roll and pitch motions.

Furthermore, even heave and pitch motions accompanying parametric rolling in regular waves are such that a significant rise of roll amplitude is associated with a small decrease in heave and a very small decrease in pitch.

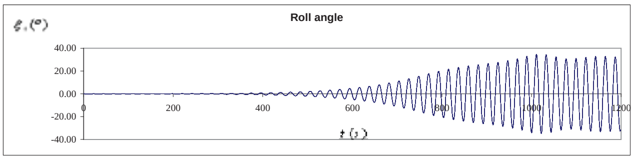


Figure 12 **Development of parametric rolling in regular waves (H_W = 7 m, T_W = 12.95 s, speed 7 kn)**
Slika 12 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 7 m, T_W = 12,95 s, brzina 7 čv)**

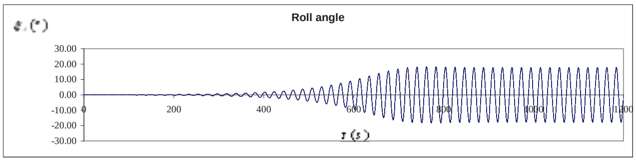


Figure 13 **Development of parametric rolling in regular waves (H_W = 7 m, T_W = 12.95 s, speed 9 kn)**
Slika 13 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 7 m, T_W = 12,95 s, brzina 9 čv)**

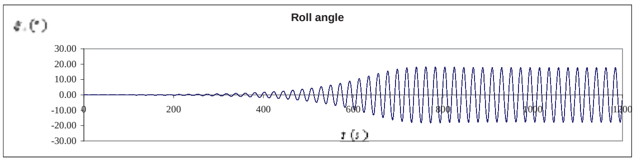
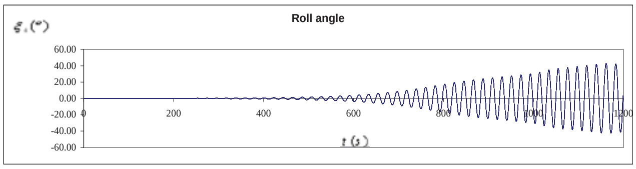


Figure 14 **Development of parametric rolling in regular waves (H_W = 7 m, T_W = 12.95 s, speed 11 kn)**
Slika 14 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 7 m, T_W = 12,95 s, brzina 11 čv)**

Figure 16 **Development of parametric rolling in regular waves (H_W = 8 m, T_W = 12.95 s, speed 9 kn)**
Slika 16 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 8 m, T_W = 12,95 s, brzina 9 čv)**



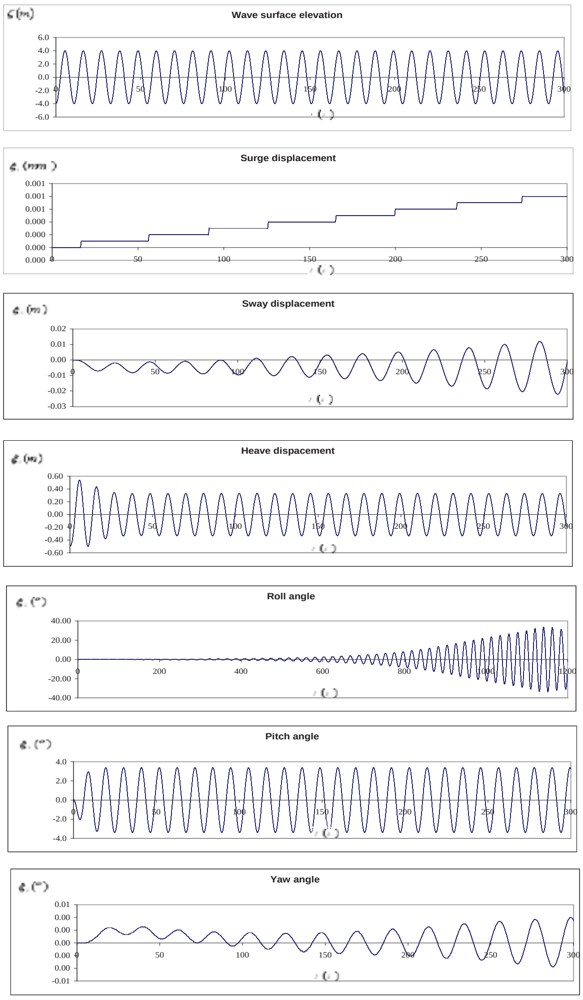


Figure 15 **Development of parametric rolling in regular waves (H_W = 8 m, T_W = 12.95 s, speed 8 kn)**
Slika 15 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 8 m, T_W = 12,95 s, brzina 8 čv)**

