Instructions for exercise 6:
Superconducting niobium cavity

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Abstract

The most prominent consequence of superconductivity is the disappearance of electrical resistance at temperatures below the superconducting transition temperature, also known as the critical temperature T_c. However, at temperatures only somewhat below T_c, remaining normal-conducting electrons can cause observable resistive ("Ohmic") losses when time-dependent electromagnetic fields are present. In this assignment, we will measure energy losses in a superconducting microwave-frequency cavity resonator at temperatures down to 4 K by dunking it into liquid helium in a test cryostat. The measurement range crosses $T_c \sim 8$ K of niobium, hence allowing for inferring a temperature-dependent surface resistance of the superconductor. Meanwhile, we study concepts in radio/microwave technology, and in cryogenics.

Contents

1 Introduction

Superconductivity is a deeply quantum-mechanical phenomenon. It usually appears only at temperatures much lower than room temperature. The phenomenon was first discovered in mercury more than a hundred years ago. The observation was enabled by the then discovered technique of liquifying helium, hence allowing to reach temperatures as low as 4.2 Kelvins which were hitherto remained unobtainable. Since then, superconductivity has been observed in very many materials, including elemental metals as well as various metallic compounds. Around one third of elemental metals have so far been discovered to exhibit superconductivity. These include e.g. common metals such as aluminum, lead, and tin. A conductor that is not in the superconducting state, either because the temperature is above the superconducting transition (critical) temperature T_c, or because it does not superconduct whatever you do, is called normal-conducting in the present context.

The critical temperature T_c varies significantly between different materials that exhibit superconductivity, but in case of elemental metals, is above the boiling point of liquid helium (4.2 Kelvins) in only a couple of cases. An important example of the latter is niobium that has a $T_c \gtrsim 8$ K, which implies it superconducts conveniently if cooled down in a liquid helium bath, which is not too difficult an exercise. The extremely low electrical resistance characteristic to superconductivity is obviously attractive for applications that benefit from the lowest possible energy losses - this in fact includes nearly everything that involves electricity. In this assignment, we focus on one such niche application: electrical resonators exhibiting very low energy losses, here realized as a superconducting three-dimensional (3D) microwave-frequency cavity resonator. A long-term application of these devices has been with particle accelerators, where the superconducting cavities act as the accelerating elements [1]. Low losses allow for creating strong electric fields that allow the little particles to go very fast. Another emerging application of superconducting 3D cavities, but also superconductivity more broadly, is with quantum technology, where superconducting resonators are used as readout or storage devices for quantum bits [2–4].

2 Theoretical background

2.1 Superconductivity

In this text, we will only very briefly mention the background and physics of superconductivity. We refer the interested reader to participate relevant courses at the Department (e.g. PHYS-E0416 - Quantum Physics, or PHYS-E0551 Low Temperature Physics V, depending on the semester), or check textbooks, e.g. by Tinkham [5], or Van Duzer [6].

It took quite some time to establish an explanation for superconductivity. Profoundly quantum-mechanical, superconductivity could not even be microscopically explained without tools of quantum many-body theory, which started to develop in early 1930's. Before this, descriptions based on deep physical intuition were put forward by Fritz London (1935), and Vitaly Ginzburg and Lev Landau (1950). In these so-called phenomenological models, one postulates physical laws emerging from the then un-known physics of electrons in the superconducting state. In the London theory, equations are heuristi-cally developed for the interaction of the supercurrent and electromagnetic fields. The Ginzburg-Landau theory takes a somewhat more microscopic approach and postulates a macroscopic wave function de-scribing a collective degree of freedom responsible for the superconductivity.

Finally in 1957, a microscopic theory of superconductivity was established by Bardeen, Cooper and Schrieffer [7]. The theory was coined the BCS theory, and is still the accepted theory of the basic mechanism of superconductivity. Superconductivity, and a closely related concept of superfluidity, have fascinated the greatest minds in science. Landau, as well as Bardeen, Cooper and Schrieffer (1972), Ginzburg (2003) were all awarded the Nobel Prize for their works on these topics.

The zero resistance of the superconducting component is strongly connected to the existence of an energy gap, denoted by Δ_0 in the spectrum of a superconductor around the Fermi level. This resembles the gap in semiconductors in the sense that it prohibits single-electron excitations at temperatures below T_c. The connection between the gap and critical temperature is

$$\Delta_0 \approx 1.76 \, k_B T_c / e \tag{1}$$

Superconductor energy gaps are small, or the order milli-Volt, as compared to those of semiconductors.

At temperatures much lower than the critical temperature, practically all electrons participate the superconducting state. However, if the temperature is only moderately below T_c, there are remaining some normal-state electrons excited across the energy gap. They are subject to resistive (Ohmic) losses. This seems to contradict the experimentally verified fact that supercurrent can steadily circulate for all practical purposes forever in a superconducting loop, even relatively near the critical temperature. The controversy is resolved by the result that the Ohmic losses have a non-vanishing contribution only if the electric and magnetic fields have time dependence. Therefore, a constant supercurrent does not exhibit Ohmis losses. The situation can be understood as two non-interacting fluids, out of which only the normal fluid exhibits losses. In a time-independent situation, the superfluid component "shorts" all electric fields, and hence there is no electric field to affect the normal fluid. Even if the latter would be moving, it would simply gradually stop. In a time-dependent case, however, electric fields do appear because of the inductive response of the superfluid. This causes a coupling channel between the two fluids, and flow of energy into the lossy normal fluid.

