Optics E-5730 Spring 2021

Geometrical Optics II

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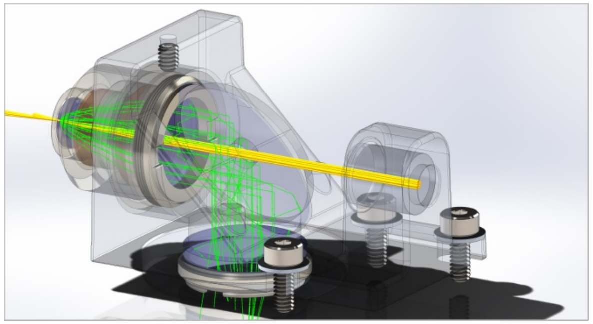
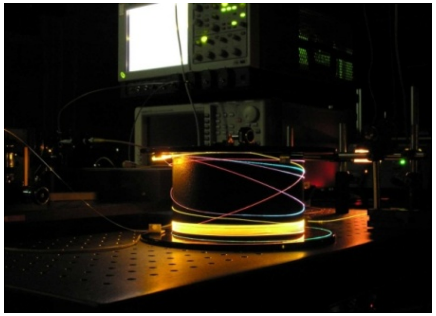
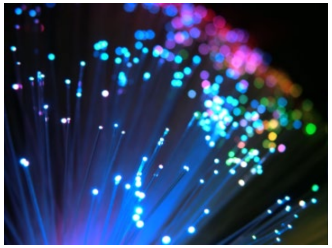
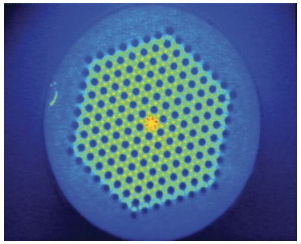
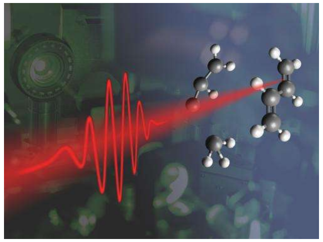



Fundamentals of Optics, Spring 2021

ELEC E-5730 lectures online using Zoom at https://aalto.zoom.us/j/8453943170

exercises online using Zoom at https://aalto.zoom.us/j/5703080612

week	day	date	time	topic	
2	Mon	11.1.2021	8-10	Lecture 1: Geometrical optics 1	
	Fri	15.1.2021	8-10	Lecture 2: Geometrical optics 2	
3	Mon	18.1.2021	8-10	Lecture 3: Wave optics 1	
	Mon	18.1.2021	10-12	Exercise 1	
	Fri	22.1.2021	8-10	Lecture 4: Wave optics 2	
4	Mon	25.1.2021	8-10	Lecture 5: Coherence 1	
	Mon	25.1.2021	10-12	Exercise 2	
	Fri	29.1.2021	8-10	Lecture 6: Coherence 2	
5	Mon	1.2.2021	8-10	Lecture 7: Radiometry	
	Mon	1.2.2021	10-12	Exercise 3	
	Fri	5.2.2021	8-10	Lecture 8: Interferometry + 30 mins mid-term exam	
6	Mon	8.2.2021	8-10	Lecture 9: Fibre optics + Optical telecom	LAB WORKS PERIOD
	Mon	8.2.2021	10-12	Exercise 4	LAB WORKS PERIOD
	Fri	12.2.2021	8-10	Lecture 10: Diffraction 1	LAB WORKS PERIOD
7	Mon	15.2.2021	8-10	Lecture 11: Diffraction 2	LAB WORKS PERIOD
	Mon	15.2.2021	10-12	Exercise 5	LAB WORKS PERIOD
	Fri	19.2.2021	8-10	NO LECTURE	LAB WORKS PERIOD
8	Mon	22.2.2021	8-10	NO LECTURE	
	Mon	22.2.2021	10-12	Exercise 6	
	Fri	26.2.2021		Examination	

Last Lecture, Geometrical Optics I – The Basics

- Light as electromagnetic radiation
- Light can treated both as photons and waves

Huygens' principle

$\updownarrow$

Fermat's principle (”principle of least time”)

$\downarrow$

1) In homogenous medium light travels in a linear fashion
2) Law of reflection: $\theta_1 = \theta_2$
3) Law of refraction or Snell's law: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

