| MODEL | $L/T$ | $B/T$ | $B_{SK}/B_L$ |
|---|---|---|---|
| SR 98 | 16.8 | 3.06 | 0.95 |
| SR 159 | 16.5 | 2.75 | 0.98 |
| SR 108 | 18.4 | 2.67 | 1.00 |
| Series 60, $C_B$ = 0.6 | 18.8 | 2.47 | 1.02 |
| Series 60, $C_B$ = 0.7 | 17.5 | 2.50 | 1.02 |
| Series 60, $C_B$ = 0.8 | 16.2 | 2.50 | 1.01 |
| SR 161 | 15.4 | 2.45 | 1.02 |

The above table clearly demonstrates that for single-screw merchant ships of normal hull form, the approach of the present paper gives roughly the same estimate of roll damping due to hull dynamic lift as the Japanese method. Including only the skeg in making this estimate is seen to be a valid approximation.

It is worthwhile discussing briefly the rationale underlying adoption of this approximation. Although the total sideforce acting on the hull at an angle of attack to the flow may be estimated with some pretense of accuracy, the distribution of this lift around the hull surface is largely a matter of conjecture, particularly for ships such as destroyers and frigates with extremely rounded bilges. The lift distribution due to rolling velocity involves an additional level of conjecture. Given the guesswork involved, the starting point chosen was to include skeg damping and ignore the rest of the hull. When this method proved to give satisfactory estimates of roll damping in practice, no further work was done on the problem. Admittedly, this is a procedure chosen on the basis of expediency, not rigor, and there is scope for further research.

Mr. Cox's point with reference to the use of the Schoenherr line to estimate $C_{DF}$ is well taken. The empirical formulation of Tamiya and Komura (footnote 11) is probably a better approximation. As regards the query on ship-fixed axes, such an axis system is described in some detail in Section 3 of reference [9].

I am grateful to Prof. Himeno for informing us of the experimental results obtained at the University of Osaka on forward speed effects on wavemaking, eddy-making and bilge keel roll damping. When these results have been sufficiently generalized, they should be incorporated into ship motion computer programs. In the present paper, only bilge keel damping is dependent on forward speed, through equation (59), a linear term which increases with forward speed in accordance with Prof. Himeno's observation. As regards nonlinear damping, one would intuitively expect this to diminish with increasing speed, as lifting effects become more important.

Before we leave the subject of roll damping, I would like to make one final point. Good estimates of roll damping may be obtained in practice by thorough application of fundamental and well-established hydrodynamic principles, primarily to the appendages and secondarily to the hull. No further fancy theoretical developments are required.

With regard to Mr. McCallum's comment on the direction of maximum rolling, the statement he refers to should read "on or near the beam." This would then include sea directions from roughly 70 to 110°. Clearly, the direction of maximum rolling depends on roll natural frequency, wave spectral content and ship speed, so making general statements is a risky business.

With regard to rolling moment of inertia, this has been estimated from the roll natural frequency. When the latter is unknown, it is recommended to use a value of total roll radius of gyration (structural plus hydrodynamic) between 35 and 40 percent of beam.

Mr. McCallum's observation regarding Figs. 7 and 8 is correct. This is rectified in the final text for TRANSACTIONS. As regards use of the Canadian weathership data for further correlation studies, this would be an interesting and useful project, but at present no effort is available at DREA for this purpose.

Dr. Salvesen's comment on sway and yaw prediction is very appropriate. Strip theory is known to have two major deficiencies with regard to sway and yaw. One is the phenomenon cited by Dr. Salvesen, due to nonlinear wave excitation. The other is unrealistically high prediction of sway and yaw displacement near zero wave encounter frequency. However, if velocities and accelerations are of interest rather than accelerations, strip theory may be adequate.

Dr. Eda introduces an interesting problem of practical importance for certain ship types. The implication is that for these ships $C_{24}\eta_4$ and $C_{64}\eta_4$ terms, especially the latter, should be added to the equations of motion, as well as rudder terms and a rudder system equation. It is worth noting that the latter are included in the DREA computer program referred to earlier.

I wish to thank Messrs. Mathisen and Lindemann for contributing several points of information on roll response of barges. Although barges are outside my range of experience, I certainly concur that knowledge is deficient regarding barge roll damping. Work on this subject should address wavemaking damping as well as viscous effects.

Finally, Mr. Ankudinov asks why the long-wave approximations to the forcing functions were introduced. The only reason for doing this was to facilitate the analysis of lateral motions in beam seas of long wavelength and thereby demonstrate theoretically that for this situation roll may be described by an uncoupled equation. In performing numerical calculations, these approximations are not used.

In closing, I wish to thank all the discussers for their very kind and illuminating submissions.



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# Wave Statistics for the Design of Ships and Ocean Structures

**Michel K. Ochi,**[1] Member

This paper presents wave information which plays a significant role in predicting responses of ships and ocean structures in a seaway, and discusses methods of application specifically for design consideration. A series of wave spectra to be used for estimating design values for the short term is developed, as well as a series for the long-term (lifetime) approach. Several factors which may seriously affect the magnitude of predicted values (including extreme values) are discussed in detail, and results of numerical computations carried out on a semisubmersible-type ocean platform are presented. In the short-term response prediction approach, it is found from the results of the computations that the upper and lower bounds of responses established by using the series of wave spectra cover satisfactorily the variation of responses computed by using wave spectra measured at various oceanographic locations in the world. It is also found that the design extreme value estimated from the long-term prediction method agrees with that estimated from the short-term prediction method.

## Introduction

THE PREDICTION of responses of ships and ocean structures in a seaway has been practiced in design for many years since the application of the linear superposition principle in stochastic processes was first introduced in the profession [1].[2] Although significant progress has been made since the original pioneering work, there still remains an area for which serious consideration has to be given in practical application of the prediction technique for design.

The area the present study addresses is that concerned with wave information such as spectra and frequency of occurrence of seas of different severities, which play significant roles in predicting responses of ocean systems. In particular, for estimating the design value, wave information for both the short term and the long term (lifetime) is required.

Although considerable attention has been given to wave spectra to be used for design consideration in the short-term prediction, the question always remains as to how realistic the predicted responses are if we use the commonly available simple spectral formulations such as Pierson-Moskowitz, Bretschneider's two parameter, the International Towing Tank Conference (ITTC), or the International Ship Structures Congress (ISSC), which have been developed for some idealized conditions. In reality, the shape of wave spectra observed in the ocean varies considerably (even though the significant wave heights are same) depending on the geographical location, duration and fetch of wind, stage of growth and decay of a storm, and existence of swell. Since a ship (or ocean structure) encounters an infinite variety of wave conditions, and since the magnitude of the responses is significantly influenced by the shape of the wave spectrum, there is some reservation on the reliability of the predicted responses unless the variability of wave conditions is reflected in the prediction technique.

There is also a question as to the assurance with which we may make predictions for ship or ocean structure operation throughout the world, when the predictions are made using spectral formulations which are developed from data obtained primarily in the North Atlantic Ocean. It is therefore highly desirable to develop a technique by which the predicted design value will reasonably cover the variation of responses expected in various oceanographic locations in the world.

In regard to the estimation of the lifetime (long-term) responses of ships and ocean structures for design, considerable improvement is required over the currently available prediction method. First, we currently have little information on how wave statistics should be used for prediction. It has been a commonly accepted practice to use wave statistics which provide information on wave height and period based on data accumulated over many years, examples of which are those given by Hogben-Lumb [2], Roll [3], Walden [4], Yamanouchi [5], etc. Although these wave statistics are extremely useful for prediction, severe seas data, which are indeed necessary for design, are unreliable since they are, without exception, sparse. One way to solve this problem is to represent data given in wave statistics by a certain probability law which governs the data, and then obtain necessary information for design from the probability function.

Furthermore, in the long-term response prediction, several factors which may seriously affect the magnitude of predicted values have to be taken into consideration. These include the frequency of occurrence of various wave spectra, various sea severities, and the number of response cycles associated with the encounter with waves.

It is the purpose of this paper to provide solutions to the problems cited in the foregoing so that a more rational and accurate evaluation of responses can be made for the design of ships and ocean structures.

The paper consists of two parts. Part 1 presents wave statistics for short-term prediction. In order to cover a variety of spectral shapes which a ship (or ocean platform) may encounter in her lifetime, two sets of wave spectra are presented, and the method for predicting extreme values of responses for design consideration is discussed. One of the sets is a six-parameter

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[2] Numbers in brackets designate References at end of paper.

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wave spectral family consisting of 11 members for an arbitrarily specified sea severity; the other is a two-parameter family consisting of nine members. As an example of a practical application, computations are carried out on a semisubmersible-type ocean platform to evaluate extreme values of the transverse force in the bridging structure associated with the wave-induced forces acting on the two hulls. Extreme values computed using these two families of wave spectra are compared with those obtained by using available wave spectra measured in various oceanographic locations throughout the world.

Part 2 presents wave statistics to be used for the long-term prediction. A method to estimate the frequency of occurrence of seas of various severities from available data on wave statistics is discussed, including a method to estimate the severest sea condition. Also, the probability function which represents the combined statistical characteristics of wave height and period is developed from analysis of measured as well as visually observed data. From this probability function, it is possible to obtain information on wave statistics in severe seas where data are always sparse. Then, a series of wave spectra for use in the long-term response prediction is presented, and its application to design is discussed. As an example of practical application, computations are made on the lifetime wave-induced loading (transverse force) of an ocean platform, the results of which may be used to assess possible fatigue failure of the platform. Also, the extreme value for the design consideration is evaluated from the long-term prediction method, and the result is compared with that evaluated from the short-term prediction approach.

## Part 1: Wave statistics for short-term prediction

Prediction of responses of ships and ocean structures in a seaway is most commonly carried out in individual seas of a specified severity. Here, sea severity is generally expressed in terms of the significant wave height.

As mentioned in the Introduction, the shape of wave spectra observed in the ocean varies considerably even though the significant wave height is the same. Hence, one way to cover a variety of spectral shapes which a ship (or ocean structure) may encounter in the sea is to develop a systematic series of wave spectra consisting of several members (called a family of wave spectra) for each sea severity.

The concept of a family of wave spectra was considered by several researchers, references [6] and [7] for example. However, the two families of wave spectra presented in this paper are developed from a statistical analysis of available data, and each member of the family is weighted following the frequency of its occurrence. One of these expresses wave spectra with two parameters (significant wave height and period), and a family of spectra for a given sea is generated from a statistical analysis of the wave period in the sea. The other represents wave spectra by multiple parameters, and a family is generated in each sea from a statistical analysis of the parameters, taking into account the correlation between them. A detailed discussion on each family is given in the following.

### Family of two-parameter wave spectra

#### Basic concept

The idea of expressing wind-generated wave spectra in terms of two parameters was first presented by Bretschneider [8]. His original spectral formulation is given as a function of the nondimensional average wave height and period:

$$S(\omega) = 3.437 \frac{F_1^2}{F_2^2} \frac{g^2}{\omega^5} e^{-0.675(g/F_2 U \omega)} \qquad (1)$$

where

$F_1$ = nondimensional wave height = $g\overline{H}/U^2$
$F_2$ = nondimensional wave period = $g\overline{T}/2\pi U$
$\overline{H}$ = average wave height
$\overline{T}$ = average wave period

$$= \frac{\int_0^\infty T \cdot S(T) dT}{\int_0^\infty S(T) dT}$$

$S(T)$ = period spectrum
$U$ = wind speed
$g$ = gravity constant

It is noted that the wind speed, $U$, involved in equation (1) essentially disappears and the spectrum can be expressed by two parameters: average wave height, $\overline{H}$, and average period $\overline{T}$. However, the area under the spectrum given by equation (1) is equal to $4\,\overline{H}^2/\pi$, which is eight times $\overline{H}^2/(2\pi)$ defined for the narrow-band spectrum. Hence, for consistency, the spectrum $S(\omega)$ should be divided by eight. Furthermore, the average wave period, $\overline{T}$, is defined from the period spectrum, $S(T)$, instead of the frequency spectrum, $S(\omega)$, in equation (1). If these modifications are incorporated into equation (1), then the spectrum can be written as

$$S(\omega) = 0.278 \frac{\overline{\omega}^4}{\omega^5} \overline{H}^2 e^{-0.437(\overline{\omega}/\omega)^4} \qquad (2)$$

where

$$\overline{\omega} = \text{average frequency} = \frac{\int_0^\infty \omega S(\omega) d\omega}{\int_0^\infty S(\omega) d\omega}$$

The spectrum can be further modified to be expressed in terms of significant wave height, $H_s$, and modal frequency, $\omega_m$. From the narrow-band spectrum assumption, we have, in general

$$H_s = \text{significant wave height} = \sqrt{\frac{8}{\pi}}\,\overline{H} = 1.60\,\overline{H}$$

$$\omega_m = \text{modal frequency} = \frac{(0.8)^{1/4}}{\Gamma(3/4)}\,\overline{\omega} = 0.77\,\overline{\omega} \qquad (3)$$

Thus, we can derive the following two-parameter spectral formulation

$$S(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} H_s^2 e^{-1.25(\omega_m/\omega)^4} \qquad (4)$$

In applying the two-parameter wave spectrum for design consideration of ships and marine structures, it is necessary to specify the values of modal period, $\omega_m$, for each sea condition. If the modal frequency is specified by using the following formula for each sea severity

$$\omega_m = 0.4\sqrt{g/H_s} \qquad (5)$$

then the two-parameter spectrum becomes the Pierson-Moskowitz spectrum, which is developed specifically for fully developed seas. In reality, however, the magnitudes of the modal frequency and number of occurrences in a given sea are random. Therefore, statistical data on wave height and period are necessary to determine the modal frequency in a given sea. Wave statistics given in references [2–5, 10, 11] are extremely valuable for this purpose; however, data in severe seas are,



**Table 1 Statistical data on wave height and period (data from reference [10])**

ZERO-CROSSING WAVE PERIOD (SEC.); SIGNIFICANT WAVE HEIGHT

| FT | M | 6.0–6.5 | 6.5–7.0 | 7.0–7.5 | 7.5–8.0 | 8.0–8.5 | 8.5–9.0 | 9.0–9.5 | 9.5–10.0 | 10.0–10.5 | 10.5–11.0 | 11.0–11.5 | 11.5–12.0 | 12.0–12.5 | 12.5–13.0 | 13.0–13.5 | 13.5–14.0 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 48 | 14.6 | | | | | | | | | | | | | | | | | |
| 46 | 14.0 | | | | | | | | | | | | | | | | 1 | 1 |
| 44 | 13.4 | | | | | | | | | | | | | | | | | 0 |
| 42 | 12.8 | | | | | | | | | | | | | | | | | 0 |
| 40 | 12.2 | | | | | | | | | | | | | | | 1 | | 1 |
| 38 | 11.6 | | | | | | | | | | | | 1 | 1 | | | | 2 |
| 36 | 11.0 | | | | | | | | | | 1 | | | 1 | | | | 2 |
| 34 | 10.4 | | | | | | | | | | 1 | 1 | 1 | 1 | 1 | | 1 | 6 |
| 32 | 9.8 | | | | | | | | | | | 2 | 1 | 1 | | | | 4 |
| 30 | 9.2 | | | | | | | | | 1 | 2 | | 1 | | | 1 | | 5 |
| 28 | 8.5 | | | | | | | | 1 | 1 | 1 | 2 | 2 | 2 | 1 | | 1 | 11 |
| 26 | 7.9 | | | | | | | | | 1 | | 4 | 1 | 1 | 1 | | | 8 |
| 24 | 7.3 | | | | | | | | 1 | 2 | 4 | 7 | 1 | 1 | | | 1 | 17 |
| 22 | 6.7 | | | | 1 | | | 1 | 3 | 4 | 3 | 3 | 1 | | 2 | | | 18 |
| 20 | 6.1 | | | | | 1 | | 1 | 1 | 5 | 5 | 4 | 1 | 1 | | | | 19 |
| 18 | 5.5 | | | | | 1 | 2 | 6 | 5 | 9 | 3 | 6 | 1 | 2 | 1 | | | 36 |
| 16 | 4.9 | | | 1 | | 2 | 4 | 14 | 7 | 10 | 6 | 4 | 1 | 3 | 1 | 1 | 1 | 55 |
| 14 | 4.3 | | | | 1 | 4 | 4 | 13 | 9 | 10 | 10 | 8 | 4 | 3 | 1 | | 1 | 68 |
| 12 | 3.7 | | | | 2 | 8 | 8 | 19 | 14 | 15 | 9 | 5 | 4 | 2 | 1 | | | 87 |
| 10 | 3.1 | | 2 | 2 | 4 | 13 | 18 | 24 | 22 | 16 | 8 | 5 | 3 | 1 | | | | 118 |
| 8 | 2.4 | 1 | 1 | 2 | 12 | 21 | 26 | 39 | 24 | 12 | 8 | 3 | 1 | 1 | | | | 151 |
| 6 | 1.8 | | 3 | 5 | 15 | 39 | 23 | 42 | 20 | 13 | 5 | 2 | | | | | | 167 |
| 4 | 1.2 | | 4 | 7 | 24 | 39 | 24 | 24 | 12 | 6 | 1 | 1 | | | | | | 142 |
| 2 | 0.6 | 3 | 4 | 3 | 12 | 17 | 16 | 12 | 5 | 1 | | | | | | | | 73 |
| 0 | 0 | | 1 | | | 2 | | 1 | 2 | | 1 | | 1 | 1 | | | | 9 |
| | | 4 | 5 | 20 | 71 | 147 | 125 | 196 | 126 | 106 | 68 | 57 | 25 | 22 | 9 | 3 | 6 | 1000 |

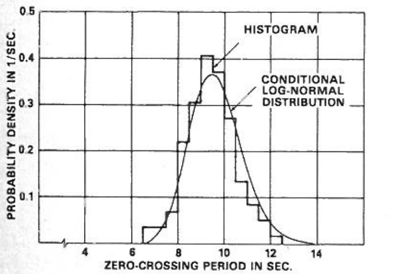


Fig. 1 Comparison of histogram and conditional log-normal distribution of zero-crossing wave period for a significant wave height of 3.35 m (11.0 ft) (data from reference [10])

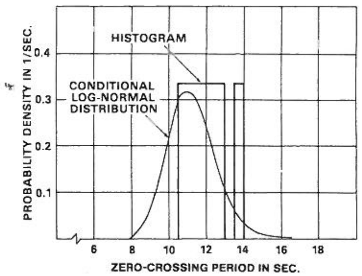


Fig. 2 Comparison of histogram and conditional log-normal distribution of zero-crossing wave period for a significant wave height of 10.7 m (35.0 ft) (data from reference [10])

without exception, sparse and no reliable information can be obtained.

As an example, Table 1, taken from reference [10], shows the tabulation of significant wave height and zero-crossing period, both analyzed from records obtained at Weather Station I (59°N, 19°W) in the North Atlantic. As can be seen in the table, statistical information of the zero-crossing period can be obtained fairly well in relatively mild seas for significant wave height up to 5.5 m (18 ft). However, it is not possible to obtain from the table reliable information on severe seas, for which information is indeed required for the design of ships and marine structures.