Theoretical considerations predict that at very reachable temperatures much below the critical tem-perature, Ohmic losses in superconducting ac systems become negligibly small. Experimentally, how-ever, the case (regrettably) is not so. It turns out there are other small sources of energy losses present in ac electrical systems that do not mind of superconductivity. The very actual mechanism(s) respon-sible for the other losses are not well understood, although it is known that they are *dielectric losses*, i.e., the same thing that happens in microwave oven. Reducing the dielectric losses some little orders of magnitude from the current state of the art would enable building the quantum computer.

2.2 Electrical resonators

Resonance is one of the most important model concepts in physics. Resonator is often understood as some physical realization of a harmonic oscillator. Sometimes, it is thought that a resonator involves ex-

ternal forcing (driving) the harmonic oscillator. We use the former wording, although misunderstandings rarely would arise.

With electrical components, resonators are easily build. This needs inductors or capacitors, which have the capability of energy storage that is needed to sustain oscillations. Electrical losses are described by a resistor contained within the resonator circuit. Typically there is no intentionally put resistor component, but the resistance comes from the wiring, or is just a model for other types of energy losses, e.g. dielectric losses. One speaks of LCR or LC circuits, or -resonators.

The description of resonators works in terms of the (complex) electrical impedance in a straightforward manner. The inductor and capacitor can be connected either in series or in parallel. We discuss parallel resonators. The circuit model is shown in Fig. 1 (a). One obtains for the impedance

$$Z = \left(\frac{1}{R_{int}} + \frac{1}{i\omega L} + i\omega C \right)^{-1} . \tag{2}$$

The impedance shows a strongly frequency-dependent behaviour around the resonance angular frequency ω_0 given by (in the limit the resistance becomes infinite)

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 . \tag{3}$$

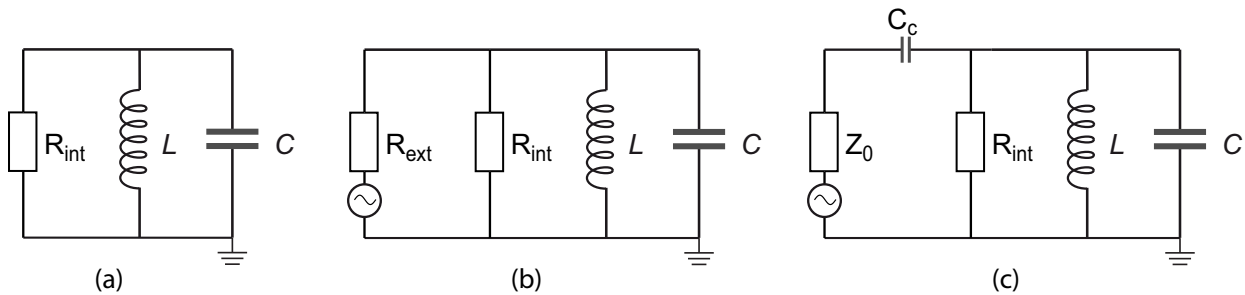Here, we also made explicit the common labelling of angular frequencies, versus frequencies. Another

Figure 1: (a) Parallel LC resonator circuit. (b) The same circuit *loaded* by external dissipation described by the resistor R_{ext}. (c) The circuit coupled to the external circuit via a small coupling capacitance C_c. Notice that the circuit is well approximated by that in (b), with R_{ext} given in terms of Z_0 and C_c.

important characteristic of a resonator is its Q ("quality") value. It describes the damping rate of the oscillations - the higher Q, the longer the system keeps oscillating once released from a displaced position. Often, high Q is a desired property in science and technology. The Q value is defined in terms of the total energy E of the oscillator, and the energy loss ΔE in one cycle:

$$Q \equiv 2\pi \frac{E}{\Delta E} = \frac{f_0}{\kappa} . \tag{4}$$

The latter equality relates Q to the (energy) damping rate κ of the oscillator[1]. For a parallel LCR resonator having a generic resistance R, one can derive the following expression:

$$Q = \frac{R}{\omega_0 L} = R\omega_0 C . \tag{5}$$

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[1] It is easy to get confused with angular frequencies and frequencies. First of all, "frequency" is ubiquitously used interchangeably for either. For κ we do not have (in accordance to literature) a separate symbol depending on which units are used. We use the convention that κ is given in the same units as other frequencies in a given equation. If no other frequencies are explicit, either units work. In case SI values are used, the units are seen from the SI unit. Notice, however, that the convention used here is not universally valid in literature.

