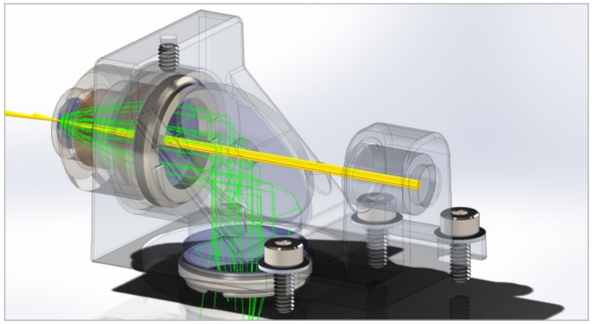
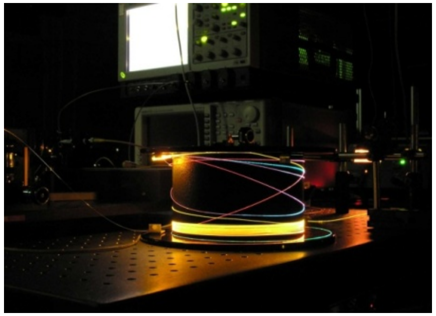
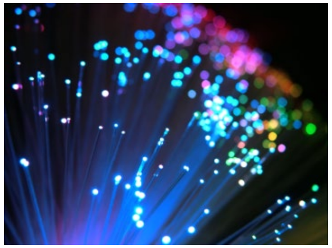
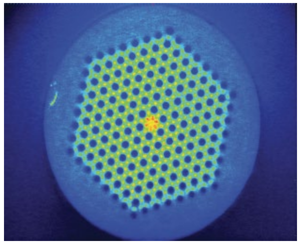
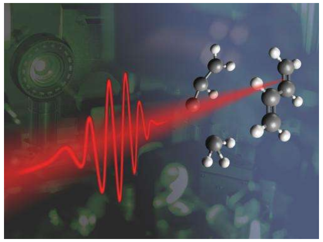
Optics E-5730 Spring 2021

Diffraction II

Lectures: Toni Laurila
Email: toni.k.laurila@aalto.fi
Tel. 050-358 3097

Last Lecture – Diffraction I

- Diffraction
- Diffraction integral
- Fraunhofer approximation
- Fraunhofer diffraction intensity for one, two and N slits
- Diffraction grating and grating spectrometer

Today's Lecture – Diffraction II

- Diffraction limit
- Fresnel approximation
- Fourier optics

Fraunhofer approximation

$$|\vec{r}-\vec{r}'| \approx z - \frac{xx'}{z} - \frac{yy'}{z} + \frac{x^2+y^2}{2z}$$

$$E(\vec{r}) = \frac{1}{i\lambda} \iint_{S'} E_{inc}(\vec{r}') \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dS'$$

$$|\vec{r}-\vec{r}'| \approx z$$

$$E(x,y,z) = \frac{e^{ikz+\frac{ik}{2z}(x^2+y^2)}}{i\lambda z} \iint_{S'} E_{inc}(x',y')\, e^{-ik\frac{xx'+yy'}{z}} dx'dy'$$

Using notation $\nu_x = \frac{x}{\lambda z}$ $\quad \nu_y = \frac{y}{\lambda z}$

$$E(x,y,z) = \frac{e^{ikz+\frac{ik}{2z}(x^2+y^2)}}{i\lambda z} \iint_{S'} E_{inc}(x',y')\, e^{-i2\pi(\nu_x x'+\nu_y y')} dx'dy'$$

2D Fourier-transformation

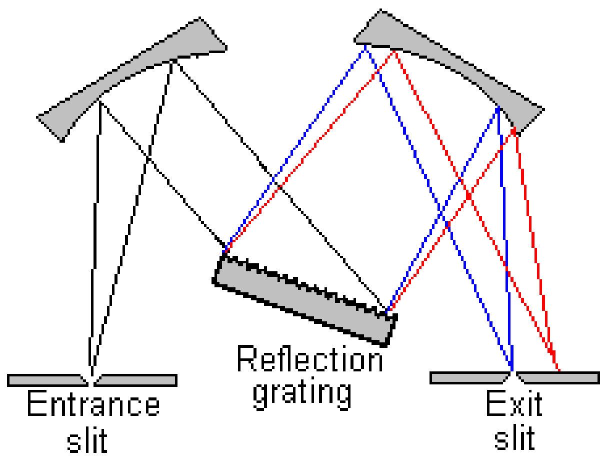
$$\mathcal{F}[E_{inc}(x',y')](\nu_x,\nu_y) = \iint_{S'} E_{inc}(x',y')\, e^{-i2\pi(\nu_x x'+\nu_y y')} dx'dy'$$

$$I(\nu_x,\nu_y) = \frac{1}{\lambda^2 z^2}\left[F[E_{inc}(x',y')](\nu_x,\nu_y)\right]^2$$

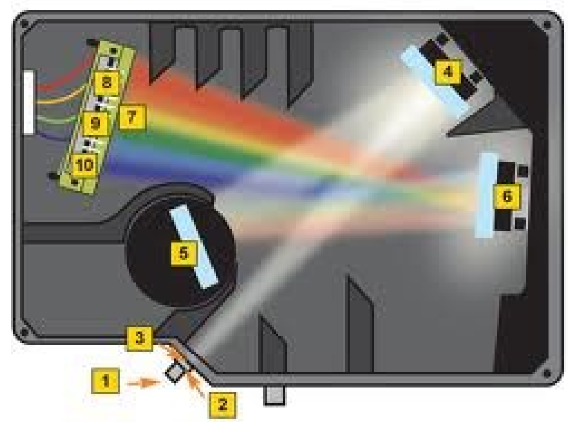
Intensity of the diffraction pattern results from 2D-Fourier transformation of the incident field.

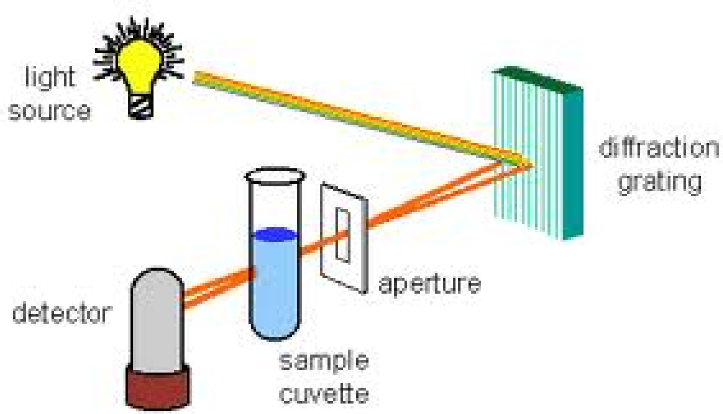
Grating Monochromator andGrating Spectrometer

monochromator



spectrometer





Fraunhofer Diffraction of a Circular Aperture

- electric field E_0 hits a circular aperture having a radius of r_0
- diffraction integral expressed in spherical coordinates (pp. 98-99)

$$E(x,y,z)=\frac{e^{ikz+\frac{ik}{2z}\left(x^2+y^2\right)}}{i\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_{inc}(x',y')e^{-i2\pi\left(\nu_x x'+\nu_y y'\right)}dx'dy'$$

$$=E_0\frac{e^{ikz+\frac{ik}{2z}\left(x^2+y^2\right)}}{i\lambda z}\int_0^{2\pi}\int_0^{r_0}e^{-i2\pi\left(\nu_x r\cos\varphi+\nu_y r\sin\varphi\right)}r\,d\varphi\,dr$$

- solutions are Bessel functions $J_1(t)$

$$\int_0^{2\pi}\int_0^{r_0}e^{-i2\pi\left(\nu_x r\cos\varphi+\nu_y r\sin\varphi\right)}r\,d\varphi\,dr=2\pi r_0^2\left(\frac{J_1(t)}{t}\right),$$

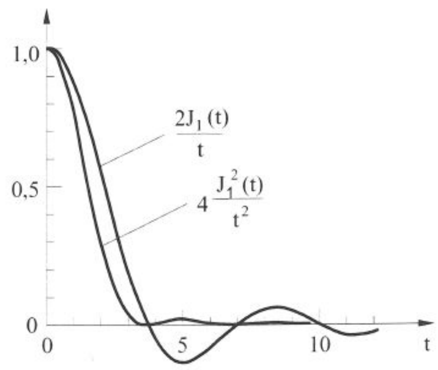
$$\text{where } t=2\pi r_0\nu \text{ ja } \nu=\sqrt{x^2+y^2}/(\lambda z)$$

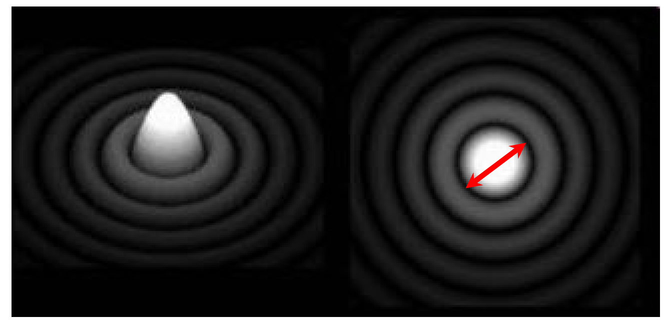
Fraunhofer Diffraction of a Circular Aperture