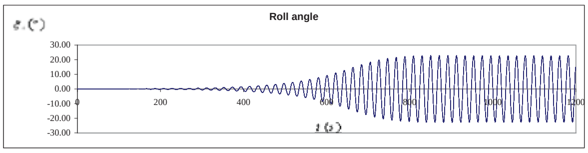
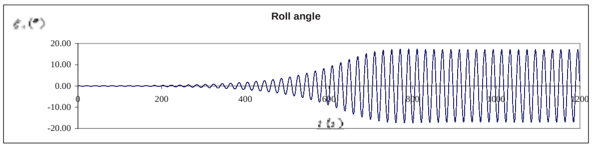


Figure 17 **Development of parametric rolling in regular waves (H_W = 8 m, T_W = 12.95 s, speed 10 kn)**
Slika 17 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 8 m, T_W = 12,95 s, brzina 10 čv)**

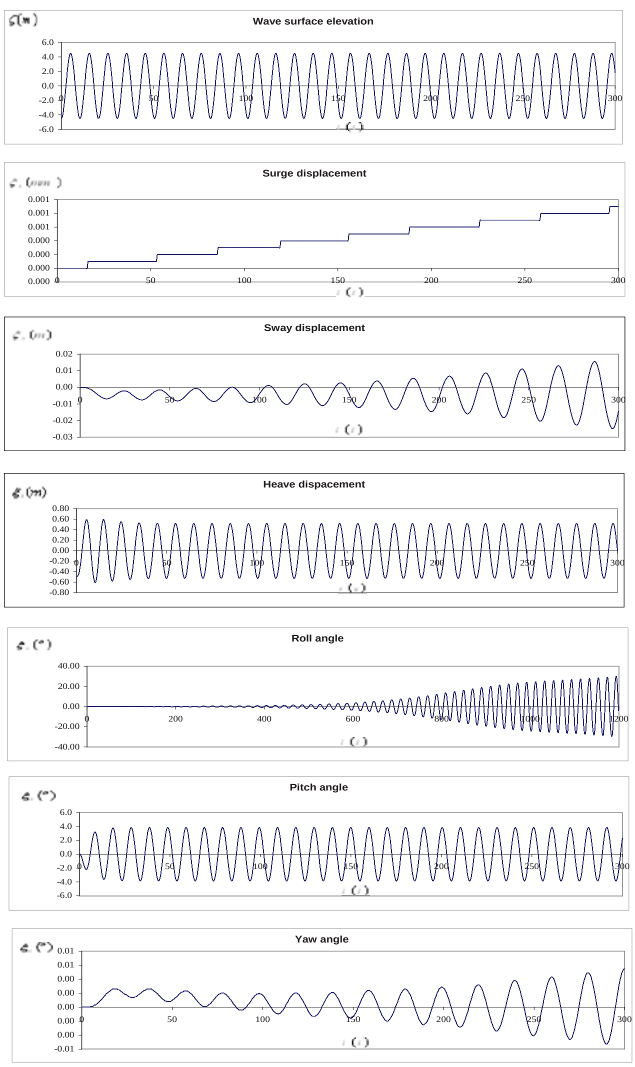
Figure 18 **Development of parametric rolling in regular waves (H_W = 8 m, T_W = 12.95 s, speed 11 kn)**
Slika 18 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 8 m, T_W = 12,95 s, brzina 11 čv)**



Similarly, the results presented in Figures 15-18 are obtained by varying ship speed from 8 to 11 knots and retaining the other parameters (wave amplitude 4 m, period 12.95 s) the significant roll amplification is decreasing (η_4 from 42° to 13°) up to the point where at 13 knots it becomes insignificant. This is relevant for establishing a pattern upon which the range of parametric conditions for such a resonant effect can be sustained and monitored.