What happens to the energy dissipation if the resonator is not sitting isolated from the rest of the universe? Information extraction from a system comes through extracting energy from it - for the measurement, the resonator needs to be connected to an external electric circuit, where the energy is inevitably lost. The measurement is modeled as shown in Fig. 1 (b), by including an external resistor R_{ext}, in series with a voltage source represented by the circle and a wavy line (remember the voltage source in a circuit has a zero impedance). One can define a corresponding *external* quality factor:

$$Q_{ext} = \frac{R_{ext}}{\omega_0 L} = R_{ext} \omega_0 C \,. \tag{6}$$

Similarly, the *internal* quality factor is:

$$Q_{int} = \frac{R_{int}}{\omega_0 L} = R_{int} \omega_0 C \,. \tag{7}$$

The total resistance in the circuit is the parallel sum $R = \left(R_{int}^{-1} + R_{ext}^{-1} \right)^{-1}$ of the two resistors, and Eq. (5) holds for the *loaded* quality factor

$$Q^{-1} = Q_{ext}^{-1} + Q_{int}^{-1} \,, \tag{8}$$

which is smaller than the internal or external quality factors. The corresponding internal and external damping rates (see Eq. (4)) add up to the total damping rate

$$\kappa = \kappa_{ext} + \kappa_{int} \,. \tag{9}$$

In a typical practical situation, it turns out that the scheme in Fig. 1 (b) is not applicable as exactly in that kind of circuit. This is because in radio frequency/microwave measurements (see section 2.3), the external resistance $\approx 50\Omega$, and $\kappa_{ext} \gg \kappa_{int}$, and the (often) desired high-Q property would be heavily compromised by just being able to read out the system. Therefore, the measurement apparatus needs to be suitably "weakly" coupled to the resonator. This is simply accomplished by inserting a coupling capacitor $C_c \ll C$ into the circuit as shown in Fig. 1 (c). One can show that in this case, the external dissipation appears as an effective resistance, whose value is tunable by the choice of C_c, given as

$$R_{ext} \simeq \frac{1}{\omega_0^2 C_c^2 Z_0} \,. \tag{10}$$

What is a suitable ratio of $\kappa_{ext}/\kappa_{int}$? In a simple-minded picture, the only fixed frequency scales in the system are ω_0, and κ_{int}. The former is out of question since it would mean $Q \approx 1$. Therefore, the scale of κ_{ext} has to be linked to κ_{int}, and indeed, usually it is preferred that $\kappa_{ext} \approx \kappa_{int}$. This is a good compromise between able to acquire information (a decent coupling to the measurements), and not too much compromise Q.

From Eq. (2) one obtains:

$$(C\kappa)^2 |Z|^2 \simeq \frac{(\kappa/2)^2}{(\omega - \omega_0)^2 + \left(\frac{\kappa}{2}\right)^2} \,, \tag{11}$$

which is accurate close to resonance $|\omega - \omega_0|/\omega_0 \ll 1$. Although not totally evident, here ω means frequency of the external driving, that is, the frequency of the voltage source. Resonance itself means that the system responds clearly the strongest to a driving at the resonance frequency. In Eq. (11) one sees that indeed when driven on resonance, the current and energy are maximized, and are limited by the losses.

The function on the right hand side in Eq. (11) is called a *Lorentzian*, and it is sharply peaked at ω_0. The full width at half maximum (FWHM) of the Lorentzian peak equals the (total) damping rate κ.

2.3 Microwave techniques

People like to label things, for example, to classify electromagnetic radiation into a couple of ranges defined by the wavelength. Needless to say, such classification is fuzzy at best. Microwaves have been said to belong to the electromagnetic spectrum in the frequencies say between 1 GHz and 100 GHz, or in terms of wavelength in free space, between on the order one meter down to a millimeter. The range of radio frequencies is even less well defined, sometimes covering nearly everything up to high microwave frequencies, although often radio frequencies can be thought as lower than those of microwaves.

Notwithstanding the labels, there is a fundamental difference between the propagation of electro-magnetic fields in electrical circuits at "low" or "high" frequencies. At low frequencies, we are used to describe circuits containing wires, resistors, inductors and capacitors, in terms of impedances of these *lumped* components. This description implicitly entails that the components are zero-dimensional, and the connecting wires may be thought such as well. At a certain low enough wavelength scale, this description *completely brakes down*. This scale is given by the physical size of the circuit or component in question. The reason is qualitatively simple: in the said limit, the fields have spatial dependence within the wire, or even inside the circuit components that in a typical case are much smaller than the connecting wires.

We note in passing that in typical cases, the fact that lumped components themselves do not function well at too high frequencies, is due to another reason. The components have parasitic impedances, for example, an inductor has some capacitance between the ends. The parasitics will modify the behavior, the more the higher the frequency. In the current exercise, we do not need to consider these cases, because we do not have any lumped components involved.

2.3.1 Transmission line

Here we very briefly cover the phenomenological description of the propagation of electromagnetic fields in a piece of wire that is so long that the fields have spatial dependence. The final result is fundamentally important, as well as counterintuitive. An exhaustive discussion is given in the standard textbook by Pozar [8]. The starting point of the description is is shown in Fig. 2. The wire has the resistance, inductance, and capacitance per unit length of R_T, L_T, and C_T, respectively. For simplicity, we ignore the losses and set $R_T = 0$ from now on. Typical values for the other parameters are $L_T \approx 0.3\,\mu$H/m, and $C_T \approx 100$ pF/m.

The line consisting of the segments as in Fig. 2 is called a transmission line. In order to describe wave propagation in it, one uses the Kirchhoff laws for each segment and takes the limit $\Delta x \to 0$, finally obtaining the wave equation for the voltage wave:

$$\frac{\partial^2 v(x,t)}{\partial t^2} - C^2 \frac{\partial^2 v(x,t)}{\partial x^2} = 0 \,, \tag{12}$$

where C is the speed of the wave, or, phase velocity. A similar equation can be obtained for the current wave.