- diffraction intensity for a circular aperture becomes Airy function (J_1 = Bessel function):

$$I(x,y,z)=\frac{4\pi^2 r_0^4 E_0^2}{\lambda^2 z^2}\left(\frac{J_1(t)}{t}\right)^2$$

$$t = 2\pi r_0 \nu \text{ ja } \nu = \sqrt{x^2+y^2}/(\lambda z)$$



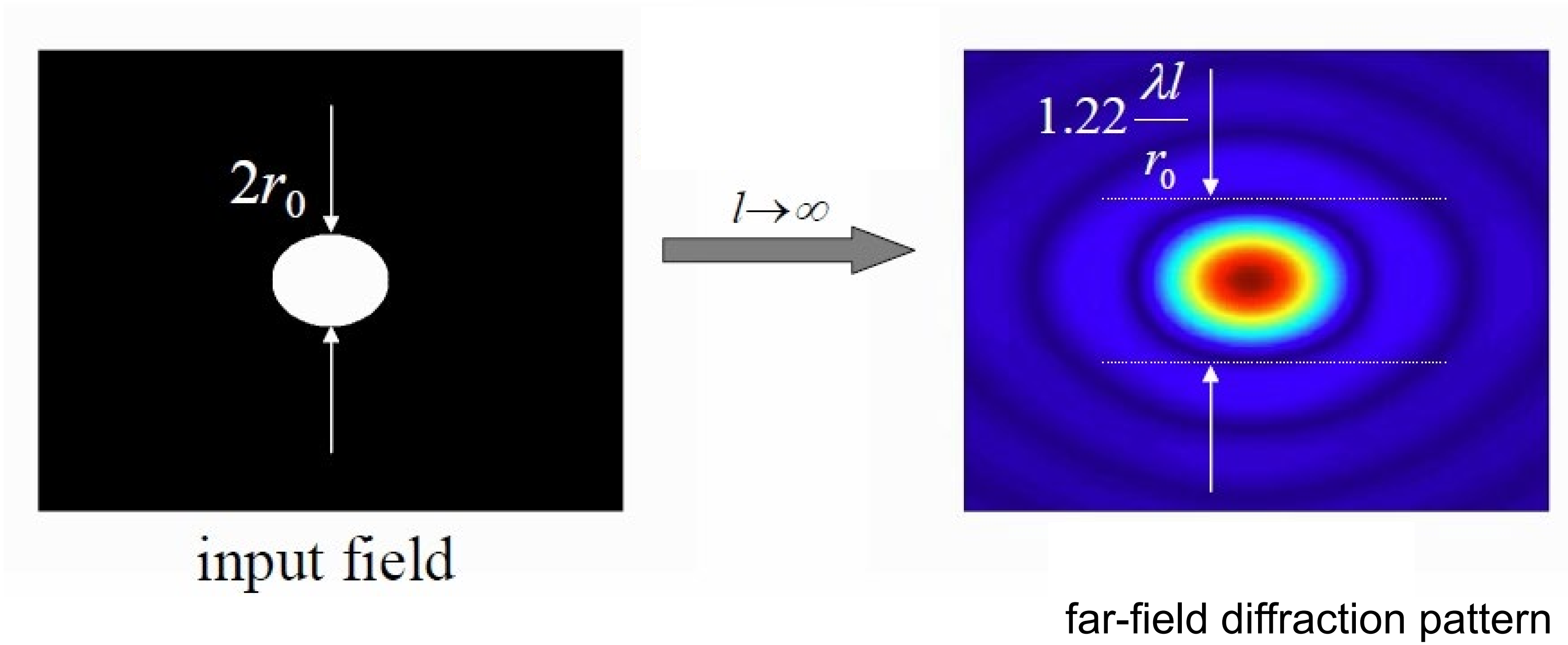


- the first zero value of J_1 is obtained at t = 3.83:

$$t = 2\pi r_0 \sqrt{x^2+y^2}/(\lambda z) = 3.83$$

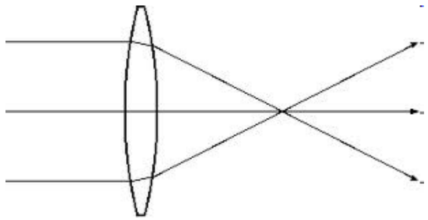
$$2\text{r} = 2\sqrt{x^2+y^2} = \frac{3.83\lambda z}{\pi r_0} = \frac{1.22\lambda z}{r_0}$$

Fraunhofer Diffraction of a Circular Aperture

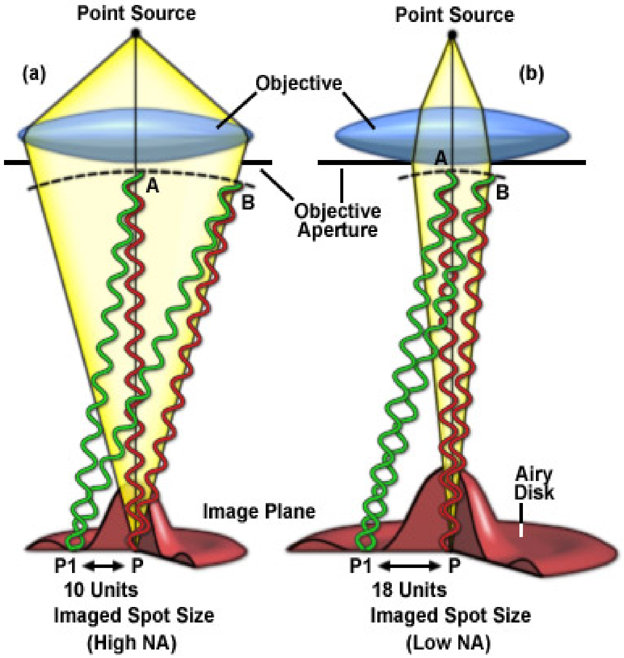


Diffraction Limited Spot Size

- Diffraction limit: focused ideal plane wave yields the minimum spot size (diameter $\varnothing_{MIN}$) of (f = focal lenght of the lens, D = lens diameter):



$$\varnothing_{MIN} = \frac{2.44\lambda f}{D}$$

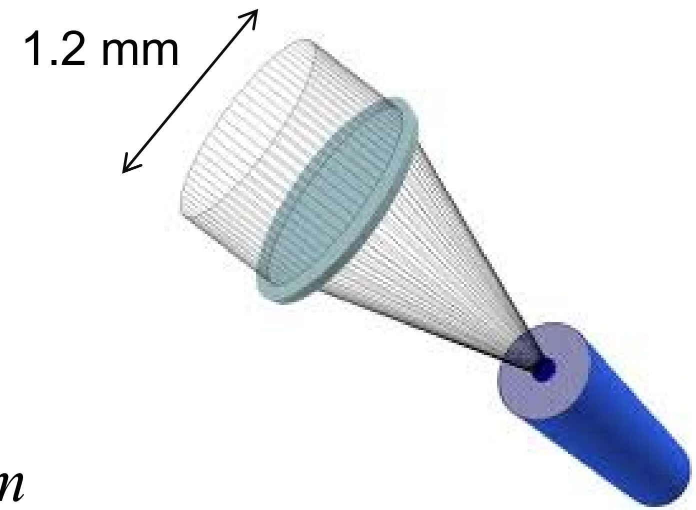


Example: Coupling a Laser Beam Into a Single-mode Fibre (SMF)

- laser beam diameter 1.2 mm
- laser wavelength 1550 nm
- SMF-28 mode field diameter 10.4 µm
- numerical aperture 0.14

$$\emptyset_{MIN} = \frac{2.44\lambda f}{D}$$

$$f \approx \frac{\emptyset_{MIN} D}{2.44\lambda} = \frac{10.4\ \mu m\, 1.2\ mm}{2.44\, 1550\ nm} = 3.3\ mm$$



EFL = 3.xx mm

e.g. www.thorlabs.com

Item # (Unmounted/ Mounted)	Info	EFL[a]	NA	OD	CA	WD[b]	DW	AR Range	M	Glass	Performance	Thread	Suggested Spanner Wrench
354330-C[c]	i	3.10 mm	0.68	6.3 mm	5.00 mm	1.76 mm	830 nm	1050 - 1700 nm	∞	D-ZK3	Focal Shift Spot Size Cross Section	-	-
C330TMD-C[c]				9.2 mm		1.76 mm						M9 x 0.5	SPW301
A414-C	i	3.30 mm	0.47	4.50 mm	3.52 mm	1.94 mm	670 nm	1050 - 1620 nm	∞	N-SF57	A414_Asph.pdf	-	-
A414TM-C				6.22 mm		1.81 mm						M6 x 0.5	SPW306
N414-C	i	3.30 mm	0.47	4.50 mm	3.52 mm	1.94 mm	670 nm	1050 - 1620 nm	∞	H-ZLAF52	N414_Asph.pdf	-	-
N414TM-C				6.22 mm		1.83 mm						M6 x 0.5	SPW306

a. EFL is specified at the design wavelength for the unmounted lens.
b. WD is specified at the design wavelength.
c. These Geltech Lenses feature an improved AR coating range of 1050 - 1700 nm.

EFL = Effective Focal Length
NA = Numerical Aperture
CA = Clear Aperture
WD = Working Distance
DW = Design Wavelength
OD = Outer Diameter
M = Magnification

Based on your currency / country selection, your order will ship from European warehouse

+1	Qty	Docs	Part Number - Universal		Price ex VAT	Available / Ships	
+1			**354330-C**	f = 3.10 mm, NA = 0.68, Unmounted Geltech Aspheric Lens, AR: 1050-1700 nm	€ 54,09	√	Today
+1			**C330TMD-C**	f = 3.10 mm, NA = 0.68, Mounted Geltech Aspheric Lens, AR: 1050-1700 nm	€ 67,23	√	2-3 Days
+1			**A414-C**	f = 3.3 mm, NA = 0.47, Unmounted Rochester Aspheric Lens, AR: 1050-1620 nm	€ 73,40	√	2-3 Days
+1			**A414TM-C**	f = 3.3 mm, NA = 0.47, Mounted Rochester Aspheric Lens, AR: 1050-1620 nm	€ 77,90	√	2-3 Days
+1			**N414-C**	f = 3.3 mm, NA = 0.47, Unmounted Rochester Aspheric Lens, AR: 1050-1620 nm	€ 75,60	√	Today
+1			**N414TM-C**	f = 3.3 mm, NA = 0.47, Mounted Rochester Aspheric Lens, AR: 1050-1620 nm	€ 80,28	√	Today

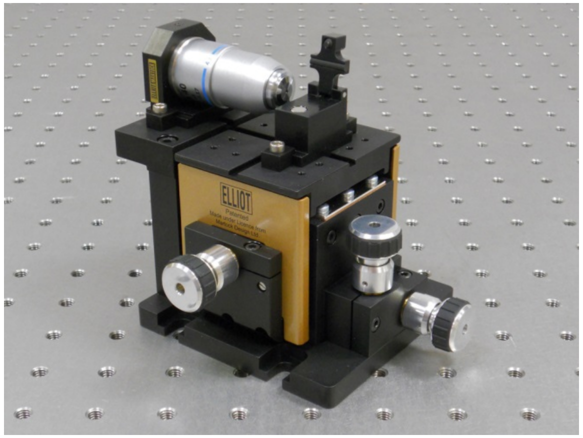
Add To Cart


A375TM-C


A375-C

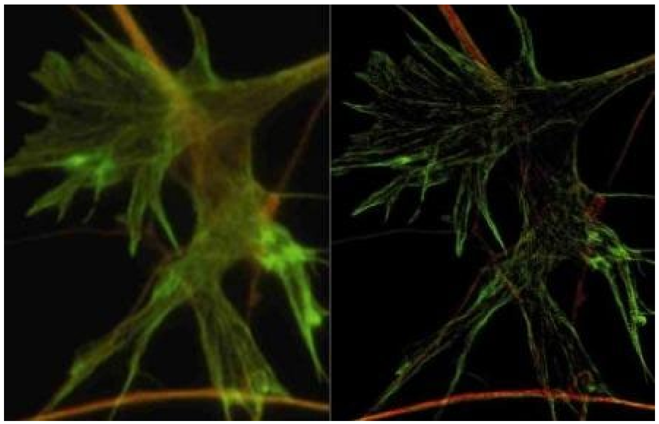

C140TMD-C

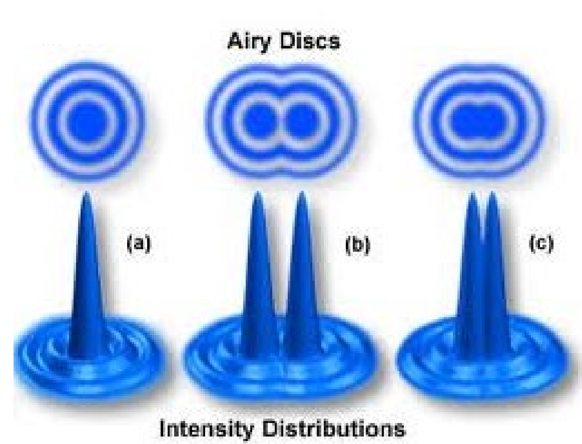

354140-C



Optical Resolution

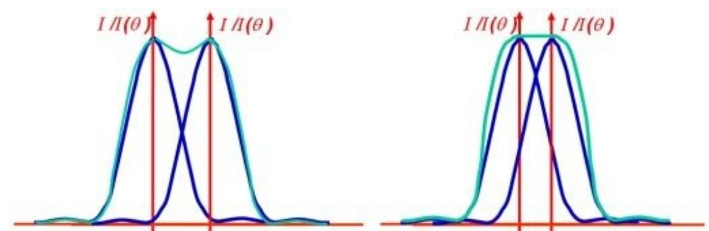
- When two spot-like light sources can be separated from each other? Question is timely in modern microscopy where the target is to distinguish single molecules, e.g., in biological samples.