Finally, the third set of conditions that have shown sustained parametric rolling is the one for wave amplitude of 4.5 m and period of 12.95 s as shown in Figures 19 and 20. The significant roll amplification is varying from η_4 = 29° at 11 knots, and subsequently decreasing η_4 = 16° (13 knots), up to the point where at 14 knots it becomes negligible.

Figure 19 **Development of parametric rolling in regular waves (H_W = 9 m, T_W = 12.95 s, speed 11 kn)**
Slika 19 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 9 m, T_W = 12,95 s, brzina 11 čv)**



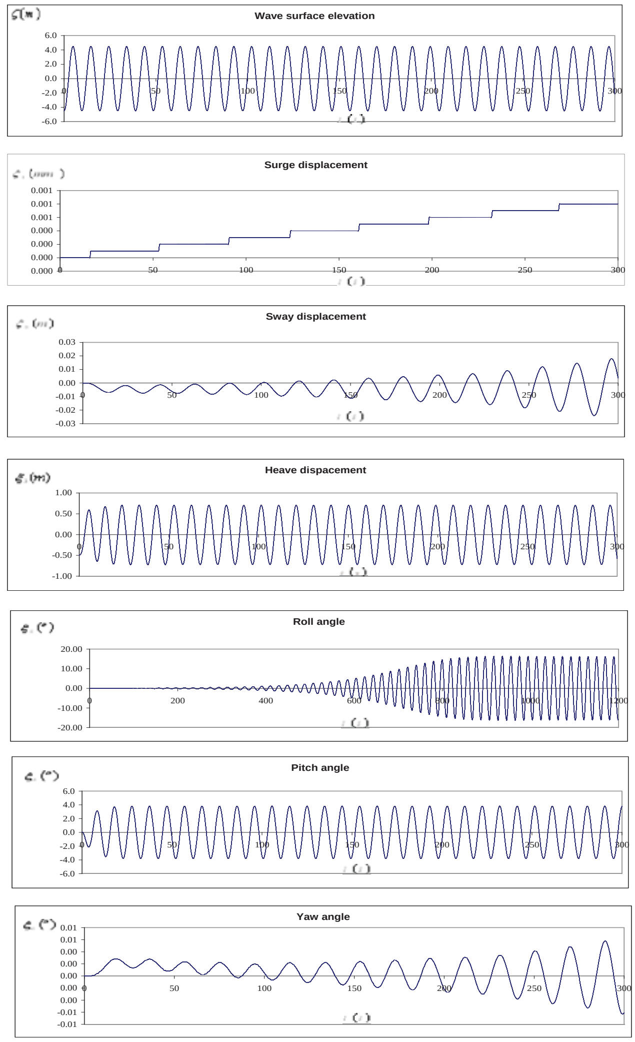


Figure 20 **Development of parametric rolling in regular waves (H_W = 9 m, T_W = 12.95 s, speed 13 kn)**
Slika 20 **Razvoj parametrijskog ljuljanja na pravilnim valovima (H_W = 9 m, T_W = 12,95 s, brzina 13 čv)**

This is in compliance with the physical behaviour of the parametric rolling phenomena. Due to the energy balance between damping and change of stability as the head sea waves travel along the hull, it is highly unlikely that such an effect can be sustained for a prolonged range of ahead speeds.

The results can also be interpreted as such that given the right combination of wave height, wave period, natural rolling period and speed, three sets of parametric rolling can be identified in head-sea. It is important to outline the parametric conditions upon which they can take place and even more important are groupiness effects which represent whether those parameters are in relative vicinity. In other words, no significant change in wave height, wave period, natural rolling period and speed between those set of conditions can be accountable for sustainable or prolonged parametric rolling given possible unfavourable ship handling tactics.

By analyzing the numerical results obtained, there are two aspects that have to be considered as the most important ones.

