

MEC-E8001 Finite Element Analysis; Mathematica solver

“A structure is a collection of elements connected by nodes. Geometry, displacement, temperature etc. of the structure are defined by the nodal values of coordinates, translation, rotation, temperature etc. of which some are known and some unknown.”

DATA STRUCTURE

$prb = \{ele, fun\}$ where

$ele = \{prt_1, prt_2, \dots\}$ elements

$fun = \{val_1, val_2, \dots\}$ nodes

Element

$prt = \{typ, pro, geo\}$ where

$typ = \text{BAR} | \text{TORSION} | \text{BEAM} | \text{RIGID} | \dots$ model

$pro = \{p_1, p_2, \dots, p_n\}$ properties

$geo = \text{Point}[\{n_1\}] | \text{Line}[\{n_1, n_2\}] | \text{Polygon}[\{n_1, n_2, n_3\}] | \dots$ geometry

Nodes

$val = \{crd, trn, rot\} | \{crd, trn, rot, tmp\}$ where

$crd = \{X, Y, Z\}$ structural coordinates

$trn = \{u_X, u_Y, u_Z\}$ translation components

$rot = \{\theta_X, \theta_Y, \theta_Z\}$ rotation components

$tmp = \mathcal{G}$ temperature

DISPLACEMENT ANALYSIS

Constraint

{JOINT, { } { { u_X, u_Y, u_Z } }, Point[{ n_1 }] }displacement constraint
{JOINT, { }, Line[{ n_1, n_2 }] }displacement constraint
{RIGID, { } { { u_X, u_Y, u_Z }, { $\theta_X, \theta_Y, \theta_Z$ } }, Point[{ n_1 }] }displacement/rotation constraint
{RIGID, { }, Line[{ n_1, n_2 }] }rigid constraint
{SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
{FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] }point force/moment
{FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] }distributed force
{FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] }distributed force

Beam model

{BAR, { { E }, { A }, { f_x } }, Line[{ n_1, n_2 }] }bar mode
{TORSION, { { G }, { A }, { m_x } }, Line[{ n_1, n_2 }] } torsion mode
{BEAM, { { E, G }, { A, I_{yy}, I_{zz} }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam
{BEAM, { { E, G }, { $A, I_{yy}, I_{zz}, \{j_X, j_Y, j_Z\}$ }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam

Plate model

{PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
{PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode
{PLATE, { { E, G, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } bending mode

Solid model

{SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] }solid
{SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }] }solid

Functions

$prb = \text{REFINE}[prb]$ refine structure representation
$\text{Out} = \text{FORMATTED}[prb]$display problem definition
$\text{Out} = \text{STANDARDFORM}[prb]$display virtual work expression
$\text{sol} = \text{SOLVE}\{\{\text{DISP}\}, prb\} \mid \text{SOLVE}[prb]$solve the unknowns

VIBRATION ANALYSIS

Constraint

{JOINT, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ } }, Point[{ n_1 }] } displacement constraint
{JOINT, { }, Line[{ n_1, n_2 }] } displacement constraint
{RIGID, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ }, { $\underline{\theta}_X, \underline{\theta}_Y, \underline{\theta}_Z$ } }, Point[{ n_1 }] } displacement/rotation constraint
{RIGID, { }, Line[{ n_1, n_2 }] } rigid constraint
{SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
{FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] } point force/moment
{FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] } distributed force
{FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] } distributed force
{FORCE, { { m } }, Point[{ n_1 }] } inertia effect
{FORCE, { { m, J } }, Point[{ n_1 }] } inertia effect
{FORCE, { { m, J }, { { i_X, i_Y, i_Z }, { j_X, j_Y, j_Z } } }, Point[{ n_1 }] } inertia effect

Beam model

{BAR, { { E }, { A }, { f_x } }, Line[{ n_1, n_2 }] } bar mode
{TORSION, { { G }, { A }, { m_x } }, Line[{ n_1, n_2 }] } torsion mode
{BEAM, { { E, G, ρ }, { A, I_{yy}, I_{zz} }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam
{BEAM, { { E, G, ρ }, { A, I_{yy}, I_{zz} }, { j_X, j_Y, j_Z } }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam

Plate model

{PLANE, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
{PLANE, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode
{PLATE, { { E, G, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } bending mode

Solid model

{SOLID, { { E, ν, ρ }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] } solid
{SOLID, { { E, ν, ρ }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }] } solid

