> restart : with(LinearAlgebra) : with(VectorCalculus) : with(plots) :

>

Now for a 2 variable exmaple.

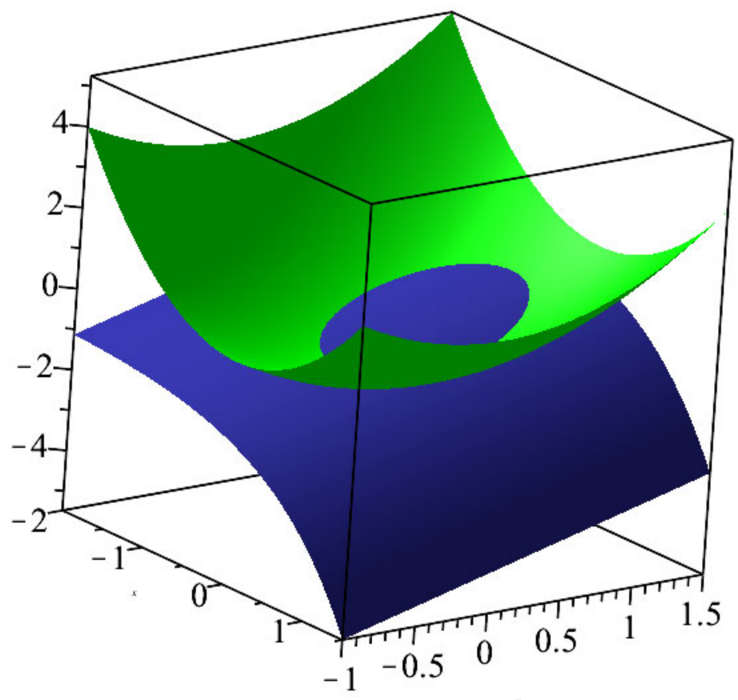
> $g := x^2 + y^2 - 1; h := y - \exp(x);$

$$g := x^2 + y^2 - 1$$

$$h := y - e^x \qquad (1)$$

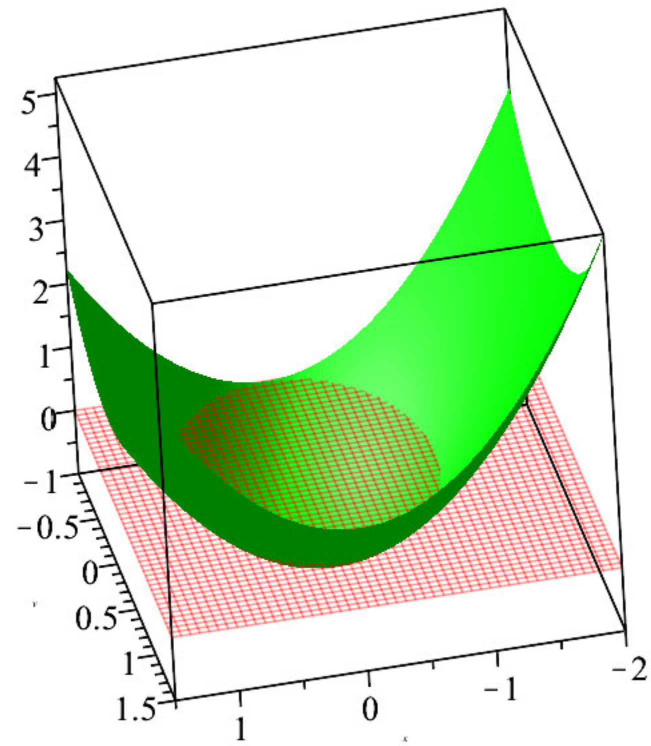
We want to find the solution(s) of [f(x,y), g(x,y)] = [0, 0]. Geometrically this is the intersection of the three surfaces z = 0, z=f(x,y) and z = g(x,y).