The interesting property is the *characteristic impedance* that is a generic property of wave propagation. In the case of electromagnetic waves, it is given as the ratio of electric and magnetic field norms. From the discussion above, one obtains the characteristic (wave-) impedance of the transmission line:
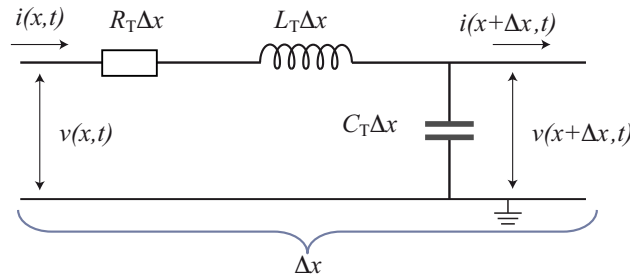
$$Z_0 = \sqrt{\frac{L_T}{C_T}} \,. \tag{13}$$

Figure 2: Modeling transmission line in terms of microscopic segments, each of length Δx. The conductor at the lower side of the figure is defined as the ground, which in a real case can be surrounding the "active" conductor as in a coaxial cable, or be physically far apart.

Notably, Z_0 is real-valued, hence it represents a *resistance*. This sounds surprising, since there were no resistors included in our analysis, but the impedances of all the sections are purely imaginary.

The parameters L_T and C_T depend on the dimensions of the cable, although very weakly. A universal standard has been selected to dimension the cables such that $Z_0 = 50\,\Omega$. This is very different from the zero-frequency impedance (milli-Ohms) of a typical piece of the same cable.

The characteristic impedance of the transmission line has a strong connection to that of another medium, namely vacuum. In vacuum, the ratio of electric and magnetic fields is the impedance of free space given as

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega\,. \tag{14}$$

2.3.2 Reflection measurement

The wave impedance describes the hardness of the material. Waves reflect when they hit a boundary between two mediums of different hardness, e.g. ocean waves reflect from a rocky shoreline. Similarly, waves propagating in a transmission line do reflect when they hit an electrical medium of different wave impedance, e.g. a wrong type of cable, or a bad connection between two cables. Let us have an incoming wave $v^+(x)$, given in complex notation and time dependence not written down for simplicity, moving to the right towards such a discontinuity:

$$v^+(x) = |v^+| \exp(-ikx)\,. \tag{15}$$

The reflected complex wave amplitude is given by

$$v^-(x) = S_{11}v^+(x)\,, \tag{16}$$

which defines the (complex) reflection coefficient S_{11} characterizing the discontinuity. The S is for scattering, and the indices mean scattering from cable 1 into cable 1.

Besides joints between cables, waves reflect when they hit the device of interest, which generally is described by a nontrivial impedance Z very different from Z_0. The reflection coefficient, no matter if the impedance is that of another transmission line or the sample, is given by

$$S_{11} = \frac{Z - Z_0}{Z + Z_0}\,. \tag{17}$$

As discussed below in section 3.1, measuring S_{11} is the basic characterization in microwave measurements, and is also carried out in this exercise. Since Z in the case of the sample is typically complex, S_{11} is complex in that case, too. As seen in Eq. (16), its phase is the phase shift experienced by the wave, and its modulus is the change in wave amplitude. One can show that

$$|S_{11}|^2 \simeq \frac{(\omega - \omega_0)^2 + \left(\frac{\kappa_{ext} - \kappa_{int}}{2}\right)^2}{(\omega - \omega_0)^2 + \left(\frac{\kappa}{2}\right)^2} \tag{18}$$

Up to a good approximation, this is of the form "constant$_1$$-$ constant$_2$$\times$ Lorentzian", c.f. Eq. (11), with the width given by κ.

In practical life, the S parameters are measured using a *Vector Network Analyzer* (VNA). The minimal version has two ports, 1 and 2, and all four S parameters can be easily measured. All the ports appear as an impedance of $Z_0 = 50\ \Omega$.

2.4 Rectangular cavity resonator

As argued above in section 2.3, the circuit in Fig. 1 built using lumped elements is usually not relevant at microwave frequencies, typically above ~ 1 GHz. Hence in some telecommunication, and foremost in quantum technology, resonators are built differently. Transmission lines, as they are described by wave propagation, naturally support confined standing wave modes that correspond to resonances. Another possibility is to make it three-dimensional (3D). The latter corresponds to a resonant cavity, which is an empty space surrounded by reflecting walls made out of metal. An acoustic analogue is the guitar echo chamber. The 3D realization of the electrical resonator offers a simple construction and usually higher Q values, but the tradeoff is a bulky component that does not fit modern mobile phones. Nonetheless, 3D cavities are in frequent use in research.

In research, 3D cavities are often made out of superconducting metal, as discussed in the introduction. In particle accelerators, they can have very high Q values (specifically $Q_{int} > 10^{10}$, implying the microwave photon lifetime of the order one second. At a low temperature, this time equals the quantum coherence time, which hence becomes something truly macroscopic. 3D cavities have indeed been used to store macroscopic "Schrödinger's cat" quantum states with superconducting quantum bits [9].

