```
> restart : with(LinearAlgebra) : with(VectorCalculus) : with(plots) :
> f := x^2 - exp(x) + sin(x);
```

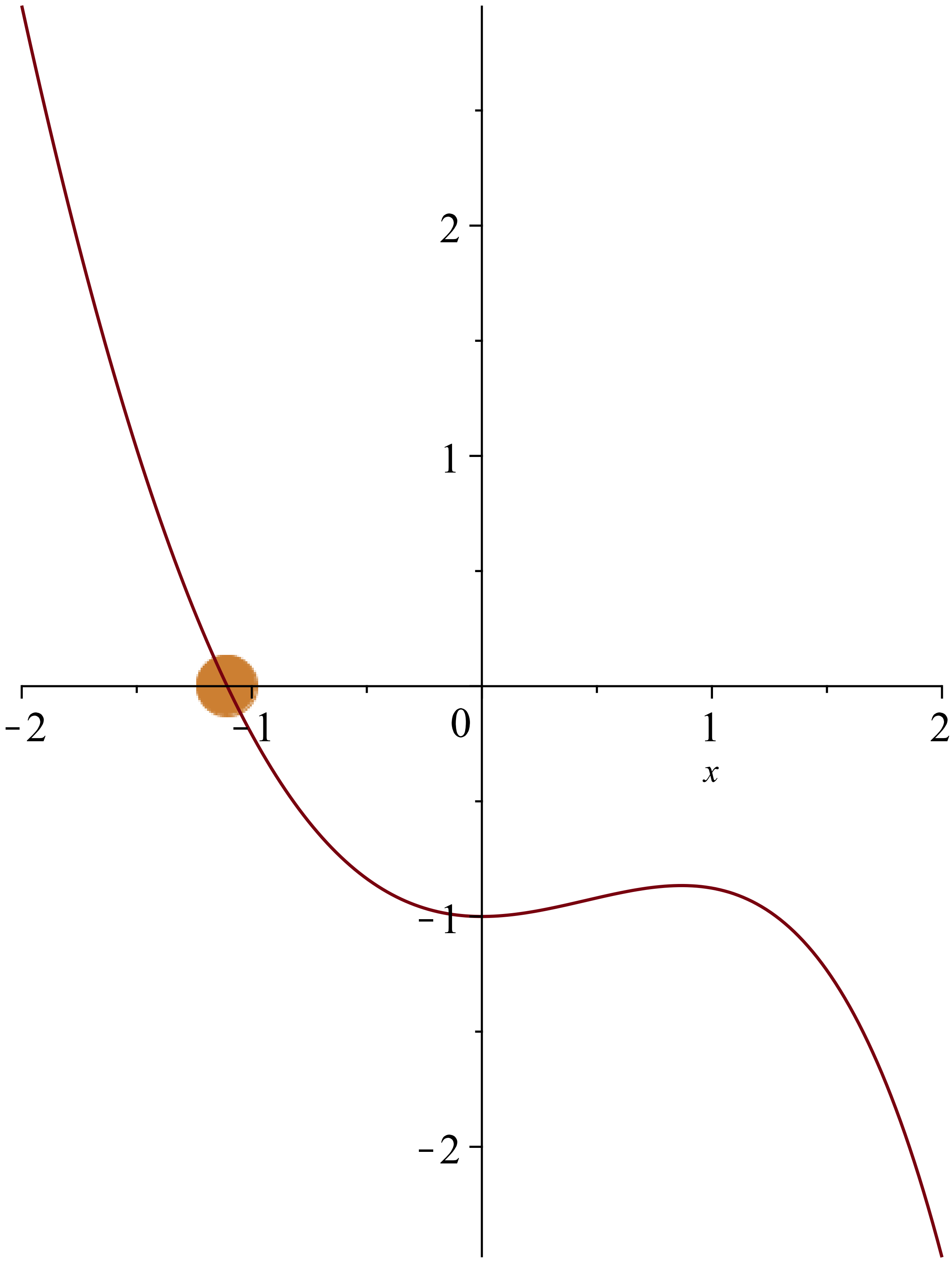
$$f := x^2 - e^x + \sin(x) \tag{1}$$

We want to find the solution(s) of f(x) = 0. Maple can do this for us.

```
> xint := fsolve(f=0, x);
```

$$xint := -1.106742853 \tag{2}$$

```
> ptp := pointplot([xint, 0], color=gold, symbolsize=40, symbol=solidcircle) :
> Pl := plot(f, x=-2..2, size=[600, 600], scaling=constrained) :
> display(ptp, Pl);
```



Now let's find this intercept using Newton's method.