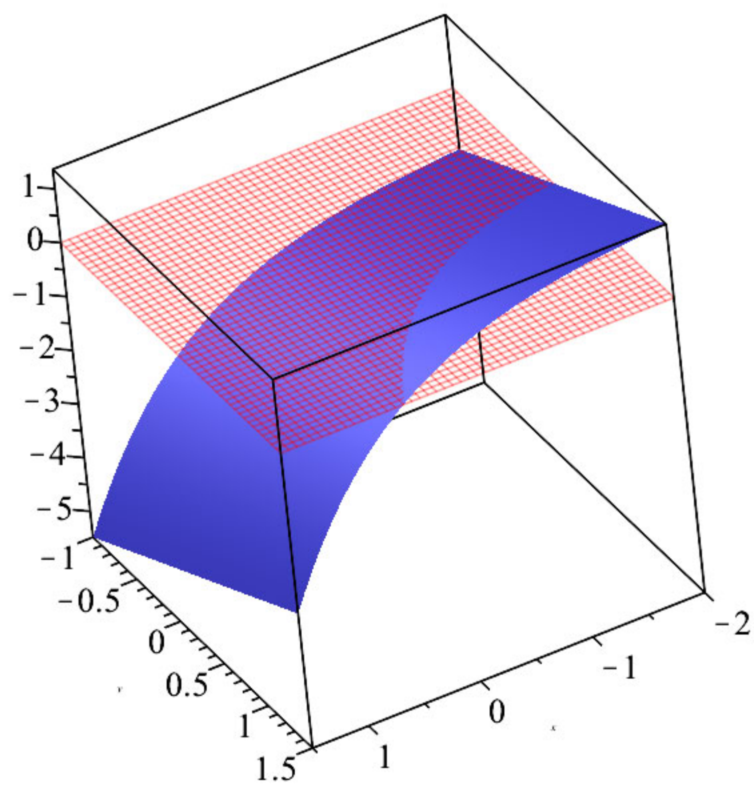
> a := plot3d(g, x = -2 ..1.5, y = -1 ..1.5, color = green, style = surface) : b := plot3d(h, x = -2 ..1.5, y = -1 ..1.5, color = navy, style = surface, transparency = 0) : display([a, b]);

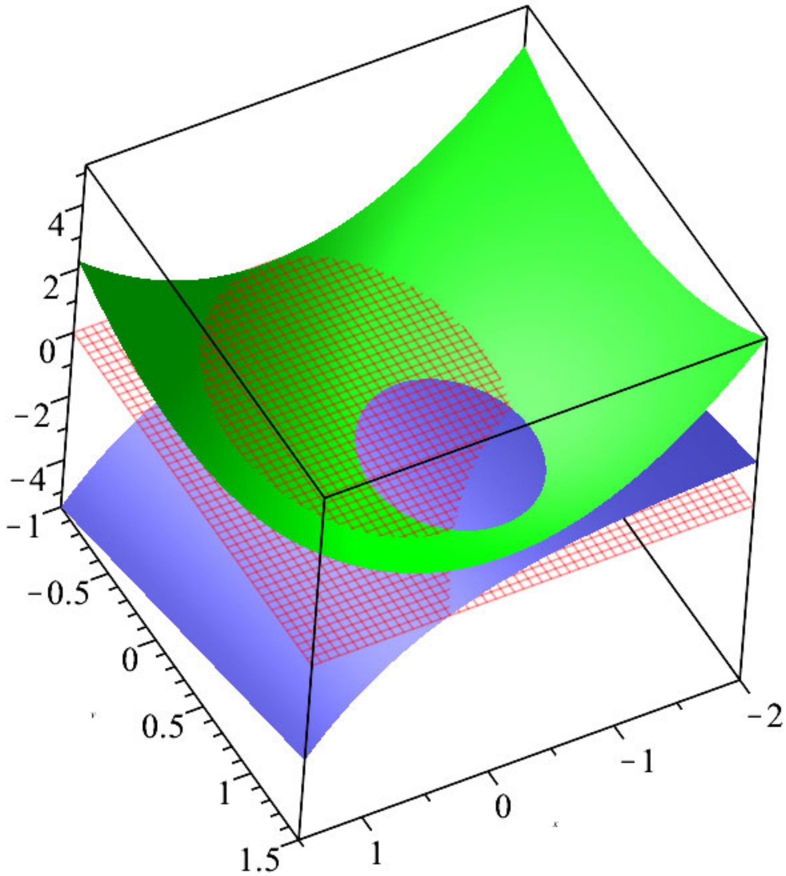


> zp := plot3d(0, x = -2 ..1.5, y = -1 ..1.5, color = red, style = wireframe) :

> display(a, zp); display(b, zp); display([a, b], zp);


5
4
3
2
1
0
−1
−0.5
0
0.5
1
1.5
1
0
−1
−2


1
0
−1
−2
−3
−4
−5
−1
−0.5
0
0.5
1
1.5
1
0
−1
−2



>

Apply Newton's Method.