In order to amplify this statement, Figs. 1 and 2 have been prepared. Figure 1 shows the histogram of zero-crossing wave periods obtained from Table 1 for a significant wave height of 3.35 m (11.0 ft). The histogram is constructed from a sample of size 118. Included also in the figure is the conditional probability density function derived from the statistical analysis of the data, which is discussed later in this section. Good agreement between the histogram and the probability density function can be seen in the figure. On the other hand, Fig. 2 shows a similar presentation for a sea of significant wave height 10.7 m (35.0 ft). Since the data are sparse (sample size of only 6) in this sea, no reliable information can be drawn from the histogram. On the other hand, the conditional probability density function derived from statistical analysis of the entire data, taking into account the correlation between wave height and period, may provide various types of information needed for design.

As these examples indicate, it is necessary to establish the conditional probability of the modal frequency or modal period for a given significant wave height. For this purpose, statistical

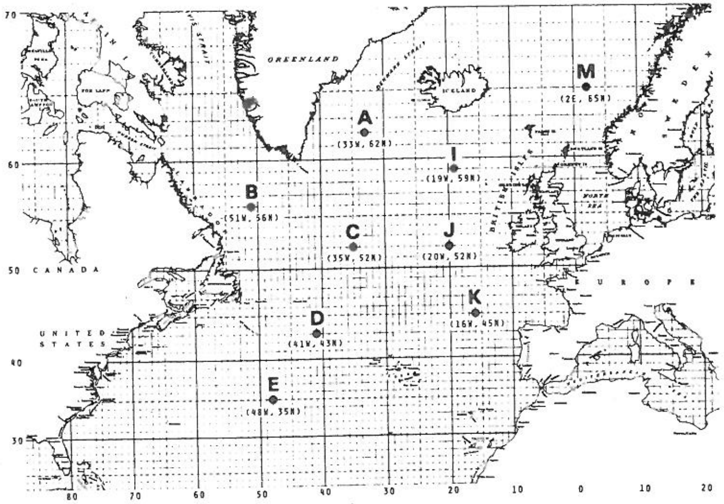


Fig. 3 Locations of nine weather stations in the North Atlantic

analysis is carried out on Draper's results obtained from records taken at Weather Station I [10] and at Weather Station J (52°N, 20°W) [11], as well as on Walden's information [4] obtained at nine weather stations (A, B, C, D, E, I, J, K, and M) in the North Atlantic shown in Fig. 3.

It is noted here that Draper's data on wave height and period are tabulations of measured significant wave height and zero-crossing period. On the other hand, Walden's data are visually observed wave heights and periods. These visually observed wave heights and periods are converted to significant wave heights and modal periods, respectively, by using a conversion factor, given in equations (38) and (39) in the Appendix, which is established from an analysis of measured and observed data taken at Stations I and J.

As discussed in detail in the Appendix of this paper, it is found from the results of the analysis that the statistical properties of both wave height and period can be evaluated based on the log-normal probability distribution. This statistical law on wave height and period appears to be valid in the range of the cumulative distribution up to 0.99 for both measured and visually observed data (see Fig. 34 in the Appendix, for example). Let us write the log-normal probability distribution applicable for wave height as

$$f(H) = \frac{1}{\sigma_H \sqrt{2\pi}\, H} \exp\left\{-\frac{1}{2}\left(\frac{\ln H - \mu_H}{\sigma_H}\right)^2\right\} \qquad (6)$$

where $\mu_H$ and $\sigma_H$ are parameters associated with the log-normal distribution of wave height. For simplicity, the log-normal distribution given in the form of equation (6) is denoted hereafter by $\Lambda(\mu_H, \sigma_H)$.

Since both wave height as well as wave period are log-normally distributed, the combined statistical properties of wave height and period follow the bivariate log-normal probability law. Hence, it can be proved that the statistical properties of the wave period for a given wave height obey the conditional log-normal probability law. These are summarized as follows:

Distribution of wave height:

$$f(H) \sim \Lambda(\mu_H, \sigma_H) \qquad (7)$$

Distribution of wave period:

$$f(T) \sim \Lambda(\mu_T, \sigma_T) \qquad (8)$$

Joint distribution of wave height and period:

$$f(H,T) \sim \Lambda(\mu_H, \mu_T, \sigma_H, \sigma_T, \rho) \qquad (9)$$

Conditional distributions of wave period for a given wave height:

$$f(T|H) \sim \Lambda\left(\mu_T + \rho \frac{\sigma_T}{\sigma_H}(\ln H - \mu_H),\ \ \sqrt{1-\rho^2}\,\sigma_T\right) \qquad (10)$$

where

$\mu_T, \sigma_T$ = parameter associated with log-normal distribution of wave period

$\rho$ = correlation coefficient between wave height and period

Thus, in summary, the conditional log-normal probability distribution yields the modal period necessary to generate the family of wave spectra in a desired sea state. Following the expression given in equation (10), the conditional distribution of modal period, $T_m$, for a specified significant wave height, $H_s$, can be written as follows:

$$f(T_m|H_s) \sim \Lambda\left(\mu_{T_m} + \rho \frac{\sigma_{Tm}}{\sigma_{H_s}}(\ln H_s - \mu_{H_s}),\ \ \sqrt{1-\rho^2}\,\sigma_{Tm}\right) \qquad (11)$$

### Derivation of family of spectra

As mentioned in the previous subsection, the joint probability of significant wave height and modal period follows the bivariate log-normal distribution given in equation (9), which carries five parameters. The values of these parameters de-



Table 2 Parameters associated with bivariate log-normal probability distribution for various locations in the North Atlantic

| Weather Station | | A | B | C | D | E | I | J | K | M |
|---|---|---|---|---|---|---|---|---|---|---|
| Significant Height | $\mu_H$ | 0.946 | 0.910 | 1.024 | 0.968 | 0.671 | 1.112 | 1.053 | 0.748 | 0.605 |
| | $\sigma_H$ | 0.619 | 0.588 | 0.571 | 0.578 | 0.577 | 0.562 | 0.565 | 0.680 | 0.571 |
| Modal Period | $\mu_T$ | 2.505 | 2.462 | 2.494 | 2.483 | 2.415 | 2.588 | 2.594 | 2.600 | 2.516 |
| | $\sigma_T$ | 0.218 | 0.218 | 0.216 | 0.209 | 0.228 | 0.142 | 0.147 | 0.174 | 0.202 |
| Correlation Coefficient | $\rho$ | 0.498 | 0.594 | 0.578 | 0.586 | 0.508 | 0.358 | 0.339 | 0.331 | 0.686 |

rived from analysis carried out at various weather stations in the North Atlantic are listed in Table 2. As can be seen in the table, the values of each parameter are rather consistent throughout all nine stations except for the following two points:

1. The parameter associated with significant wave height, $\mu_H$, for Stations E and M are much smaller in comparison with $\mu_H$ for other stations.

2. The correlation coefficient, $\rho$, for Stations I, J, and K are small in comparison with $\rho$ for other stations.

Since the magnitude of the $\rho$-values are not large for all stations, the difference in $\rho$-values does not cause significant differences in the various statistical values of wave height and period. However, the difference in $\mu_H$-values appears to result in an appreciable difference in statistical values of wave height. In illustration of this statement, Fig. 4 shows the confidence zone of significant wave height and modal period in which the probability of occurrence is 0.95. The derivation as well as detailed discussion of this confidence zone is given in Part 2 of the paper; however, it is sufficient to state here that the confidence zones for Stations E and M are substantially different from those for other stations, and that the cause of this difference may be attributed to small $\mu_H$-values of these stations.

In fact, the results of a statistical analysis have indicated that the sea severities at Stations A, B, C, D, I, J, and K are not significantly different, though there are some minor differences. However, the severities at Stations E and M are substantially low in comparison with other stations. For this reason, the results of the statistical analysis obtained from data at Stations A, B, C, D, I, J, and K are averaged and called the "mean North Atlantic" values in the following presentations.

In order to generate a family of wave spectra through the probability density function of the modal frequency given in equation (11), we may consider (i) the modal frequency which is most likely to occur (most probable value) and (ii) the upper and lower values of modal frequency for a specified confidence coefficient. These are obtained from the conditional distribution function as follows:

Most probable modal period, $T_m(m)$:

$$T_m(m) = \exp\left\{\mu_{T_m} + \rho \frac{\sigma_{Tm}}{\sigma_{H_s}}(\ln H_s - \mu_{H_s}) - (1-\rho^2)(\sigma_{T_m})^2\right\} \qquad (12)$$

Upper and lower values of the modal period, $T_m(\gamma)$ for a given confidence coefficient $\gamma$:

$$T_m(\gamma) = \exp\left\{\mu_{T_m} + \rho \frac{\sigma_{Tm}}{\sigma_{Hs}}(\ln H_s - \mu_{H_s}) \pm C\sqrt{1-\rho^2}\,\sigma_{T_m}\right\} \qquad (13)$$

where

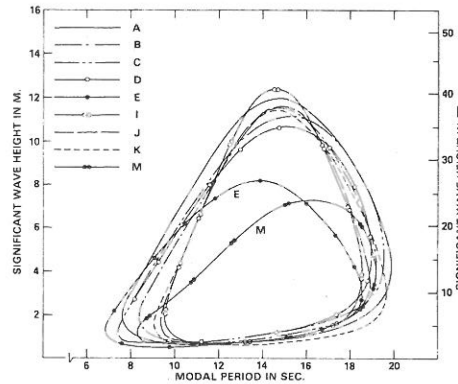


Fig. 4 Comparison of domains for confidence coefficient 0.95

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{C} e^{-x^2/2}\, dx = \gamma$$

Specifically

c = 1.96 for $\gamma$ = 0.95
1.44 0.85
1.15 0.75
0.67 0.50

Thus, by choosing confidence coefficients of 0.95, 0.85, 0.75, and 0.50, a total of nine modal periods (including the most probable value) is determined for a specified significant wave height, $H_s$.

As an example, Fig. 5 shows the conditional probability density function and the modal periods determined from equations (12) and (13). The figure pertains to a significant wave height of 4.6 m (15.1 ft) at Station I. The figure also indicates the weighting factor for each modal period. As can be seen in the figure, the weighting factors are as follows:

| | |
|---|---|
| Most probable modal period | 0.2500 |
| Modal period | |
| for confidence coefficient 0.50 | (each) 0.1875 |
| for confidence coefficient 0.75 | (each) 0.0875 |
| for confidence coefficient 0.85 | (each) 0.0500 |
| for confidence coefficient 0.95 | (each) 0.0500 |

These weighting factors are applicable irrespective of sea severity, and they play a significant role in evaluating the long-term response of ships and ocean structures in a seaway.

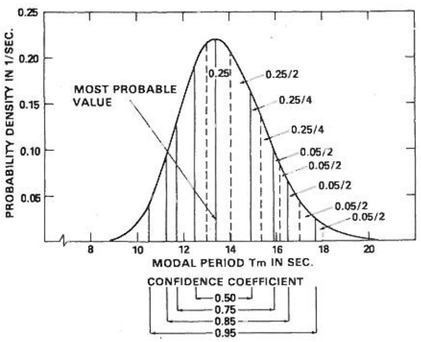


Fig. 5 Probable period for various confidence coefficients

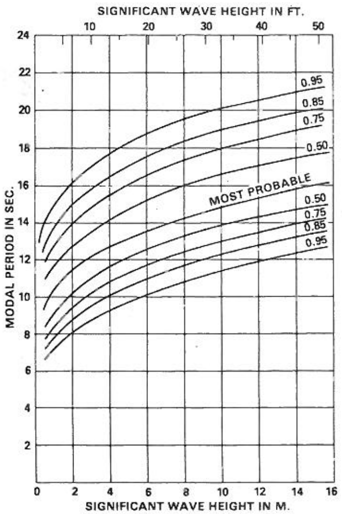


Fig. 6 Modal periods for the (mean) North Atlantic as a function of significant wave height

The modal period most likely to occur and those for various confidence coefficients in seas of various severities are shown in Fig. 6. The figure pertains to the mean North Atlantic. Although the modal frequencies, $\omega_m$, required to establish the family of spectrum can be evaluated from the figures, the frequencies are expressed in terms of significant wave height and are given in Table 3. Thus, a family of wave spectra consisting of nine members for an arbitrarily specified significant wave height can now be generated from equation (4) using the values given in Table 3.

The modal frequencies for each sea state of the family of two-parameter wave spectra thus developed are somewhat different from those presented earlier in reference [12]. The present series is developed using the new calibration factor in converting visually observed data to significant wave height

**Table 3 Modal frequencies for the (mean) North Atlantic wave spectra as a function of significant wave height ($\omega_m$ in rps, $H_s$ in metres)**

| | | $\omega_m$ - value |
|---|---|---|
| Lower $\omega_m$ | 0.95 | $0.048(8.75 - \ell n\ H_s)$ |
| | 0.85 | $0.054(8.44 - \ell n\ H_s)$ |
| | 0.75 | $0.061(8.07 - \ell n\ H_s)$ |
| | 0.50 | $0.069(7.77 - \ell n\ H_s)$ |
| Most probable | | $0.079(7.63 - \ell n\ H_s)$ |
| Upper $\omega_m$ | 0.50 | $0.099(6.87 - \ell n\ H_s)$ |
| | 0.75 | $0.111(6.67 - \ell n\ H_s)$ |
| | 0.85 | $0.119(6.65 - \ell n\ H_s)$ |
| | 0.95 | $0.134(6.41 - \ell n\ H_s)$ |

as discussed in the Appendix. Also, as stated earlier, the data obtained at Stations E and M are not included in the mean North Atlantic data.

Examples of the family of wave spectra for the mean North Atlantic are shown in Figs. 7 and 8. These pertain to families of spectra for significant wave heights of 3.0 m (9.8 ft) and 9.0 m (29.5 ft), respectively.

## Family of six-parameter wave spectra

### Basic concept

In order to cover a variety of shapes of wave spectra associated with the growth and decay of a storm, including the existence of swell, a series of wave spectra which involves six parameters is developed in reference [13]. From the results of a statistical analysis of measured wave spectra, it was found that spectral shapes similar to a typical example shown in Fig. 9 have been most frequently observed. In a broad sense, the shape of the most commonly observed spectra has a peak at the lower frequencies which decreases exponentially to a plateau at the higher frequencies.

Although the wave energy at the higher frequencies is usually much less than that at the lower frequencies, its contribution to responses of marine vehicles and structures may be significant depending on their size and speed.

Thus, it is highly desirable to represent the shape of the entire spectrum as closely as possible, and this may be achieved by separating the spectra into two parts. This concept of decomposition of wave spectra is particularly useful in representing wave spectra when a swell exists in the wind-generated local sea.

Figure 10 shows a pictorial sketch illustrating the decomposition of wave spectra into two parts, one which includes primarily the lower-frequency components of the energy and the second which covers primarily the higher-frequency components of the energy. Then, the spectrum of each part is expressed by a mathematical formula with three parameters, namely, significant wave height, $H_s$, modal frequency, $\omega_m$, and shape parameter, $\lambda$:

$$S(\omega) = \frac{1}{4}\frac{\left(\frac{4\lambda+1}{4}\omega_m^{\,4}\right)^{\lambda}}{\Gamma(\lambda)}\frac{H_s^{\,2}}{\omega^{4\lambda+1}}e^{-(4\lambda+1/4)(\omega_m/\omega)^4} \qquad (14)$$

where $\Gamma(\lambda)$ is a gamma function.



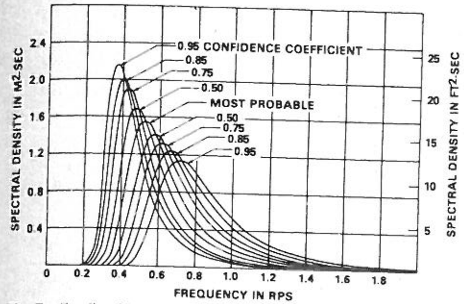


Fig. 7 Family of two-parameter wave spectra for significant wave height 3.0 m (9.8 ft)

The detailed derivation of equation (14) is given in reference [13]. The parameter $\lambda$ controls the shape (sharpness) of the spectrum when the other two parameters are held constant, and the spectral shape becomes sharper with increasing $\lambda$. In particular, by letting $\lambda = 1$, equation (14) reduces to the two-parameter wave spectrum given in equation (4).

By combining two sets of three-parameter spectra, one representing the low-frequency components and the other the high-frequency components of the wave energy, the following six-parameter spectral representation can be derived:

$$S(\omega) = \frac{1}{4}\sum_j \frac{\left(\frac{4\lambda_j+1}{4}\omega_{mj}^{\,4}\right)^{\lambda_j}}{\Gamma(\lambda_j)} \times \frac{H_{sj}^{\,2}}{\omega^{4\lambda_j+1}}e^{-(4\lambda_j+1/4)(\omega_{mj}/\omega)^4} \qquad (15)$$

where $j = 1, 2$ stands for the lower and higher frequency components, respectively.

The six parameters, $H_{s1}$, $H_{s2}$, $\omega_{m1}$, $\omega_{m2}$, $\lambda_1$, and $\lambda_2$, involved in equation (15) are determined numerically such that the difference between theoretical and observed spectra is minimal.

Examples of comparisons between measured spectra and mathematical six-parameter spectra are shown in Figs. 11 and 12. For example, Fig. 11 shows a comparison for the case when swell coexists with wind-generated waves, and the spectrum hence is double peaked. On the other hand, Fig. 12 shows a comparison for a very severe wind-generated sea of significant wave height 14.5 m (47.7 ft) in which the sea is partially developed and has a very sharp peak at the lower frequencies in the spectrum. As can be seen in these examples, the six-parameter spectra derived in equation (15) appear to represent a variety of sea conditions expected to occur in the ocean.

### Derivation of family of spectra

A total of 800 available spectra observed in the North Atlantic [14–16] is classified into ten groups depending on severity, and then for each group a statistical analysis is carried out on the parameters. In the analysis, however, the number of parameters is reduced to five—$\theta$, $\omega_{m1}$, $\omega_{m2}$, $\lambda_1$, and $\lambda_2$—where $\theta$ is the ratio of two significant wave heights, $H_{s1}$ and $H_{s2}$.

The following steps are taken in the statistical analysis of the parameters:

(*a*) Probability density functions are established for each parameter. For example, it is obtained that the parameters $\lambda_1$ and $\lambda_2$ both follow the gamma probability law for all ten groups.

