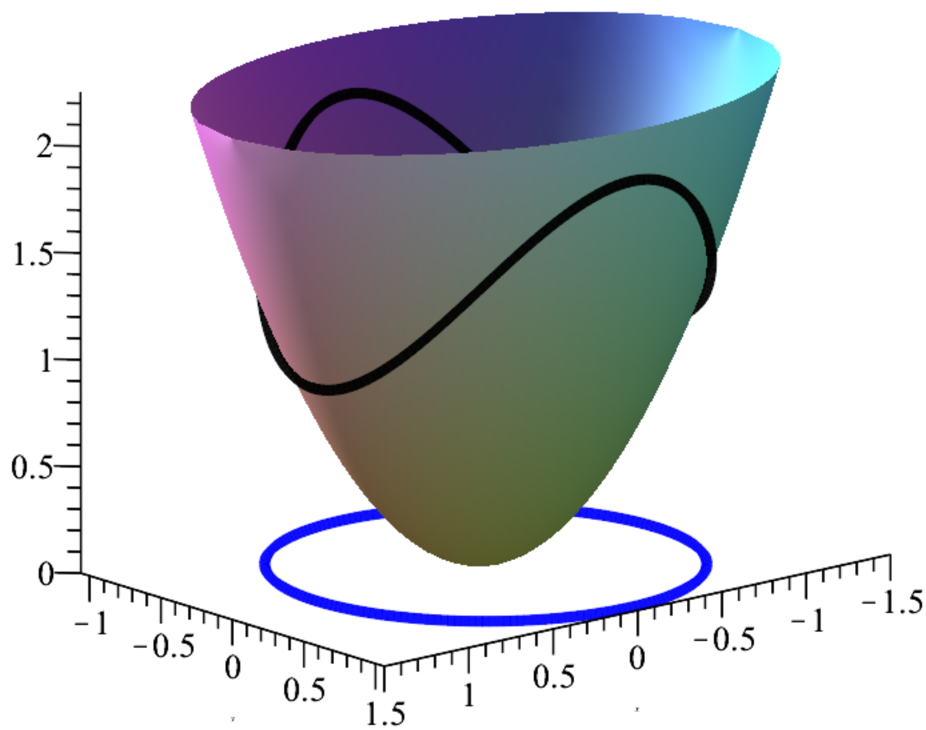
We examine the extreme values of x^2 + 2y^2 over the disk of radius one.

```
> with(plots):
> f:= x^2 + 2*y^2;
```

$$f := x^2 + 2\,y^2 \tag{1}$$

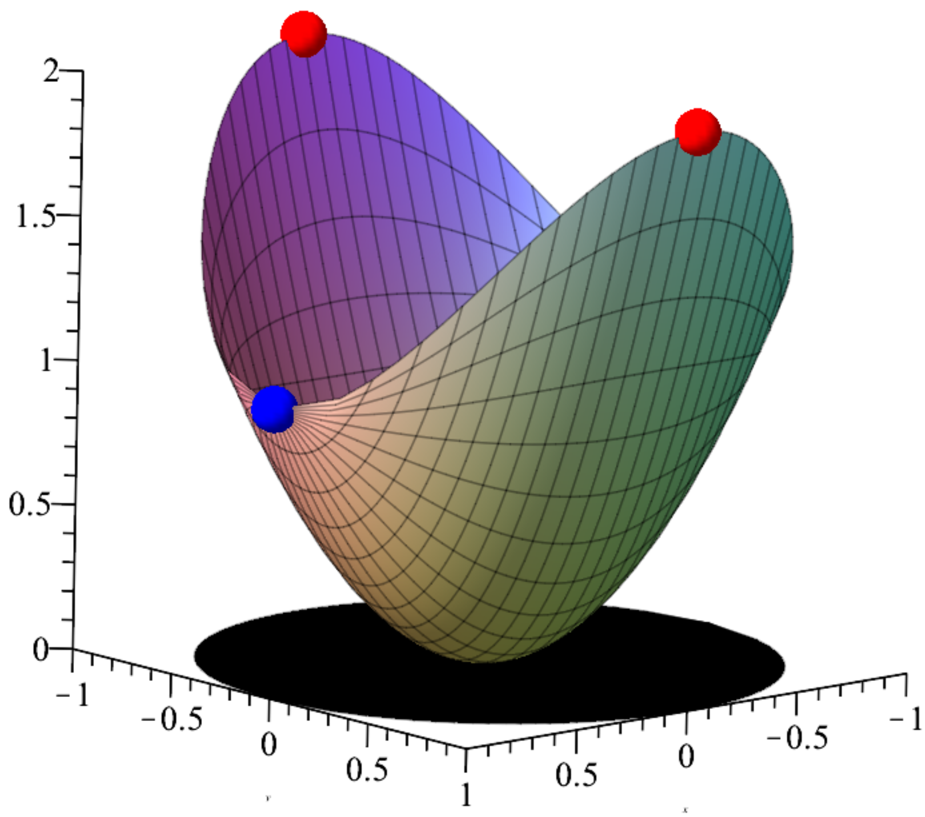
```
> r:=1.5:
> A:=plot3d(f,x=-r..r,y=-sqrt(r^2-x^2)/sqrt(2)..sqrt(r^2-x^2)/sqrt
  (2),axes=framed,style=patchnogrid, thickness=1, transparency=0):
  #plotting the curface over an ellipse to make the top look flat.
> B:=spacecurve([cos(t),sin(t),0],t=0..2*Pi,color=blue, thickness=
  8): #circle in the xy-plane
> Cin:=spacecurve([1.02*cos(t),1.02*sin(t),(cos(t))^2 + 2*(sin(t))
  ^2],t=0..2*Pi,color=black, thickness=8): #cuve on the surface
  above the circle in the xy-plane with 1.02 fudge factor to make
  the image look better
> Cout := spacecurve([0.98· cos(t), 0.98· sin(t), (cos(t))^2 + 2*(sin(t))^2], t=0..2*Pi,
     color=black, thickness=8) :
     #cuve on the surface above the circle in the xy-plane with 0.98 fudge factor to make the
     image look better
> display([A,B,Cin,Cout], scaling=constrained,title="walking around
  the blue curve loking for the maximum", orientation=[50,70]);
```

walking around the blue curve loking for the maximum



Now for a fifferent view of the same thing.

```
> a:=plot3d(f,x=-1..1,y=-sqrt(1-x^2)..sqrt(1-x^2),axes=framed):
  #Note how we are plotting the surface over the disk of radius 1
> b:=plot3d(0,x=-1..1,y=-sqrt(1-x^2)..sqrt(1-x^2),color=black):
  #this plots the disk of radius one in the xy-plane
> c:=pointplot3d({[1,0,1],[-1,0,1]},axes=framed,color=blue,symbol=
  solidcircle, symbolsize=40):
> d:=pointplot3d({[0,1,2],[0,-1,2]},axes=framed,color=red,symbol=
  solidcircle, symbolsize=40):
> display({a,b,c,d}, scaling=constrained, orientation=[50,70]);
```



The blue balls are at the minima on the boundary. The red balls are at the maxima on the boundary. The absolute minimum is f=0 which occurs at (0,0). The absolute maximum is f=2 which occurs at (0, -1) and (0,1).