- **Rayleigh's criterion**: two geometrical points can be separated if the maximum of the intensity distribution of one point source falls at the minimum of the other intensity distribution; aperture diameter D (radius r_0) yields a minimum spatial resolution δ (resolution) at the distance z from the aperture:

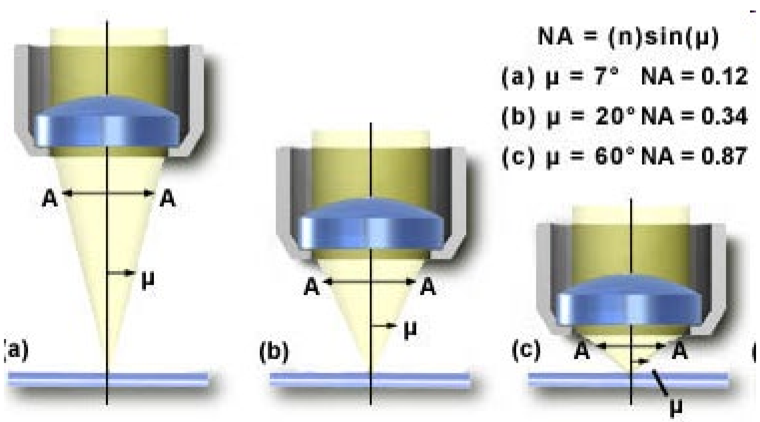
$$\frac{spatial\ resolution}{distance} = \frac{\delta}{z} = 1.22\frac{\lambda}{D}$$

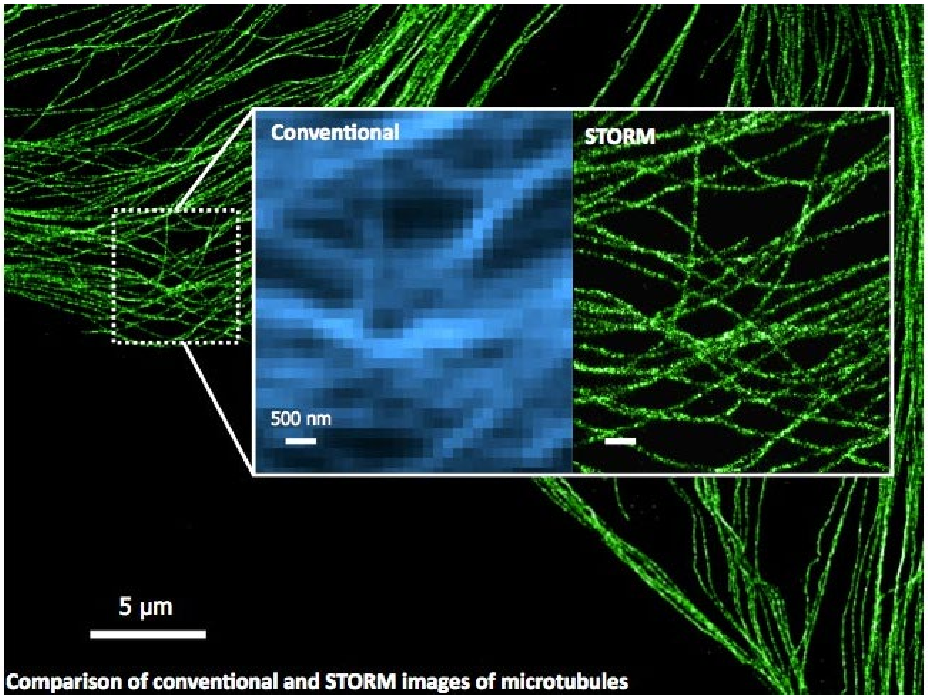


Optical Resolution

For resolution at the focal point of a lens:

$$spatial\ resolution\ \delta = 1.22\frac{\lambda f}{D}$$





http://www.microscopyu.com/tutorials/java/imageformation/airyna/index.html

Fraunhofer Approximation Creates Useful Outcomes:

- intensity distribution of N slits (diffraction grating)
- smallest possible spot size of a lens and optical resolution

”observation far from the aperture”: 2z >> k(x’²+y’²)

$$E(\vec{r}) = \frac{1}{i\lambda}\int_{S'} E_{inc}(\vec{r}\,')\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dS'$$

exponent:

$$|\vec{r}-\vec{r}\,'| = \sqrt{z^2+(x-x')^2+(y-y')^2}$$

$$\approx z\left(1+\frac{(x-x')^2}{2z^2}+\frac{(y-y')^2}{2z^2}\right)$$

$$= z-\frac{xx'}{z}-\frac{yy'}{z}+\frac{x^2+y^2}{2z}+\frac{x'^2+y'^2}{2z}$$

$$\approx z-\frac{xx'}{z}-\frac{yy'}{z}+\frac{x^2+y^2}{2z}$$

denominator:

$$|\vec{r}-\vec{r}\,'| \approx z$$

$$E(x,y,z) = \frac{e^{ikz+\frac{ik}{2z}(x^2+y^2)}}{i\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{inc}(x',y')e^{-i2\pi(\nu_x x'+\nu_y y')}dx'dy'$$

$$\propto \Im\{E_{inc}(x',y')\}$$

Fresnel Approximation (More Accurate than Fraunhofer)

$$E(\vec{r}) = \frac{1}{i\lambda} \int_{S'} E_{inc}(\vec{r}\,') \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} dS'$$

exponent

$$|\vec{r}-\vec{r}\,'| = \sqrt{z^2 + (x-x')^2 + (y-y')^2}$$
$$\approx z\left(1 + \frac{(x-x')^2}{2z^2} + \frac{(y-y')^2}{2z^2}\right)$$

denominator $|\vec{r}-\vec{r}\,'| \approx z$

$$\Rightarrow \quad E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{inc}(x',y') e^{\frac{ik}{2z}\left[(x-x')^2 + (y-y')^2\right]} dx'\,dy'$$
$$= E_{inc}(x',y') \otimes h_z(x',y')$$

Diffracted field is the 2D convolution between the incident field and the transfer function of an empty space h_z

$$h_z(x',y') = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}\left(x'^2 + y'^2\right)}$$

Fresnel Diffraction of a Rectangular Aperture

- In the Exercises the Fraunhofer approximation for a rectangular aperture is derived (aperture size $2x_0 \bullet 2y_0$):

$$I(\nu_x, \nu_y) = \frac{16 x_0^2 y_0^2 E_0^2}{\lambda^2 z^2} \mathrm{sinc}^2(2\pi \mathrm{x}_0 \nu_x)\mathrm{sinc}^2(2\pi \mathrm{y}_0 \nu_x)$$

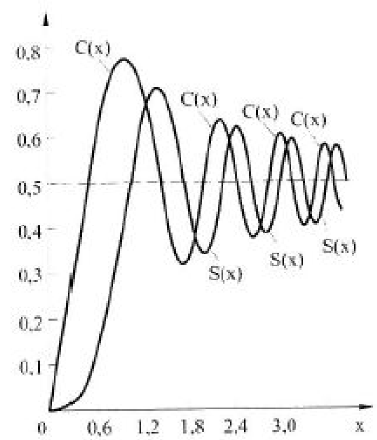
- In Fresnel approximation diffraction integral is separated into x- and y-dependent parts:

$$E(x, y, z) = E_0 \frac{e^{ikz}}{i\lambda z} \mathrm{I}_{\mathrm{x}_0}(\mathrm{x}) \mathrm{I}_{\mathrm{y}_0}(\mathrm{y})$$

$$\mathrm{I}_{\mathrm{x}0}(\mathrm{x}) = \int_{-x_0}^{x_0} e^{\frac{ik}{2z}(x-x')^2} dx' = \sqrt{\frac{\lambda z}{2}} \left[C(\eta_1) - C(\eta_2) + iS(\eta_1) - iS(\eta_1) \right]$$

$$C(\eta) = \int_0^\eta \cos\left(\frac{\pi u^2}{2}\right) du, \;\; S(\eta) = \int_0^\eta \sin\left(\frac{\pi u^2}{2}\right) du$$

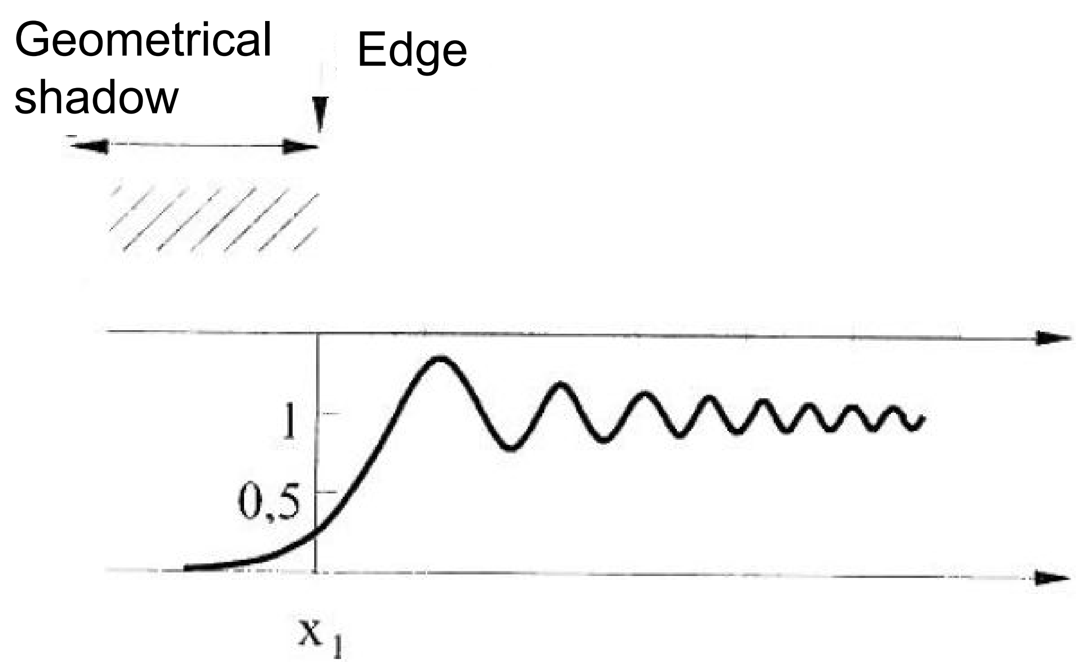
$$\eta_1 = \sqrt{\frac{2}{\lambda z}}(x + x_0), \;\; \eta_2 = \sqrt{\frac{2}{\lambda z}}(x - x_0)$$



Fresnel Diffraction of a Rectangular Aperture

By default, Fresnel approximation gives more accurate results than Fraunhofer approximation so the Fresnel approximation is valid closer to the diffracting aperture. The downside is that the solutions are not as straigthforward to interpret and numerical analysis is required.