(*b*) Three values are determined from the probability density function for each parameter, namely, the modal value, and the

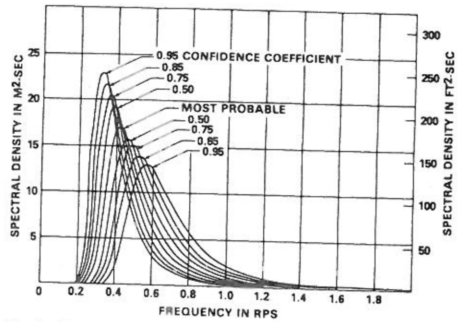


Fig. 8 Family of two-parameter wave spectra for significant wave height 9.0 m (29.5 ft)

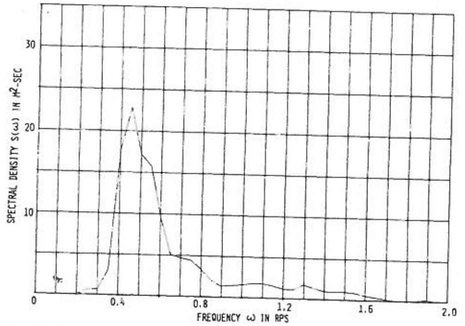


Fig. 9 Example of wave spectrum [significant wave height 10.1 m (33.1 ft)]

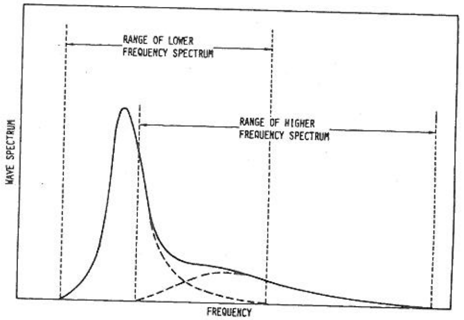


Fig. 10 Decomposition of wave spectra

upper and the lower values are determined from a confidence band for a confidence coefficient of 0.95.

(*c*) For each value of a parameter, values of each of the other parameters are determined from the original data by taking their respective averages in the region of ±5 percent. Thus, a total of 15 spectra is established for a given sea severity.

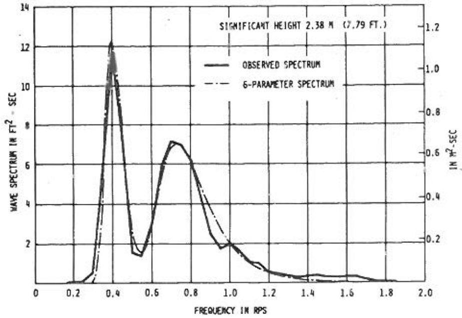


**Fig. 11** Comparison of measured and six-parameter spectrum [significant wave height 2.38 m (7.8 ft)] (from reference [13])

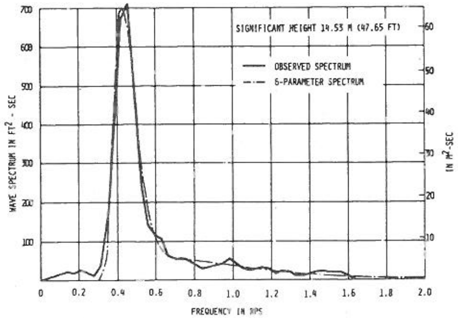


**Fig. 12** Comparison of measured and six-parameter spectrum [significant wave height 14.5 m (47.7 ft)] (from reference [13])

**Table 4 Values of six parameters as a function of significant wave height ($H_s$ in metres)**

| | $H_{s1}$ | $H_{s2}$ | $\omega_{m1}$ | $\omega_{m2}$ | $\lambda_1$ | $\lambda_2$ |
|---|---|---|---|---|---|---|
| Most Probable Spectrum | $0.84\ H_s$ | $0.54\ H_s$ | $0.70\ e^{-0.046\ H_s}$ | $1.15\ e^{-0.039\ H_s}$ | 3.00 | $1.54\ e^{-0.062\ H_s}$ |
| 0.95 Confidence Spectra | $0.95\ H_s$ | $0.31\ H_s$ | $0.70\ e^{-0.046\ H_s}$ | $1.50\ e^{-0.046\ H_s}$ | 1.35 | $2.48\ e^{-0.102\ H_s}$ |
| | $0.65\ H_s$ | $0.76\ H_s$ | $0.61\ e^{-0.039\ H_s}$ | $0.94\ e^{-0.036\ H_s}$ | 4.95 | $2.48\ e^{-0.102\ H_s}$ |
| | $0.84\ H_s$ | $0.54\ H_s$ | $0.93\ e^{-0.056\ H_s}$ | $1.50\ e^{-0.046\ H_s}$ | 3.00 | $2.77\ e^{-0.112\ H_s}$ |
| | $0.84\ H_s$ | $0.54\ H_s$ | $0.41\ e^{-0.016\ H_s}$ | $0.88\ e^{-0.026\ H_s}$ | 2.55 | $1.82\ e^{-0.089\ H_s}$ |
| | $0.90\ H_s$ | $0.44\ H_s$ | $0.81\ e^{-0.052\ H_s}$ | $1.60\ e^{-0.033\ H_s}$ | 1.80 | $2.95\ e^{-0.105\ H_s}$ |
| | $0.77\ H_s$ | $0.64\ H_s$ | $0.54\ e^{-0.039\ H_s}$ | 0.61 | 4.50 | $1.95\ e^{-0.082\ H_s}$ |
| | $0.73\ H_s$ | $0.68\ H_s$ | $0.70\ e^{-0.046\ H_s}$ | $0.99\ e^{-0.039\ H_s}$ | 6.40 | $1.78\ e^{-0.069\ H_s}$ |
| | $0.92\ H_s$ | $0.39\ H_s$ | $0.70\ e^{-0.046\ H_s}$ | $1.37\ e^{-0.039\ H_s}$ | 0.70 | $1.78\ e^{-0.069\ H_s}$ |
| | $0.84\ H_s$ | $0.54\ H_s$ | $0.74\ e^{-0.052\ H_s}$ | $1.30\ e^{-0.039\ H_s}$ | 2.65 | $3.90\ e^{-0.085\ H_s}$ |
| | $0.84\ H_s$ | $0.54\ H_s$ | $0.62\ e^{-0.039\ H_s}$ | $1.03\ e^{-0.030\ H_s}$ | 2.60 | $0.53\ e^{-0.069\ H_s}$ |

(*d*) Of these 15 spectra, five are associated with the modal value of the five parameters. It was found, however, that the shapes of these five spectra are nearly the same; therefore, the spectrum associated with the modal value of the parameter $\theta$ is chosen as representative, and this spectrum is called the most probable spectrum for a given sea severity. Thus, a total of 11 spectra is derived as a family of wave spectra for a specified sea severity.

The values of the six parameters for the family consisting of 11 members are expressed as functions of significant wave height, $H_s$, and are given in Table 4, so that a family of spectra for the desired sea can be generated from equation (15).

The weighting factor for each member of the family is as follows:

| | | |
|---|---|---|
| Most probable spectrum | | 0.50 |
| All other spectra | (each) | 0.05 |

The weight given to the most probable wave spectrum is much higher than that for the other spectra. This is because the most probable spectrum also represents four other spectra associated with the modal value of the parameters, as stated earlier in this section.

Examples of the family of spectra are shown in Figs. 13 and 14. These are for significant wave heights of 3.0 m (9.8 ft) and 9.0 m (29.5 ft), respectively, and may be compared with the family of two-parameter spectra shown in Figs. 7 and 8.

It can be seen from a comparison of the two families that the members of the six-parameter family have a wider variety of shapes than the members of the two-parameter family. Some members of the six-parameter family have double peaks, and the majority of the spectra have sharper peaks than those of the two-parameter family.

## Application to design

The method for applying the families of wave spectra developed for short-term prediction of responses of ships and marine structures in a seaway is discussed in this section. A significant benefit in applying the short-term response prediction lies in the estimation of extreme values, which are necessary for design consideration.

Needless to say, the extreme value for design has to be determined by taking into account all sea severities, all varieties of wave spectral shapes, speeds (in the case of a ship), headings to waves, etc., expected in the ship's (or marine structure's) lifetime weighted by the frequency of occurrence of each factor. This gives an impression that the extreme value should be evaluated only through the long-term prediction approach. However, as is shown later, the results of computations to evaluate extreme values through both short- and long-term prediction approaches clearly indicate that the short-term prediction in severe seas is much to be desired for estimation of extreme values. The estimation procedure as well as the accuracy of the computation for the short term appears to be much superior to that for the long-term prediction. This subject is discussed in detail in connection with the application of wave statistics for the long-term prediction to design.



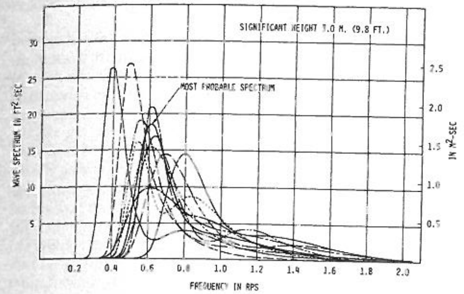


**Fig. 13** Family of six-parameter wave spectra for significant wave height 3.0 m (9.8 ft) (from reference [13])

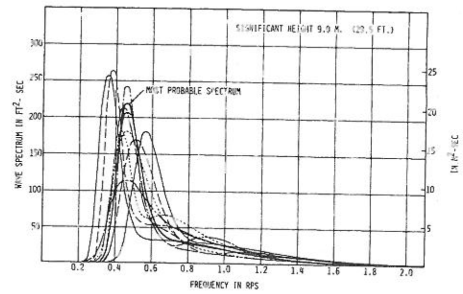


**Fig. 14** Family of six-parameter wave spectra for significant wave height 9.0 m (29.5 ft) (from reference [13])

In using either the two-parameter or the six-parameter family of wave spectra for the short-term response prediction for each sea severity, one of the family members yields the largest response, while another yields the smallest response with confidence coefficient of 0.95. Hence, by connecting the points obtained in each sea severity, the upper and lower bound responses can be established.

For estimating the extreme values of the responses, various factors such as operation (or exposure) time, risk parameter, frequency of encounter with seas, and speed (in case of a ship), all of which affect the magnitude of the extreme values, have to be considered for each sea severity. A more detailed discussion on each of these subjects is given in the following:

(*a*) *Operation* (*or exposure*) *time*. The extreme values of responses are a function of the number of encounters with waves, and hence the persistence of each sea state has to be considered in the estimation. This implies that for a ship the estimation of extreme responses is made assuming that the ship maintains a certain speed in a given sea as long as the sea condition is unchanged.

Figure 15 taken from reference [17] shows the longest persistence of every 1.52-m (5 ft) interval of significant wave height estimated from analysis of data given in reference [14]. For example, significant wave heights between 6 m (19.7 ft) and 7.5 m (24.6 ft) can be expected to persist for a maximum of 40 hours. For seas of significant wave height of 4 m (13.1 ft) or less, a 45-hour (h) duration is used by extending the curve in the figure. It is noted that the extreme values increase significantly during the first several hours and thereafter increase very slowly with time. Hence, the extension of the curve given in the figure would not cause any significant error in predicting the extreme values for mild sea states.

(*b*) *Risk parameter*. Estimation of extreme response such as wave-induced bending moment, etc., for design consideration is made using the concept of a risk parameter [18]. The

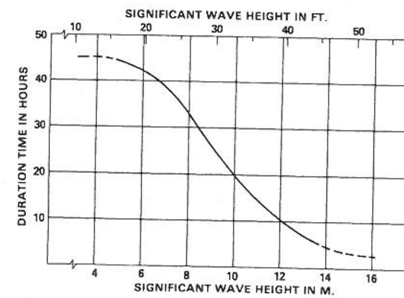


**Fig. 15** Significant wave height and its persistence in the North Atlantic every 1.52-m interval

risk parameter, denoted by $\alpha$, represents the probability that the extreme response in a given sea will exceed the estimated design load. This implies that if $\alpha$ is chosen as 0.01, then the response of one ship (or marine structure) in 100 sister ships operating in the same environment may exceed the estimated extreme value.

It is noted that the ship may encounter seas of the same severity $k$-times in her lifetime. Thus, for each sea state it is necessary to divide the risk parameter $\alpha$ by $k$ for evaluating the design response so that the value which is unlikely to exceed with $(1 - \alpha)$ percent assurance can be estimated for all sea states.

Although the value of risk parameter can be chosen at the designer's discretion, the results of computations made on various types of marine vehicles have indicated that $\alpha = 0.01$ appears to be appropriate in practice.

(*c*) *Frequency of encounter with seas*. As mentioned in the discussion on risk parameter, information concerning the frequency of encounter for a marine system with each sea severity in her lifetime is necessary in order to evaluate the design extreme response. For this, information on the frequency of occurrence of each sea state is necessary. Since the frequency of occurrence differs depending on the geographical location, information on the frequency in the service area(s) of the ship (or marine structure) should be considered. A method to obtain the frequency of occurrence of various sea severities, including the estimation of the severest sea condition, is discussed in Part 2 using Walden's data from the North Atlantic.

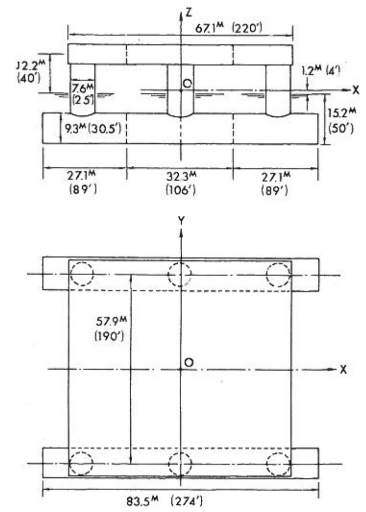


**Fig. 16** Semisubmersible-type ocean platform used for computations

(*d*) *Ship speed in a seaway.* For evaluating the extreme ship responses in a seaway, the speed in each sea severity has to be estimated since the number of encounters with waves depends on ship speed. Two different types of speed reduction in a seaway should be considered: one due to the added resistance and reduced propulsive efficiency associated with waves and ship motions, and the other due to voluntary slowdown by the ship's master in order to ease severe ship motions. The methods to evaluate the former speed reduction are discussed in references [19–23], while the method to estimate the latter is presented in [17]. For a quick evaluation of ship speed loss in a seaway at the design stage, Aertssen's formula [24, 25] derived from the results obtained from full-scale trials is recommended.

By taking these various factors which affect the magnitude of extreme responses into consideration, the probable extreme value and the design extreme value for a sea of specified severity can be evaluated by the following formulas:

Probable extreme value (amplitude):

$$\bar{y}_n = \sqrt{2 \ln \left\{ \frac{(60)^2 T}{2\pi} \sqrt{\frac{m_2}{m_0}} \right\}} \sqrt{m_0} \qquad (16)$$

Design extreme value (amplitude):

$$\hat{y}_n(\alpha) = \sqrt{2 \ln \left\{ \frac{(60)^2 T}{2\pi(\alpha/k)} \sqrt{\frac{m_2}{m_0}} \right\}} \sqrt{m_0} \qquad (17)$$

where

- $T$ = longest duration of specified sea in hours
- $m_0$ = area under response spectrum
- $m_2$ = 2nd moment of response spectrum
- $\alpha$ = risk parameter
- $k$ = number of encounters with a specified sea in ships's (or marine structure's) lifetime

It is noted that the extreme values given in equations (16) and (17) include the effect of the bandwidth of the response spectrum on extreme values [18].

As a numerical example, the results of computations carried out on a semisubmersible-type ocean platform are presented. The semisubmersible-type platform considered in the present study has vertical and horizontal cylinders as the supporting structure as shown in Fig. 16. For this type of floating structure, the wave-induced forces and moments acting on the structure can be approximated by the summation of the forces and moments acting on each individual unit, assuming the interaction between units to be negligible. That is, each segmented column-hull assembly is considered as an individual unit, and wave-induced forces and moments are evaluated for each unit. Then, the resulting forces and moments on the entire structure are evaluated by a summation of force and moment components. The detailed method of the computations is given in references [26, 27].

The most severe wave-induced loadings which have to be considered for design occur at the crossbeam bridge structure when the platform encounters beam waves [26]. In particular, if the length of the wave is approximately equal to twice the hull separation, the forces on the two horizontal cylinders are about 180 deg out of phase, and this results in a severe transverse force on the structure.

Figure 17 shows the probable extreme transverse forces acting on the structure in beam seas of various severities. The two-parameter wave spectra family is used in this computation. The total service time of the structure is assumed to be 20 years, and for one quarter of its lifetime it is assumed to be exposed to beam seas. Included also in the figure are the upper and lower bounds of the probable extreme values as well as the most probable extreme values.

Figure 18 shows the results of computations similar to those presented in Fig. 17, but the computations are carried out by using the six-parameter wave spectra family. It can be seen in Figs. 17 and 18 that use of the two-parameter family results in a wider range between upper and lower bounds of the response than that obtained by use of the six-parameter family in relatively severe seas. This is a general trend observed in many other responses computed by using these two families of wave spectra.

It may be of considerable interest to see how well the upper and lower bounds of the responses computed using the two families of spectra cover the variation of responses computed using wave spectra measured at various locations in the world. Since both the two-parameter and six-parameter families are developed using the data measured in the North Atlantic, the bounds may well cover the responses to various wave spectra which are measured in the North Atlantic. However, the question remains as to how well we may predict the responses using these families of wave spectra when the structure is operating throughout the world.

In order to provide an answer to this question, Figure 19 has been prepared. The figure shows the probable extreme transverse forces computed by using 157 available spectra measured at various oceanographic locations. These include the spectra measured in the North Pacific, those near Argus Island, the Mediterranean Sea, and near the British Isles (data which are not used in the development of the wave spectral families), and in the South Atlantic. Included also in the figure for comparison purposes are the upper and lower bounds of the responses using the two families of wave spectra.

As can be seen in Fig. 19, the bounds satisfactorily cover the variation of responses computed using the measured spectra in various locations in the world. Some other examples of comparison made on various types of ships may be seen in reference [12], though the two-parameter family of wave spectra used in reference [12] is somewhat different from that developed in the present study, as mentioned earlier. It appears, as a conclusion, that the bounds established with confidence coefficient 0.95 reasonably cover the variation of responses of marine systems in various seas throughout the world, even though these families of wave spectra are developed from an analysis of data obtained primarily in the North Atlantic.

As an example of extreme values for design consideration, Fig. 20 shows the results of computations carried out following equation (17) using the two- and six-parameter families of wave spectra. The risk parameter, $\alpha$, is taken as 0.01, and the frequency of occurrence of each sea state is taken into consideration.