Functions

prb = REFINE[*prb*] refine structure representation
Out = FORMATTED[*prb*] display problem definition

Out = STANDARDFORM[{VIBR}, *prb*] display virtual work expression
sol = SOLVE[{VIBR}, *prb*] solve angular speeds and modes
sol = SOLVE[{VIBR, *ini*}, *prb*] solve unknowns (does not work for a DAE)

STABILITY ANALYSIS

Constraint

{JOINT, { } | { { u_X, u_Y, u_Z } }, Point[{ n_1 }] }displacement constraint
 {JOINT, { }, Line[{ n_1, n_2 }] }displacement constraint
 {RIGID, { } | { { u_X, u_Y, u_Z }, { $\theta_X, \theta_Y, \theta_Z$ } }, Point[{ n_1 }] }displacement/rotation constraint
 {RIGID, { }, Line[{ n_1, n_2 }] }rigid constraint
 {SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
 {FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] }point force/moment
 {FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] }distributed force
 {FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] }distributed force

Beam model

{BAR, { { E }, { A }, { f_x } }, Line[{ n_1, n_2 }] }bar mode
 {TORSION, { { G }, { A }, { m_x } }, Line[{ n_1, n_2 }] } torsion mode
 {BEAM, { { E, G }, { A, I_{yy}, I_{zz} }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam
 {BEAM, { { E, G }, { $A, I_{yy}, I_{zz}, j_X, j_Y, j_Z$ } }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] } beam

Plate model

{PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
 {PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode
 {PLATE, { { E, G, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } bending mode

Solid model

{SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] }solid
 {SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }] }solid

Functions

prb = REFINE[*prb*] refine structure representation
 Out = FORMATTED[*prb*]display problem definition
 Out = STANDARDFORM[{STAB}, *prb*]display virtual work expression
sol = SOLVE[{STAB, *p*}, *prb*]find the critical values of *p* and the modes

NONLINEAR ANALYSIS

Constraint

{JOINT, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ } }, Point[{ n_1 }] }displacement constraint
{JOINT, { }, Line[{ n_1, n_2 }] }displacement constraint
{RIGID, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ }, { $\underline{\theta}_X, \underline{\theta}_Y, \underline{\theta}_Z$ } }, Point[{ n_1 }] }displacement/rotation constraint
{RIGID, { }, Line[{ n_1, n_2 }] }rigid constraint
{SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
{FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] }point force/moment
{FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] }distributed force
{FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] }distributed force

Beam model

{BAR, { { E }, { $A, \{j_X, j_Y, j_Z\}$ }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] }bar mode

Plate model

{PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
{PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode

Solid model

{SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] }solid
{SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }] }solid

Functions

prb = REFINE[*prb*] refine structure representation
Out = FORMATTED[*prb*]display problem definition
Out = STANDARDFORM[{NONL}, *prb*] display virtual work expression
sol = SOLVE[{NONL}, *prb*] find the likely numerical solution
sol = SOLVE[{NONL, ALL}, *prb*] find all the solutions

THERMOMECHANICAL ANALYSIS

Constraint

{JOINT, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ } }, Point[{ n_1 }] }displacement constraint
{JOINT, { }, Line[{ n_1, n_2 }] }displacement constraint
{RIGID, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ }, { $\underline{\theta}_X, \underline{\theta}_Y, \underline{\theta}_Z$ } }, Point[{ n_1 }] }displacement/rotation constraint
{RIGID, { }, Line[{ n_1, n_2 }] }rigid constraint
{SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
{FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] }point force/moment
{FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] }distributed force
{FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] }distributed force

Beam model

{BAR, { { E, α, k }, { A }, { f_x, s, \mathcal{G}_0 } }, Line[{ n_1, n_2 }] }bar mode

Plate model

{PLANE, { { E, ν, α, k }, { t }, { { f_X, f_Y, f_Z }, s, \mathcal{G}_0 } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
{PLANE, { { E, ν, α, k }, { t }, { { f_X, f_Y, f_Z }, s, \mathcal{G}_0 } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode

Solid model

{SOLID, { { E, ν, α, k }, { { f_X, f_Y, f_Z }, s, \mathcal{G}_0 } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] }solid

Functions

prb = REFINE[*prb*] refine structure representation
Out = FORMATTED[*prb*]display problem definition
Out = STANDARDFORM[{TMEC}, *prb*] display virtual work expression
sol = SOLVE[{TMEC}, *prb*]solve the unknowns