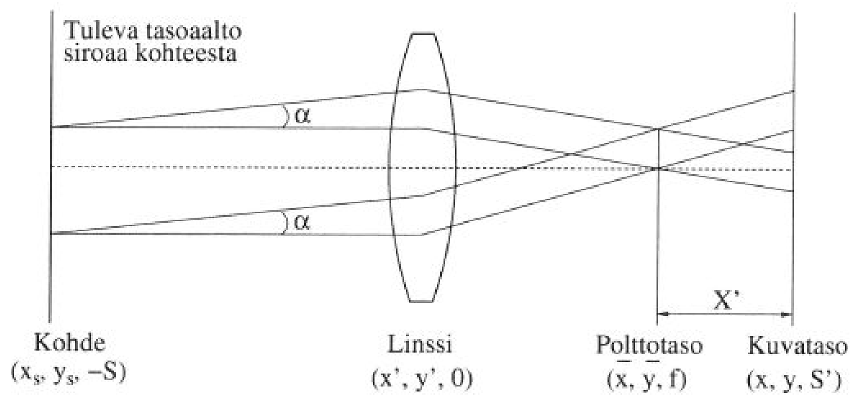
Sharp edge creates intensity oscillation that can be calculated using Fresnel approach near the edge.



Fourier Optics

Fourier Optics

- Let us investigate the following case where at plane z = –S there is a diffractive object which is illuminated by a plane wave E_0e^{ikz}
- Diffractive object has an amplitude transmission function $\tau(x_S,y_S)$. According to Fraunhofer approximation far away from the object we see intensity that is the Fourier transformation of the field $E_0\tau(x,y)$.

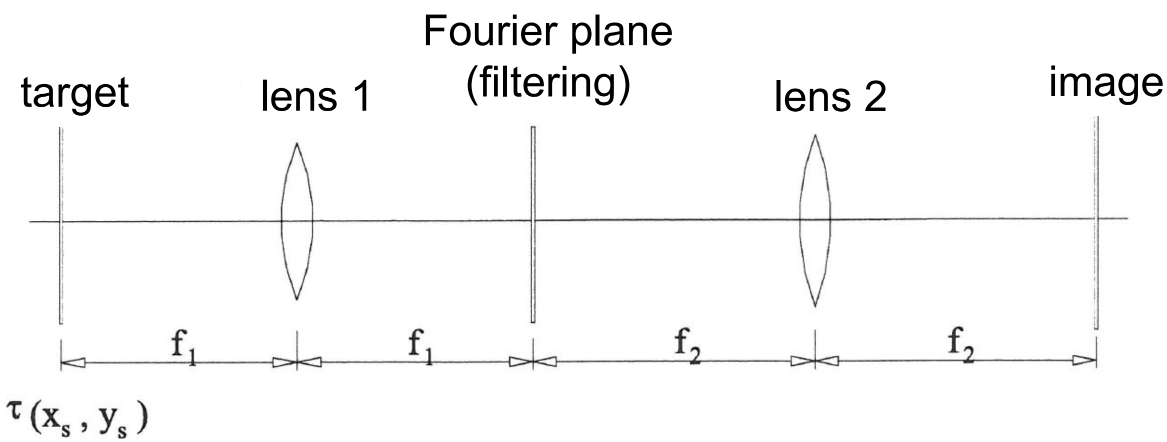


- A lens collects the diffracted rays together and the result is a Fourier transformation at the focal plane of the lens (image position determined by the thin lens equation).
- It can be formally shown that intensity at the focal plane is 2D Fourier transformation:

$$I(\bar{x},\bar{y},f)=\left|E(\bar{x},\bar{y},z)^2\right|=\frac{\left|E_0\right|^2}{\lambda^2 z^2}\left|\Im\{\tau(x,y)_S(\nu_{\bar{x}},\nu_{\bar{y}})\}\right|^2$$

Fourier Optics – 4F System

Optical Fourier signal can be filtered like electronic Fourier signal.
Below two consecutive Fourier transformations form an image (x,y) of $\tau(x_S,y_S)$



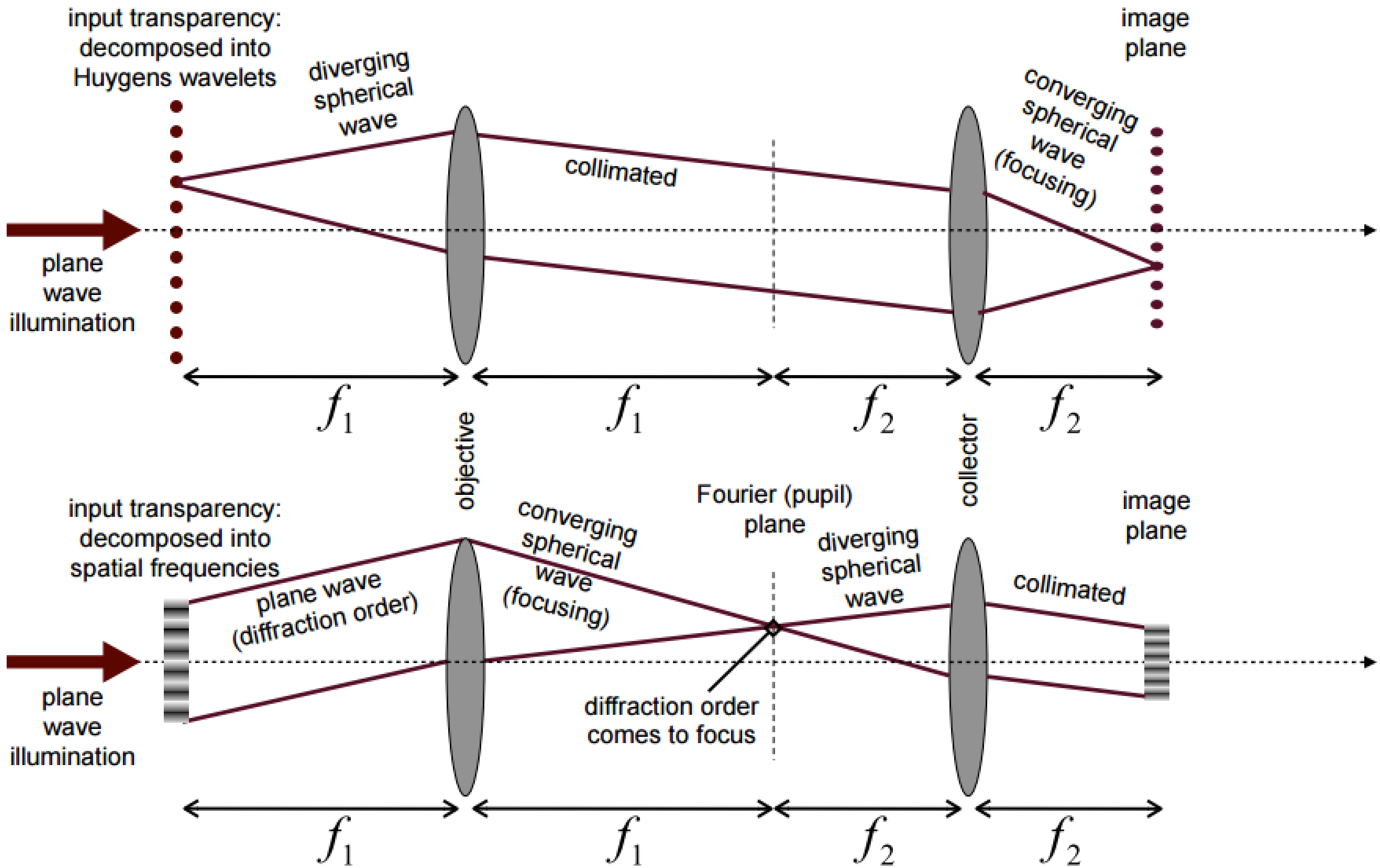
$$I(\bar{x},\bar{y},f)=\left|E(\bar{x},\bar{y},z)^2\right|=\frac{\left|E_0\right|^2}{\lambda^2 z^2}\left|\Im\{\tau(x,y)_S(\nu_{\bar{x}},\nu_{\bar{y}})\}\right|^2 \qquad \nu_x = x/(\lambda z), \nu_y = y/(\lambda z)$$

ν's dimension ∝ k
that is "frequency"