The upper bound of the extreme values shown in Fig. 20 is considered as the design load. As can be seen in the figure, the design extreme values obtained from the two families of wave

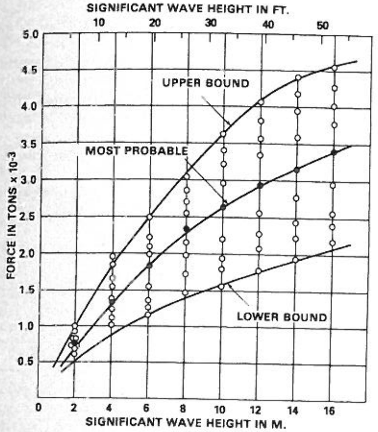


**Fig. 17** Probable extreme values of transverse force in various sea states computed by using two-parameter family of wave spectra

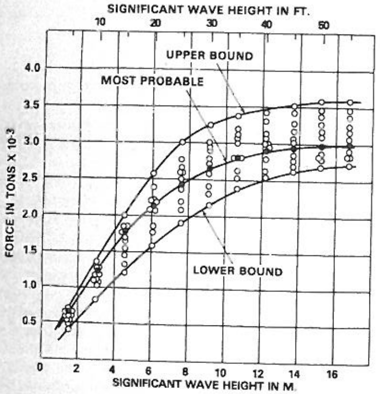


**Fig. 18** Probable extreme values of transverse force in various sea states computed by using six-parameter family of wave spectra

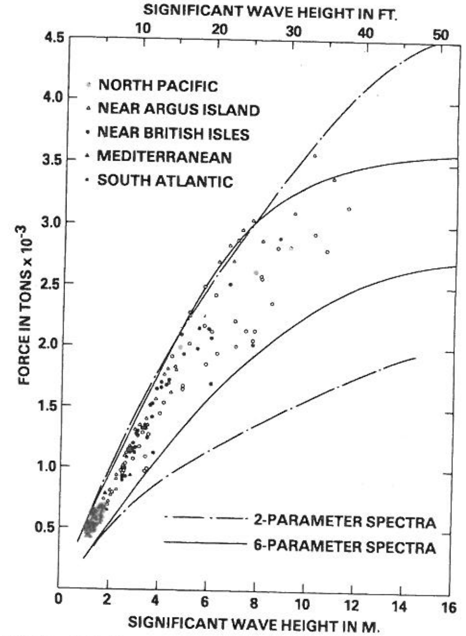


**Fig. 19** Probable extreme values of transverse force computed in worldwide measured wave spectra and upper and lower bounds using the two-parameter and six-parameter families of wave spectra

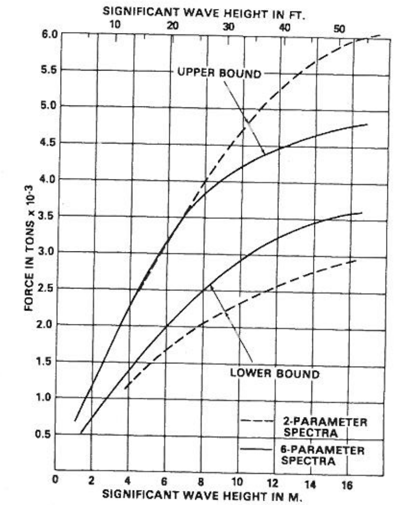


**Fig. 20** Bounds of extreme values of transverse force for design consideration

spectra agree well up to a sea of significant wave height of 8.0 m (26.2 ft), but the two-parameter family yields much larger design values than the six-parameter family in severe seas. This is a general trend observed in evaluating the design extreme values carried out on other types of marine systems, including ships.

The example of the extreme values for design consideration shown in Fig. 20 is based on the concept of short-term response prediction. These design values are compared later with those evaluated based on the concept of long-term response prediction.

## Part 2: Wave statistics for long-term prediction

In contrast to the short-term prediction of ship and marine structure response, the long-term prediction considers all variations of the response at every cycle of wave encounter irrespective of their magnitude. The long-term prediction provides valuable information for evaluating possible fatigue failure for which repeated loadings play a significant role even though the magnitudes are not large. It may be used also to evaluate the extreme value expected to occur in the lifetime of a ship (or marine structure). In this case, however, extreme care has to be given, as is discussed later.

For evaluation of the long-term response, a series of wave spectra may be developed from two different approaches; one is to incorporate the frequency of occurrence of various sea severities with either the two-parameter or the six-parameter wave spectral family, and the other is to generate a series from the joint probability density function of significant wave height and wave period. The latter approach, however, is applicable only for the two-parameter wave spectral family. These two different approaches are discussed in detail in the following with some numerical examples of their application.

### Estimation of severest sea condition

Prior to discussing the frequency of occurrence of various sea severities, it may be well to estimate the severest sea condition expected to occur in the North Atlantic based on the extreme-value statistics. Although the significant wave height follows the log-normal probability distribution, this holds only for the cumulative probability distribution up to 0.99 as discussed in the Appendix. Therefore, it is not appropriate to estimate the severest sea state (namely, the extreme significant wave height), based on the log-normal distribution, since the cumulative distribution for the extreme value will be much greater than 0.99. Instead, it may be evaluated by applying the concept of asymptotic distribution of extreme values, which is applicable for any probability function if certain conditions are met [28].

For this purpose, let us assume that the cumulative distribution function can be expressed asymptotically in the following form:

$$F(x) = 1 - e^{-q(x)} \tag{18}$$

On the other hand, the probability density function of the extreme value in $n$-observations, denoted by $g(y_n)$, is given by

$$g(y_n) = n[f(x)(F(x))^{n-1}]_{x=y_n} \tag{19}$$

Hence, the probable extreme value, denoted by $\bar{y}_n$, is obtained by letting the derivative of $g(y_n)$ with respect to $y_n$ be zero. It can be derived with the aid of equation (18) that the probable extreme value for large $n$ satisfies the following condition:

$$e^{q(\bar{y}_n)} = n \tag{20}$$

Hence, from equations (18) and (20), we have

$$\frac{1}{1 - F(\bar{y}_n)} = n \tag{21}$$

It is then possible to estimate the probable extreme value, $\bar{y}_n$, from the relationship given in equation (21). For the present problem, the cumulative distribution function involved in equation (21) is evaluated from the data, particularly for large significant wave heights.

As an example, Fig. 21 shows the left-hand side of equation (21), which is often called the return period, in logarithmic form using the significant wave height data obtained at Stations I and J [10, 11]. Since there is no appreciable difference between the cumulative distribution functions for high significant wave heights obtained at Stations I and J, the results are expressed by a single line as shown in the figure.

In estimating the probable extreme values from the line drawn in the figure, the number of observations involved in the data has to be considered since the magnitude of the extreme value depends on this number. It is stated in references [11] and [12] that the data are based on random samples from 10 years of observation and that the numbers $n$ = 2400 for Station I and $n$ = 1440 for Station J. Using these numbers in equation (21), it can be estimated from Fig. 21 that the probable extreme significant wave heights in 10 years are 15.4 m (50.5 ft) for Station I and 14.6 m (47.9 ft) for Station J.

It should be noted that the magnitudes of probable extreme significant wave heights thus estimated from Fig. 21 are much smaller than the values known to date. This is not surprising, since Draper's data shown in Fig. 21 are for the numbers $n$ = 2400 and 1400 in 10 years. This implies that the sampling rates for the data presented in Fig. 21 are one observation in every 37- and 61-h interval, respectively. Hence, it is obvious that very severe seas which do not persist for a long period (see Fig. 15) are missed in the data.

In order to estimate the probable extreme significant heights at Stations I and J more precisely, Figs. 22 and 23 have been prepared using Walden's visually observed data at these stations. The figures present results similar to that shown in Fig. 21, but the numbers are significantly larger: $n$ = 23 620 for Station I and $n$ = 24 947 for Station J in 10 years. Walden's data, therefore, provide information on the sea condition every 3.7- and 3.5-h intervals, sufficient to cover all severe sea conditions.

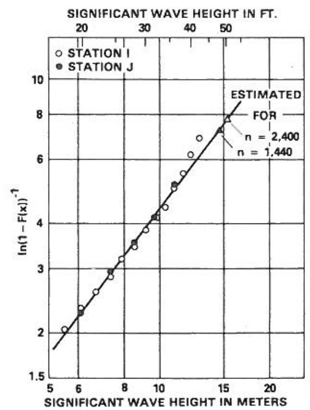


**Fig. 21** Probable extreme significant wave heights in 10 years at Stations I and J (data from references [10, 11])



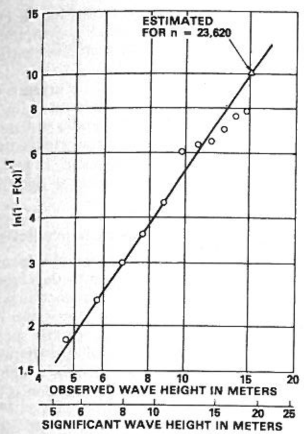


**Fig. 22** Probable extreme significant wave height in 10 years at Station I (data from reference [4])

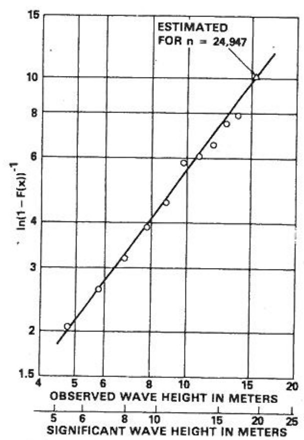


**Fig. 23** Probable extreme significant wave height in 10 years at Station J (data from reference [4])

As can be seen in Figs. 22 and 23, the estimated extreme (visually observed) wave heights in 10 years are 15.1 m (49.4 ft) and 15.4 m (50.5 ft) for Stations I and J, respectively. Using the conversion factor given in equation (38) in the Appendix, the probable extreme significant wave heights expected to occur in 10 years become 18.7 m (61.5 ft) and 19.2 m (62.9 ft) for Stations I and J, respectively.

It is of interest to compare the extreme significant wave heights thus estimated with the measured ones. A difficulty involved in the comparison is that information concerning the length of time (in years) in which the extreme heights were measured is not clearly known. However, from available information given in references [29] and [30], the extreme significant wave heights obtained through spectral analysis of records taken at Stations I and J are as follows:

Station I: 16.7 m (54.7 ft) in 8 years [29]
Station J: 15.3 m (50.1 ft) in 8 years [29]
16.8 m (55.0 ft) in 14 years [30]

Thus, the difference between the estimated and measured extreme significant wave heights is approximately 15 percent as an average.

It may also be of interest to compare the results presented in Fig. 21 with those shown in Figs. 22 and 23. For this purpose, let us evaluate the extreme height for $n$ = 2400 from Fig. 22, and then convert the value to the significant wave height by using equation (38) in the Appendix. The extreme significant wave height thus evaluated from Fig. 22 becomes 15.6 m (51.0 ft) for Station I as compared with 15.1 m (49.5 ft) estimated from Fig. 21. A similar comparison may be made for Station J, and it is found that the significant wave height is 14.9 m (48.9 ft) from Fig. 23 as compared with 15.4 m (50.5 ft) from Fig. 21. As these comparisons indicate, the extreme wave heights estimated from Draper's analysis of measured data and Walden's observed data (convert to significant height) agree very well if the estimation is made for the same sample size.

**Table 5 Probable extreme significant wave heights expected in 10 years at various locations in the North Atlantic**

| Station | Sample size | Probable extreme significant height |
|---|---|---|
| A | 24,696 | 19.0 m (62.4 ft) |
| B | 24,988 | 18.0 (58.9 ) |
| C | 25,639 | 15.6 (51.1 ) |
| D | 25,951 | 17.5 (57.6 ) |
| E | 24,788 | 14.6 (48.0 ) |
| I | 23,620 | 18.7 (61.5 ) |
| I(Winter) | 5,510 | 18.9 (62.0 ) |
| J | 24,947 | 19.2 (62.9 ) |
| K | 26,508 | 19.3 (63.3 ) |
| M | 26,359 | 13.1 (42. 9 ) |

The probable extreme wave heights expected in 10 years are evaluated for various locations in the North Atlantic following the same procedure presented in Figs. 22 and 23, and the results are summarized in Table 5. The probable extreme significant wave height estimated using the data obtained during the winter season at Station I is also included in the table. The sample size for the winter season at Station I is 5510 in 10 years in contrast to 23 620 for all-year-round data at Station I. As can be seen in the table, the extreme significant wave height estimated from the winter season data agrees very well with that estimated from the all-year-round data. This indicates the significance of the sample size in estimating the extreme values.

The question arises as to the estimation of the probable extreme significant wave height expected in 20 years, which is considered to be a reasonable lifetime for ships and marine structures. Note that Walden's data used in the present analysis are accumulated over a 10-year period. Hence if we assume that the statistical characteristics of the extreme values expected

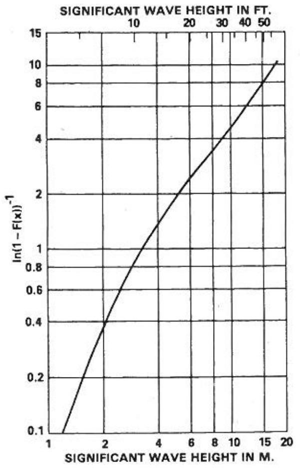


**Fig. 24** Cumulative distribution of significant wave height for the (mean) North Atlantic

to occur in 20 years are the same as those observed in the data accumulated in 10 years, then the estimation for 20 years is possible from the line shown in Figs. 22 and 23. That is, by doubling the sample size, the extreme significant wave heights in 20 years become 19.6 m (64.4 ft) and 20.2 m (66.4 ft) for Stations I and J, respectively. These values indicate that the probable extreme significant wave height in 20 years is only 5 percent greater than that in ten years.

## Frequency of occurrence of various sea severities

The frequency of occurrence of various sea severities can be obtained based on the log-normal probability distribution. However, the frequencies for severe sea states should be modified, taking into consideration the probability distribution used for estimating the extreme significant wave heights. In other words, the log-normal distribution is used for the cumulative distribution up to about 0.99, and then the asymptotic extreme distribution is used for estimating the extreme significant wave height.

For convenience, the frequency of occurrence of various significant wave heights is expressed in terms of the logarithm of the return period, $\ln(1 - F(x))^{-1}$, and is presented in Fig. 24. The information in this figure pertains to the mean North Atlantic with the exceptions of Stations E and M. The frequencies for each one-meter interval of significant wave height are given in Table 6.

### Series of wave spectra for long-term prediction

For evaluating the long-term responses of ships and marine structures, the families of wave spectra developed for the short-term prediction can be used in conjunction with the frequency of occurrence of various sea states given in Table 6. That is, let the weighting factor of a specific spectrum in a certain sea be $p_j$. From Table 6, obtain the frequency of occurrence of this particular sea $p_i$. Then the weighting factor of the spectrum for the long-term prediction is simply the product of $p_i$ and $p_j$.

Since the frequency of occurrence of 18 different sea severities for the mean North Atlantic are given in Table 6, the two-parameter and the six-parameter spectra families produce 162 and 198 spectra, respectively, which can be used for the long-term response prediction.

There is another way to generate a series of wave spectra from the joint probability density function of the significant wave height and modal period. Although this approach is applicable only for the family of two-parameter wave spectra, a benefit is that as many spectra as desired can be generated in the series. The following is the principle underlying the development of the series of wave spectra by this approach.

Let us transform the joint probability density function of wave height, $H$, and period, $T$, into new random variables $r$ and $\theta$ as illustrated in Fig. 25. That is

$$\begin{cases} H = H_0 + r\cos\theta \\ T = T_0 + r\sin\theta \end{cases} \qquad (22)$$

where

$H_0$ = modal value of $H = \exp\{\mu_H - \sigma_H{}^2\}$
$T_0$ = modal value of $T = \exp\{\mu_T - \sigma_T{}^2\}$

Since the point $(H_0, T_0)$ shown in Fig. 25 is the mode of the two-dimensional probability density function, the combination of wave height, $H_0$, and wave period, $T_0$, yields the spectrum most likely to occur in the long term. By carrying out the transformation, the joint probability density function of $r$ and $\theta$, $f(r,\theta)$, becomes

$$f(r,\theta) = \frac{r}{\sigma_1\sigma_2(2\pi)\sqrt{1-\rho^2}\,(H_0 + r\cos\theta)(T_0 + r\sin\theta)} \times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln(H_0 + r\cos\theta) - \mu_H}{\sigma_H}\right)^2 - 2\rho\left(\frac{\ln(H_0 + r\cos\theta) - \mu_H}{\sigma_H}\right)\left(\frac{\ln(T_0 + r\sin\theta) - \mu_T}{\sigma_T}\right) + \left(\frac{\ln(T_0 + r\sin\theta) - \mu_T}{\sigma_T}\right)^2\right]\right\} \qquad (23)$$

**Table 6** Frequency of occurrence of various sea states in the (mean) North Atlantic

| Significant Wave Height (in Meters) | Frequency of Occurrence | Significant Wave Height (in Meters) | Frequency of Occurrence |
|---|---|---|---|
| < 1 | 0.0503 | 9 - 10 | 0.0079 |
| 1 - 2 | 0.2665 | 10 - 11 | 0.0054 |
| 2 - 3 | 0.2603 | 11 - 12 | 0.0029 |
| 3 - 4 | 0.1757 | 12 - 13 | 0.0016 |
| 4 - 5 | 0.1014 | 13 - 14 | 0.00074 |
| 5 - 6 | 0.0589 | 14 - 15 | 0.00045 |
| 6 - 7 | 0.0346 | 15 - 16 | 0.00020 |
| 7 - 8 | 0.0209 | 16 - 17 | 0.00012 |
| 8 - 9 | 0.0120 | 17 < | 0.00009 |



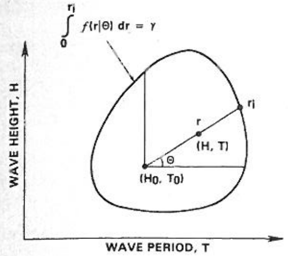


**Fig. 25** Pictorial sketch illustrating transformation of random variables

From the joint probability density function given in equation (23), the conditional probability density function of $r$ for a given $\theta$ can be derived:

$$f(r\,|\,\theta) = \frac{f(r,\theta)}{\int_0^\infty f(r,\theta)dr} \qquad (24)$$

Then, a point $r_i$ can be determined for each angle, $\theta$, such that

$$\int_0^{r_i} f(r\,|\,\theta)dr = \gamma \qquad (25)$$

where $\gamma$ is the confidence coefficient.

The inside of the boundary line obtained by connecting points $r_i$ thus determined is the confidence domain for a specified confidence coefficient $\gamma$. It is noted that every point inside the domain provides information on wave height and wave period, and hence the two-parameter spectrum can be drawn for each point.

Figure 26 shows the confidence domain obtained, following the method described in the foregoing, using Draper's measured data shown in Table 1. The numbers given in the table are also included in Fig. 26. As seen in the figure, the theoretical confidence domain for the confidence coefficient 0.99 obtained based on the bivariate log-normal probability law covers the original data very well.

The confidence domains are obtained for all nine stations in the North Atlantic using Walden's data. As an example, Fig. 3, shown earlier, is the confidence domain for the confidence coefficient 0.95 for all nine stations. As can be seen in Fig. 3, the confidence domains for Stations A, B, C, D, I, J, and K are approximately equal. While there is a substantial difference between the confidence domains for these seven stations and Stations E and M.

The confidence domains for the mean North Atlantic are obtained for various confidence coefficients (0.50, 0.75, 0.85, 0.95, and 0.99), and the results are shown in Fig. 27. Included also in the figure is a line connecting the largest significant wave height for each domain.