The resonant frequencies of the 3D cavity intuitively corresponds to wavelengths of the electric and magnetic fields that somehow match the physical dimensions of the box. The configuration of the fields corresponding a given resonant frequency is called an eigenmode, or mode for short. There are infinitely many modes in a system - one can always add e.g. one more wave period such that the resulting wave still matches the dimensions.

Let us look at the scheme of the rectangular 3D resonant cavity in Fig. 3. The dimensions are labeled with the symbols X, Y, and Z, which describe the extent of the volume in x, y, and z directions in our coordinate system. One can label the modes by the letters m, n, and l, which denote the number of half periods of the wave in x, y, and z directions, respectively.

Here we will just quote the result for the mode frequencies:

$$f_{mnl} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m\pi}{X}\right)^2 + \left(\frac{n\pi}{Y}\right)^2 + \left(\frac{l\pi}{Z}\right)^2} \tag{19}$$

Here, c is the speed of light in vacuum, and μ_r and ϵ_r are the relative permeability and permittivity of the medium inside the cavity (usually, the cavity is empty). Notice that not all combinations of the

indices correspond to a mode. One can classify the the modes in two families: TE (Transverse Electric) and TM (Transverse Magnetic) modes. TE modes have the electric field transverse to the direction that would correspond to propagation of the wave, if the cavity would be stretched into a long waveguide. TM modes have an analogous property. In our case, that is $Y < X < Z$, the lowest-frequency mode is the TE_{101} mode.
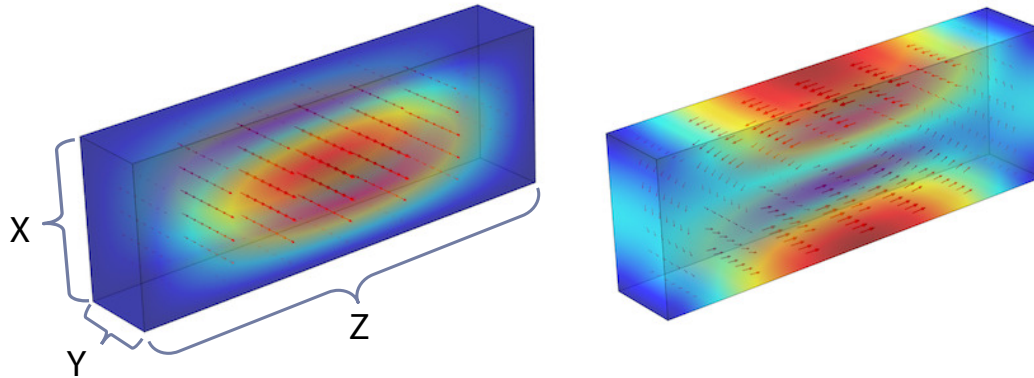

Figure 3: Electric (left) and magnetic (right) field profiles corresponding to the lowest eigenmode at $f_0 \simeq 11.34$ GHz of the studied rectangular cavity resonator. The colors code the normalized absolute value of the field inside the volume. Red is the maximum, dark blue the minimum. The arrows indicate the direction of the field, with the length of the arrow proportional to the field norm. The fields have been solved in Comsol finite element method software. The labels indicate the dimensions of the cavity.

Measuring the properties of the cavity occurs via a reflection measurement, namely probing the reflection coefficient S_{11} given in Eq. (17). What is then Z for our cavity? To begin with, one could use some physical intuition. Our cavity is, of course, and electromagnetic resonator. But so is the lumped element resonator in Fig. 1, described by inductance and capacitance (and resistance). So, one could expect that the cavity would also allow for description in terms of lumped elements. This turns out to be the case indeed. Regrettably, obtaining the result is a pretty complicated exercise, out of one obtains equivalent parameters L, C and R for the cavity resonator. One also obtains that if the cavity is coupled to the measurement system (essentially transmission line) through the electric field little bit leaking out from the cavity via a measurement port, the coupling can be described by a coupling capacitance as in Fig. 1(c). The reflection measurement is then described exactly by the circuit in Fig. 1 (c). The incoming and reflected waves are hidden into the transmission line characterized by Z_0 alone.

The losses of the cavity again consist of external and internal losses, as in Eq. (9). The above result for the Lorentzian lineshape of the $|S_{11}|^2$ measurement, Eq. (18), holds true allowing for inferring the total loss rate notwithstanding the internals of the resonator.

2.5 Ohmic losses in cavities

Cavities that are not superconducting naturally exhibit resistive losses, because ac currents flow in the metal that has resistance. In fact, the effective ac resistance can be substantially larger than that determined by the bulk resistivity of the metal. This is because ac currents flow in a very thin layer on top of the metal, an effect known as the skin effect (reminding that this has nothing to do with the characteristic impedance concept!). The corresponding skin depth δ_s is down to only a micron thick at high GHz frequencies.

The losses of cavities, whether normal conducting or superconducting, are characterized by a con-

cept known as surface resistance (or -resistivity), or sheet resistance R_s, having the units of Ohms. It is connected to the bulk resistivity ρ and the thickness of the "surface" (here δ_s) as

$$R_s = \frac{\rho}{\delta_s} \tag{20}$$

The Q value of our cavity limited by the surface resistance becomes

$$Q_{int} = \frac{(kXZ)^3 Y \eta_0}{2\pi^2 R_s} \frac{1}{2X^3 Y + 2YZ^3 + X^3 Z + XZ^3}. \tag{21}$$

Here, η_0 is the impedance of free space, Eq. (14), and the wave number $k = \omega_0 \sqrt{\mu_0 \epsilon_0}$. The corresponding internal damping κ_{int} rate is obtained according to the general expression in Eq. (4).