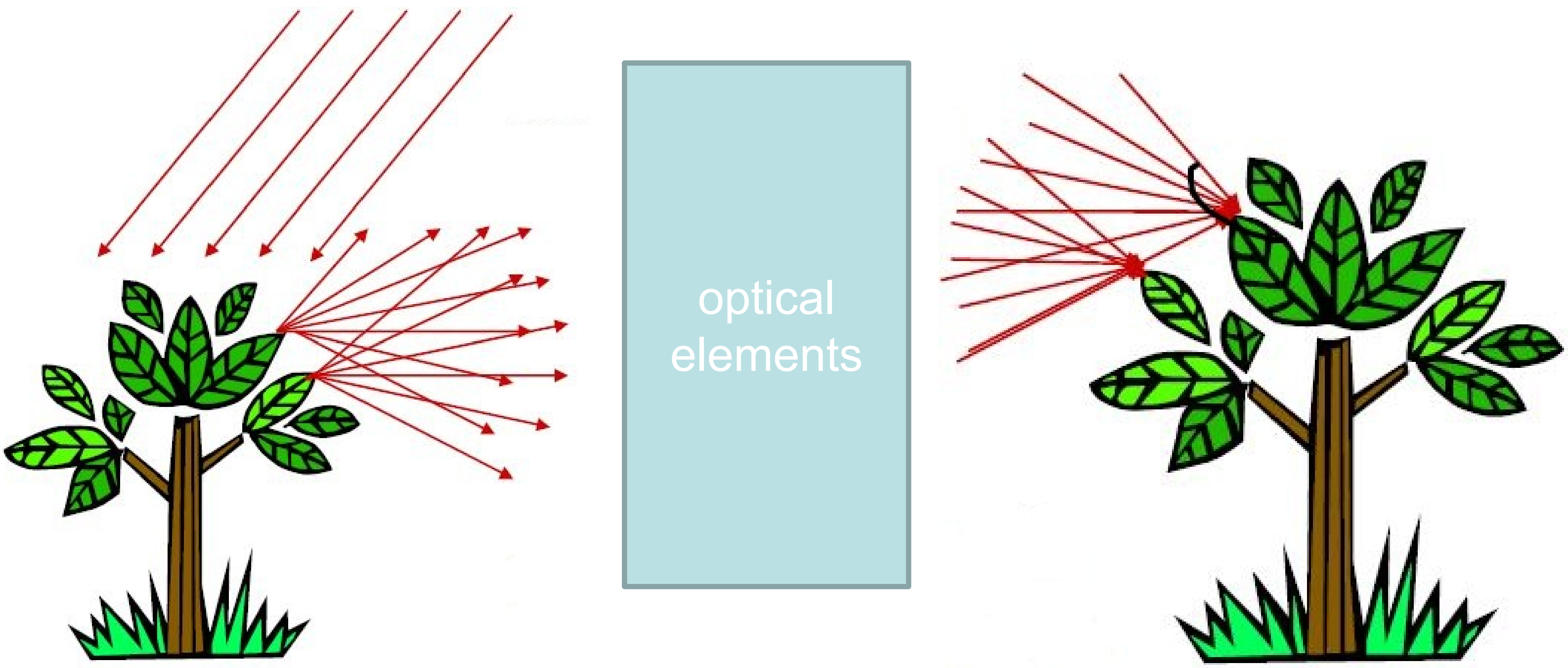
- Total internal reflection

Today’s Lecture, Geometrical Optics II

Powerful Tools for Optical Design

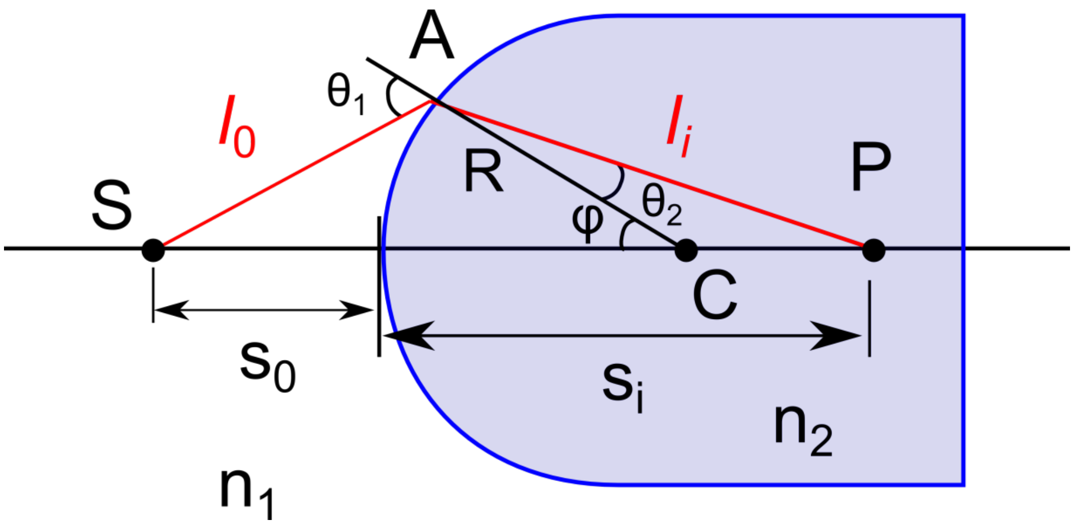
- Lens maker’s formula and thin lens equation
- Ray tracing in optical systems
- Lenses: magnification, numerical aperture, f-number
- Non-ideal lenses - aberrations
- Matrix formalism for ray tracing

Why Imaging Optics Is Needed?