In order to develop a series of wave spectra from the confidence domains of various confidence coefficients, $\gamma$, each segment and the origin (mode) are weighted as follows:

| | |
|---|---|
| Domain between $\gamma$ = 0.99 (inclusive) and 0.95 | 0.05 |
| $\gamma$ = 0.95 (inclusive) and 0.85 | 0.10 |
| $\gamma$ = 0.85 (inclusive) and 0.75 | 0.10 |
| $\gamma$ = 0.75 (inclusive) and 0.50 | 0.25 |
| inside $\gamma$ = 0.50 (inclusive) | 0.25 |
| Origin (mode) | 0.25 |

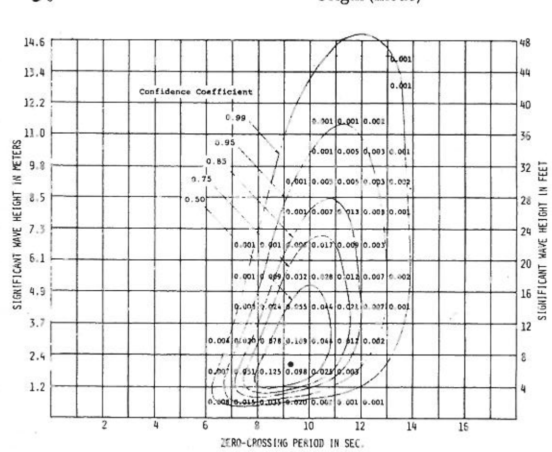


**Fig. 26** Comparison of domains of significant wave height and zero-crossing period for various confidence coefficients (data from reference [10])

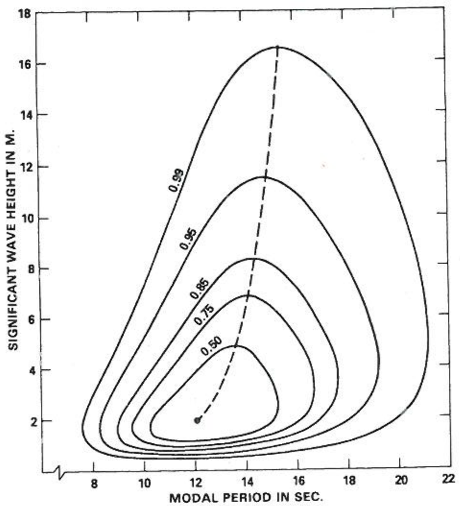


**Fig. 27** Confidence domains of significant wave height and modal period for the (mean) North Atlantic

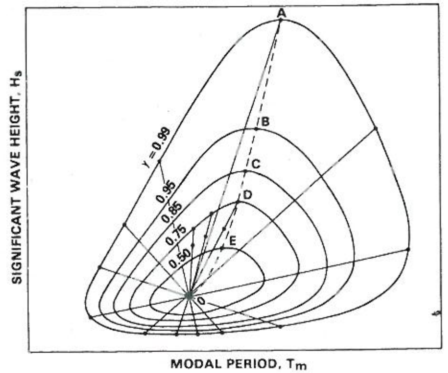


**Fig. 28** Pictorial sketch illustrating a method to establish a series of wave spectra for long-term prediction

The total number of wave spectra comprising the series is distributed in proportion to the weighting factor given above. For example, suppose it is desired to generate a total of 240 spectra for evaluating the long-term response, then the 240 points are distributed among the various domains in accordance with the weighting factors: 12, 24, 24, 60, and 60. In addition, 60 points are allotted to the origin (mode).

Figure 28 is a pictorial sketch illustrating the method to distribute the points in each domain. The procedure to determine a total of 240 spectra is as follows:

(*a*) *For the domain* $\gamma = 0.99$. Connect the origin 0 (the point which yields the most probable wave height and period) with a point A, which corresponds to the highest significant

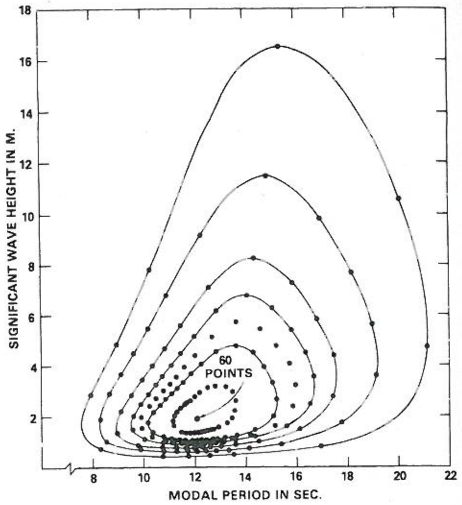


**Fig. 29** 240 combinations of significant wave height and modal period for a series of wave spectra for long-term prediction

wave height for $\gamma = 0.99$. Using the line OA as the baseline, draw lines every 30 degree-intervals, and determine 12 points as shown in the figure.

(*b*) *For the domain* $\gamma = 0.95$. Using the line OB as the baseline, determine 24 points by drawing lines every 15 degree-intervals.

(*c*) *For the domain* $\gamma = 0.85$. Using the line OC as the baseline, determine 24 points by the same procedure given in Item (*b*).

(*d*) *For the domain* $\gamma = 75$. Using the line OD as the baseline, draw lines every 12 degree-intervals. Then, 30 points are determined along the line for $\gamma = 0.75$, and another 30 points are determined between two lines for which $\gamma = 0.75$ and $\gamma = 0.50$. As an example, only six points are shown in Fig. 28.

(*e*) *For the domain* $\gamma = 0.50$. Using the line OE as the baseline, determine 60 points by the same procedure as described in Item (*d*).

(*f*) *A total* of 180 points obtained in the foregoing yields 180 two-parameter wave spectra. All these spectra should be considered to carry the same weight, 1/240 each. In addition, a spectrum corresponds to the modal value of the bivariate log-normal distribution (the solid circle in Fig. 28), and the response to this spectrum is weighted as 60/240. Figure 29 shows the thus-determined 240 combinations of significant wave height and modal period for the mean North Atlantic.

## Application to design

Various factors, each of which is weighted according to its occurrence in the lifetime of a marine system, have to be considered in the long-term response prediction. These include (*a*) seas of various severities, (*b*) a variety of wave spectral shapes, (*c*) speeds (in the case of a ship), (*d*) various headings to waves, and, in addition, (*e*) the expected number of cycles of the response for a given sea, wave spectral shape, speed, and heading. The last factor, the number of response cycles, has



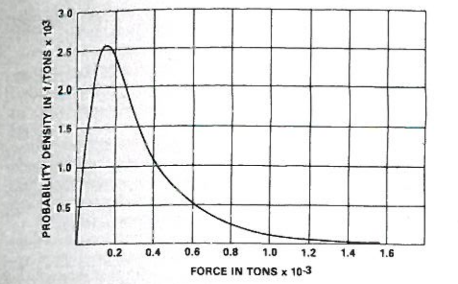


**Fig. 30** Probability density function of long-term transverse force obtained by using six-parameter family of wave spectra

not been considered in most of the long-term prediction methods developed to date.[3]

It is noted that the number of cycles of each short-term response plays a significant role in establishing the probability density function for the long term, since the long-term prediction considers all variations of the response at every cycle of wave encounter irrespective of their magnitude. It should also be noted that the accumulated number of response cycles for each short-term response has to be used for evaluating the extreme value based on the long-term prediction approach.

Taking various factors discussed in the foregoing into consideration, the probability density function applicable to the long-term response can be written as follows:

$$f(x) = \frac{\sum_i \sum_j \sum_k \sum_l n_* p_i p_j p_k p_l f_*(x)}{\sum_i \sum_j \sum_k \sum_l n_* p_i p_j p_k p_l} \qquad (26)$$

where

$f_*(x)$ = probability density function for short-term response

$n_*$ = average number of responses per unit time of short-term response

$= \frac{1}{2\pi}\sqrt{(m_2)_*/(m_0)_*}$

$(m_0)_*$ = area under short-term response spectrum

$(m_2)_*$ = second moment of short-term response spectrum

$p_i$ = weighting factor for sea condition

$p_j$ = weighting factor for wave spectrum

$p_k$ = weighting factor for heading to waves in a given sea

$p_l$ = weighting factor for speed in a given sea and heading

The total number of responses expected in the lifetime of a marine system becomes

$$n = \left(\sum_i \sum_j \sum_k \sum_l n_* p_i p_j p_k p_l\right) \times T \times (60)^2 \qquad (27)$$

where $T$ is the total exposure time to the sea.

As an example of application of this methodology, the long-term probability function is computed for a transverse force acting on the ocean platform used for the short-term response prediction. For the ocean platform, the weighting factor for speed, $p_l$, can be neglected. Furthermore, in order to simplify the problem, the computation is carried out for 20

[3] This was first pointed out by Dr. M. Chang of the David W. Taylor Naval Ship Research and Development Center.

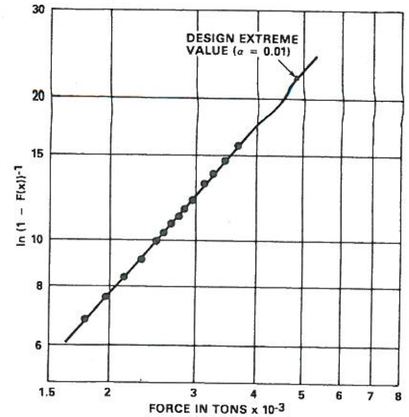


**Fig. 31** Design extreme value of transverse force evaluated by using six-parameter family of wave spectra

years assuming that the platform will be exposed to beam seas for one quarter of its lifetime, namely, $p_k = 1/4$ (constant). The severest sea state considered in the computation has a significant wave height of 17 m (55.8 ft).

Figure 30 shows the long-term probability density function obtained by using the six-parameter wave spectra family. Since the total number of responses, $n$, can be obtained from equation (27), the number of cycles of various loadings necessary for evaluating possible fatigue failure of the structure can readily be evaluated from the figure.

In order to evaluate the extreme value for design based on the long-term prediction approach, the asymptotic distribution of extreme values is considered. That is, analogous to equation (21), the design value, $\hat{y}_n$, with the risk parameter, $\alpha$, can be determined from

$$\frac{1}{1 - F(\hat{y}_n)} = \frac{n}{\alpha} \qquad (28)$$

The cumulative distribution function involved in equation (28) is obtained by integrating equation (26) with respect to $x$, and the left-hand side of equation (28) is shown in Fig. 31 in the logarithmic scale. With $n = 2.078 \times 10^7$ evaluated from equation (27), and by choosing $\alpha = 0.01$, Fig. 31 yields the design extreme value of 4800 tons. This value agrees with the design extreme value obtained for the severest sea state of significant wave height 16.5 m (16.0 to 17.0 m) in the short-term prediction using the same six-parameter wave spectral family (see Fig. 20).

Computations of the long-term transverse force are also carried out using the two-parameter wave spectral family. In this case, computations are made for the two approaches discussed earlier. One method is the same as that used in conjunction with the six-parameter family; the other method uses the series of spectra determined from the confidence domains shown in Fig. 29. These two are identified as the "weighted sea approach" and the "confidence domain approach," respectively, in the following presentation of the computed results.

Figure 32 shows the long-term probability density functions. The probability density function obtained from the "confidence domain approach" indicates that the lifetime response contains a larger number of small-magnitude responses in relatively mild

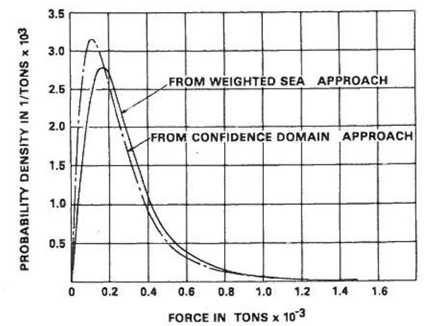


**Fig. 32** Probability density function of long-term transverse force obtained by using two-parameter family of wave spectra

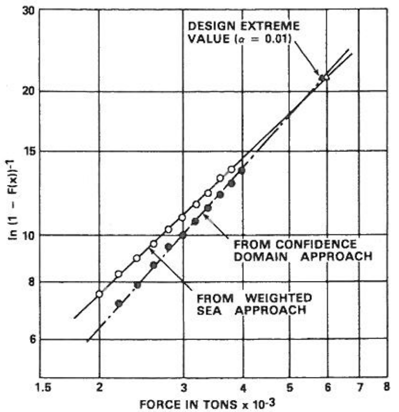


**Fig. 33** Design extreme value of transverse force evaluated by using two-parameter family of wave spectra

seas than are obtained from the "weighted sea approach." The probability density function from the latter approach agrees reasonably with that obtained by using the six-parameter wave spectral family shown in Fig. 30.

The extreme values for design consideration estimated by using the two-parameter family are shown in Fig. 33. The number of response cycles in 20 service years is the same for both approaches, $n = 1.864 \times 10^7$ in the present example. The design values by choosing $\alpha = 0.01$ then become 6000 and 5870 tons for the "weighted sea approach" and "confidence domain approach," respectively.

It should be noted that the extreme design value estimated from the "weighted sea approach" agrees well with the extreme design value estimated for the highest significant wave height (16.5 m or 54.1 ft) considered in the short-term prediction method given earlier in Fig. 20. This same property was observed in the results computed by using the six-parameter wave spectral family. Hence, it may safely be concluded that the extreme value for design consideration estimated from the long-term response prediction method agrees with that estimated from the short-term response prediction method.

The results of the computations clearly indicate that a considerable percentage of small magnitudes of response in relatively mild seas (which do not contribute to the extreme value) is included in the long-term probability density function. In other words, the magnitude of response does not reach a level critical for operation or structural safety irrespective of how long a marine system operates in mild seas, while the magnitude reaches the critical level within a few hours in severe seas. This suggests that the short-term prediction in severe seas is much to be desired for estimation of extreme values.

It should also be pointed out that the estimation procedure of the extreme values through the short-term prediction approach is extremely simple in comparison with that through the long-term approach, and that the accuracy of the computation for the former appears to be much superior to that of the latter approach. That is, extreme values in the short-term prediction approach can be estimated by the formulas given in equations (16) and (17) as contrasted with the estimation by extending the cumulative distribution function for the long term.

Thus, in summary, as far as estimation of extreme values is concerned, it is appropriate to consider severe seas expected in the service area and apply the short-term prediction method.

## Summary and conclusions

This paper presents wave information which plays a significant role in predicting responses of ships and ocean structures in a seaway, and discusses methods of application specifically for design consideration. For this, a series of wave spectra to be used for short-term prediction as well as a series for long-term (lifetime) prediction is developed. In applying the series to evaluate responses of marine systems, several factors which may seriously affect the magnitude of predicted values (including extreme values) are discussed, and results of numerical computations carried out on a semisubmersible-type ocean platform are presented. Conclusions derived from the results of the present study are summarized in the following:

### Short-term response prediction

1. Two series of wave spectra are developed for the short-term response prediction of a marine system—one a six-parameter wave spectra family consisting of 11 members for a specified sea state and the other a two-parameter wave spectra family consisting of nine members for each sea. Each of these members carries a weighting factor depending on its frequency of occurrence.

2. In the development of the two-parameter family, it was found from the results of analysis that both wave height and wave period obey the log-normal probability law for a cumulative distribution up to 0.99. Hence, the combined statistical properties of wave height and period may be evaluated based on the bivariate log-normal distribution.

3. In the development of the six-parameter family, a variety of wave spectral shapes associated with the growth and decay of a storm, including the existence of swell, is analyzed. Hence, members of this family have a wider variety of shapes than members of the two-parameter family.

4. In using either the two-parameter or the six-parameter family for the response prediction for each sea severity, one of the family members yields the largest response, while another yields the smallest response with confidence coefficient of 0.95. Hence, by connecting the values obtained in each sea severity, the upper- and lower-bound responses can be established.



5. Comparisons of responses computed using these two families of spectra and those computed using spectra measured at various oceanographic locations have indicated that the bounds established with confidence coefficient 0.95 reasonably cover the variation of responses of marine systems in various sea throughout the world, even though the families of wave spectra were developed from an analysis of data obtained primarily in the North Atlantic. Use of the two-parameter family, however, results in a wider range between upper and lower bounds of the response than that obtained using the six-parameter family in relatively severe seas.

6. For estimating extreme values of the responses for design consideration, various factors such as operation (or exposure) time, risk parameter, frequency of encounter with seas, and speed (in the case of a ship) should be considered. The results of computations have shown that the design extreme values obtained from the two families of wave spectra agree well in seas of mild to moderate severity, but the two-parameter family yields much larger values than the six-parameter family in severe seas. This is a general trend observed in evaluating the design extreme values carried out on various types of marine systems, including ships.

### Long-term response prediction

7. A series of wave spectra for evaluation of the long-term response is developed from two different approaches, one which incorporates the frequency of various sea severities with either the two-parameter or the six-parameter wave spectra family (weighted sea approach) and the other which generates a series from the joint probability function of significant wave height and period (confidence domain approach). The latter approach is applicable only to the two-parameter wave family.

8. The severest sea state (extreme significant wave height) in the long term may be estimated by applying the concept of asymptotic distribution of extreme values. In estimating the extreme values, however, the sample size of the data has a significant effect on the magnitude of extreme values. It is estimated that the most probable extreme significant wave height at Station I in the North Atlantic is 18.7 m (61.5 ft) in 10 years as compared with 16.7 m (54.7 ft), the highest significant height measured in eight years. The probable extreme value in 20 years is estimated to be approximately 5 percent greater than that in 10 years.

9. In the long-term response prediction, various factors, each of which is weighted according to its occurrence in the lifetime of a marine system, have to be considered. These include seas of various severities, a variety of wave spectral shapes, various speeds (in the case of a ship), various headings to waves, and, in addition, the expected number of cycles of the response. The accumulated number of response cycles for each short term has to be used for evaluating the statistical properties of the response based on the long-term prediction approach.

10. Results of numerical computations have shown that the extreme value for design consideration estimated from the long-term prediction method agrees with that estimated from the short-term prediction method. However, the estimation procedure of the extreme values through the short-term prediction approach is extremely simple in comparison with that through the long-term approach. Greater care has to be taken in the use of the long-term approach in order to obtain the same accuracy as that provided by the short-term approach.

## Acknowledgments


The author would like to express his sincere thanks to Dr. J. E. Whalen, Operations Research Inc., for his assistance in developing the computer programs to evaluate the long-term probability distribution and confidence domains of the bivariate log-normal probability distribution.


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# Appendix

## Joint probability distribution of wave height and period

In order to obtain the combined statistical characteristics of wave height and period, available measured and observed data are independently analyzed. Since both measured and observed data taken at Stations I and J in the North Atlantic are available, a statistical analysis is made to obtain the relationship between them. It was found from the results of the analysis that both the measured and observed wave heights follow the same probability law, namely, the log-normal probability law. The measured and observed wave periods were also found to follow the log-normal probability law. Hence, it is possible to evaluate the combined statistical characteristics of wave height and period by the bivariate log-normal probability law. It is also possible to find the conversion factor by which the wave information for design use can be obtained from the visually observed data. A detailed discussion is given in the following.

### Analysis of measured data

From the records obtained at Station I (19° W, 59° N) in the North Atlantic, Draper and Squire [10] present statistical data of significant wave height, $H_s$, and zero-crossing period, $T_0$, which is duplicated in Table 1 of this paper. In order to obtain the combined statistical characteristics of the two random variables, $H_s$ and $T_0$, from Draper's data, we may first consider the marginal probability distribution of the significant wave height from Table 1.

Figure 34 shows the cumulative distribution function of the significant wave height plotted on the log-normal probability paper, while Fig. 35 shows it plotted on Weibull probability paper. The comparison between the histogram constructed from Table 1 and the two probability density functions is shown in Fig. 36. As seen in this figure, the data are approximately represented by the log-normal probability distribution over the range for the cumulative distribution up to 0.99.