> $x[0] := 1;$

$$x_0 := 1 \qquad (3)$$

> **for** i **from** 0 **to** 7 **do** $x[i+1] := evalf\left(x[i] - \frac{subs(x = x[i], f)}{subs(x = x[i], diff(f, x))}\right)$ **end**;

$$x_1 := -3.926470376$$
$$x_2 := -2.049552680$$
$$x_3 := -1.370389794$$
$$x_4 := -1.140046777$$
$$x_5 := -1.107398620$$
$$x_6 := -1.106743115$$
$$x_7 := -1.106742853$$
$$x_8 := -1.106742853 \quad \textbf{(4)}$$

We see that x_8 = xint

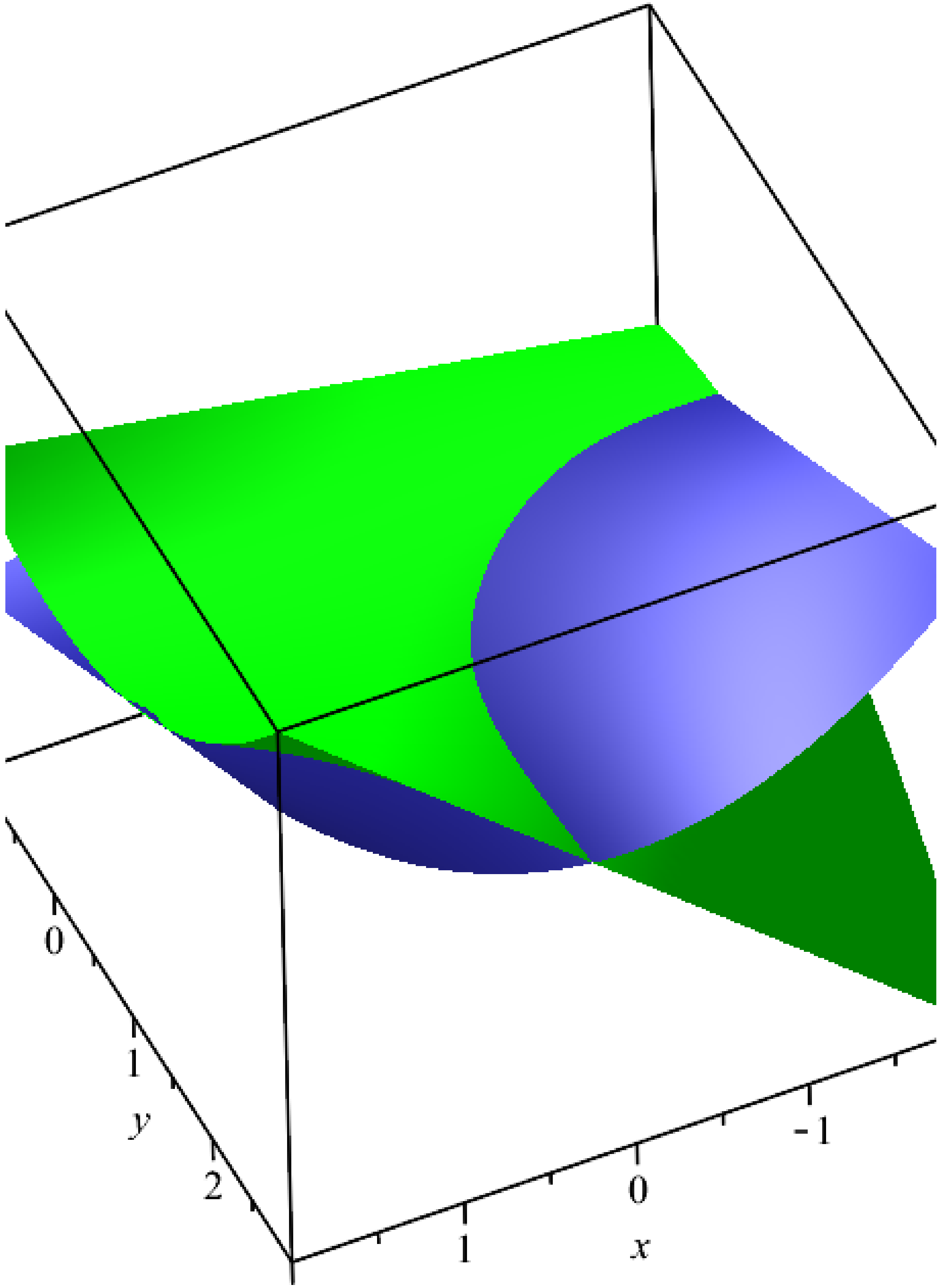
Now for a 2 variable exmaple.

```
> g := x·(1 + y^2) − 1; h := y·(1 + x^2) -2;
```