Optics is required ”to collect” light scattered from the object to form a sharp, focused image

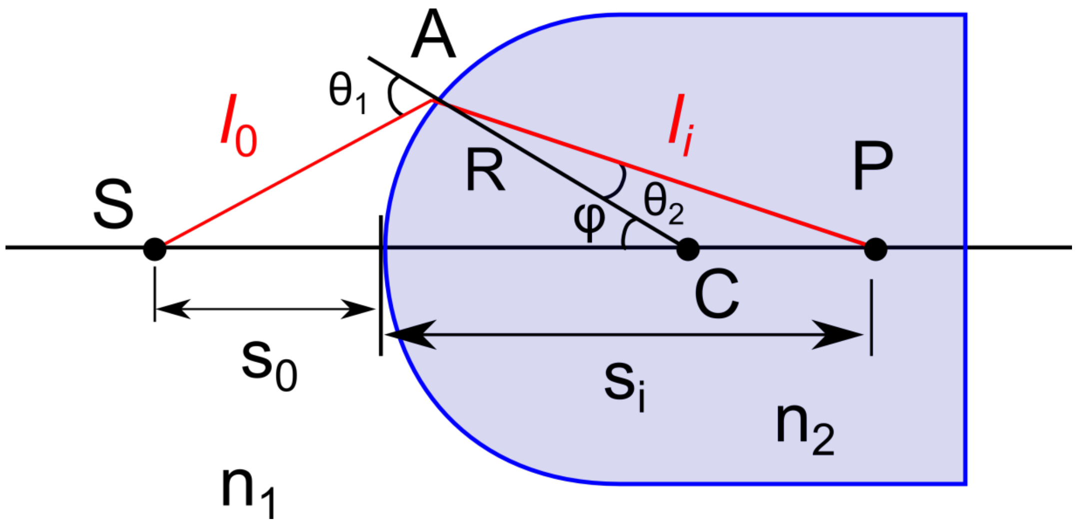
Refractive Power of a Spherical Surface



$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Result: all rays refract through point P at s_i which is independent on the angle φ.

Deriving Refractive Power of a Spherical Surface 1/5

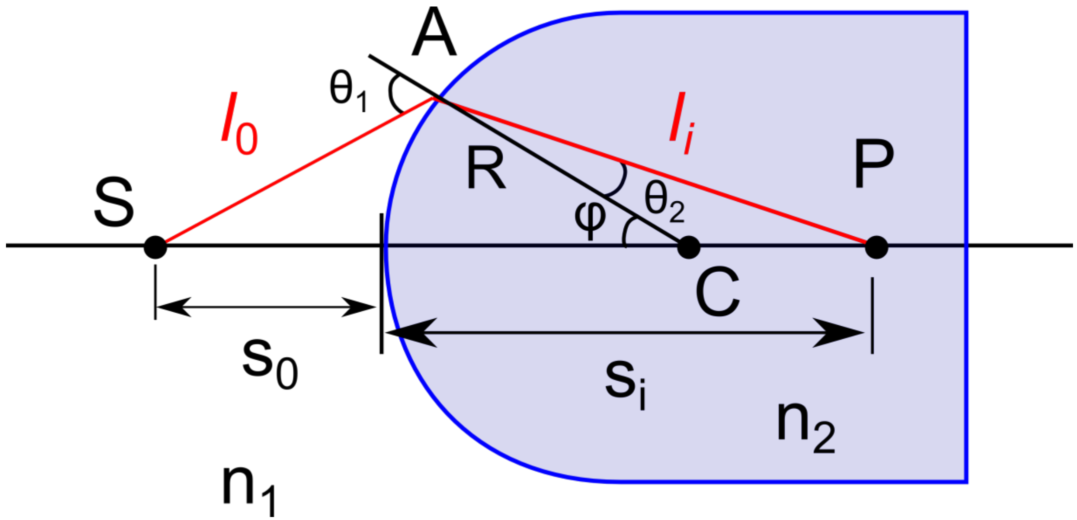


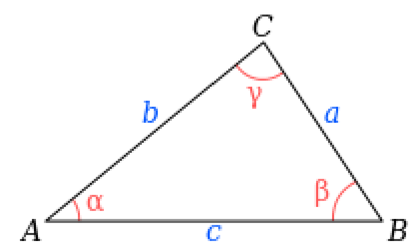
optical path length $L = n_1 l_0 + n_2 l_i$

according to the Fermat's principle the variation of the path equals zero

$$dL/d\varphi = 0$$

Deriving Refractive Power of a Spherical Surface 2/5





$c^2 = a^2 + b^2 - 2ab\cos\gamma$

law of cosines applied to triangles SAC and ACP

SAC: $l_0{}^2 = R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi$

ACP: $l_i{}^2 = R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi$

$\cos\varphi = -\cos(\pi - \varphi)$

$${l_0}^2 = R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi$$

$${l_i}^2 = R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi$$

$$L = n_1 l_0 + n_2 l_i$$

$$\mathrm{L} = n_1\sqrt{R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi} + n_2\sqrt{R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi}$$

dL/dφ = 0

$$n_1\frac{1}{2}\frac{2\mathrm{R}(s_0 + R)\sin\varphi}{\sqrt{R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi}} - n_2\frac{1}{2}\frac{2\mathrm{R}(s_i - R)\sin\varphi}{\sqrt{R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi}} = 0$$

$$n_1 \frac{1}{2} \frac{2\mathrm{R}(s_0 + R)\sin\varphi}{\sqrt{R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi}} -$$

$$n_2 \frac{1}{2} \frac{2\mathrm{R}(s_i - R)\sin\varphi}{\sqrt{R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi}} = 0 \quad \Big\| \cdot \frac{1}{sin\varphi}$$

$$\Rightarrow \quad \frac{n_1 R s_0}{l_0} + \frac{n_1 R^2}{l_0} - \frac{n_2 R s_i}{l_i} + \frac{n_2 R^2}{l_i} = 0$$

$$\left(\frac{n_1}{l_0} + \frac{n_2}{l_i}\right) R^2 = \frac{n_2 R s_i}{l_i} - \frac{n_1 R s_0}{l_0} \quad \Big\| \cdot \frac{1}{R^2}$$

