5A Confidence intervals

Class problems

5A1 (Soda fountain) A soda fountain dispenses soft drinks in cups. The volume of a drink (in milliliters) is approximately normally distributed with mean μ and a known standard deviation of $\sigma = 3$, caused by random variations in the dispensing process. In nine cups the following volumes were observed: 304, 298, 301, 302, 301, 300, 305, 300, 306.

(a) Find a confidence interval for μ at 95% confidence level.

(b) Find a confidence interval for μ at 99% confidence level.

(c) When performing an experiment like in (a) — dispensing nine cups and then calculating a confidence interval — what is the probability that the calculated interval will (i) contain μ, (ii) be completely below μ, (iii) be completely above μ?

(d) How many cups do we need to take in order to obtain a 95% confidence interval that has length at most 1 milliliter (0.5 ml on each side of point estimate)?

(e) How many cups for a 95% confidence interval of length 0.1 milliliters?

5A2 (Opinion poll) An opinion poll reported in July 2016 by Helsingin Sanomat, 89% of Finns think that president Niinistö has performed his duties well or extremely well. The poll was conducted by recording the opinions of 1002 Finns of ages 15–79 years, and the margin of error was reported as approximately 3 percentage points (in both directions). Let us assume that the margin of error was calculated by using the conservative interval for the binary model (see e.g. Lecture 4B).

(a) From the reported numbers, deduce what confidence level was used.

(b) How many Finns should have been recorded, in order to obtain a margin of error of 1 percentage point (in both directions), at the same confidence level?

Home problems

5A3 (Multiparty opinion poll) From a large population, a random sample of $n = 100$ persons were asked which of the four parties A,B,C,D they support. The numbers of the supporters were 70, 28, 2 and 0.

(a) For each party separately, consider the *binary* question (supporting party X or not), and calculate a 95% confidence interval for the proportion of party-X supporters in the population. Give the results as intervals like $[0.500, 0.600]$, with both endpoints expressed in 3 decimals (or 1 decimal in percent form).

(b) Repeat the previous, now using the conservative length of confidence intervals (see e.g. Lecture 4B).

(c) Do the calculated intervals seem reasonable and meaningful? If not, explain in what way, and what you think might be the reason. Note. Here a small amount of thinking is enough. We do not require a complete solution. Solutions for such situations exist in the literature, but they are somewhat more complicated.

(d) If the true proportion of some party X in the population is nonzero, can the sample proportion be zero? Apply common sense, not calculations.

(e) If the true proportion of some party X in the population is zero, can the sample proportion be nonzero? Apply common sense, not calculations.

5A4 In a physical experiment, particle decays are observed at random intervals. The intervals are assumed independent and each of them exponentially distributed with unknown mean μ (so the rate parameter of the distribution is $\lambda = 1/\mu$). 30 consecutive intervals were observed, and their mean was 12.09 seconds and standard deviation 11.47 seconds.

(a) Calculate a confidence interval for μ at 90% confidence level. Use the general method for estimating an unknown mean (Lecture 4B).

(b) The individual intervals were not normally distributed. Do you think the general method is still reasonable? Why / why not? Explain in words.

(c) Would the same method work well if we had $n = 3$ observations? Why / why not?