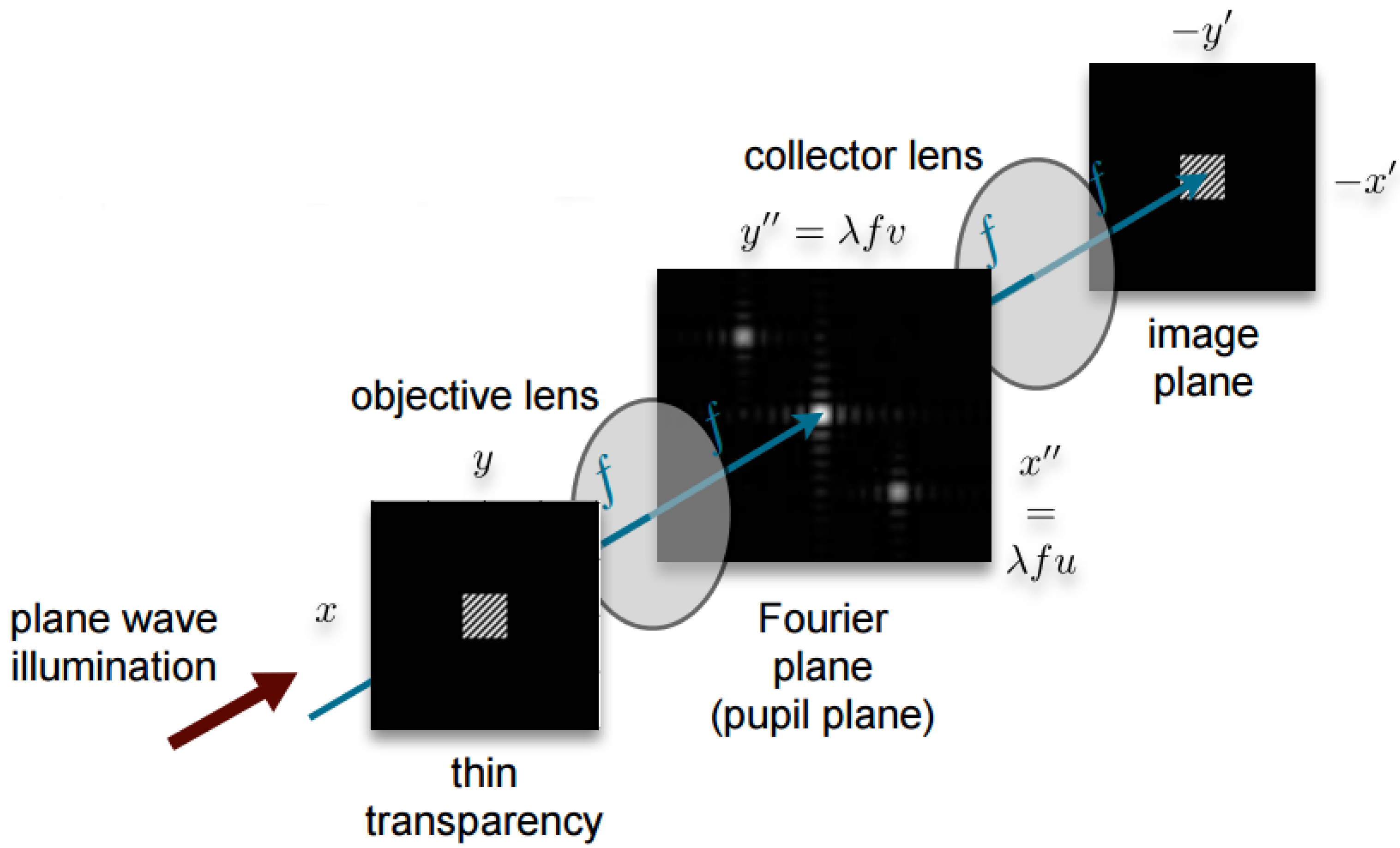
<u>In the Fourier plane</u> with a suitable aperture one can remove:

a) high spatial frequencies ν_x and ν_y (low-pass filtering)
b) low spatial frequencies (high-pass filtering)

4F System – Physical Justification



Examples of Optical Fourier Filtering



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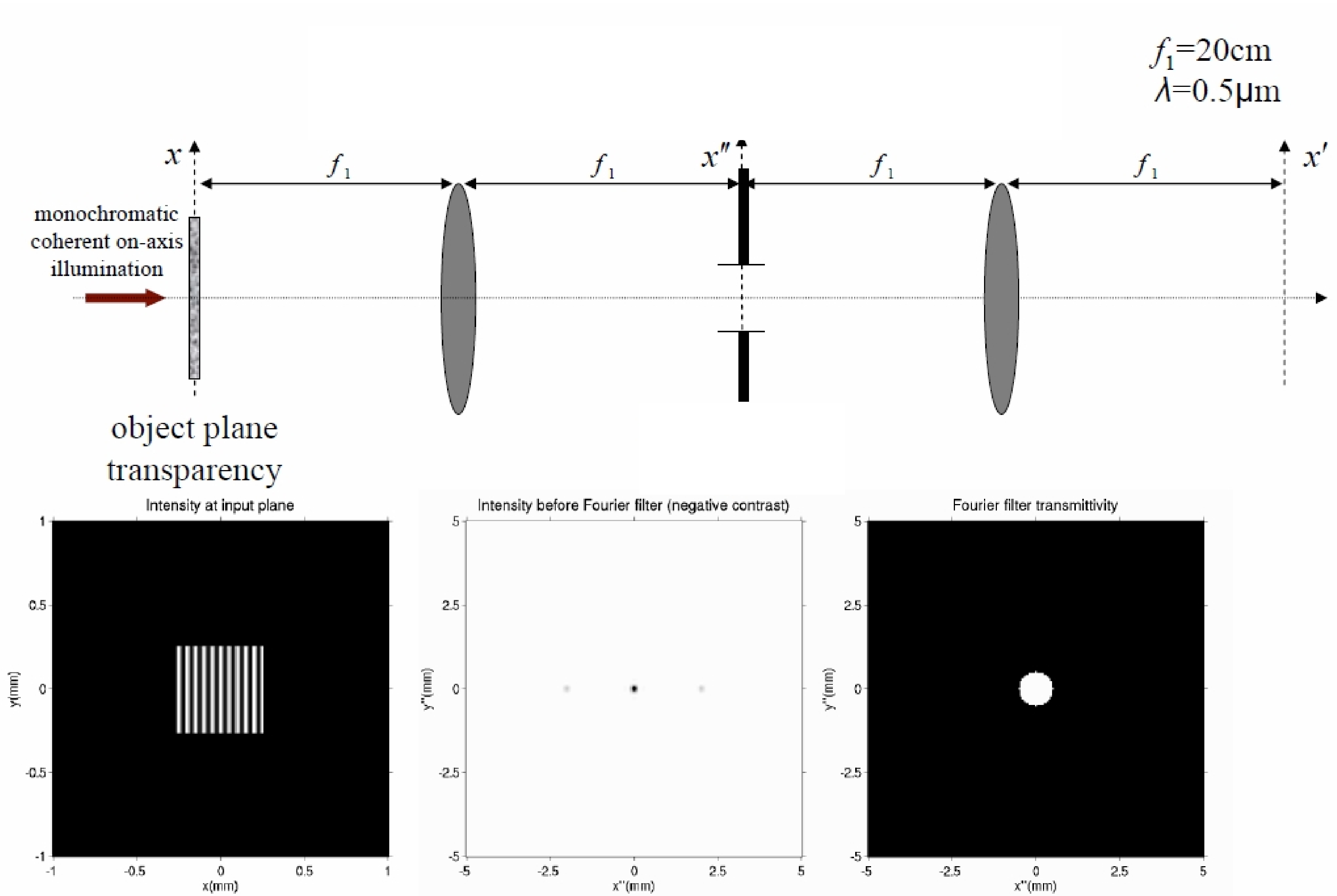
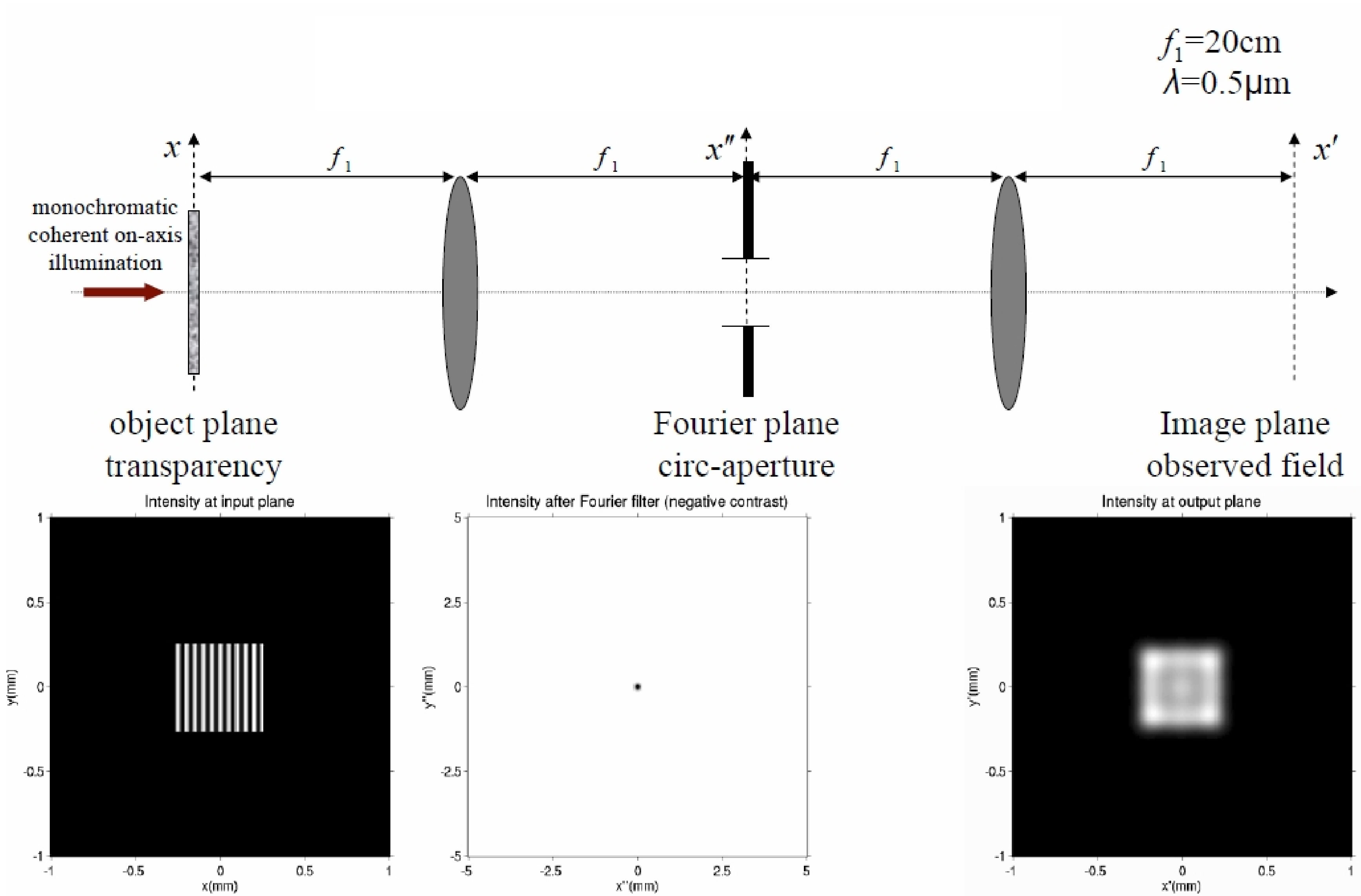