> $F := Matrix(2, 1, [g, h]);$

$$F := \begin{bmatrix} x^2 + y^2 - 1 \\ y - e^x \end{bmatrix} \tag{2}$$

> $J := Jacobian([g, h], [x, y]);$

$$J := \begin{bmatrix} 2\,x & 2\,y \\ -e^x & 1 \end{bmatrix} \tag{3}$$

> $Jinv := MatrixInverse(J);$

$$Jinv := \begin{bmatrix} \frac{1}{2\,(y\,e^x + x)} & -\frac{y}{y\,e^x + x} \\ \frac{e^x}{2\,(y\,e^x + x)} & \frac{x}{y\,e^x + x} \end{bmatrix} \tag{4}$$

> $x[0] := -2; y[0] := 2;$

$$x_0 := -2$$
$$y_0 := 2 \tag{5}$$

```
> for i from 0 to 5 do A := Matrix(2, 1, [x[i], y[i]]) - subs(x = x[i], y=y[i] , Jinv) . subs(x
      = x[i], y=y[i] , F); x[i + 1] := evalf(A[1, 1]); y[i + 1] := evalf(A[2, 1]); end;
```

$$A := \begin{bmatrix} -2 - \frac{7}{2\,(2\,e^{-2} - 2)} + \frac{2\,(2 - e^{-2})}{2\,e^{-2} - 2} \\ 2 - \frac{7\,e^{-2}}{2\,(2\,e^{-2} - 2)} + \frac{2\,(2 - e^{-2})}{2\,e^{-2} - 2} \end{bmatrix}$$

$$x_1 := -2.132611768$$

$$y_1 := 0.117388233$$

$$A := \begin{bmatrix} -1.29198221880000 \\ 0.218164893800000 \end{bmatrix}$$

$$x_2 := -1.29198221880000$$

$$y_2 := 0.218164893800000$$

$$A := \begin{bmatrix} -0.991062977488216 \\ 0.357395918780530 \end{bmatrix}$$

$$x_3 := -0.991062977488216$$

$$y_3 := 0.357395918780530$$

$$A := \begin{bmatrix} -0.921287101930850 \\ 0.397081466899635 \end{bmatrix}$$

$$x_4 := -0.921287101930850$$

$$y_4 := 0.397081466899635$$

$$A := \begin{bmatrix} -0.916584692161867 \\ 0.399878025769967 \end{bmatrix}$$

$$x_5 := -0.916584692161867$$

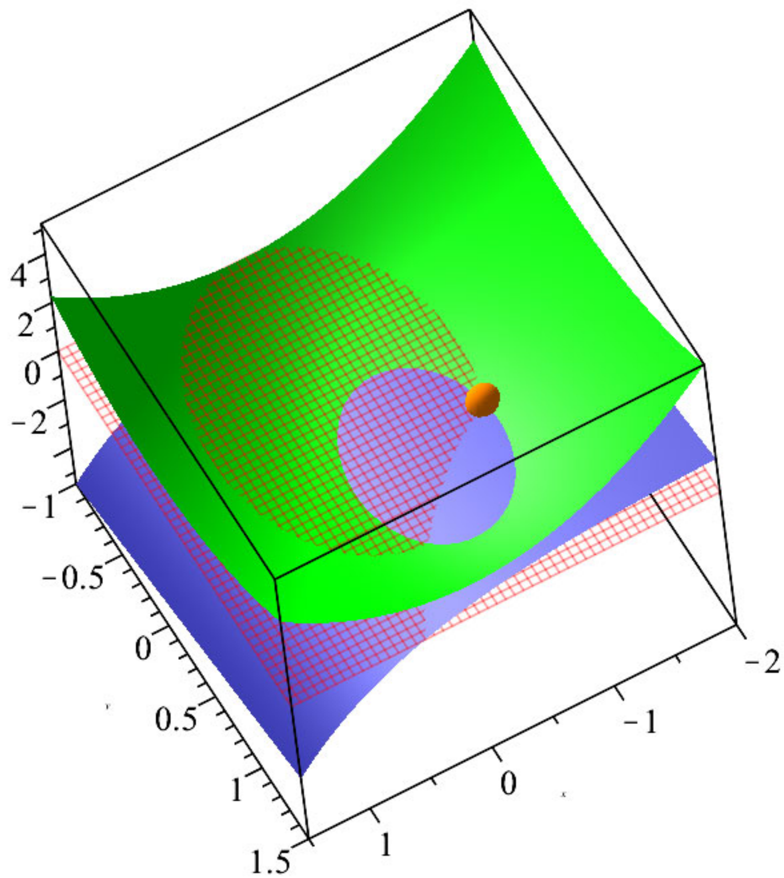
$$y_5 := 0.399878025769967$$

$$A := \begin{bmatrix} -0.916562583596326 \\ 0.399891273992734 \end{bmatrix}$$

$$x_6 := -0.916562583596326$$

$$y_6 := 0.399891273992734 \quad \textbf{(6)}$$

```
>
> p := pointplot3d( [x[6], y[6], subs(x = x[6], y = y[6], h) ], color = coral, symbol = solidcircle) :
> display( [a, b, zp, p], symbolsize = 40 );
```