$$g := x\left(y^2 + 1\right) - 1$$
$$h := y\left(x^2 + 1\right) - 2 \quad \textbf{(5)}$$

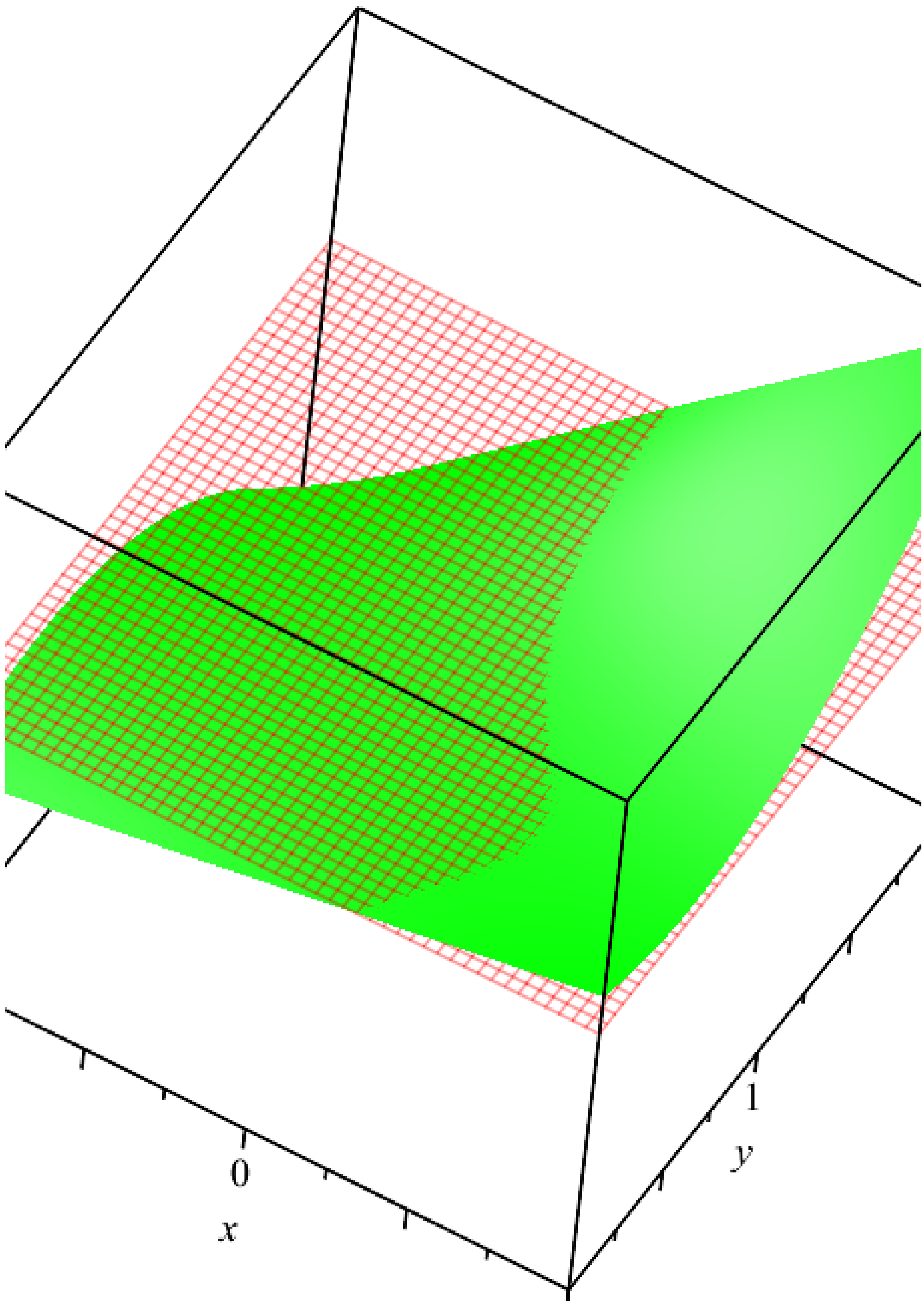
We want to find the solution(s) of [f(x,y), g(x,y)] = [0, 0]. Geometrically this is the intersection of the three surfaces z = 0, z=f(x,y) and z = g(x,y).