In superconductors, as discussed in section 2.1, there are also generally small Ohmic losses at finite frequencies, as well as dielectric losses that ultimately limit the highest quality factors. The corresponding surface resistance has the following temperature dependence:

$$R_s \simeq \alpha \left(\frac{T}{T_c}\right)^4 \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-3/2} + R_0 \tag{22}$$

where α is a constant, and R_0 is the unspecified dielectric loss contribution.

3 Instructions for the lab exercise

The exercise involves plenty of details on handling the instruments. Describing all of them here would clutter the message. Therefore below we list the main process flow, with some details included. Assistant will guide you through details on e.g. how to assemble indium seal, operate the pumping system, connect the cables, and so forth.

3.1 Description of the equipment

For the cryogenic part, we use a test *cryostat* suitable for the purpose. It is equipped with microwave cables for the measurements. Illustrative photographs of the setup are shown in Fig. 4. The sample (cavity in the present case) is attached to the copper cold finger, which provides heat conduction from the cryoliquid bath to the sample. The sample space is surrounded by a removable vacuum can, which prevents cryoliquids from entering the sample space.

Temperature at the cold finger of the cryostat is measured using a DT-670 silicon diode thermometer from Lake Shore [10]. Its operation is based on the strongly temperature dependent electrical conduction in semiconductors. A current bias of 10μA is applied in the forward direction through the diode, and the corresponding voltage drop gives the temperature signal.

The cabling scheme in shown in Fig. 5. The cables are connected to another using SMA connectors. The cavity is probed via a coupling pin attached to an SMA feedthrough in the cavity wall. The coupling pin is positioned such that it sees an appreciable electric field amplitude, since the measurement couples to the cavity via electric field (cf. the coupling capacitor C_c in Fig. 1(c)). The impedance of the circuitry, although not explicitly indicated, is $Z_0 = 50\,\Omega$. The directional coupler allows for separating incoming and reflected waves.
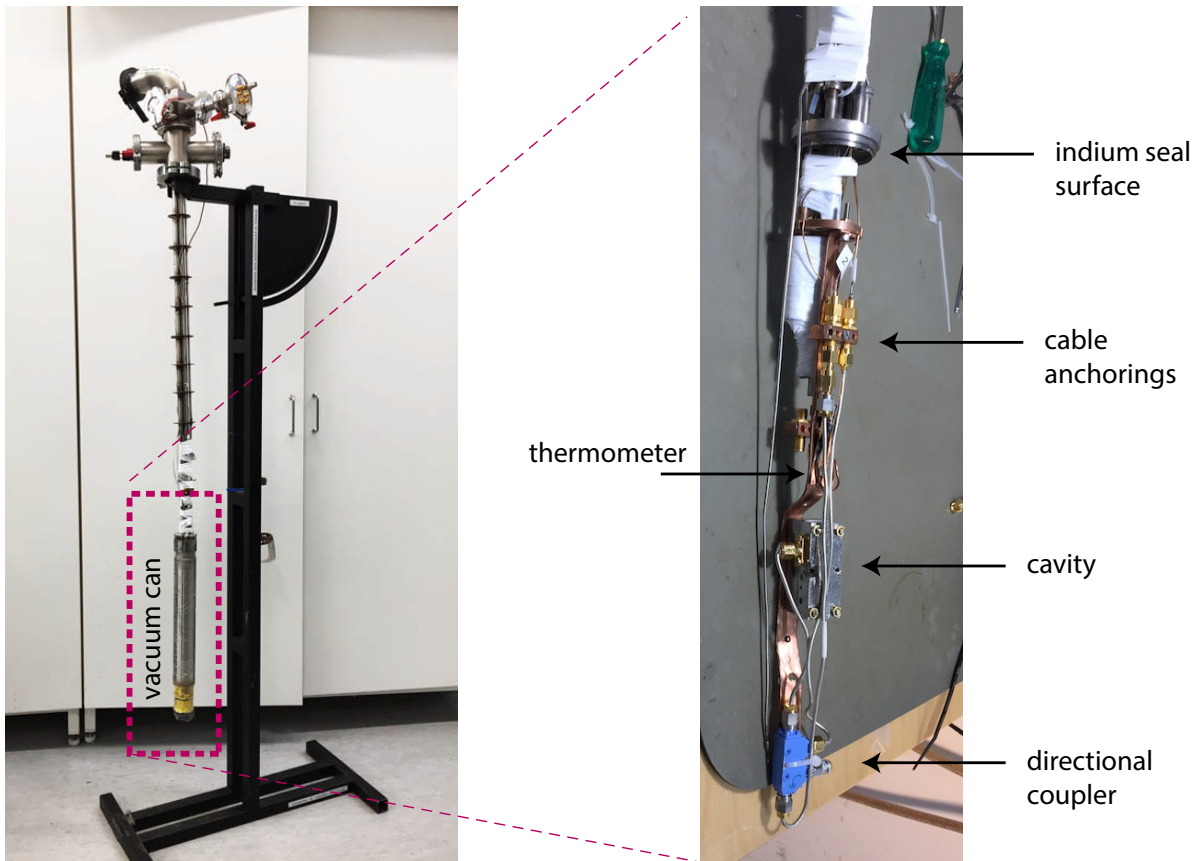
Figure 4: Photographs of the cryostat. Left: overall view, vacuum can installed. Right: view showing the contents inside the vacuum can.