The first one is the parametric condition that precedes all the other ones upon which the parametric rolling effect develops (H_W = 6 m, T_W = 12.95 s, speed 7 knots). The second one is the parametric set-up where for a particular speed the highest amplifications of roll motion are observed and preferably for an extended range of wave heights (for instance, H_W = 7-8 m, T_W = 12.95 s, speed 7-9 knots).

This represents crucial information. On the one hand, it coincides with the effort to get a speed of the ship which will most likely cause parametric rolling for a given condition. That speed is in a way a "design speed" for parametric rolling and thus can be taken as a starting point for the preparation of model tests that will be taking place. It is imperative that prior to the experiment itself, such conditions are predicted in order to calibrate test instruments and devices, wave generation etc. for measuring the movements of the ship.

On the other hand, as mentioned above, simulations are needed in conjunction with the polar diagrams as a written version of the operational guidelines for the Master to mitigate or avoid parametric rolling. Polar diagrams can be produced for different sea states and loading conditions. Aside from severe rolling in head seas, other significant results of the investigations were decreased roll response with greater vessel speed. Taking into account the results obtained, translated into ship handling tactics, the vessel should have remained in beam seas during the storm or, if headed into the seas, should have increased speed to reduce rolling. All of these manoeuvres are contrary to normal heavy weather ship handling practice and counter-intuitive for masters.

It has to be noted that this simulation corresponds well with information reported onboard this vessel in Oct. 1998. Furthermore, the conditions upon the parametric rolling took place and the amplitudes of the roll itself are in agreement with the experimental results published in [5]. As a part of a HydraLab III project there is an ongoing experimental investigation taking place in CEHIPAR (*Canal de Experiencias Hidrodinámicas del Pardo*), Spain where the above presented numerical results can be evaluated and verified. The results have showed that the occurrence of parametric rolling depends on the wave period, the natural roll period, ship's forward speed and a threshold wave height and thus ship operators should be aware of this risk and take precaution measures to reduce it. Further work will involve comparisons between numerical and experimental results that will be performed in order to demonstrate the validity of the proposed technique for parametric roll prediction in regular as well as in irregular waves.

4 Conclusion

The theoretical background of the parametric rolling phenomena is given with emphasis on prediction of occurrence of parametric rolling (susceptibility criteria) as well as the prediction of the amplitude of parametric rolling (severity criteria). A review of one simple current technique for calculating a particular vessel's vulnerability to parametric roll is demonstrated. The aim of this paper is to describe a model for the prediction of parametric roll which is sufficiently fast to identify the effect, based on operational guidelines from classification societies (*ABS*), to avoid or to reduce the effects of that phenomenon. Classification guides regarding parametric rolling phenomena are also outlined.

The calculations and checks are done for a series of container vessels for design wave and forward speed, which will most likely lead to the development of parametric roll. The majority of the results have not indicated the likelihood to parametric roll and thus have not been displayed. However, the chosen cases have shown that even a relatively small containership, usually not known to exhibit parametric rolling behaviour, turned out to be vulnerable to parametric roll resonance as the susceptibility criteria are not satisfactory. On the other hand, for a very large Post-Panamax ship which was proven to be susceptible to parametric rolling, the procedure resulted in a negative outcome. Since the standardized procedures such as this one should be taken only as a starting point in the evaluation of parametric roll probability, the full numerical procedure presented for a Post-Panamax, C11 class containership should be conducted to ensure that the effects of parametric rolling are avoided. Special attention has to be given to the usefulness of the rolling model experiments to validate numerical prediction of roll damping.

References

[1] PAULLING, J. R., ROSENBERG, R. M.: "On Unstable Ship Motion Resulting from Non-linear Coupling", Journal of Ship Research, Vol. 3, No.1. 1959.

[2] HUA, J.: "A study of the parametrically excited roll motion of a RO-RO ship in following and heading waves", International Shipbuilding Progress, 39 (420), p.345–366, 1992.

[3] NEVES, M. A. S., PEREZ, N. A., VALERIO, L.: "Stability of small fishing vessels in longitudinal waves", Ocean Engineering 26(12), p.1389-1419, 1999.

[4] RIBEIRO e SILVA, S., GUEDES SOARES, C.: "Time Domain Simulation of Parametrically Excited Roll in Head Seas", Proceedings of the 7th International Conference on Stability of Ships and Ocean Vehicles (STAB'2000); Renilson, M. (ed) Launceston, Tasmania, Australia, p. 652-664. 2000.