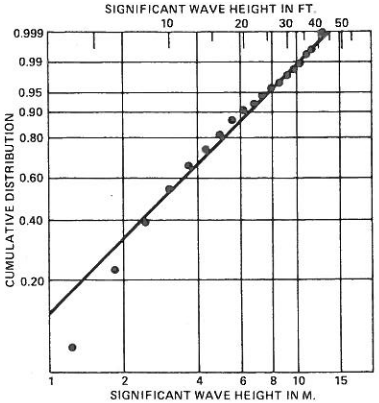


Fig. 35 Cumulative distribution function of significant wave height plotted on Weibull probability paper (data from reference [10])

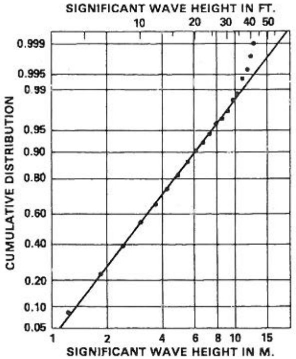


Fig. 34 Cumulative distribution function of significant wave height plotted on log-normal probability paper (data from reference [10])

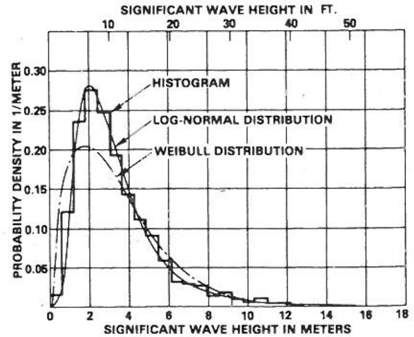


Fig. 36 Comparison between histogram of significant wave height and log-normal and Weibull probability density functions (data from reference [10])

As another example, Draper and Whitaker's data [11] obtained at Station J (20° W, 52° N) are analyzed in the same fashion as for Station I. The results indicate the same trend as observed in the analysis made for Station I, and, hence, it may safely be concluded that the significant wave height obeys the log-normal probability law expressed as follows:

$$f(H_s) = \frac{1}{\sigma_{HS}\sqrt{2\pi}}\frac{1}{H_s}e^{-1/2(\ln H_s - \mu_{HS}/\sigma_{HS})^2} \qquad (29)$$

For convenience, the probability density function given in equation (29) may be written as

$$f(H_s) \sim \Lambda(\mu_{HS}, \sigma_{HS}) \qquad (30)$$

where $\Lambda(\mu_{HS}, \sigma_{HS})$ is the log-normal probability density function with parameters $\mu_{HS}$ and $\sigma_{HS}$.

It is noted here that, although considerable attention has been given to statistical information of long-term wave height, the probability function applicable to long-term significant wave height is the focus of much criticism. Some claim that the log-normal distribution is appropriate [31–33], while others believe the data can be better fitted by the Weibull distribution [34, 35]. In the present analysis, however, a total of 12 cases of measured and observed wave height data is analyzed not only from comparison of the cumulative distribution functions but also from a comparison of the histogram and the probability density functions. The results of the analysis of all cases illustrate that the long-term wave heights follow the log-normal probability law over the range for the cumulative distribution up to 0.99.

Next, the marginal probability density function of the zero-crossing period is derived. Figure 37 shows a comparison between the histogram and log-normal probability density function of the zero-crossing period using the data obtained at Station I. Good agreement between them can be seen in the figure. The comparison made using the data obtained at Station J also shows the same trend as that obtained for Station I. It is noted that an attempt was made to represent the wave period data by the Weibull probability distribution. However, the results indicate that the representation is extremely poor for both measured and observed wave period. Thus, it is concluded that the zero-crossing period follows the log-normal probability law. That is

$$f(T_0) \sim \Lambda(\mu_{TO}, \sigma_{TO}) \qquad (31)$$

where $\Lambda(\mu_{TO}, \sigma_{TO})$ is the log-normal probability density function with parameters $\mu_{TO}$ and $\sigma_{TO}$.

From the conclusion given in equations (30) and (31), and from the property of the log-normal probability distribution, it can be derived that the combined statistical properties of significant wave height and zero-crossing period follow the bivariate log-normal probability law, which may be written as

$$f(H_s, T_0) \sim \Lambda(\mu_{HS}, \sigma_{HS}, \mu_{TO}, \sigma_{TO}, \rho) \qquad (32)$$

where $\rho$ is the correlation coefficient for two random variables, $H_s$ and $T_0$.

The conclusion given in equation (32) plays a significant role in evaluating various statistical properties of significant wave height and zero-crossing period. First of all, it is possible from equation (32) to evaluate the joint probability of occurrence of a specified significant wave height and zero-crossing period, particularly for severe seas, as demonstrated in Fig. 26. It is noted that the statistical information given in the original data is sparse for severe seas, and no reliable information can be obtained from the data, in general.

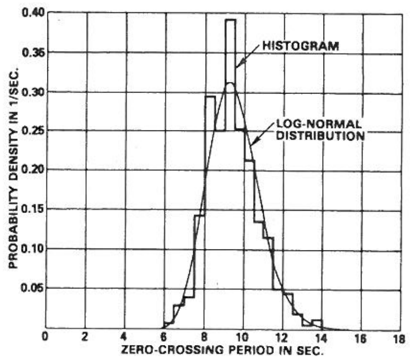


Fig. 37 Comparison between histogram of zero-crossing period and log-normal probability density function (data from reference [10])

Secondly, the statistical properties of the zero-crossing period for a specified significant wave height can be obtained from equation (32) with the aid of equation (30). It is a conditional log-normal probability distribution given by

$$f(T_0|H_s) \sim \Lambda\left(\mu_{TO} + \rho\frac{\sigma_{TO}}{\sigma_{HS}}(\ln H_s - \mu_{HS}),\ \sqrt{1-\rho^2}\cdot\sigma_{TO}\right) \qquad (33)$$

The values of modal period necessary to establish a family of wave spectra for a specified significant wave height are determined from the conditional probability density function shown in equation (33).

Thirdly, the confidence domain for a specified confidence coefficient can be evaluated based on equation (32). Then, a series of wave spectra necessary for the long-term response prediction can be established from the confidence domains as demonstrated in Fig. 29.

### Analysis of visually observed data

The same statistical analysis made on measured data of wave height and period is also carried out on observed data given in reference [4]. The data presented in [4] are the accumulated observed wave heights and periods at nine locations in the North Atlantic (see Fig. 3) over a period of 10 years. The observations were made regularly by trained oceanographers at 3½-hour intervals, as an average. Wave heights were visually observed, and wave periods were counted by a stopwatch from visually observed wave crests. Therefore, the observed wave height represents neither the significant nor the average wave height. Also, the observed wave period represents neither the zero-crossing nor the average wave period. Hence, in the analysis of the data given in reference [4], the wave height and period are defined as visually observed wave height, $H_v$, and visually observed period, $T_v$. Later, these visually observed wave heights and periods are converted to significant wave height and zero-crossing period (or modal period), respectively, so that the information based on visually observed data can be used for the design of ships and marine structures.

As an example of the statistical analysis carried out on wave height, Fig. 38 shows a comparison between the histogram and

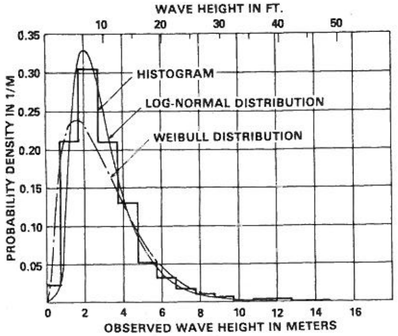


**Fig. 38** Comparison between histogram of visually observed wave height and log-normal and Weibull probability density functions (data from reference [4])

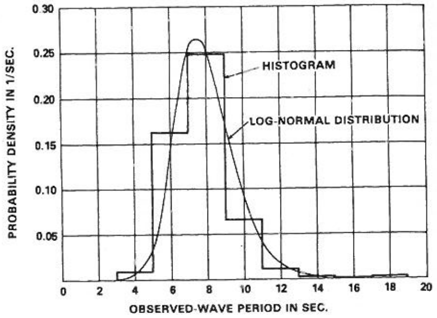


**Fig. 39** Comparison between histogram of visually observed wave period and log-normal probability density function (data from reference [4])

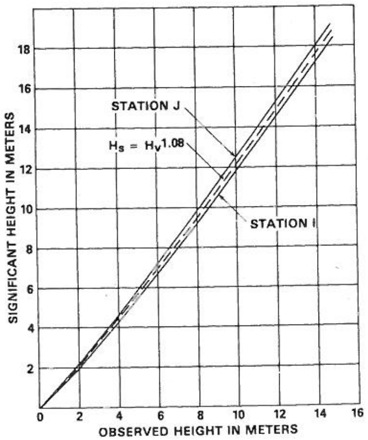


**Fig. 40** Conversion from visually observed wave height to significant wave height

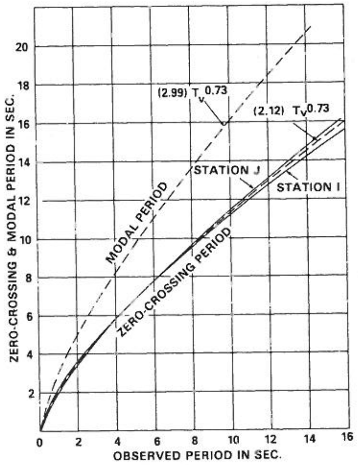


**Fig. 41** Conversion from visually observed wave period to zero-crossing period and modal period

the log-normal as well as the Weibull probability density functions using the data observed at Station I. The results of a similar analysis carried out for the remaining eight stations have indicated the same trend as observed in Fig. 38. Hence, the conclusion regarding the statistical property of the long-term wave height obtained from the measured data is also applicable to the visually observed wave height. That is

$$f(H_v) \sim \Lambda(\mu_{HV}, \sigma_{HV}) \quad (34)$$

where $\Lambda(\mu_{VH}, \sigma_{VH})$ is the log-normal probability density function with parameters $\mu_{VH}$ and $\sigma_{VH}$.

Figure 39 shows an example of a comparison between the histogram of the visually observed wave period and the log-normal probability density function using the data obtained at Station I. As this example indicates, and from the results of a similar analysis made on the remaining eight stations, it is concluded that the visually observed wave period appears to follow the log-normal probability law. That is

$$f(T_v) \sim \Lambda(\mu_{TV}, \sigma_{TV}) \quad (35)$$

where $\Lambda(\mu_{TV}, \sigma_{TV})$ is the log-normal probability density function with parameters $\mu_{TV}$ and $\sigma_{TV}$.

The conclusions given in equations (34) and (35) are the same as derived earlier for the measured significant wave height and zero-crossing period except for the values of the parameters involved in the probability density functions. This implies that the conclusion regarding the combined statistical properties of the significant wave height and period can also be applicable to the visually observed wave height and period.

Since all of the visually observed as well as the measured wave height and period follow the same probability law (log-normal probability law), it is now possible to find the relationship between observed and measured data by applying the property of the log-normal probability distribution. The procedure is as follows.

Let us assume that the relationship between the visually observed wave height, $H_v$, and the significant wave height, $H_s$, is given by

$$H_v = aH_s^{\,b} \quad (36)$$

where $a$ and $b$ are constants. It was obtained that the significant wave height follows the log-normal probability law with parameters $\mu_{HS}, \sigma_{HS}$. Then, it can be proved that the visually observed height, $H_v$, which is functionally related as given by equation (36), has to follow the log-normal probability law given by

$$f(H_v) \sim \Lambda(\ln a + b \cdot \mu_{HS},\ b\sigma_{HS}) \quad (37)$$

By equating equation (34) and (37), the constants $a$ and $b$ involved in equation (36) can be determined. The same procedures of analysis can also be applied to wave period. From an analysis of measured and observed data obtained at Stations I and J, the constants are evaluated for wave height and period individually.

Figure 40 shows the relationship between significant wave height and visually observed height determined thereby. As can be seen in the figure, there is no serious difference in the relationships derived by using the data obtained at two different locations (Stations I and J). However, it is a general trend that the visually observed wave heights are less than the significant wave height, particularly in severe seas. The average of the two lines for Stations I and J is drawn in the figure, and this average line is expressed by

$$H_s = H_v^{\,1.08} \quad (38)$$

An analysis similar to that made for wave heights is carried out for the observed wave period and the measured zero-crossing periods at Stations I and J, and the results are shown in Fig. 41. As seen in the figure, there is practically no difference in the relationships between the observed and zero-crossing period derived by using the data obtained at Stations I and J. The average curve is drawn in the figure and from this the conversion formula can be derived.

Furthermore, the zero-crossing period, $T_0$, is converted to the modal wave period, $T_m$, defined as the wave period for which the wave spectrum peaks. Information of modal period is necessary to determine the two-parameter wave spectra defined in equation (4). The conversion can be made by using the theoretical relationship between the modal period and the zero-crossing period, assuming that the spectra are narrow-banded.

Thus, the conversions from the visually observed wave period to the zero-crossing and modal period are given as

$$\begin{aligned} T_0 &= 2.12\ T_v^{\,0.73} \\ T_m &= 1.41\ T_0 = 2.99\ T_v^{\,0.73} \end{aligned} \quad (39)$$

The conversion formulas given in equations (38) and (39), and used in the present study, are assumed to be applicable to all visually observed data presented in reference [4].

## Discussion

**N. Hogben,[4] Visitor**

First I would like to express my warm appreciation to Dr. Ochi for his valuable work on spectral modeling of sea conditions and its application to structure design and to welcome the presentation of this latest addition to the outstanding series of papers which he has written on this subject. Unfortunately, due to late arrival of the preprint, my comments have been drafted in some haste and are based on a rather superficial reading of the paper.

I am particularly interested in Part 2 of the paper, which is concerned with long-term statistics and makes use of visual wave data to assist the predictions. There are two main points which I wish to make. The first is to note that long-term statistics as experienced by ships in normal service are likely to contain a substantial fair-weather bias because of their tendency to dodge the more extreme conditions. This must be borne in mind when interpreting the weather station data used in the paper in application to ship design. This point is strongly supported by data in a paper of mine[5] comparing statistics from weather ships and from voluntary observing ships. I would be interested to know if Dr. Ochi has considered making any use of voluntary ship data which are available in very large quantities for most areas of practical interest.

My second point is to draw attention to the possibilities for use of wind in predicting long-term wave statistics, noting that visual observations of Beaufort number have been reported from ships in service since 1854. I am myself currently exploring this approach and a preliminary account of progress will shortly be available.[6]

[4] National Maritime Institute, Feltham, Middlesex, England.
[5] Hogben, N., "Sea State Studies at the Ship Hydrodynamics Laboratory, Feltham," *Marine Observer*, Vol. 33, July 1963.
[6] Hogben, N., "Wave Climate Synthesis for Engineering Purposes," National Maritime Institute Report No. NMI R45 (in preparation).

**Kai Kure,[7] Visitor**

The present paper by Dr. Ochi reflects his deep understanding of stochastic phenomena for the benefit of the whole profession. We are used to that from his earlier papers.

Many valuable aspects are included in the present paper. The emphasis given to the variability of spectra referring to a given sea state is very important for design application. Either this is purely probabilistic or semiprobabilistic as advocated, for example, by Faulkner and Sadden.[8]

Another important aspect is discussed by Dr. Ochi concerning the long-term estimation of extreme values versus the short-term estimation in extreme seaways. I read Dr. Ochi's paper to say that the contribution of tail areas of the peak distribution curves from lower sea states erroneously affects the extreme loads when using the long-term approach. These tail ends do not exist in reality and would produce particularly large errors for vessels operating mainly in sheltered water and only occasionally in the open sea.

For a semiprobabilistic design philosophy, I would like Dr. Ochi's opinion on the merits of the two approaches in terms of relative values of the appropriate partial safety coefficients on a characteristic load determined either way.

Many parameters are involved in prediction of loads due to waves. These are not only ship speed and heading and exposure time, but also loading condition and geographical area. The latter two are correlated in the load-line zones. It is interesting to note the resemblance of worldwide data with the North Atlantic data in the paper. However, wave pattern differences from, say, the North Sea to the North Atlantic Ocean must result in different responses.

[7] Danish Ship Research Laboratory, Lyngby, Denmark; chairman, ISSC Design Load Committee.
[8] Faulkner, D. and Sadden, J. A., "Towards a Unified Approach to Ship Structural Safety," Paper No. 3, RINA Spring Meeting, 1978.

I would like Dr. Ochi's comments on this and even more on the effect of parameter variability on responses. Would it be possible to adapt the concept of fuzzy logic to evaluate the variability due to human factors in selecting speed and heading?

The floating structure adopted for the response examples is laterally large. It seems to me, as outlined in a 1976 paper of mine,[9] that the short-crested character of seaways is particularly important for such structures and that azimuthal correlations must be considered in load predictions. I would like comments from Dr. Ochi on that question also, but I shall probably have to tame my curiosity to future works of the author. I am looking forward to them.

**Bruce L. Hutchison**, Member

In our consulting practice we are frequently concerned with determining the probability distributions of sea states from the data bases available in the literature. The author has made a most valuable contribution to the techniques to be applied to this process. In particular, the introduction of the bivariate log-normal probability distribution for significant wave height and zero-crossing period, and the families of probability-weighted sea spectra, is most welcome and should find immediate application in our practice.

We are often concerned with analyzing special cargo movements and individual voyages. In our analysis of the overall risk of encountering a sea state, we assume that the duration of the voyage may represent several independent trial samples of the underlying sea-state probability distribution. The number of trial samples, $n$, is equal to the duration of exposure divided by the independence time for the sea-state events. In a 1978 *Marine Technology* paper[10] this discusser presented the following expression for the overall risk of encountering a sea state with cumulative probability of exceedance, $\alpha$:

$$P\{H_{1/3} > H\} = 1 - (1 - \alpha)^n \approx n\alpha$$

for small $\alpha$.

Thus the overall risk of encountering the 1 percent sea state on a five-day voyage, assuming a 24-hr independence interval, would be about 5 percent. Has the author considered the effect of repeated independent trials in the overall risk of encounter for short-term analyses? Does he know of any data available suitable for estimating the independence interval for sea-state events?

Concerning the method for establishing a series of wave spectra suitable for long-term prediction as illustrated in Fig. 28, it is my understanding that the probability distribution along $\gamma$ = constant contours is uniform. If this interpretation is correct, then it would seem that an unbiased method for selecting the spectral parameter pairs from a $\gamma$-contour would space the points at equal intervals along the contour's length. The procedure shown does not lead to equal spacing along a contour. What is the nature of the probability distribution along a $\gamma$-contour?

**J. F. Dalzell**, Member

When the author presents a paper in this forum which does not contain useful methods or data, I shall be very surprised. The present paper is very much up to the author's usual standard.

Within the present state-of-the-art of prediction of wave-going characteristics of ships and ocean structures, the designer is faced with the problem of what to do about wave statistics and, more importantly, how to interpret and weight predicted results when he gets them. The author addresses the essential needs by:

(*a*) Providing specific values for the statistics of occurrence of wave heights and periods, including an elegant approach to the joint distribution, as well as estimates of probable extremes.