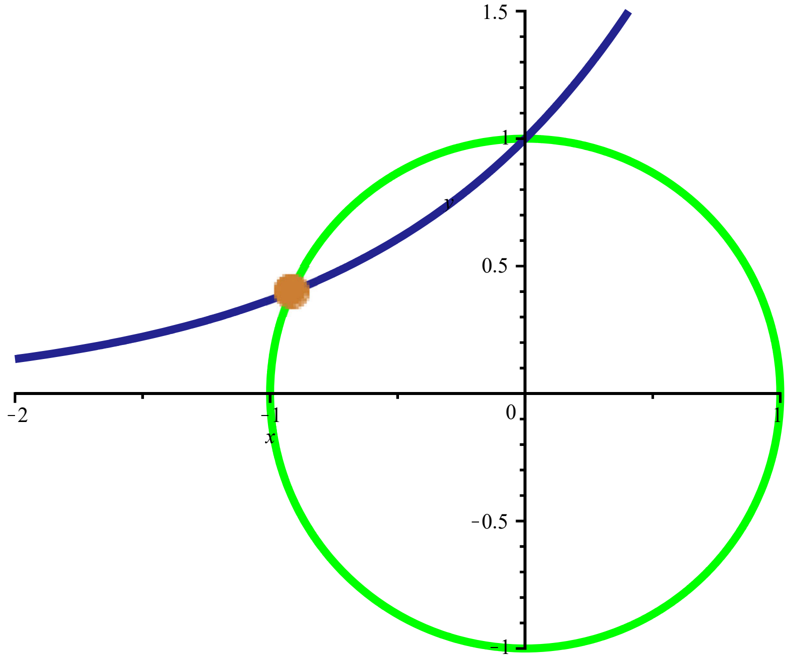


Here are plots of the intersection of the surfaces with the xy-plane.

> $xyp := implicitplot([g=0, h=0], x=-2..1.5, y=-1..1.5, grid=[100, 100], color=[green, navy], thickness=3):$

> $pp := pointplot([x[6], y[6]], color=gold, symbolsize=30, symbol=solidcircle):$

> $display([xyp, pp], size=[600, 600], scaling=constrained);$



>

> $x[0] := 2; y[0] := 2;$

$$x_0 := 2$$

$$y_0 := 2 \tag{7}$$

> **for** i **from** 0 **to** 5 **do** $A := Matrix(2, 1, [x[i], y[i]]) - subs(x = x[i], y = y[i], Jinv) . subs(x = x[i], y = y[i], F); x[i+1] := evalf(A[1, 1]); y[i+1] := evalf(A[2, 1]);$ **end**;

$$A := \begin{bmatrix} 2 - \frac{7}{2\,(2\,e^2 + 2)} + \frac{2\,(2 - e^2)}{2\,e^2 + 2} \\ 2 - \frac{7\,e^2}{2\,(2\,e^2 + 2)} - \frac{2\,(2 - e^2)}{2\,e^2 + 2} \end{bmatrix}$$

$$x_1 := 1.149003652$$

$$y_1 := 1.100996348$$

$$A := \begin{bmatrix} 0.494039131500000 \\ 1.08860343750000 \end{bmatrix}$$

$$x_2 := 0.494039131500000$$

$$y_2 := 1.08860343750000$$

$$A := \begin{bmatrix} 0.136891358936767 \\ 1.05358510381892 \end{bmatrix}$$

$$x_3 := 0.136891358936767$$

$$y_3 := 1.05358510381892$$

$$A := \begin{bmatrix} 0.0160782309758589 \\ 1.00816671840998 \end{bmatrix}$$

$$x_4 := 0.0160782309758589$$

$$y_4 := 1.00816671840998$$

$$A := \begin{bmatrix} 0.000282838819896544 \\ 1.00015677451922 \end{bmatrix}$$

$$x_5 := 0.000282838819896544$$

$$y_5 := 1.00015677451922$$

$$A := \begin{bmatrix} 9.22340954987756\ 10^{-8} \\ 1.00000005225374 \end{bmatrix}$$