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0}\right)$$

Here we do so called 'paraxial approximation':

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} + \cdots \approx 1 \; when \; \varphi \ll$$

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \cdots \approx \varphi \text{ when } \varphi <<$$

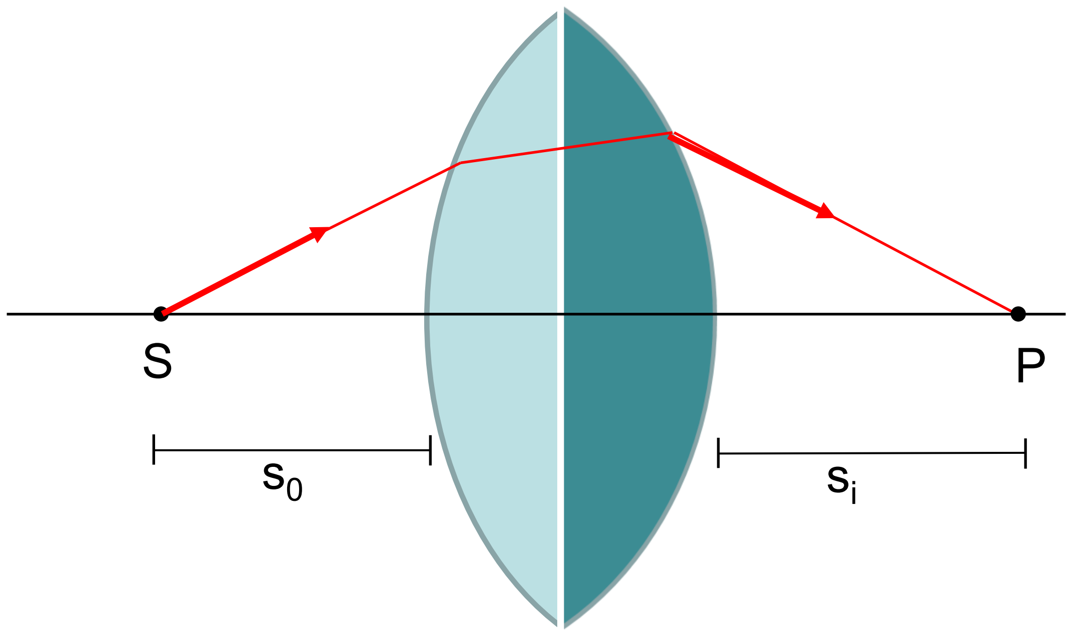
$$l_0{}^2 = R^2 + (s_0 + R)^2 - 2\mathrm{R}(s_0 + R)\cos\varphi$$

$$l_i{}^2 = R^2 + (s_i - R)^2 + 2\mathrm{R}(s_i - R)\cos\varphi$$

$$\Rightarrow \quad l_0 = s_0, \quad l_i = s_i$$

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0}\right) \quad \Rightarrow \quad \frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Thin Lens = Two Thin Spherical Surfaces