(*b*) Providing three relatively compact approaches to the variability of wave spectra as a function of significant height along with the rationally derived sets of weighting functions which are so necessary in making sense out of predictions.

With respect to the author's conclusion on the relative merits of short- and long-term approaches to extreme values, it seems obvious to this discusser that both must be sensitive to the weighting factor for the largest sea state, or, in a sense, to the validity of the statistics of the persistence of the extreme sea. In both cases it has to be assumed that the theoretical statistics of short-term response hold irrespective of the sample size. I have considerable reservations that this is true. One can only hope that the predictions resulting from such assumptions are conservative.

Although the author's emphasis in the paper is upon prediction of extremes, the material presented, especially the two families of scaler spectra, is equally if not more applicable to average performance predictions for design alternatives. The author notes that his current two-parameter family is "somewhat" different from that previously presented in reference [12]. I have to part company with the author on the "somewhat." There appears to be a quite significant difference in the context of performance predictions for smallish ships. The new two-parameter family seems much more in line with observed spectra than was the old.

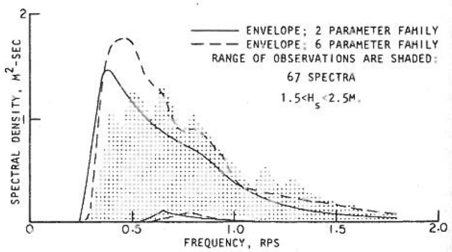


**Fig. 42** Comparison of envelopes of two- and six-parameter spectral families with observed spectra from reference [16] for significant heights between 1.5 and 2.5 m

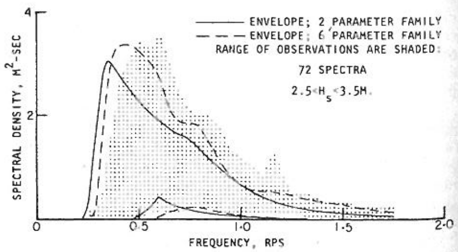


**Fig. 43** Comparison of envelopes of two- and six-parameter spectral families with observed spectra from reference [16] for significant heights between 2.5 and 3.5 m

I was initially less than convinced of the reasonableness of the two- and six-parameter spectral families by the examples shown in the paper because the comparisons involving the families as a whole are made indirectly by means of particular vehicle responses predicted for a sample of observed wave spectra and for the spectral families.

As an occasional practitioner, I was hoping to find verification that the families reasonably represent the variability of observed wave spectra, rather than the variability of particular responses computed for specific vehicles.

The accompanying Figs. 42 and 43 are the result of a brief (and incomplete) attempt to satisfy myself on this point. Since observed spectral data do not come in exact significant-wave-height sizes, some grouping is necessary. The shaded areas in the figures indicate the extreme spread of sprectral density reported in reference [16] for two ranges of significant wave height (1.5 to 2.5 m and 2.5 to 3.5 m) (4.9 to 8.2 ft and 8.2 to 11.4 ft). Upper and lower envelopes of spectral density for the two- and six-parameter families were obtained by computation of the spectral families over the range of significant wave height shown. I am unwilling to read too much detail into these results since, due to sampling variability, the extremes of observed spectral densities undoubtedly overemphasize the spread, and because the spectral families were derived from a much larger set of data. While there may be a tendency to overestimate the relative importance of long waves, the figures indicate that either spectral family reasonably well covers the range of a sample of previous spectral observations. In this connection, does the author have a personal preference for one family or the other? If both are used, can the author recommend methods of rationalizing any divergent results?

**Wolfgang Reuter**, Member

[The views expressed herein are the opinions of the discusser and not necessarily those of the Department of Defense or the Department of the Navy.]

I congratulate Dr. Ochi for dealing so well with a complex subject and I thank him for orienting his perspective toward application in design. This is particularly commendable because the transformation of R&D accomplishments into design technology have often taken our profession unduely long periods of time.

With the designs of the DD 963 and LHA-1 Class ships, the Navy took a major step toward improving its ship seakeeping design efforts. For the first time a performance specification was prepared requiring operational capabilities of certain missions in defined sea conditions and further requiring performance verification. At that time the convenient Pierson-Moskowitz spectral formulation was invoked and appropriate studies were accomplished with the eventual successful introduction of these ships. However, these design efforts did not go on without identification of several major problems specifically associated with swells and short-crested wind-generated seas. Hence, when the Naval Sea Systems Command sponsored a Seakeeping Workshop on the Integration of Seakeeping Technology into the Ship Design Process in June of 1975, the road map for a change in Navy design practice became more visible. R&D needs identified at that workshop and subsequent work have gone a long way toward resolving our design problems of the early 70's.

This brings us to the consideration of Dr. Ochi's work today and its contribution to future naval ship design. The ability to predict extreme values is but one part of our seakeeping design problems. However, the determination of the applicability of short-term versus long-term statistics to that design problem will have a significant impact on our ability to update our design practices without introducing unacceptable complexity into the design process. This will be accomplished by the application of short-term extreme-value statistics to the two-parameter spectral response data which are presently being generated for each new ship design.

However, this cannot be accomplished by Navy or industry without active participation by the users of these extreme-value data, specifically the structural and equipment designers. With improved ability to more accurately specify extreme values and limit loads, a corresponding improvement in their utilization in design must be accomplished.

I hope that next year we might see a paper on the practical application of extreme values and limit loads to the analysis of ship structures and hull-borne equipment accompanied by an analysis of the potential for reduced design margins.

**Edward N. Comstock**, Member

[The views expressed herein are the opinions of the discusser and not necessarily those of the Department of Defense or the Department of the Navy.]

In keeping with his outstanding reputation, Dr. Ochi has again provided the Society with an excellent dissertation on the subject of wave statistics for extreme-value and limit-load design. It is now dependent on the profession at large to integrate the results of this basic research into design practice through the updating of extreme-value and limit-load prediction procedures and analysis techniques. It is in this integration, however, that a number of technical questions and practical considerations arise which must be addressed.

In Dr. Ochi's discussion of the two-parameter spectrum, he describes the confidence coefficient domain (Fig. 4), an adaption of which, in conjunction with a specified cumulative probability, is presently being used at the Naval Ship Engineering Center for seakeeping performance assessment. The successful use of this approach, with the inherent weighting factor in the form of the joint probabilities of occurrence of significant wave height and modal period, raises questions regarding the weighting functions of modal period tabulated in the text and Fig. 5 of the paper. Where are these weighting functions from? What physical significance do they have, and what advantages are obtained over the use of the cumulative probabilities associated with the specified confidence coefficients?

A further question arises when comparing the boundary values of modal period for a given significant wave height derived from Fig. 6 with the confidence zone domains of Fig. 4. In effect, while the lower boundary values of modal period appear to correspond with a constant value of confidence coefficient, the upper boundary appears to cut across a substantial range, corresponding to widely varying values of joint probabilities of occurrence, an inconsistent and unsatisfactory condition. These comments should not obscure the fact, however, that the two-parameter spectrum and confidence coefficients based on observed, measured, or hindcasted spectra are most readily adapted to ship design.

Dr. Ochi's comments regarding the decomposition of wave spectra into "sea" and "swell" are wholeheartedly concurred with and have been the object of the Naval Sea Systems Command's Seakeeping R&D Program since its inception in 1976. However, the introduction of a six-parameter *point* spectrum to approximate both a "sea" with its associated spreading of energy over direction, and a "swell" with its probable variation in principal direction, is extremely cumbersome for use in design and, to some extent, illogical. At this time, there is insufficient data to justify this additional complexity while ignoring the first-order effect of short cresting.

[9] Kure, Kai, Discussion to "Prediction of Extreme Wave Induced Loads on Ocean Structure," by M. K. Ochi, First BOSS Conference, Trondheim, Norway, Vol. 2, 1976.

[10] Hutchison, Bruce L. and Bringloe, J. Thomas, "Application of Seakeeping Analysis," *Marine Technology*, Vol. 15, No. 4, Oct. 1978, pp. 416–431.

**William A. Cleary, Jr.,** Member

[The views expressed herein are the opinions of the discusser and not necessarily those of the U.S. Coast Guard.]

In the opinion of this discusser, this paper is a significant step toward efficient utilization of wave spectra in ship design and ocean engineering. All designers are faced with a choice as to the limits of design. Ocean or ship designers have the added difficulty of trying to describe the ocean as a support medium for their ship, which also creates variable loads on it. While the long-term seaway prediction method has proved to be excellent for the 1960's and 1970's, it has not offered direct proof that a single severe storm was sufficiently covered in design calculations. Dr. Ochi's paper seems to fill that gap.

The only constructive criticism that I have at this time is to question the use of 20 years for the highest significant height of seaway. It is perhaps only hearsay evidence but I have several times listened to comments that designers of offshore platforms for the North Sea have often been surprised that their design values for a 50- or 100-year storm have been exceeded in the first year or two of actual operation.

In the same area of concern, it is noted that the short-term extreme-value calculation produces design estimates similar to the long-term method. The author is using a confidence coefficient of 0.95 and he states that this reasonably covers the responses in ocean areas throughout the world. If the service time of the structure is assumed to be 50 years or more, will this increase the design extreme value in equation (17)?

In the section "Application to design," the author notes the influence of exposure time, risk over ship's lifetime, occurrence of high seas, and vessel speed in seaway, all of which are used in equation (16). The philosophy of ship design currently in force under international law and more specifically in the International Load Line Convention would not allow the approach in equation (16) unless "all oceans" data only were used in the formula. The reason for this is that when any ocean structure receives an international load line certificate from any administration it is allowed to go anywhere on the ocean for the next five years.

Formula (16) may be of great use in evaluating future international proposals for limited ocean area fixed structures. Before it is used for evaluating movable ocean structures, a new international legal philosophy must be conceived and adopted.

**Walter H. Michel,** Member

Once again Dr. Ochi has produced a work of great scope and valuable content. It is apparent that a tremendous amount of data evaluation and analysis was required and that we are only seeing the tip of the iceberg. One is tempted to ask for more details out of curious interest, but, as with the iceberg, it is more prudent to stand off, in full assurance that the rest of it is there.

Of particular interest and extreme value for engineering application is the author's synthesis of the vast amount of North Atlantic data into a well-defined range of sea spectra, for both the more commonly used two-parameter formula and his own more comprehensive six-parameter family, as given in Tables 3 and 4, respectively. It is also of interest to see that available worldwide spectra fit within the same confines, as illustrated in Fig. 19 of the paper. Can Dr. Ochi affirm that this latter observation is true in a general sense, and not only for such particular cases as the design example used herein?

In regard to the design example, it is noted that the bounds of probable force are narrower for the six-parameter spectrum and within the bounds of the two-parameter spectrum, particularly in the range of the higher waves. A study of the spectral shapes as given in Figs. 8 and 14 will support this finding, in consideration of the fact that the forcing function for the particular semisubmersible in beam seas is maximum at a frequency of about 0.8 rps. It should be realized, however, that the situation can be reversed for cases where the maximum response would be closer to the peak frequency of the spectra, insofar as the members of the six-parameter family are seen to have higher energy content in this region.

As to the six-parameter spectrum itself, it is a more accurate representation than the two-parameter spectrum and is particularly useful for properly analyzing average vessel response in moderate seas and swells where double-peaked spectra are more likely to occur. It is noted that for high to extreme seas, however, that the influence of the second peak greatly diminishes, until one can reasonably adopt a single three-parameter relationship for such spectra as shown in Fig. 12. That makes life a little easier for the designer.

The paper is convincing in the development of spectral values that are as close to real life as one can presently expect for the ocean areas studied. Unfortunately, the same cannot be said for the determination of extreme values for design, as addressed in the second half of the paper. Use is made of theoretical statistics for a linear process, which have had little if any confirmation by data comparison for the gravity wave system that exists, which is highly nonlinear in the range of extreme heights. If the proposed factors of risk and lifetime encounters are included, along with the upper bound of 95 percent spectrum probability, we have a situation that is definitely overkill.

For illustration, consider a significant wave height of 50 ft (15.24 m) for the two-parameter spectrum. If we apply equation (17) and use values of its parameters as given in the paper, the design extreme wave height would be 125 ft (38.1 m) and at a steepness beyond the breaking wave limit. To my knowledge, there are no marine vessels designed to such severe criteria and no data, experimental or otherwise, to confirm its existence.

Let me say that this situation is not the fault of Dr. Ochi. The sea data that oceanographers and statisticians compile almost never cite the singular value of the maximum wave height recorded. Apparently it is simply thrown into the general averages that define the basic spectrum, and thereafter a theoretical projection is made that tells what the maximum wave height should be, with almost complete disregard of what it was found to be.

It is hoped that this situation will be rectified, and also that when sufficient data on maximum wave heights are compiled, Dr. Ochi will be commissioned to use his masterful techniques to give the rest of us as meaningful an evaluation of extreme values as he has given here on spectra.

**Manley St. Denis,** Member

I am pleased that the author has made the most of what is still, in spite of active developments in the field due in large part to his enterprise and enthusiasm, a rather poorly developed area of research. This is due in part because of the complexity of the problem, but mostly because of the sparsity of observations required to validate the underlying theory and to turn speculations into a practical technique for making useful predictions.

The finding that in the North Atlantic Zone of the westerlies the significant wave height and zero-crossing period are closely fitted by the bivariate logarithmic-normal distribution is perhaps fortuitous but certainly welcome, for this results in a powerful simplification in the analysis of the observational material and in the prediction technique. However, the observations fitted by such a distribution span an interval of a decade, and such a relatively short time interval is insufficient to provide a firm statistical fit. The author is wise to avoid arguing the validity of the logarithmic-normal law beyond the range of the data (as, for example, Jasper did in his 1956 paper); for reason tells us that the long-term distribution of seaway characteristics must eventually be bounded, and the logarithmic-normal distribution is not. The author does not discuss very much the tail of the probability distribution, that is, the range in probabilities of occurrence greater than 0.99, reserving, I suspect, the pleasure of doing this in a future paper. Yet it is this range that must interest the ship designer, for the ship must be endowed with the capability to survive with a high degree of reliability the severest wave she is likely to encounter during her lifetime. There is much to develop as yet to achieve a reliable prediction of the characteristics of seaways of extreme intensities.

This brings me to my second point: The author transforms the wave characteristics of significant height and zero-crossing period into a variance spectrum by the original two-parameter formulation due to Bretschneider. He is aware, of course, that such a two-parameter spectrum is not reliably representative of any seaway: that is, indeed, the reason he developed his more flexible six-parameter spectrum. Ultimately, he must extend the present work so as to employ his six-parameter spectrum, but this promises to be a most laborious task, and the instrumented observation of waves to yield the six-parameter spectrum promises to be one of rather formidable difficulty.

But I must observe that the representation of the seaway by a variance spectrum is valid only when a set of three conditions is fulfilled, namely, that the sea is composed of linear waves, that it is steady, and that it is Gaussian. Only light seaways fulfill these conditions, but when the severity of the seaway is high, none of these conditions is fulfilled and the variance spectrum may be at best only a partial representation of the seaway, or perhaps may have to be discarded completely.

A final point: For design analysis it is necessary to go beyond significant wave height and some related characteristic period (zero-crossing, modal, etc.) and to derive the long-term extreme values of these characteristics. These involve in part a transformation from significant wave height to short-term maximum height. The aggradation factor by which the former must be multiplied to obtain the latter depends on the persistence of the sea state and on the confusion level of the sea. The author has treated each of these aspects in previous papers, but has included only the first of these in the present paper. I am sure that in a future paper he will include both aspects.

To summarize, the paper is a remarkable one in that it satisfies in part our needs but also in that it whets our desire for more knowledge.

In closing, I should like to claim partial credit for this paper and for the preceding ones that the author has presented to the Society, for it was I who was instrumental in keeping Dr. Ochi in America. And I hope that our esteemed presiding officer and vice president take notice of this and see to it that the Society issue me a Certificate of Appreciation for having helped Dr. Ochi remain in America.

**Rodney T. Schmitke,** Member

Dr. Ochi has again made a significant contribution to the field of seakeeping research. I would like to raise one point.

Dr. Ochi has shown a number of slides comparing analytical spectral formulations with measured spectra. In general, the six-parameter spectrum gives a much better representation of the measurement than the two-parameter spectrum. I would like to point out, however, that there are other two-parameter formulations which may well represent measured spectra better. In particular, I cite the spectrum developed by Gospodnetic and Miles by regression analysis of data obtained at Station India in the North Atlantic. I invite the author's comments on this point.

**Edward V. Lewis,** Member

This is an outstanding paper because it provides a comprehensive comparison of two distinct approaches to obtaining design loads of ocean structures: considering the complete population of load cycles and considering the load cycles only for very severe sea conditions.

The latter approach has several dangers:

1. We may not identify the most extreme sea.
2. The extreme load may not occur in the extreme sea.

This paper shows how to avoid these pitfalls and concludes that the two approaches give comparable results.

Dr. Ochi shows a number of comparisons of the shapes of different ocean wave spectra (for example, Figs. 11–14). I urge that he—and others engaged in such studies—investigate the use of slope (or acceleration) spectra for such purposes. Spectra plotted in this form may be easier to study and interpret, since they do not have the usual sharp peak of an exponential curve.

The six-parameter spectrum family is a very ingenious development, but I should like to offer one word of caution. The existence of double peaks suggests the presence of two separate storm and/or swell systems. Hence, it is likely that they are moving from two different directions. We are led inevitably to the need for further refinement by dealing with directional spectra, which will have a significant effect on response. This approach has become a real possibility through the use of FNWC spectral hindcasts, which are directional in character.

In determining long-term probabilities, it should be noted that another approach not clearly spelled out is to derive the long-term cumulative distribution of individual load cycles from the density function—integrating Fig. 30, for example. This will yield an estimate of the load expected to be exceeded once in a ship's lifetime—or in the lifetimes of 100 or 1000 ships—as I have discussed elsewhere. The result of this other approach should perhaps be compared with the methods discussed in the paper. Furthermore, I believe this approach is essential not only for fatigue considerations but for reliability evaluations leading to an estimate of the failure probability.

Finally, using the extreme-value approach, I think it is essential to give practical significance to the parameter $\alpha$. It is my understanding that it can be considered as a lifetime probability that is considered acceptable—that is, 0.01 corresponds to one ship in 100.

**Stanley G. Stiansen,** Member

The author is to be congratulated for his comprehensive presentation of the North Atlantic wave data analysis and the new approach for predicting responses of ships and marine structures in a seaway. Of particular interest is the example where the probable extreme transverse force, computed for a semisubmersible-type ocean platform using worldwide measured wave spectra, falls between the upper and lower bounds computed based on the wave spectral families and the short-term extreme methods suggested in the paper. It is interesting to note that the author's use of North Atlantic Ocean data to establish the wave spectral family used for design is consistent with the ABS application of the so-called H-family wave data for evaluating a ship's long-term response.