[5] FRANCE, W.N., LEVADOU, M., TREAKLE, T.W., PAULLING, J.R., MICHEL, R.K., MOORE, C.: "An investigation of head-sea parametric rolling and its influence on container lashing systems", Marine Technology 40:1, p. 1-19, 2003.

[6] BELENKY, V. L., WEEMS, K. M., LIN, W. M., PAULLING, J. R.: "Probabilistic Analysis of Roll Parametric Resonance in Head Seas", Proc. of 8th Int. Conf. on Stability of Ships and Ocean Vehicles STAB, p.325-340. 2003.

[7] SHIN, Y., BELENKY, V.L., PAULLING, J.R., WEEMS, K.M., LIN, W.M.: "Criteria for parametric roll of large containerships in head seas", SNAME Annual Meeting, 2004.

[8] UMEDA, N., HASHIMOTO, H., VASSALOS, D., URANO, S., OKOU, K.: "Nonlinear Dynamics on Parametric Roll Resonance with Realistic Numerical Modelling", International Shipbuilding Progress 51:2/3, p. 205-220, 2004.

[9] SURENDRANA, S., LEEB, S.K., SOHN K.H.: "Simplified model for predicting the onset of parametric rolling", Ocean Engineering 34, p. 630–637, 2007.

[10] …: "ABS Guide for the Assessment of Parametric Roll Resonance in the Design of Container Carriers", American Bureau of Shipping, 2004.

[11] KREUZER, E. J.; SICHERMANN, W.M.; Investigation of Large Amplitude Roll Motions and Capsizing. Proc. 9th Int. Symp. on Practical Design of Ships and other Floating Structures PRADS, 2, Pages 689-696. 2004.
[12] KRÜGER, S., HINRICHS, R., CRAMER, H.: "Performance Based Approaches for the Evaluation of Intact Stability Problems, Proc. of 9th Int. Symp. on Practical Design of Ships and other Floating Structures PRADS, 2, p. 697-703, 2004.
[13] …: "ITTC – Recommended Procedures and Guidelines; Predicting the Occurrence and Magnitude of Parametric Rolling", 2006.
[14] NEVES, M., PÉREZ, N., LORCA, O., RODRIGUEZ, C.: "Hull design considerations for improved stability of fishing vessels in waves, Proc. of STAB'03, 8th International Conference on Stability of Ships and Ocean Vehicles, Spain, 2003.
[15] RIBEIRO e SILVA, S., SANTOS, T.A., GUEDES SOARES, C.: "Parametrically excited roll in regular and irregular head seas", Int. Shipbuild. Progr., 52, p. 29-56. 2005.
[16] RIBEIRO e SILVA, S., GUEDES SOARES, C: "Non-Linear Time Domain Simulation of Dynamic Instabilities in Longitudinal Waves", Proceedings of the 27th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), Estoril, Portugal, Paper OMAE2008-57973, 2008.
[17] SALVESEN, N., TUCK, E. O., FALTINSEN, O.: "Ship Motions and Sea Loads", Transactions of SNAME, Vol. 78, p. 250-287, 1970.
[18] …: Maritime Reporter Magazine, April 2008, p. 55.
[19] …: "Head-sea parametric rolling of container ships", Marine Services Information Sheet, Lloyd's Register, 2003.
[20] …: Bureau Veritas Technical Note: "Parametric roll - Bureau Veritas approach", 2005.
[21] …: IMO Document SLF48/4/12, "On the development of performance-based criteria for ship stability in longitudinal waves", Submitted by Italy, July, 2005.
[22] TAYLAN, M.: "On the parametric resonance of container ships, Ocean Engineering, Volume 34, Issue 7, p. 1021-1027, 2007.
[23] MILLER, E. R.: "Roll Damping, Technical Report 6136-74-280, NAVSPEC, 1974.
[24] IKEDA, Y., HIMENO, Y., TANAKA, N.: "A prediction method for ship roll damping", Report of Department of Naval Architecture, Osaka, No. 405. 1978.