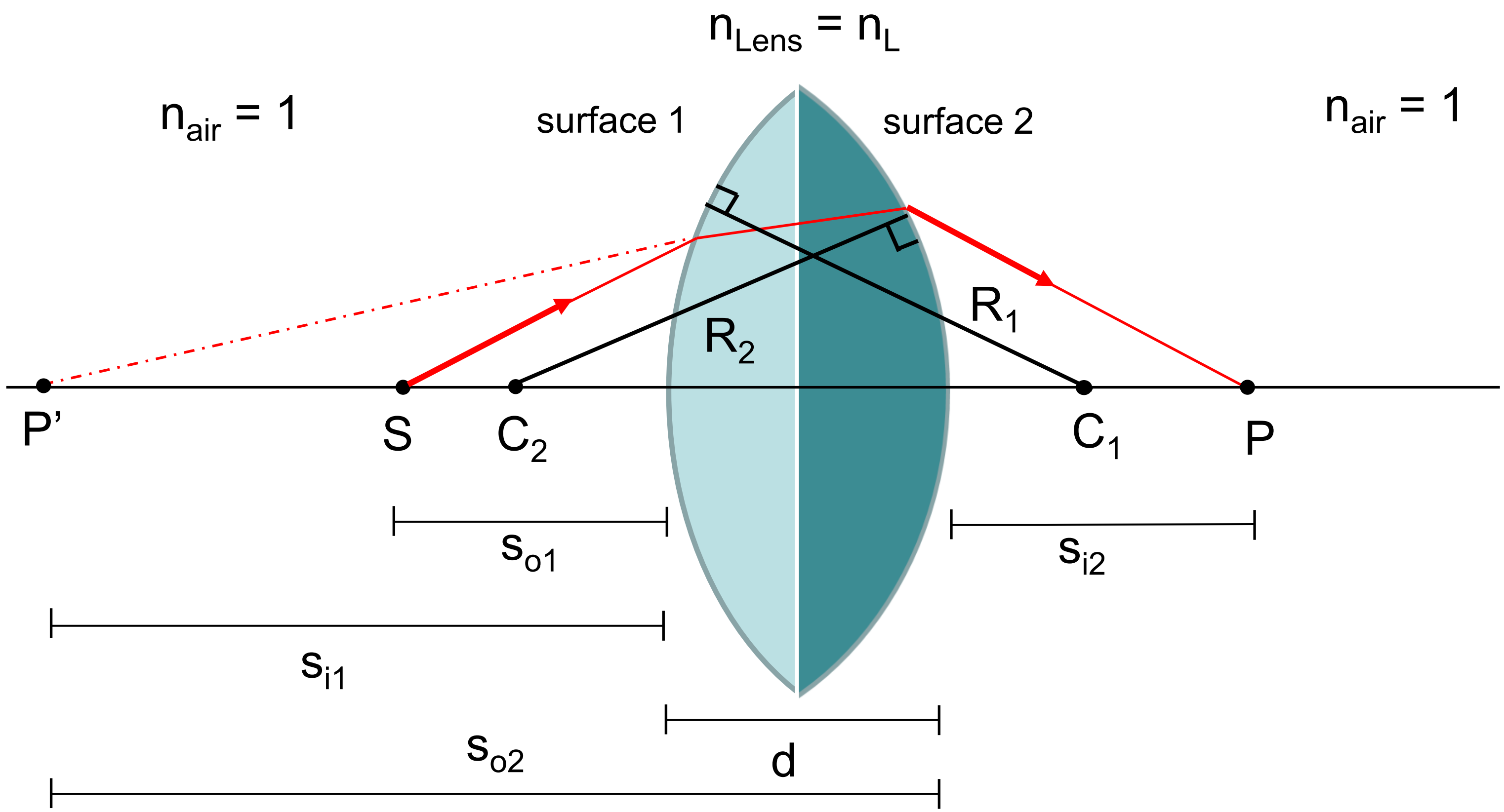
Lens Maker's Formula (Eq. 1.7)

$$\frac{1}{f} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin Lens Equation (Eq. 1.10)

$$\frac{1}{S_O} + \frac{1}{S_i} = \frac{1}{f}$$

Deriving Lens Maker's Formula and Thin Lens Equation 1/5



After surface 1 the rays emerging from S seem to emerge from virtual point P':

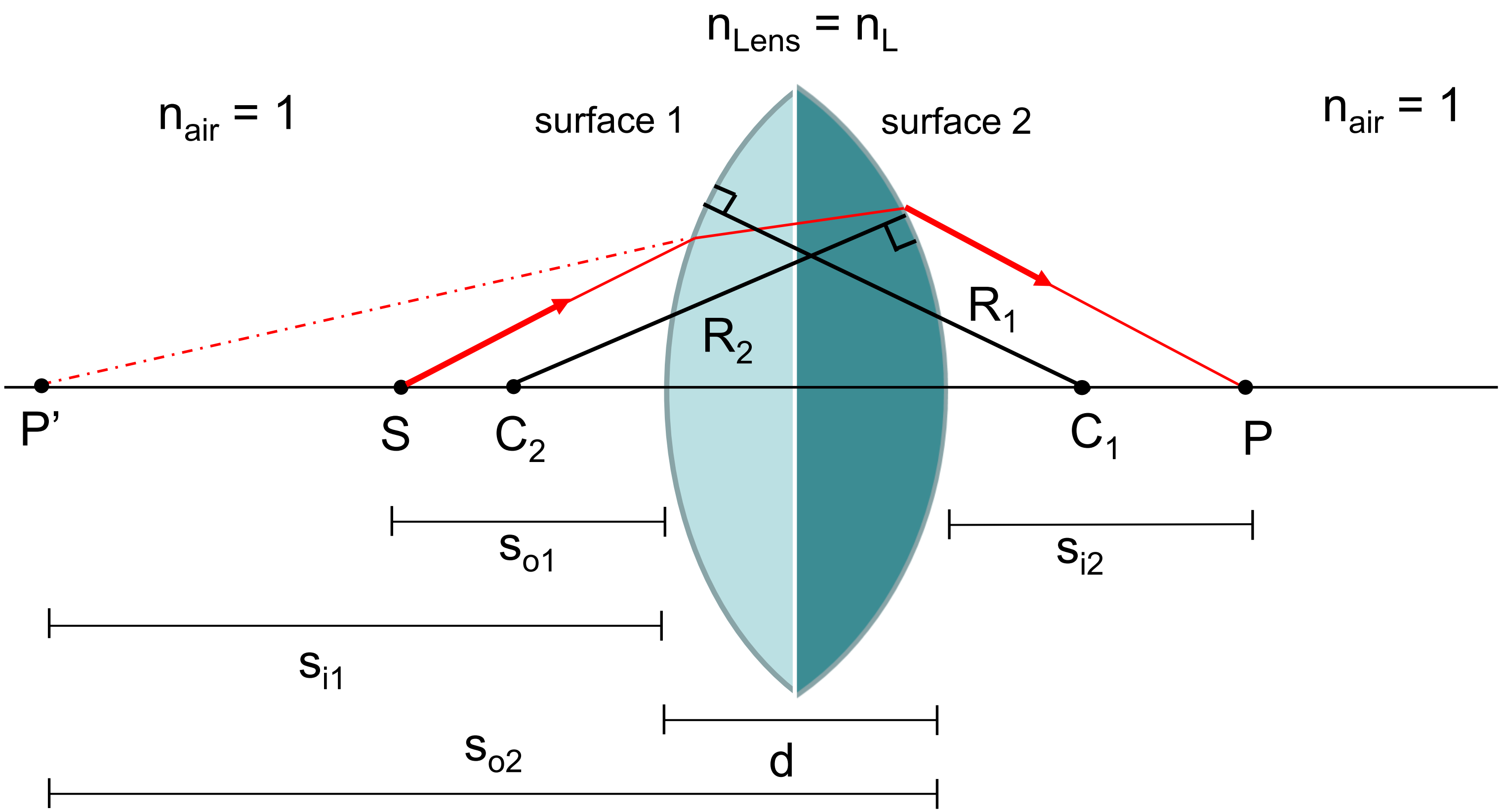
surface 1; R > 0 $$\frac{1}{S_{O1}} + \frac{n_L}{S_{i1}} = \frac{n_L - 1}{R_1}$$