The cavity response is recorded using the VNA also displayed in Fig. 5. It gives both the amplitude and phase shift of the wave exhibited as it traverses through the system, although in this exercise we are interested in only the amplitude. Off resonance, the cavity exhibits an impedance very much different from $Z_0 = 50\,\Omega$ (the cavity acts as a mirror), and the wave will be fully reflected. Therefore off resonance we would naively be expected to measure $|S_{11}| \simeq 1$. However, as mentioned, VNA probes the entire circuit, which attenuate the waves. Therefore, the "baseline" of the resonance dip has $|S_{11}| \ll 1$. This is easy to take into account in the analysis, since the attenuation acts as a pure scaling factor for S.

One should bear in mind also the units of the S parameters. In microwave engineering, one is typically interested in ratios of signals as they travel though the system. The relative change is due to either attenuation or amplification. Let us denote original signal power (or some other value of interest) by P_{ref}, and the measured signal power by P. The power ratio is usually expressed in decibels, defined as

$$10\log_{10}\left(\frac{P}{P_{ref}}\right) \Rightarrow 10\log_{10}\left(|S|^2\right) \tag{23}$$

The benefit is that if there are e.g. two amplifiers with gains specified as 12 dB and 23 dB, one easily calculates on top of the head that the total gain is 35 dB. The VNA by default gives S parameters in dB scale, and for the analysis it should be converted into linear units according to Eq. (23).

One more possibly confusing thing is that, as one could see in Fig. 5, VNA sends the signal out

from port 1, and records the signal in port 2. Therefore, we have to "fool" the VNA to actually record S_{21}, although it is actually measuring S_{11} of the cavity.
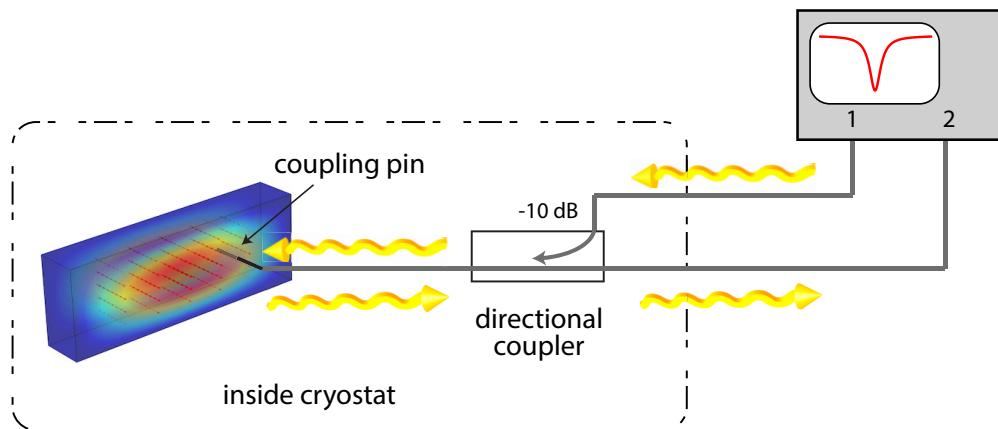

Figure 5: Cabling schematics of the experiment. The gray solid lines are 50 Ω coaxial cables. The wavy arrows denote the signal traveling directions in the system. The waves reflect from the cavity. See text for further information.

3.2   Safety precautions

Face shield and *leather gloves* are required for safety when handling cryoliquids. The latter are liquid nitrogen, 77 Kelvins, and liquid helium, 4.2 Kelvins. When the cryoliquids are boiling heavily during the cooldown, jets of cold gases do appear. These can cause swift low-temperature burns, although this is rare and unlikely.

The exhaust gases from our boiling cryoliquids are not poisonous. However, in a closed space, the gases could cause danger by replacing oxygen. The exercise room has sufficient ventilation in order to fan out the exhaust gases.

3.3   Steps of the exercise

1. Put on fresh rubber glows, and wear them throughout the exercise.

2. Check the cryostat is firmly attached to the rack with a clamp, vacuum can open.

3. Measure the inner dimensions of the cavity with a caliper.

4. Close the cavity by putting the screws and nuts in place, and tightening them modestly.

5. Temporarily attach cavity to port 1 of VNA, and check you see a faintly visible cavity resonance dip in the S_{11} parameter centered at approximately 11.37 GHz. Write down the frequency at 0.1 MHz accuracy.

6. Attach cavity by a screw into the copper cold finger. Tighten the screw modestly.

7. Connect the flexible SMA cable to the cavity, and tighten with torque wrench.

8. Check the thermometer is connected and gives reasonable readings.

9. Take about 14 cm piece of indium wire. Wipe it in cleanroom paper tissue wetted in acetone or isopropanol. Take tiny drop of vacuum grease to your glove, and rub it to the indium.