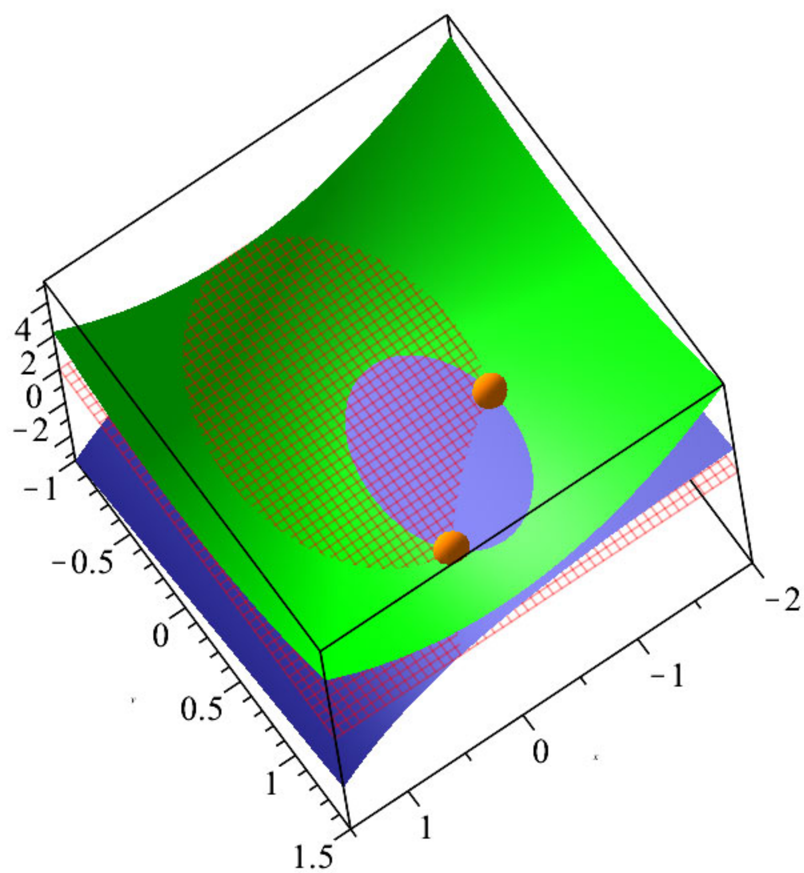
$$x_6 := 9.22340954987756\ 10^{-8}$$

$$y_6 := 1.00000005225374 \quad \textbf{(8)}$$

\>

\> *p2* := *pointplot3d*([*x*[6], *y*[6], *subs*(*x* = *x*[6], *y* = *y*[6], *h*)], *color* = *coral*, *symbol* = *solidcircle*) :

\> *display*([*a*, *b*, *zp*, *p*, *p2*], *symbolsize* = 40);

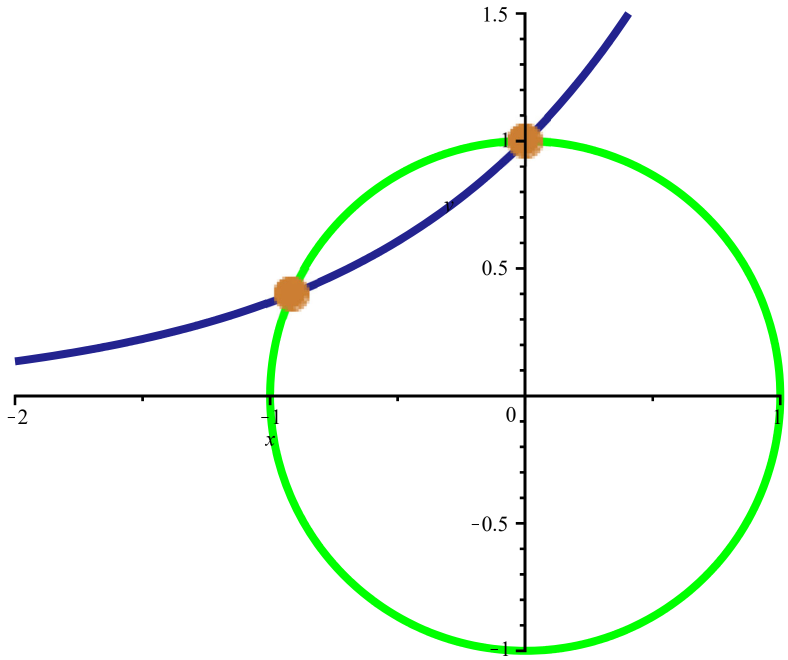


Here are plots of the intersection of the surfaces with the xy-plane.

> $xyp := implicitplot([g=0, h=0], x=-2..1.5, y=-1..1.5, grid=[100, 100], color=[green, navy], thickness=3):$

> $pp2 := pointplot([x[6], y[6]], color=gold, symbolsize=30, symbol=solidcircle):$

> $display([xyp, pp, pp2], size=[600, 600], scaling=constrained);$



>