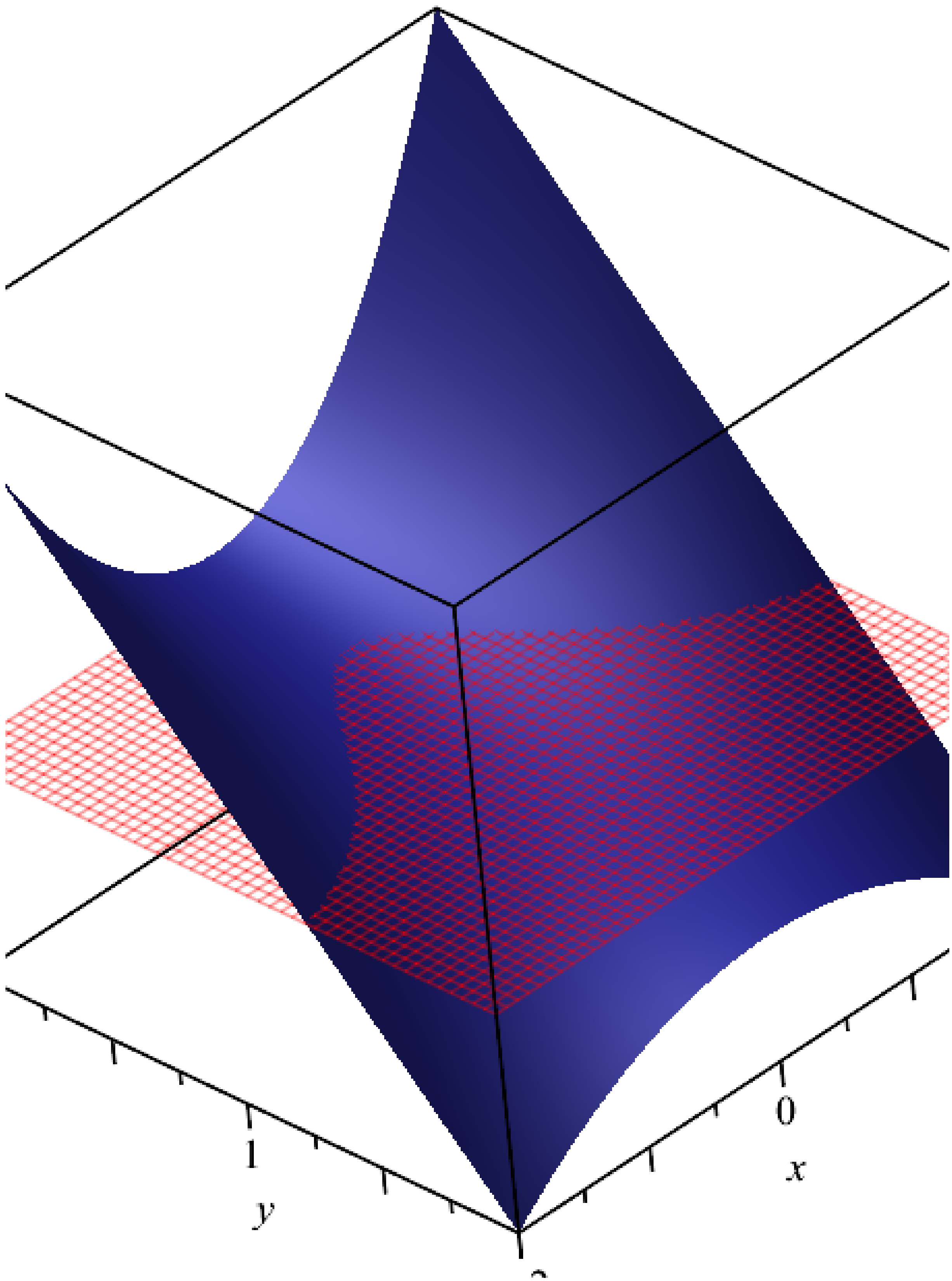
```
> a := plot3d(g, x=-2 ..2, y=-1 ..3, color=green, style=surface ) : b := plot3d(h, x=-2 ..2, y=
    -1 ..3, color=navy, style=surface, transparency=0) : display([a, b]);
```