Examples of Optical Fourier Filtering



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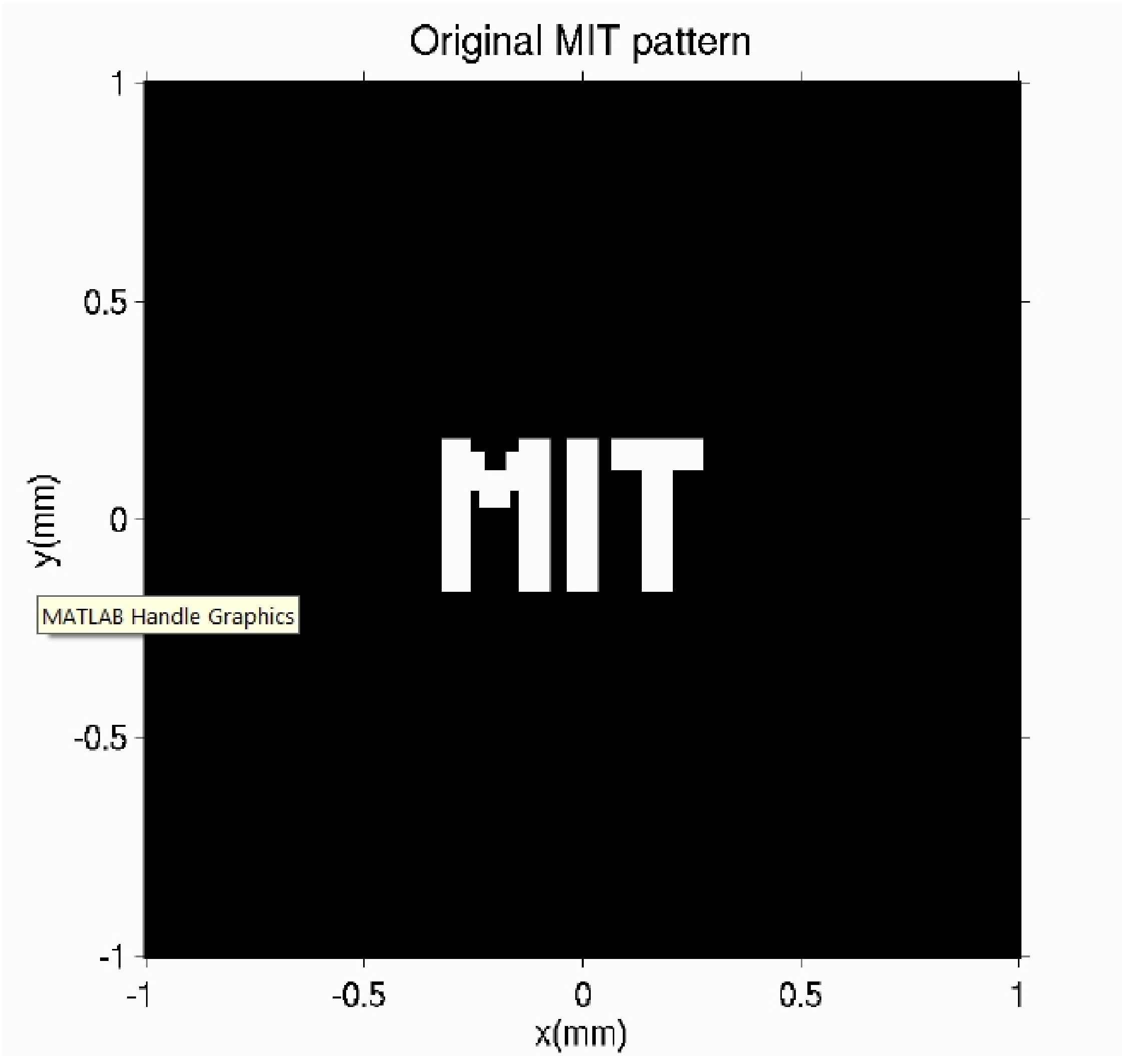

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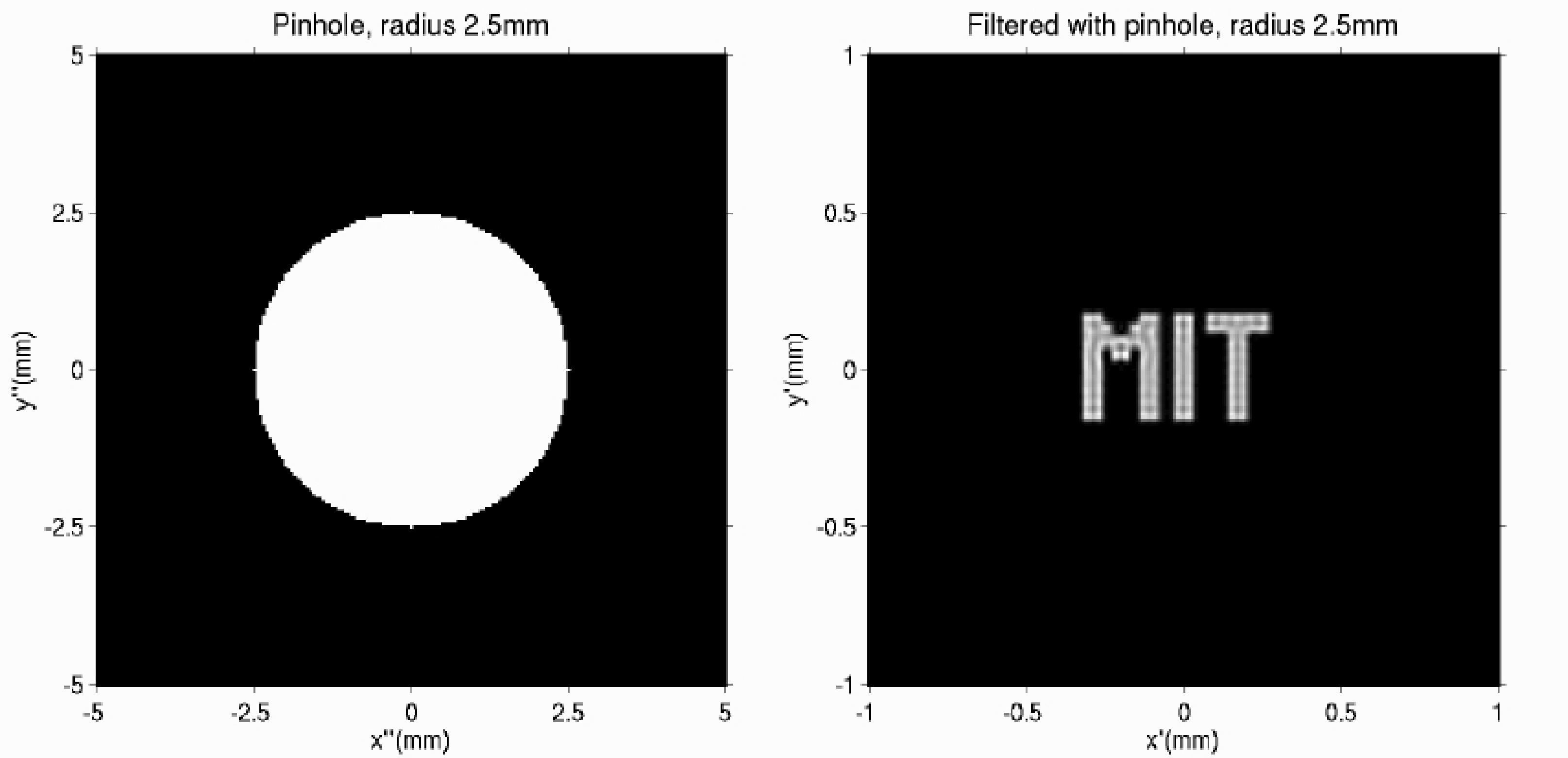
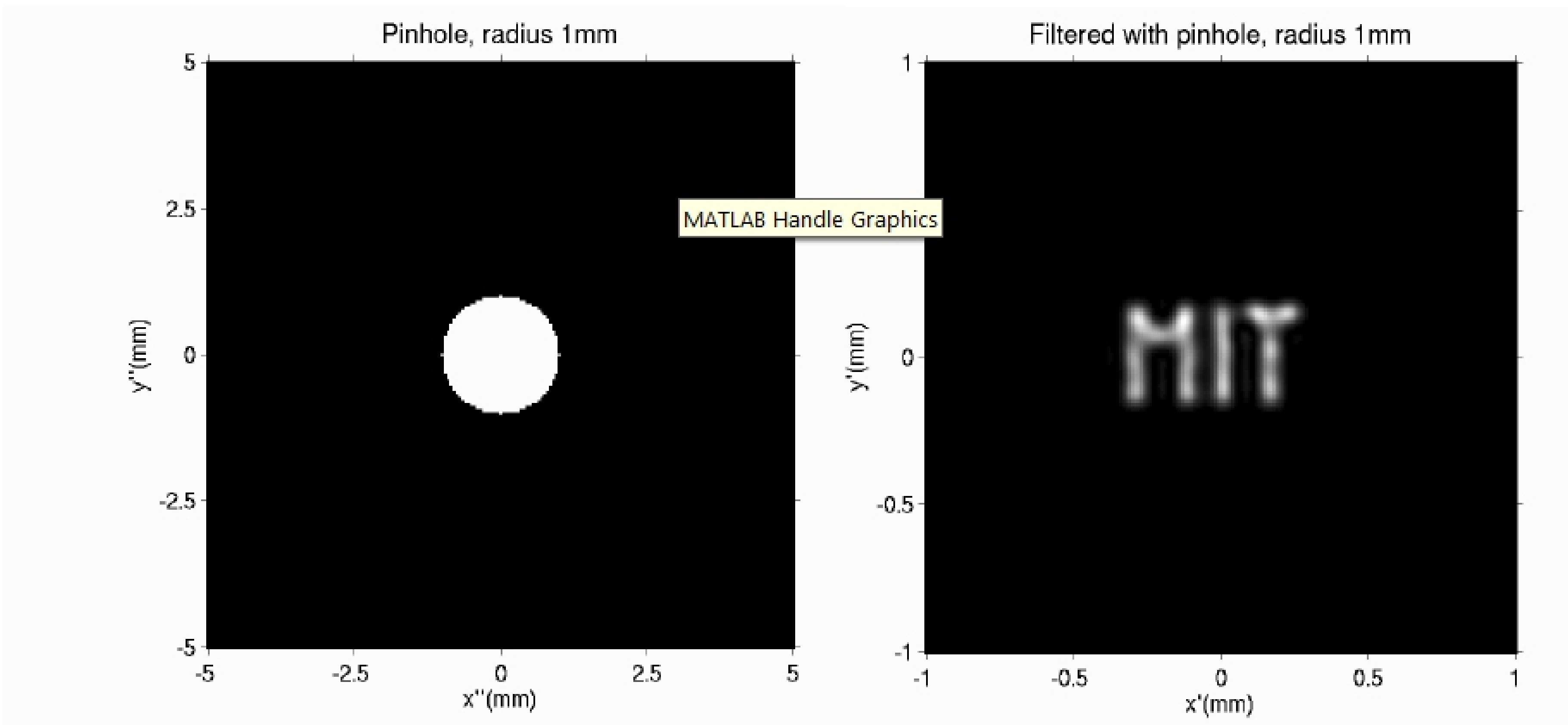


f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Examples of Optical Fourier Filtering