also $$\lfloor S_{o2} \rfloor = \lfloor S_{i1} \rfloor + d$$

$-s_{i1}$

$s_{o2} = -s_{i1} + d$

Deriving Lens Maker's Formula and Thin Lens Equation 2/5



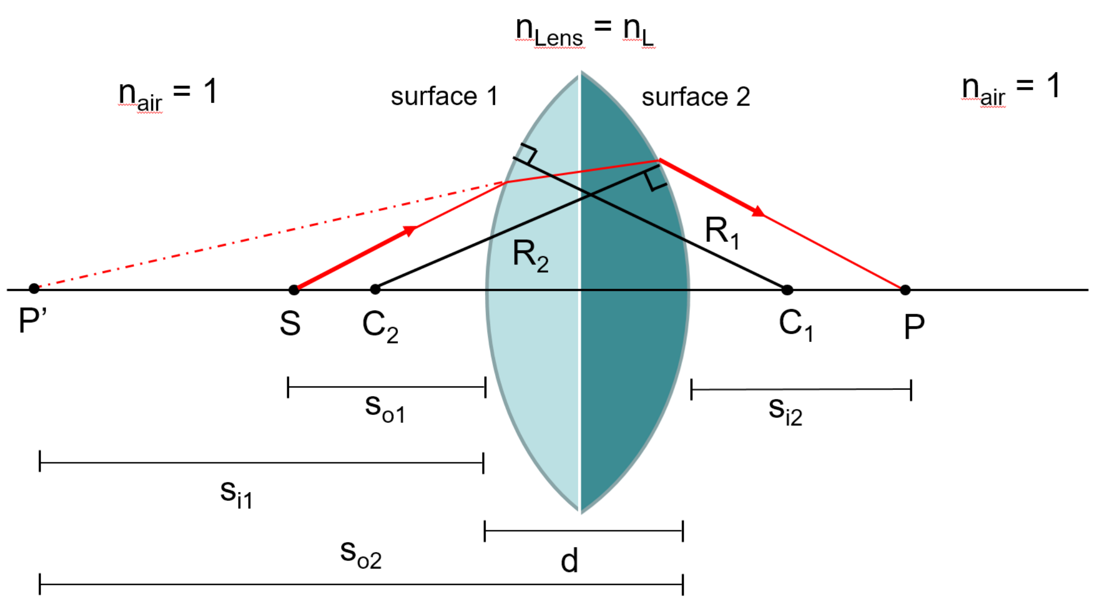
surface 2 $$\frac{n_L}{-S_{i1}} + \frac{1}{S_{i2}} = \frac{1-n_L}{R_2}$$

