



Aalto University
School of Electrical
Engineering

ELEC-E8126: Robotic Manipulation

Kinematic redundancy

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16.3.2020

Learning goals

- Understand modeling and characteristics of redundant kinematic chains.
- Understand how redundancy can be used to address e.g. singularities, joint limits or obstacles.

Kinematic redundancy

- *Kinematically redundant* manipulator has more than minimal number of degrees of freedom to complete its task.
 - Thus, same task configuration can be achieved with infinitely many joint configurations.
- Why are kinematically redundant manipulators interesting?

Kinematic redundancy

- *Kinematically redundant* manipulator has more than minimal number of degrees of freedom to complete its task.
 - Thus, same task configuration can be achieved with infinitely many joint configurations.
- Why are kinematically redundant manipulators interesting?
 - Secondary tasks: e.g. avoid singularities, avoid joint limits, avoid obstacles, optimize motion.

Example: 6-DOF manipulator, translation task

- 6-DOF serial manipulator
- Only translation of e-e needs to be controlled in position.
 - Orientation can be ignored.
- How many degrees of motion does the robot have?
- How many are constrained by task?
- Is the system redundant?

Inverse differential kinematics

- Remember: Forward differential kinematics

$$\dot{\mathbf{x}} = J(\boldsymbol{\theta}) \dot{\boldsymbol{\theta}}$$

- What is the inverse of this?
- When is it non-unique?
- What are the other solutions?

Inverse differential kinematics

- Remember: Forward differential kinematics

$$\dot{\mathbf{x}} = J(\boldsymbol{\theta}) \dot{\boldsymbol{\theta}}$$

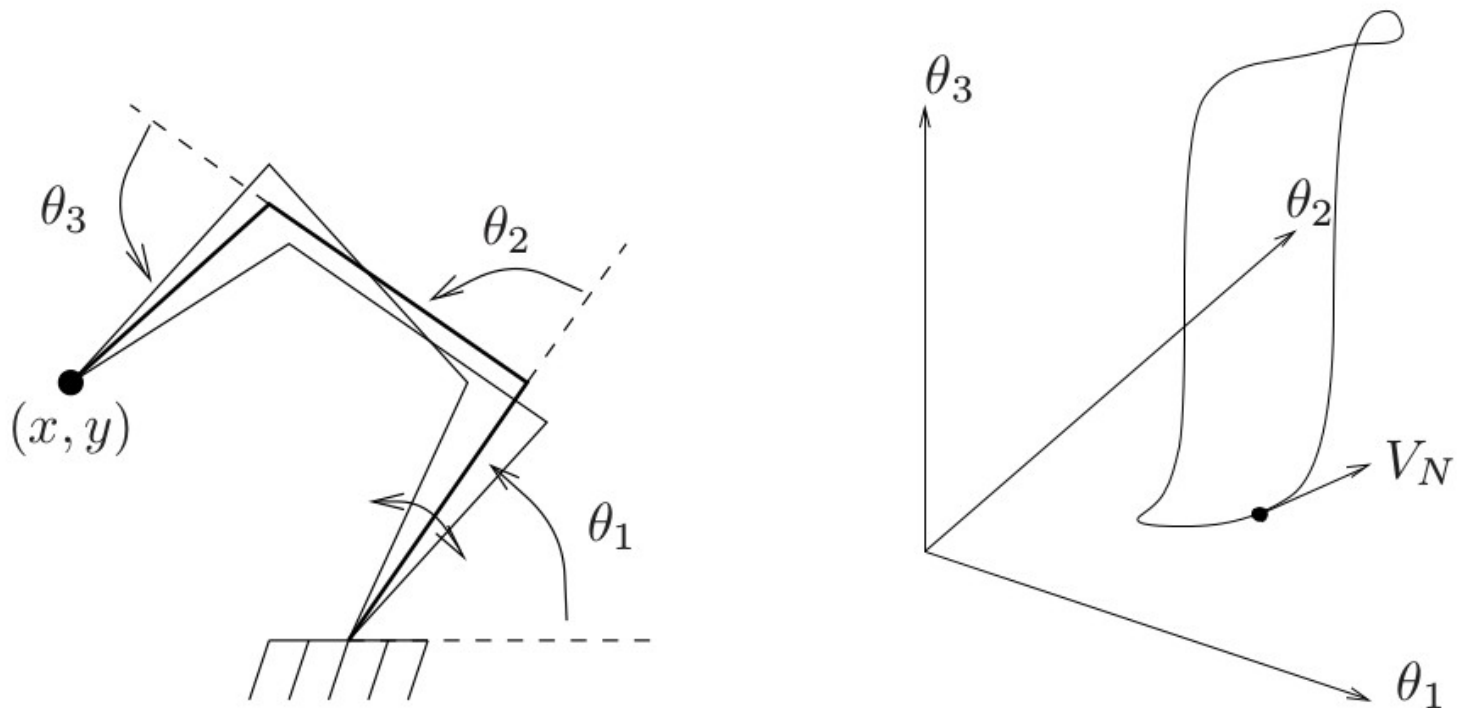
- What is the inverse of this? $\dot{\boldsymbol{\theta}} = J^{-1}(\boldsymbol{\theta}) \dot{\mathbf{x}}$ $\dot{\boldsymbol{\theta}} = J^+(\boldsymbol{\theta}) \dot{\mathbf{x}}$
- When is it non-unique?
- What are the other solutions?