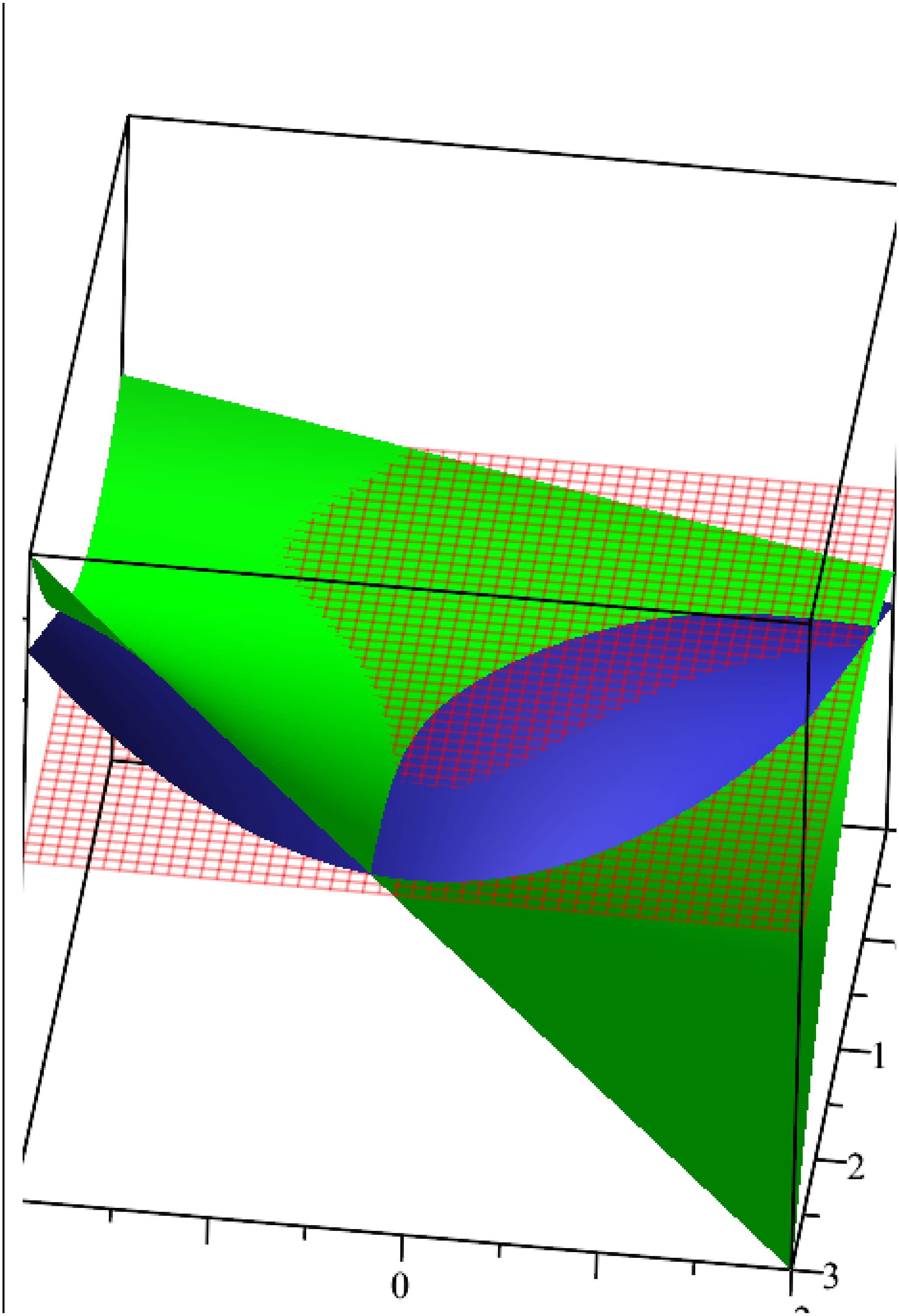
> $zp := plot3d(0, x = -2 ..2, y = -1 ..3, color = red, style = wireframe)$:

> $display(a, zp)$; $display(b, zp)$; $display([a, b], zp)$;


0
x
1
y



1
y
0
x



0
1
2
3

>

Apply Newton's Method.

> $F := Matrix(2, 1, [g, h]);$

$$F := \begin{bmatrix} x\left(y^2+1\right)-1 \\ y\left(x^2+1\right)-2 \end{bmatrix} \quad \textbf{(6)}$$

> $J := Jacobian([g, h], [x, y]);$

$$J := \begin{bmatrix} y^2+1 & 2\,x\,y \\ 2\,x\,y & x^2+1 \end{bmatrix} \quad \textbf{(7)}$$

> $Jinv := MatrixInverse(J);$

$$Jinv := \begin{bmatrix} -\frac{x^2+1}{3\,x^2\,y^2-x^2-y^2-1} & \frac{2\,x\,y}{3\,x^2\,y^2-x^2-y^2-1} \\ \frac{2\,x\,y}{3\,x^2\,y^2-x^2-y^2-1} & -\frac{y^2+1}{3\,x^2\,y^2-x^2-y^2-1} \end{bmatrix} \quad \textbf{(8)}$$

> $x[0] := 3; y[0] := 2;$

$$x_0 := 3$$
$$y_0 := 2 \quad \textbf{(9)}$$

> **for** i **from** 0 **to** 5 **do** $A := Matrix(2, 1, [x[i], y[i]]) - subs(x = x[i], y = y[i], Jinv) . subs(x = x[i], y = y[i], F); x[i+1] := evalf(A[1, 1]); y[i+1] := evalf(A[2, 1]);$ **end**;

$$A := \begin{bmatrix} \frac{103}{47} \\ \frac{55}{47} \end{bmatrix}$$
$$x_1 := 2.191489362$$
$$y_1 := 1.170212766$$
$$A := \begin{bmatrix} 2.17221898774593 \\ 0.361704894993136 \end{bmatrix}$$
$$x_2 := 2.17221898774593$$
$$y_2 := 0.361704894993136$$
$$A := \begin{bmatrix} 0.115617630778300 \\ 0.914876325299808 \end{bmatrix}$$
$$x_3 := 0.115617630778300$$
$$y_3 := 0.914876325299808$$
$$A := \begin{bmatrix} 0.429997962360412 \\ 1.90798741626009 \end{bmatrix}$$