$\} < 0$

$\} R_2 < 0$

$s_{o2} = -s_{i1} + d$

Deriving Lens Maker's Formula and Thin Lens Equation 3/5



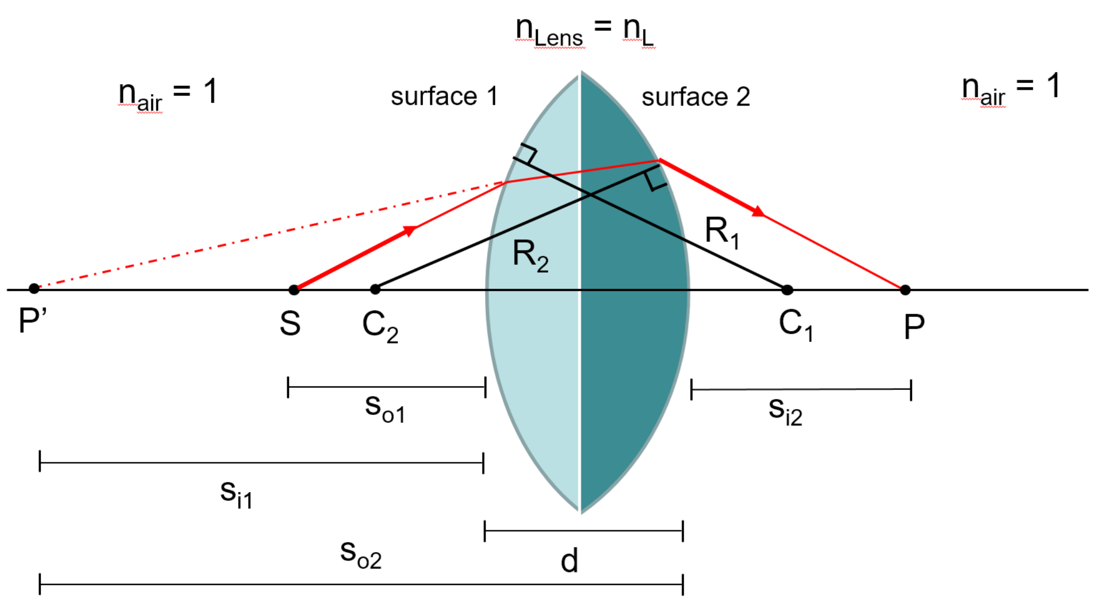
Let's combine equations for surfaces 1 and 2:

$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} + \frac{n_L(d - \cancel{s_{i1}}) + \cancel{n_L s_{i1}}}{s_{i1}(d - s_{i1})} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{S_{o1}} + \frac{1}{S_{i2}} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_L d}{s_{i1}(s_{i1} - d)} \longrightarrow 0$$

when d $\longrightarrow$ 0

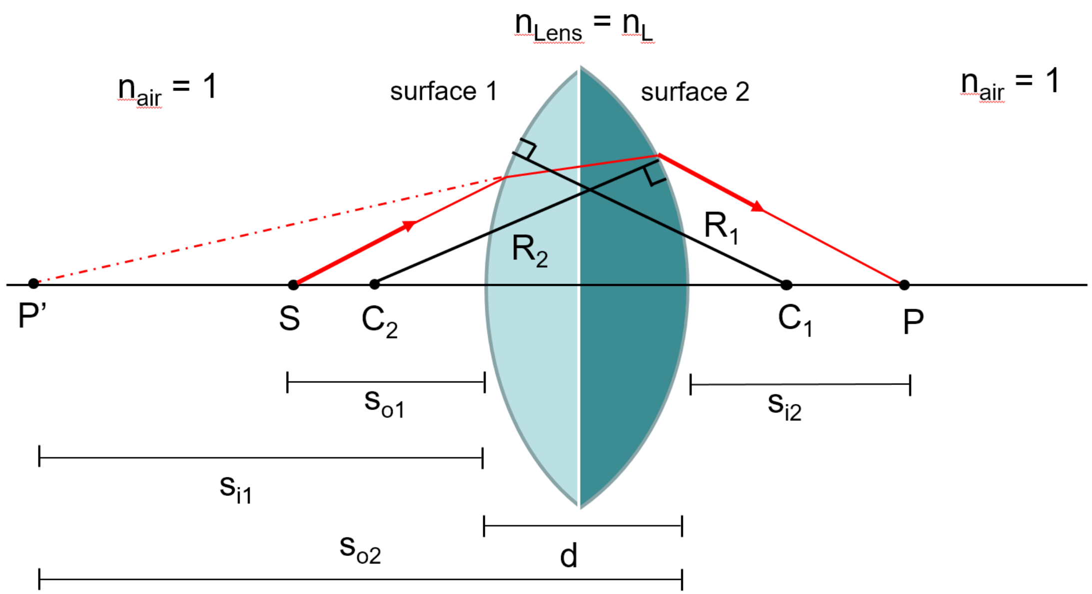
Deriving Lens Maker's Formula and Thin Lens Equation 4/5



Thin lens approximation (d → 0):

$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Deriving Lens Maker's Formula and Thin Lens Equation 5/5



$$\lim_{S_o \to \infty} S_i = f_i \qquad \lim_{S_i \to \infty} S_o = f_o \qquad f_i = f_o = f$$

$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} = \frac{1}{f}$$

Thin Lens Equation (Eq. 1.10) where

$$\frac{1}{f} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Lens Maker's Formula (Eq. 1.7)

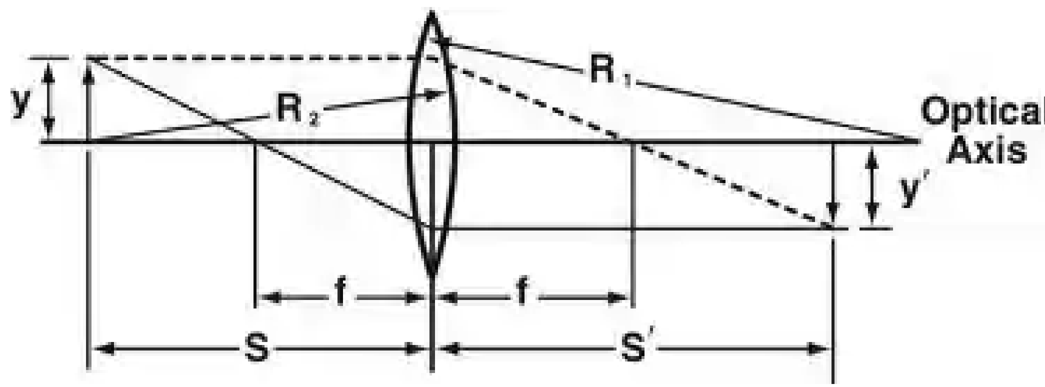
Lens Maker's Formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin Lens Equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