10. Assemble the indium inside the vacuum can - assistant will give guidance.

11. Put the vacuum can in place. One person holds the can, while others tighten the bolds up to a firm tightness.

12. Connect the pumping tube, open the valves, and start pumping the vacuum can for about 5 min.

13. Dress up a *face shield*, and well as *leather gloves* on top of your rubber gloves.

14. Lift the liquid nitrogen dewar in place below the cryostat. Lift it up with the lab jack until the cryostat vacuum can is more or less inside liquid.

15. Keep pre-cooling in liquid nitrogen until the thermometer shows about 100 K. This takes around 45 minutes. Meanwhile, you can take a *coffee break*, or persuade the assistant to give a *lab tour* in the Low Temperature Laboratory premises.

16. When cold enough, lift the cryostat into the liquid helium dewar. At this point, cold helium gas jets out from the dewar. Check your skin is protected.

17. Tighten the cryostat upper flange to the dewar.

18. Open the measurement script C:/APLcourse/SuperconductingCavity.m in Matlab. Type in as the center frequency the room-temperature cavity frequency plus 24.5 MHz (the increase is due to thermal contraction during cooldown). Check the span is 5 MHz. Run the script in order to check things are working.

19. When thermometer shows about 1.2 V (~ 20 K), put the script running. Cooldown through T_c down to 4.2 K is pretty fast as compared to rest of the process, so data needs to be taken at a fast rate in order to obtain enough data points.

20. If there seems to be only a few curves obtained right below T_c, or if the resonance was outside the span, assistant will repeat the final stages of the cooldown.

21. When ready, lift the cryostat from the helium dewar, and firmly attach it to the rack. Close the helium dewar.

4 Reporting

4.1 Parameter values

The dimensions of the cavity are as follows: $X = 14.1$ mm, $Y = 8.2$ mm, $Z = 40.1$ mm.

The frequency of the cavity is $f_0 \simeq 11.37$ GHz at room temperature, depending on the bolt tightening force. At lower temperature, the frequency creeps up by ~ 30 MHz because of thermal contraction. The external loss rate of the cavity is $\kappa_{ext} \simeq 9.6$ kHz. The internal loss rate at the base temperature (4.2 K) should be in the range $\kappa_{int} \approx 5$ kHz ... 40 kHz, depending on the amount contamination inside the cavity (=dielectric losses).

4.2   No error analysis required

In this exercise, error analysis is not required to be carried out. Nonetheless, one has to observe that the results are sensible, as usual.

4.3   Data analysis

The model data Matlab-datafile includes the following variables:

- `dfreq`: frequency values.

- `Voltage`: Thermometer voltage readings, which should be converted to temperatures.

- `datamag`: $|S_{11}|$ in decibels in 2D array. The first index is the running index of the measurement, which corresponds to different temperature values. The second index is frequency.

Additionally, the original Matlab-script used in the measurement is provided just for your information.

In the analysis, let us investigate only the temperatures below roughly 20 K. Otherwise, there will be plenty of un-interesting data points measured in the non-superconducting state. The most important data is that measured just while passing T_c. The latter event is quite evident as the temperature value when the resonance comes crystal clearly visible in the measurement.

1. Use Eq. (19) and the measured dimensions of the cavity to predict the mode frequency, and compare it to the stated value.

2. Convert the thermometer reading voltages into temperature according to the diode data sheet, or the ready-made Matlab function "TempCal.m" provided in the exercise material.

3. Fit (inverted) Lorentzian functions to the measured resonance dips in order to find κ_{int}, according to Eq. (18). Use a fixed value for κ_{ext} as provided above. Be cautious whether you are using decibels, power, or voltage!

4. Plot the internal loss rate κ_{int} as a function of temperature (in the temperature range from 4.2 K up to clearly above T_c, e.g. 20 K). Show the vertical axis in logarithmic scale, because the values vary quite a lot.

5. Fit temperature dependence of κ_{int} to theory, that is, Eqs. (21,22). Use as fitting parameters the constant α, superconducting gap Δ_0, and the residual resistance R_0.

   - Make sure you operate only in the temperature region where you are superconducting, because Eq. (22) works only in that case!
   - The values of κ_{int} span several orders of magnitude, and fitting can be therefore challenging. It might be beneficial to run the fitting routine with the logarithm of κ_{int} in order to limit the dynamic range.
   - Plot the fitted curve together with the data.

6. Based on the result from step 5, recover the critical temperature of the cavity material, and compare to a literature value.

4.4 For the report

(a) Briefly describe the motivation and background of the work, as well as the equipment and methods. It is acceptable to use pictures and graphs from the material - however, in this exercise, it is considered a handicap *if most of the graphical material is copied*, and can lead to point deduction in borderline cases.

(b) Derive equation (12). Start from Kirchhoff laws used in the circuit segment in Fig. 2, and combine the results to obtain Eq. (12). From the result, write down expression for the speed C of the wave in terms of the given wire parameters, evaluate the value using the given parameter values, and compare to the corresponding quantity in free space.

(c) Do all steps of the data analysis as listed in section 4.3. Explain in your own words, more in detail than in 4.3, what you are doing and why you are doing as you are doing.

(d) Present any graphs used in the data analysis.

(e) Attach all the analyzed resonance curves, complete with fit functions plotted on top of the data, as an appendix.

4.5 For the *lomake*

(i) Do all steps of the data analysis as listed in section 4.3.

(ii) Do steps (b),(d),(e) listed for the report in section 4.4.

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