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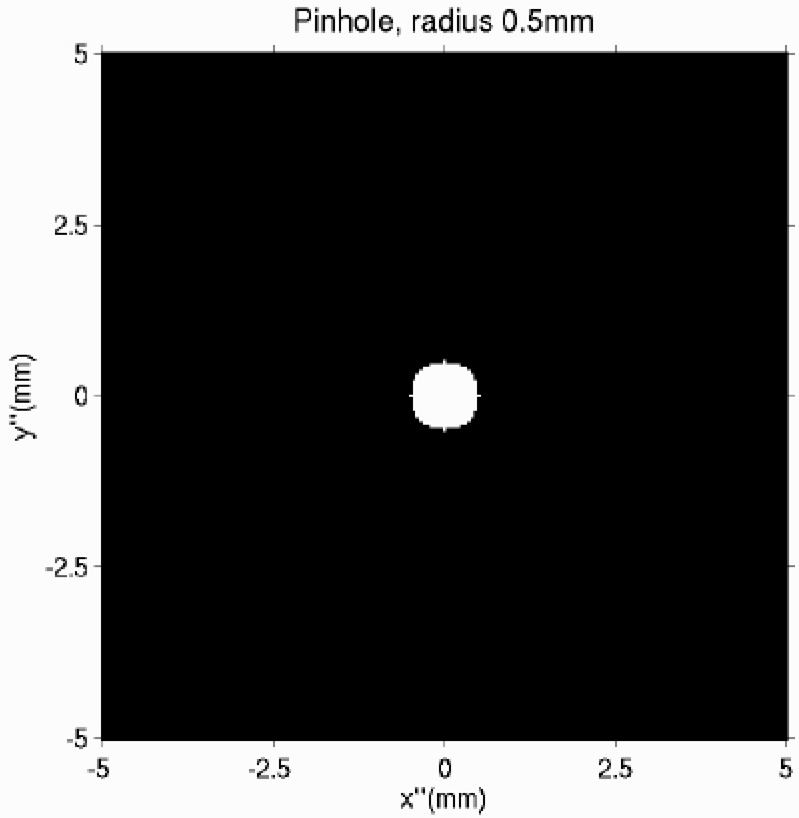
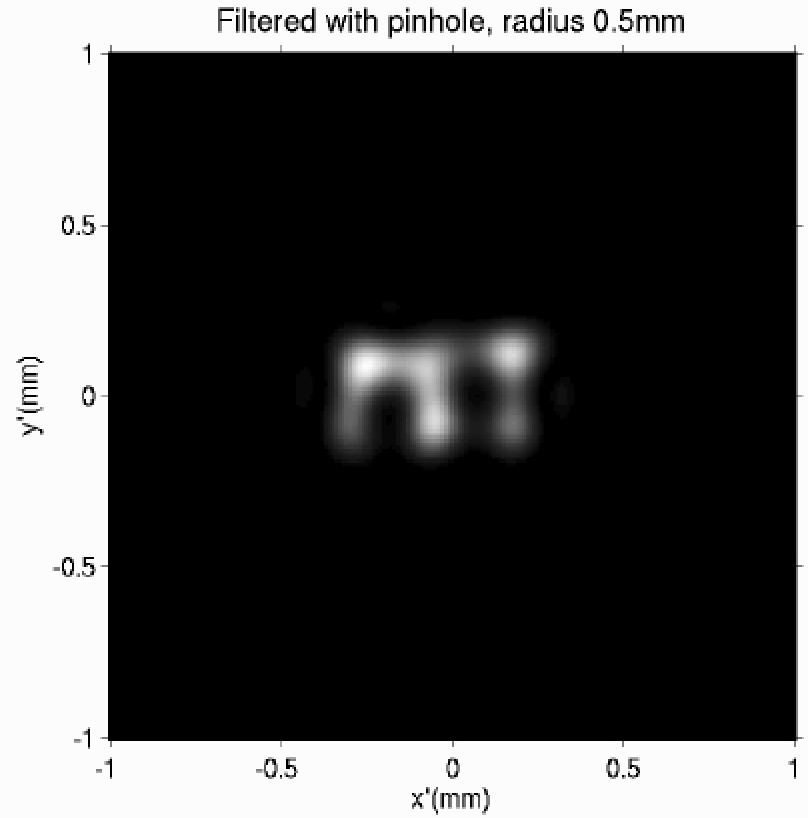
Examples of Optical Fourier Filtering



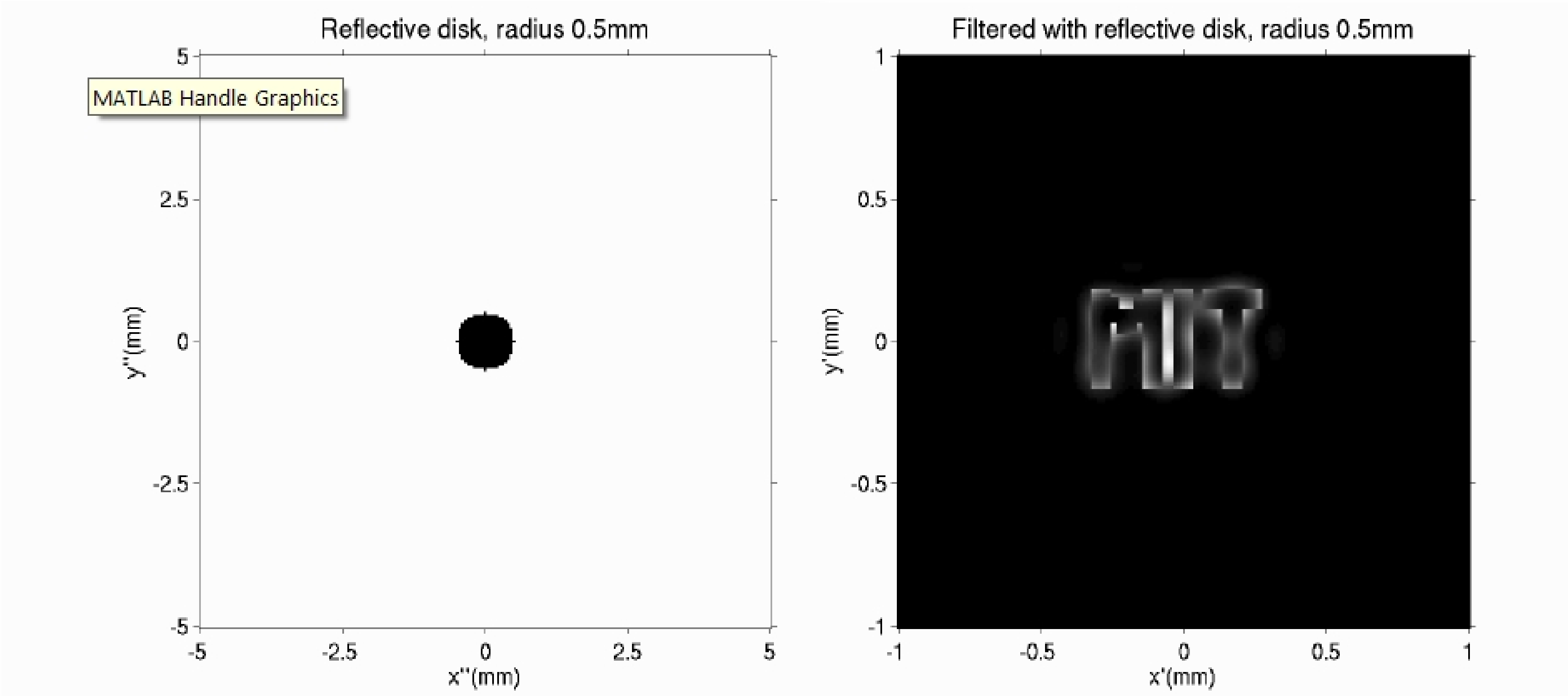


f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Examples of Optical Fourier Filtering



f_1=20cm
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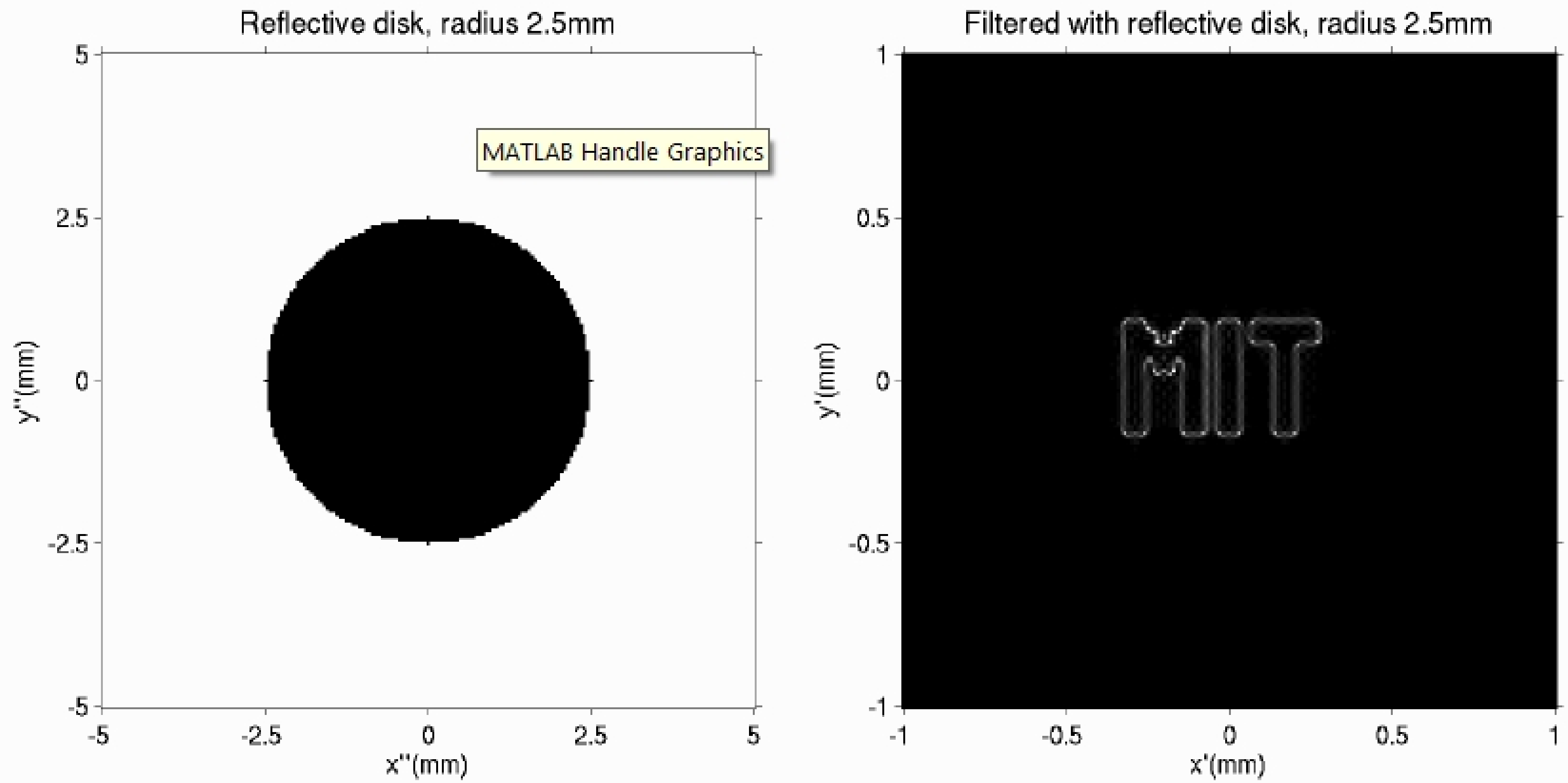
Fourier filter