The H-family, which is also based on North Atlantic data, consists of five significant wave heights groups, namely, 10, 20, 30, 40, and 48.2 ft (3, 6, 9, 12, and 14.6 m). There are 10 spectral members in each group except the last group, which has 12 member spectra. The probability of occurrence (weighting factor for sea condition) of these five wave groups is 84.54, 13.30, 2.01, 0.14, and 0.01 percent, respectively. This is somewhat different from the wave group and percentage of occurrence given in Table 6 of the paper.

**Table 7**

| Ships | M. K. Ochi's Short-Term Extreme: Probable Value (ton-M) | M. K. Ochi's Short-Term Extreme: Design Value (ton-M) | M. K. Ochi's Long-Term Response: Probable Value (ton-M) | M. K. Ochi's Long-Term Response: Design Value (ton-M) |
|---|---|---|---|---|
| 1 | $0.82 \times 10^5$ | $1.12 \times 10^5$ | $0.90 \times 10^5$ | $1.19 \times 10^5$ |
| 2 | $4.00 \times 10^5$ | $5.44 \times 10^5$ | $4.90 \times 10^5$ | $6.40 \times 10^5$ |
| 3 | $9.43 \times 10^5$ | $12.85 \times 10^5$ | $8.60 \times 10^5$ | $12.00 \times 10^5$ |

**Table 8**

| Ships | ABS Long-Term Prediction: Short-Crested Sea (ton-M) | ABS Long-Term Prediction: Long-Crested Sea (ton-M) |
|---|---|---|
| 1 | $0.66 \times 10^5$ | $0.76 \times 10^5$ |
| 2 | $3.42 \times 10^5$ | $3.88 \times 10^5$ |
| 3 | $7.90 \times 10^5$ | $8.92 \times 10^5$ |

**Table 9**

| Ships | ABS Long-Term Short-Crested Sea (tons/cm²) | M. K. Ochi's Short-Term Design Extreme (tons/cm²) | ABS Permissible Stress (tons/cm²) |
|---|---|---|---|
| 1 | 1.30 | 1.83 | 1.61 |
| 2 | 1.33 | 1.95 | 1.67 |
| 3 | 1.37 | 1.95 | 1.68 |

The wave spectral family and the method for calculating extreme responses of ships and other marine structures suggested by the author have been applied to three existing oceangoing ships, designated as Ships 1, 2 and 3. Ship 1 is a Mariner class cargo ship 161 m (528 ft) long, Ship 2 is a bulk carrier 249 m (817 ft) long, and Ship 3 is an oil tanker 310 m (1017 ft) long. In addition, ABS long-term prediction for these three ships has also been calculated using H-family wave data. A comparison of the two methods was made and is summarized as follows:

1. Using the author's six-parameter wave spectral family and a risk parameter of 0.01, the short-term extreme and the long-term response techniques suggested in the paper, the vertical bending moments amidships calculated for the three ships are given in Table 7, where an equally distributed weighting factor for heading to waves is assumed. Also, two thirds of ships' 20-year lifetime is assumed to be at sea.

2. The vertical bending moments amidships of the three ships, calculated using the H-family wave data and the ABS long-term prediction, are given in the Table 8 for two cases: one with the cosine-square spreading function to simulate the short-crestedness of the seaway, and the other for long-crested waves. The vertical bending moments amidships at a probability level of $10^{-8}$ are given in the table. In these calculations, each spectral member in a significant wave height group is equally weighted. The calculated results of the short-crested sea agree very well with ABS Rule requirements, while those of the long-crested sea are comparable with the results for the probable values in Table 7, in which the long-crested sea is considered.

3. The stillwater bending moments for these ships are 0.47, 0.97, and $3.63 \times 10^5$ ton-M, respectively. Combining the stillwater bending moments with the results in Tables 7 and 8, we arrive at the total vertical bending stresses amidships as given in Table 9.

In conclusion, as shown by the calculations in Tables 7 and 8, the results based on the probable extreme approach suggested in the paper are comparable to the ABS long-term prediction for similar sea conditions. As evident from the comparison, the question of whether a design should be based on the probable extreme value or the "design extreme value" should be considered in conjunction with a predetermined allowable stress. It should be noted that the current longitudinal strength standard has proven successful with many ship-years of service experience. A new approach in estimating the extreme loads could not be utilized as a basis to justify an increase of strength requirements.

## Author's Closure

I would first like to take up the subject concerning the estimation of design values through the short-term prediction approach as opposed to that through the long-term approach, since this subject was apparently of interest to many discussers.

I am grateful to Mr. Reuter for his remarks that, as far as the evaluation of extreme responses is concerned, we may update our design practices without introducing unacceptable complexity into the design process.

The disadvantage of applying the long-term prediction method in estimating the extreme value is that a considerable percentage of small-magnitude responses in relatively mild seas (which apparently do not contribute to the extreme value) is included in the estimation. The magnitude of ship response will not reach a level critical for safe navigation irrespective of how long the ship operates in mild seas, while the magnitude of responses will reach the critical level within a few hours in severe seas. In other words, severe seas mask (or nullify) the contribution of mild seas to extreme values. Therefore, as far as estimation of extreme values is concerned, it appears to be appropriate to consider only severe seas, and apply the short-term prediction method.

I would like to add the following remark to the section "Application to design" of Part 2 of the paper. At the end of the section it is stated that the design extreme value should be estimated in severe seas expected in the service area of a ship (or ocean platform). This should read that the short-term prediction should be carried out in seas *up to* the severest seas expected in the service area. This precaution is necessary since the extreme response may not necessarily occur in the severest sea condition as is pointed out by Professor Lewis. For instance, the extreme value of the bow vertical acceleration of the Mariner occurs in seas of significant wave height 25 ft (7.6 m), far below the severest sea condition used in the computation [12]. This is due to speed loss and the response characteristics in severe seas.

Mr. Stiansen shows a comparison of design extreme bending stress of the Mariner of 1.83 ton/cm² following the author's short-term approach versus an extreme bending stress of 1.30 ton/cm² computed by the ABS long-term short-crested sea approach. The Mariner's design extreme midship bending moment was computed earlier by the author, where a value of $0.95 \times 10^5$ ton-M was obtained.[11] This leads to a design extreme bending stress of 1.55 ton/cm², about 15 percent less than that given in Mr. Stiansen's discussion. This value is quite reasonable in comparison with the ABS permissible stress of 1.61 ton/cm². The extreme bending moment computed by the ABS long-term prediction method is determined at probability level of $10^{-8}$, which is an arbitrarily chosen number. Equation (27) of the paper provides a precise number of responses expected in the lifetime of a ship. The author would like to suggest that the ABS long-term prediction method should be modified incorporating the number of response cycles, which has not been

[11] Ochi, M. K., "Concept of Probabilistic Extreme Values and its Applications," *Proceedings*, International Symposium on Practical Design in Shipbuilding, 1977.



considered in most of the long-term prediction methods developed to date.

Dr. Hogben raises a question concerning the utilization of wave information accumulated by voluntary observations made by ships on service routes as well as the utilization of wind data for predicting long-term wave statistics. In general, data provided by voluntary ships seem to be unreliable due to a substantial fair-weather bias because of their tendency to avoid severe seas. In this regard, data accumulated by weather ships stationed at various locations in the ocean are not biased, and thereby it is appropriate to statistically analyze these data. In connection with the possibility of using wind data in predicting long-term wave statistics, it seems to be necessary to accumulate data involving simultaneous observations of wind and waves before any prediction is attempted. There is certainly a general statistical trend between wind and waves as is demonstrated by Bouws.[12]

Mr. Kure questions the relative merits of the short- and long-term approaches in terms of the safety coefficients from the viewpoint of a semiprobabilistic design philosophy. In the numerical example, the same risk parameter, $\alpha$, is used for both short- and long-term approaches, and exactly the same design values are derived. Hence, the relative merits of the two approaches in terms of the safety coefficients appear to be the same.

Mr. Kure raises a question concerning the effect of differences in wave patterns for some geographical areas on responses. This is certainly an important subject to be considered. For instance, the ship responses may differ in the North Sea from those in the North Atlantic Ocean because the effect of fetch has to be considered for wave patterns in the North Sea. The families of wave spectra presented in this paper are for open seas. The author has been developing another family of wave spectra applicable for fetch-limited seas, and it is anticipated that the results will be published in the very near future.

Mr. Kure suggests that since the width of an ocean structure is large, the evaluation of responses should be made in short-crested seas, taking into account possible correlation between directional components of the sea surface. Apart from the complexity involved in the evaluation, there is no doubt that his suggested approach is valid for a more sophisticated and accurate evaluation of responses of an ocean structure in a seaway.

Mr. Hutchison questions whether or not the effect of repeated encounters with a specified sea is included in the short-term prediction. The number of encounters with a specified sea in the ship's lifetime is considered in the evaluation of the design extreme value. This is the factor, $k$, in equation (17) of the paper, and this factor is newly added to the formula which was derived previously by the author. The value of $k$ is determined from information on the longest persistence of a given significant wave height which may be found in Fig. 15.

In regard to the determination of a series of wave spectra for long-term prediction, Mr. Hutchison requests elaboration of the probability distribution along the $\gamma$-contour line given in Fig. 28 of the paper. The $\gamma$-line in the figure provides the information that the probability of occurrence of the combination of wave height and period *inside* the line is a certain value, 0.99 for example. However, no probability can be assigned to the $\gamma$-line. Polar coordinates are used for the joint probability function of wave height and period as shown in equations (22) through (25). Therefore, pairs of significant wave heights and zero-crossing periods are chosen not at equal intervals along the $\gamma$-line; instead, they are chosen for equal angles around the origin which represents the modal values of significant wave height and period.

Mr. Dalzell comments that the two-parameter family of wave spectra presented in this paper is significantly (not "somewhat") different from the two-parameter family previously presented by the author in 1977. I acknowledge that there is a substantial difference between the families, particularly in mild seas; hence, the magnitude of predicted responses of small-size ships by using the newly developed two-parameter family may differ significantly from those predicted by using the previous one.

As stated in the paper, the two-parameter family presented in this paper uses a newly developed conversion factor from the visually observed data to significant wave height. For the previously developed two-parameter family, Nordenstrom's conversion factor was used. The new conversion factor yields a series of spectra much more in line with observed spectra than the previous one as commented on by Mr. Dalzell.

In connection with verification that the two-parameter and six-parameter families reasonably cover the variability of observed wave spectra, Mr. Dalzell has shown comparison of the envelopes of the two- and six-parameter spectral families with observed spectra. The results of his comparison indicate that either spectral family reasonably well covers the range of variation of the observed spectra. This result may give the impression that these two families are equally applicable to be used for design consideration. I would like to note, however, that the good coverage of the envelope of both the two-parameter and six-parameter families does not mean good representation of the individual spectra by both the two- and six-parameter spectral formulations. In fact, more than 70 percent of a total of 800 observed spectra cannot be represented adequately by the two-parameter spectral formulations, as contrasted with almost all of the observed spectra being represented satisfactorily by the six-parameter formulation. In other words, due to the physical complexity involved in the generation of wind-generated waves, the shape of wave spectra observed in the ocean varies considerably (even though the sea severities are the same), and hence it appears that the variability of spectral shapes cannot be represented satisfactorily in terms of only two parameters.

Mr. Comstock raised a question regarding the derivation of the weighting factor for each modal period. The derivation is a routine method in statistics. For example, in the case of the two modal periods for a confidence coefficient 0.95, each has a weight 0.05/2 at the tail portion of the density function, and each has a weight 0.05/2 which is shared with the period for a confidence coefficient 0.85. Hence, each modal period for a confidence coefficient of 0.95 has a total weighting factor of 0.05. The physical meaning of the factor is that a total of 10 percent of the shapes of wave spectra observed in a given sea state may have modal periods very close to the two values corresponding to the confidence coefficient of 0.95.

Mr. Comstock believes an inconsistency exists in the value of the modal periods from a comparison of Figs. 4 and 6. Unfortunately, this question is raised due to his misunderstanding of the figures. Figure 4 shows the confidence domains established for the joint distribution of significant wave height and modal period, while Fig. 6 shows confidence domains of the modal period for a given significant wave height, namely, a conditional distribution. Hence, these two figures cannot be compared directly.

In regard to the family of six-parameter wave spectra, some discussers believe that the spectra are decomposed into two parts: sea and swell. This is not correct. In the development of the six-parameter spectral representation, each spectrum is decomposed into two parts, one which includes primarily the lower-frequency components of the energy, the other which covers primarily the higher-frequency components of the en-

[12] Bouws, E., "Wind and Wave Climate in the Netherlands Sector of the North Sea Between 53° and 54° North Latitude," Koninklijk Nederlands Meterologisch Instituut, Scientific Report 78–9, 1978.

ergy. The lower-frequency part is not necessarily associated with swell. A detailed discussion on this subject is given in reference [13].

Mr. Cleary questions the use of 20 years for the highest significant wave height of the seaway, and he suggests that the service time of the structure should be 50 years or more for an ocean structure. He asks if this would increase the design extreme value given in equation (17). The numerical example given in this paper is evaluated for 20 years of service, and indeed it appears to be too short a service time for an ocean structure. The design extreme value given in equation (17) carries a parameter, $k$, which reflects the number of encounters with a specified sea in the ocean structure's lifetime. Therefore, the $k$-value for 50 years of service is 2.5 times of that for the 20 years of service, and this leads to increase in the design value. In the case of the ocean structure for which the numerical example is presented in this paper, the design value for 50 years of service will be approximately 6 percent more than that for 20 years of service.

Mr. Cleary introduces an interesting practical problem. He states that the designers of offshore platforms for the North Sea have often been surprised that their design value for a 50-year or 100-year storm has been exceeded in the first one or two years of actual operation. I would like to remark that the value used in their design has been evaluated based on the severest sea state expected in a 50- or 100-year storm, but their estimation is based on the most probable extreme value defined in this paper. Theoretically, it can be proved that the largest value of the response will exceed the estimated probable extreme value with a probability of 63.2 percent. Therefore, it is not surprising that the design value for a 50- or 100-year storm was exceeded in the first one or two years of operation. To overcome this problem, the formula to evaluate the extreme value for design use presented in this paper carries two parameters, $\alpha$ and $k$, as given in equation (17), and the design extreme value is significantly larger than the probable extreme value which has been considered to date.

Mr. Michel asks if the results shown in Fig. 19 hold in a general sense. The trend observed in Fig. 19 appears to be generally true judging from many other examples presented in reference [12], although the modal frequencies of the two-parameter family used in reference [12] are different from those given in this paper.

Mr. Michel raises an interesting discussion on extreme wave height. He considers the incidental extreme wave height for design and points out that it would be as high as 125 ft in a sea of significant wave height of 50 ft. It is not surprising that a single extreme wave height of 100 ft or higher may actually occur in open seas of significant wave height of 50 ft. However, the author would like to point out that it is not appropriate to consider an incidental extreme wave height alone; instead, the wavelength associated with this extreme wave height should be considered for design.

Professor Lewis suggests the use of the cumulative distribution of individual load cycles in determining long-term probabilities, since this yields an estimate of the load expected to be exceeded once in a ship's lifetime. I cannot concur with this concept of the necessity of the cumulative distribution function. This is because the load expected to be exceeded once in a ship's lifetime is merely equal to the most probable extreme value, which is simply estimated through the short-term prediction approach as is discussed in the paper.

Mr. Schmitke asks if some spectral formulations other than the two presented in the paper were considered in the study. He points out the Gospodnetic and Miles spectral formulation as an example. The author has considered various spectral formulations in the course of this study. However, the Gospodnetic and Miles spectral formulation is not appropriate for establishing a family of wave spectra since it contains too many parameters.

Finally, the author sincerely appreciates Professor St. Denis's discussion, which comprehensively summarizes all key problems required to be clarified in the future.



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# Analysis and Control of Distortion in Welded Aluminum Structures

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Results of recent investigations at M.I.T. on the subject of analysis and control of distortion in welded aluminum structures are summarized. After a brief overview of the distortion problem as a whole, three kinds of welding distortion are analyzed: longitudinal bending distortion, out-of-plane angular distortion, and buckling distortion. Experimental results are presented and compared with predictions obtained by computer programs developed at M.I.T. Methods for the control of these kinds of distortion are proposed. They include the elastic prestraining method, the differential heating methods, and the application of tensile stresses during welding. It is concluded that the computer programs developed so far can, with proper modifications, be very useful to designers and fabricators in dealing with the distortion problem. They can be part of an integrated system capable of determining the optimum values for the parameters of the problem, taking into account structural hydrodynamics and fabrication considerations.

## Introduction

ONE of the most troublesome problems a welding fabricator faces today is that of weld distortion. The nonuniform heating and cooling cycle, which occurs in the weld and the adjacent base metal, causes the development of complex strains during welding. Their respective stresses combine and react to produce internal forces that can cause bending, rotation, or buckling, or all three. These dimensional changes are collectively known as welding shrinkage distortion.

During the past 30 years, most critical structures such as ships and submarines were built of steel. As a consequence, huge amounts of empirical information was gathered on the various kinds of welding distortion encountered during the fabrication of steel structures. At the same time, analytical investigations were carried out in an effort to gain better understanding of the problems.

Lately, however, the interest in using aluminum extensively as a structural material has increased. As examples, one can mention the building by the U. S. Navy of the surface-effect ships (SES) or the construction of cryogenic tanks for LNG carriers. This occurred due to the many desirable properties aluminum possesses, such as high strength-to-weight ratio and excellent fracture toughness at low temperatures. Unfortunately, however, the distortion problems in welding aluminum are much more severe than those of steel, for the following reasons:

1. Compared with steel, aluminum has a higher heat conductivity.
2. Aluminum has about twice the coefficient of thermal expansion of steel.
3. The modulus of elasticity of aluminum is one-third that of steel.

Little information can, accordingly, be transferred directly from the knowledge of welding distortion of steel structures.

The foregoing facts gave the incentive to investigators at M.I.T. to carry out a thorough fundamental study on distortion in welded aluminum structures. The three-year investigation was first directed toward an understanding of the problems through the conduction of experiments and the development of analytical means to predict distortion. After successful completion of this task, the effort was then directed toward finding and analyzing ways of distortion control. Several methods were tried such as clamping, elastic-plastic prestraining, and differential heating.

This paper focuses on the problem of distortion in welded structures, especially in aluminum welded structures. An effort is first made to analyze the overall distortion problem and to consider its causes and consequences. Various methods for the determination of distortion are outlined and then the analytical simulation method which was used throughout the investigation is explained. Six different computer programs developed for this study are summarized.

Three different weld distortion problems are selected to illustrate the method of analysis. They are the following: longitudinal bending distortion of built-up beams, out-of-plane distortion of panel structures, and buckling distortion of thin rectangular plates. These types of distortion were selected because they are typically found in ship structures. Experimental results are presented and compared with predictions made by the various computer programs. Finally, various ways for controlling distortion are proposed and tested against experimental evidence.

To avoid any duplication, it is assumed that the readers of this paper are familiar with, or have access to, Welding Research Council Bulletins 149 and 174, published in 1970 and 1972, respectively [1, 2].[3]

## General overview of the distortion problem

In analyzing the structural and material strength of a specific structure, it is a common practice to make various assumptions which usually lead to a substantial simplification of the problem

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[3] Numbers in brackets designate References at end of paper.