QMS Problem set 1 (due 21.1.2021)

Warm-up: (Each warm-up is connected to one of the problems (first warm-up to first problem etc.))

W1: Answer problem E7B.3(a) (2 points)

E7B.3(a) Which of the following functions can be normalized (in all cases the range for x is from $x = -\infty$ to ∞, and a is a positive constant): (i) e^{-ax^2};; (ii) e^{-ax}. Which of these functions are acceptable as wavefunctions?

W2: Answer problem E7C.2(a) (2 points)

E7C.2(a) Identify which of the following functions are eigenfunctions of the operator d/dx: (i) $\cos(kx)$; (ii) e^{ikx}, (iii) kx, (iv) e^{-ax^2}. Give the corresponding eigenvalue where appropriate.

W3: Answer problem E7D.7(a) (2 points)

E7D.6(a) For a particle in a box of length L sketch the wavefunction corresponding to the state with the lowest energy and on the same graph sketch the corresponding probability density. Without evaluating any integrals, explain why the expectation value of x is equal to $L/2$.

W4: Fill in the conceptual questionnaire for the course. The link will be posted on the course page after Friday's review session. (3 points)

Problems:

P1: Explain why normalization is important in quantum mechanics? Then answer P7B.6 (5 points)

P7B.6 Suppose that in a certain system a particle free to move along x (without constraint) is described by the unnormalized wavefunction $\psi(x)=e^{-ax^2}$ with a = 0.2 m^{-2}. Use mathematical software to calculate the probability of finding the particle at $x \geq 1$ m.

P2: Explain what physical consequences does the non-commutation of operators have? Then answer P7C.15 (a) [You don't need to do (b)!) (5 points)

P7C.15 Evaluate the commutators (a) $[\hat{H},\hat{p}_x]$ and (b) $[\hat{H},\hat{x}]$ where $\hat{H}=\hat{p}_x^2/2m+\hat{V}(x)$. Choose (i) $V(x) = V_0$, a constant, (ii) $V(x) = \frac{1}{2}k_f x^2$. (*Hint:* See the hint for Problem P7C.13.)

P3: Answer P7D.6 (5 points)

P7D.6 Consider a particle of mass m confined to a one-dimensional box of length L and in a state with normalized wavefunction ψ_n. (a) Without evaluating any integrals, explain why $\langle x \rangle = L/2$. (b) Without evaluating any integrals, explain why $\langle p_x \rangle = 0$. (c) Derive an expression for $\langle x^2 \rangle$ (the necessary integrals will be found in the *Resource section*). (d) For a particle in a box the energy is given by $E_n = n^2h^2/8mL^2$ and, because the potential energy is zero, all of this energy is kinetic. Use this observation and, without evaluating any integrals, explain why $\langle p_x^2 \rangle = n^2h^2/4L^2$.

BONUS (3 points)

P7D.7 This problem requires the results for $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p_x \rangle$, and $\langle p_x^2 \rangle$ obtained in Problem P7D.6. According to Topic 7C, the uncertainty in the position is $\Delta x = (\langle x^2 \rangle - \langle x \rangle^2)1/2$ and for the linear momentum $\Delta p_x = (\langle p_x^2 \rangle - \langle p_x \rangle^2)^{1/2}$. (a) Use the results from Problem P7D.6 to find expressions for Δx and Δp_x. (b) Hence find an expression for the product $\Delta x \Delta p_x$. (c) Show that for $n = 1$ and $n = 2$ the result from (b) is in accord with the Heisenberg uncertainty principle, and infer that this is also true for $n \geq 1$.