Intensity @ image plane

Examples of Optical Fourier Filtering

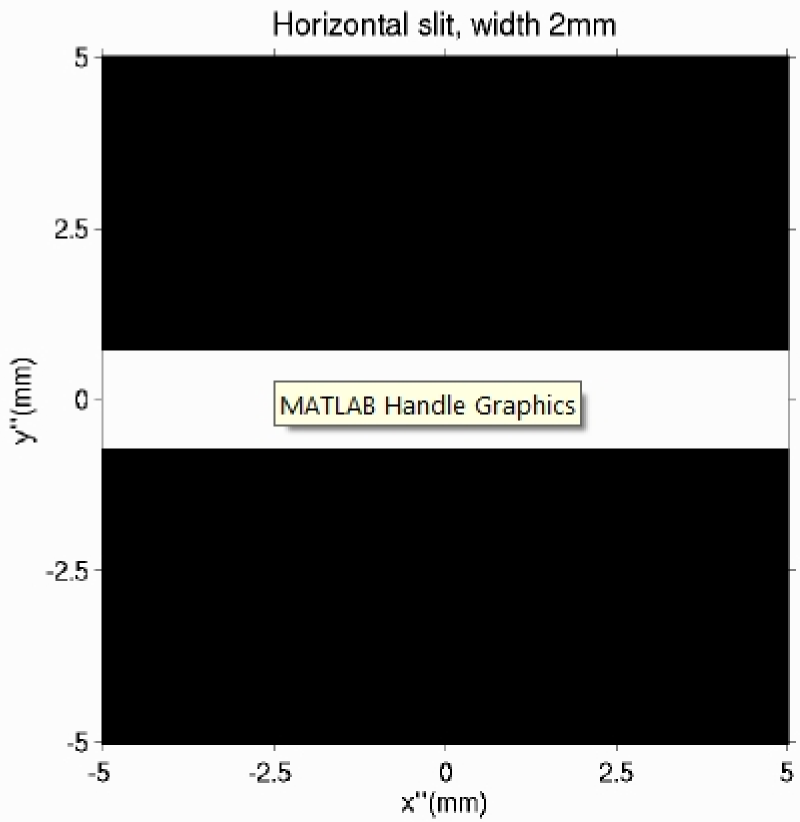


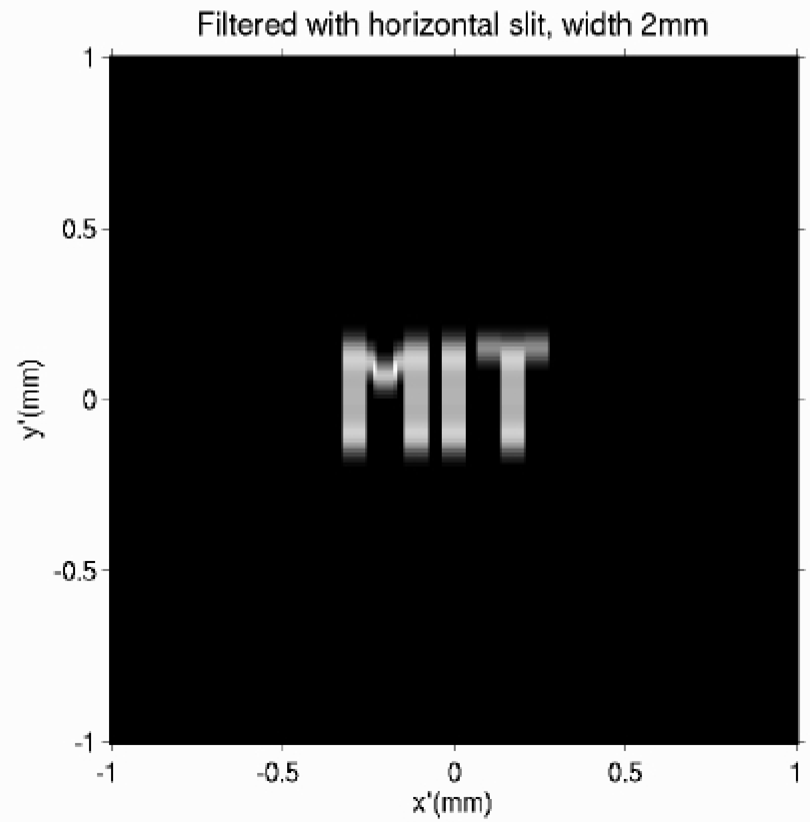
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Examples of Optical Fourier Filtering



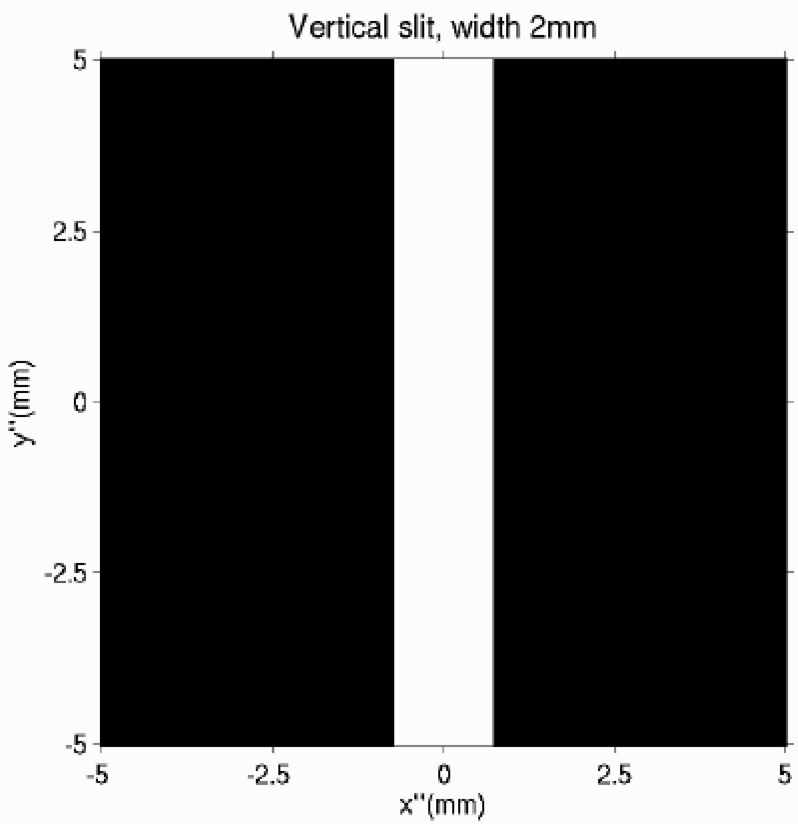


f_1=20cm
λ=0.5μm

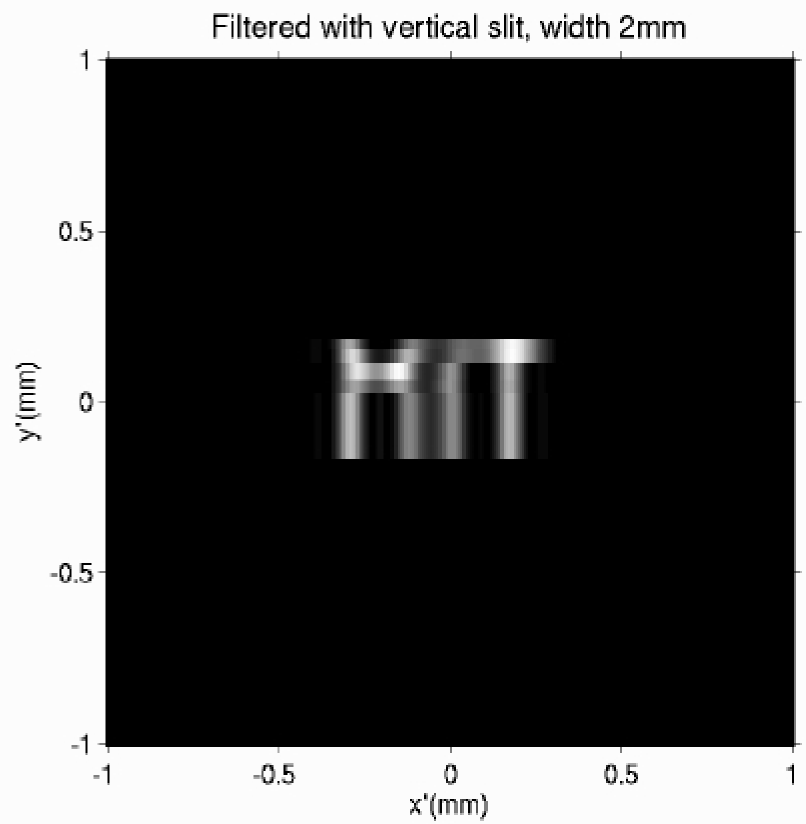
Fourier filter

Intensity @ image plane

Examples of Optical Fourier Filtering



Fourier filter



Intensity @ image plane

f_1=20cm
λ=0.5μm

Optics Course Contents

Geometrical optics and ray tracing

- lens calculations
- transfer matrices

Wave optics

- Fresnell reflection and transmission factors for amplitude and intensity
- polarisation states of light
- polarising components and waveplates

Radiometry

- definitions
- blackbody radiation

Interference, coherence and introduction to lasers

- different interferometers and their applications
- coherence length and time
- Fourier optics

Diffraction

- diffraction integral and Fraunhofer&Fresnell approximations
- diffraction grating
- diffraction limit

Optical fibres and optical telecom

- basics of optical fibres
- WDM, EDFA

Optics E-5730 Spring 2021

Thank You!