$$\dot{\boldsymbol{\theta}} = J^+(\boldsymbol{\theta}) \dot{\mathbf{x}} + (I - J^+(\boldsymbol{\theta}) J(\boldsymbol{\theta})) \dot{\boldsymbol{\theta}}_0$$


anything

Internal (self) motion example

- Task: 2-D position.



Using internal motions

- Why did we want internal motions?
- How? Two approaches:
 - Optimize performance criteria.  We'll look at this a bit closer.
 - Add more tasks.
- Both approaches only move in null space of primary task.

$$\dot{\theta} = J^+(\theta)\dot{x} + \boxed{(I - J^+(\theta)J(\theta))\dot{\theta}_0}$$

Optimizing performance criteria

- Consider we want to minimize some joint-dependent criterion $H(\theta)$ that can be expressed analytically
- How to write a controller to move joints towards minimum of H ?

$$\dot{\theta} = J^+(\theta)\dot{x} + (I - J^+(\theta)J(\theta))\dot{\theta}_0$$

Optimizing performance criteria

- Consider we want to minimize some joint-dependent criterion $H(\theta)$ that can be expressed analytically
- How to write a controller to move joints towards minimum of H ?

$$\dot{\theta} = -k_H \nabla H(\theta)$$

- Now substitute to velocity controller:

$$\dot{\theta} = J^+(\theta)\dot{x} - k_H (I - J^+(\theta)J(\theta))\nabla H(\theta)$$

Performance criteria examples

- Joint-limit avoidance
 - Propose criteria!
- Singularity avoidance
 - E.g. manipulability

$$H(\boldsymbol{\theta}) = \sqrt{|J(\boldsymbol{\theta})J^T(\boldsymbol{\theta})|}$$

Connection: In-hand motions / Kinematic and actuator redundancies

- Remember the grasping constraint?

$$J \dot{\theta} = G^T V_o$$

- Kinematic redundancy – null space of J.
 - Internal motions.
- Actuator redundancy – null space of G.
 - Internal forces.

Summary

- Redundancies can be used to resolve additional tasks without sacrificing primary task.
- Redundancies are especially useful to avoid joint limits and singularities.

Next time: Learning in manipulation

- Readings:
 - Peters et al., “Robot Learning”, in Springer Handbook of Robotics, 2nd ed., secs. 15-15.1.2, 15.4
 - Brock et al., “Mobility and Manipulation”, in Springer Handbook of Robotics, 2nd ed., secs. 40, 40.4, 40.4.2-40.4.3
 - Handbook freely available through library webpage lib.aalto.fi. Log-in first and then search for “Springer Handbook of Robotics”.