$$x_4 := 0.429997962360412$$

$$y_4 := 1.90798741626009$$

$$A := \begin{bmatrix} 0.162170472277894 \\ 2.05879939129760 \end{bmatrix}$$

$$x_5 := 0.162170472277894$$

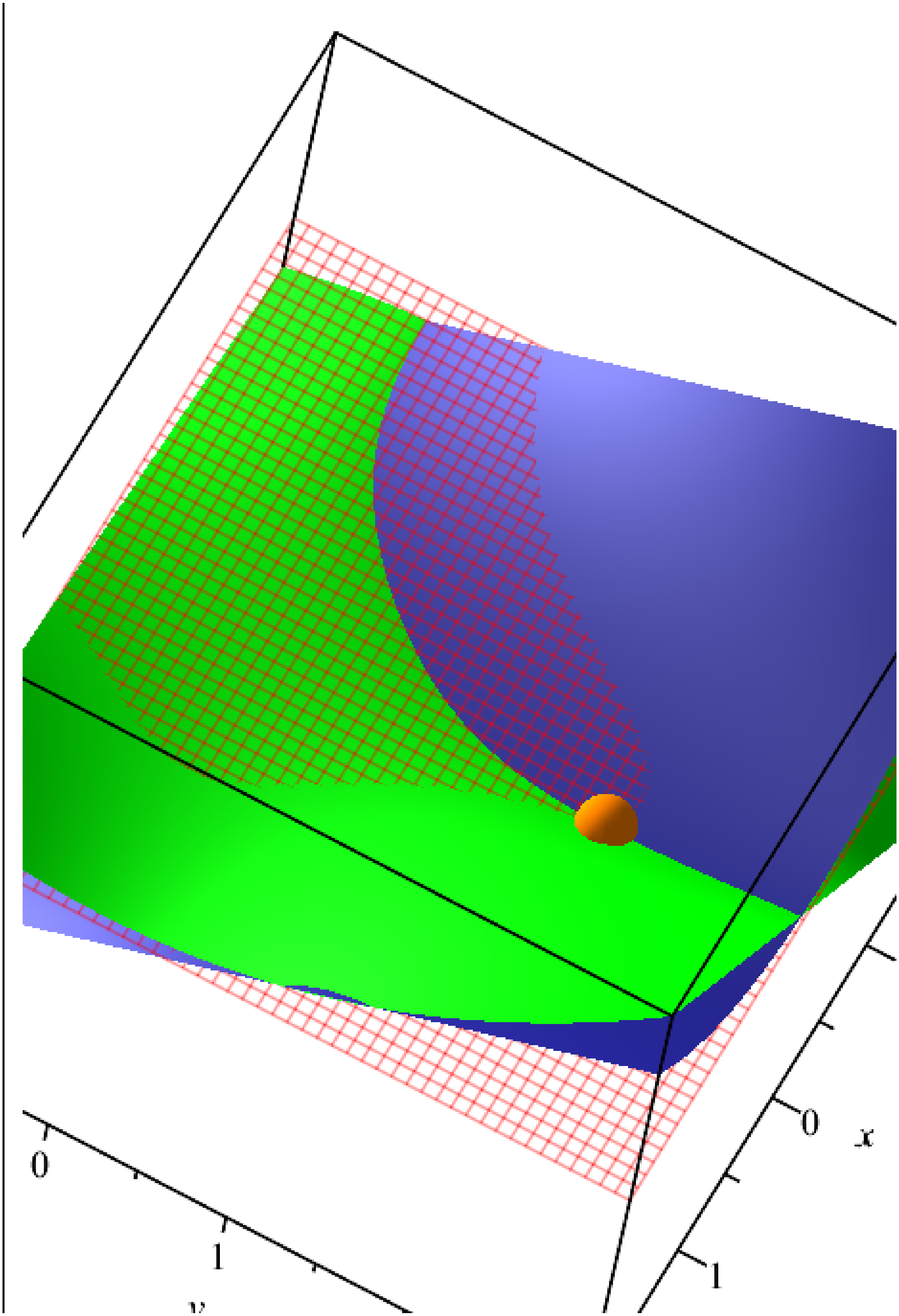
$$y_5 := 2.05879939129760$$

$$A := \begin{bmatrix} 0.208782125988151 \\ 1.91842185216502 \end{bmatrix}$$

$$x_6 := 0.208782125988151$$

$$y_6 := 1.91842185216502 \quad \textbf{(10)}$$

```
> p := pointplot3d([x[6], y[6], subs(x = x[6], y = y[6], h)], color = coral, symbol = solidcircle) :
> display([a, b, zp, p], symbolsize = 40);
```


0
1
0
x
1

Here are plots of the intersection of the surfaces with the xy-plane.

> $xyp := implicitplot([g = 0, h = 0], x = -2..2, y = -1..3, grid = [100, 100], color = [green, navy], thickness = 3)$:

> $pp := pointplot([x[6], y[6]], color = gold, symbolsize = 30, symbol = solidcircle)$:

> $display([xyp, pp], size = [600, 600], scaling = constrained)$;