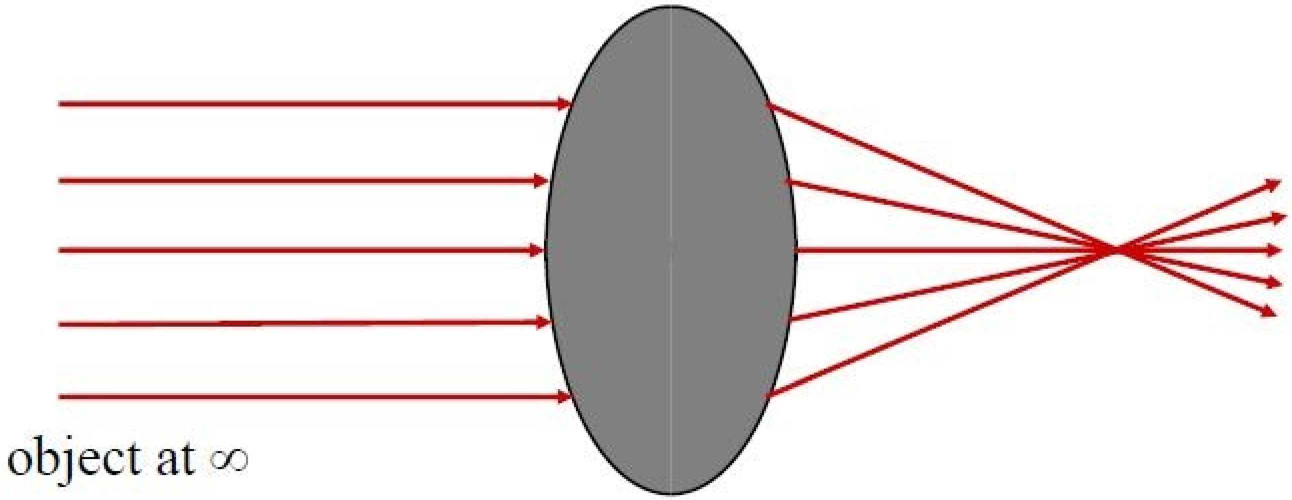
($s=s_o$ and $s'=s_i$)



Assumptions made during the derivation:

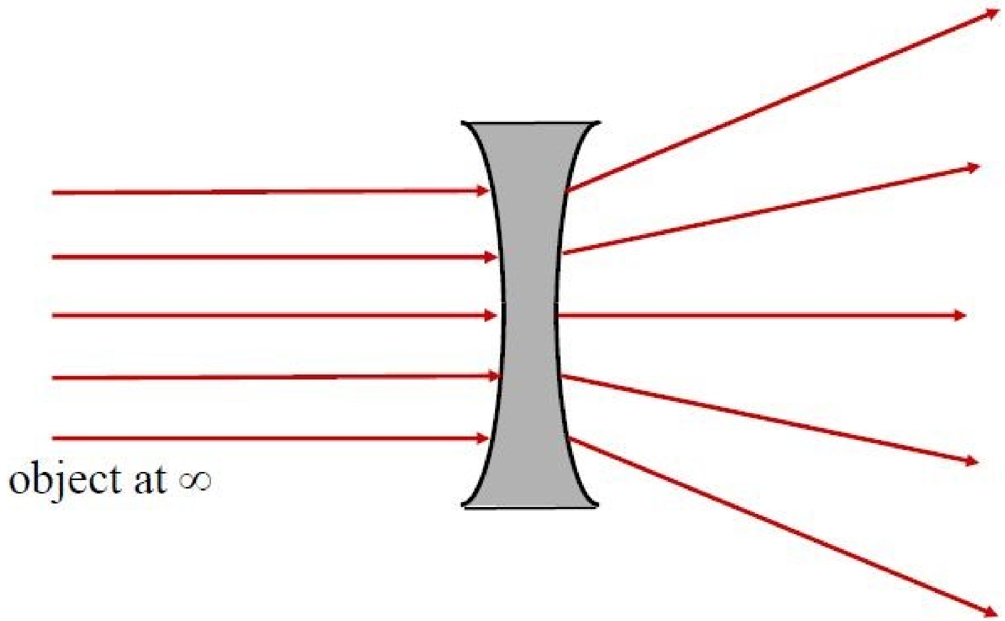
1. paraxial approximation ($\sin\theta \approx \theta$ and $\cos\theta \approx 1$; 1st terms in the series expansions)
2. lens is thin

Lens Types
Positive Lens



Refraction of a ray is proportional to the distance from the optical axis.

Lens Types
Negative Lens

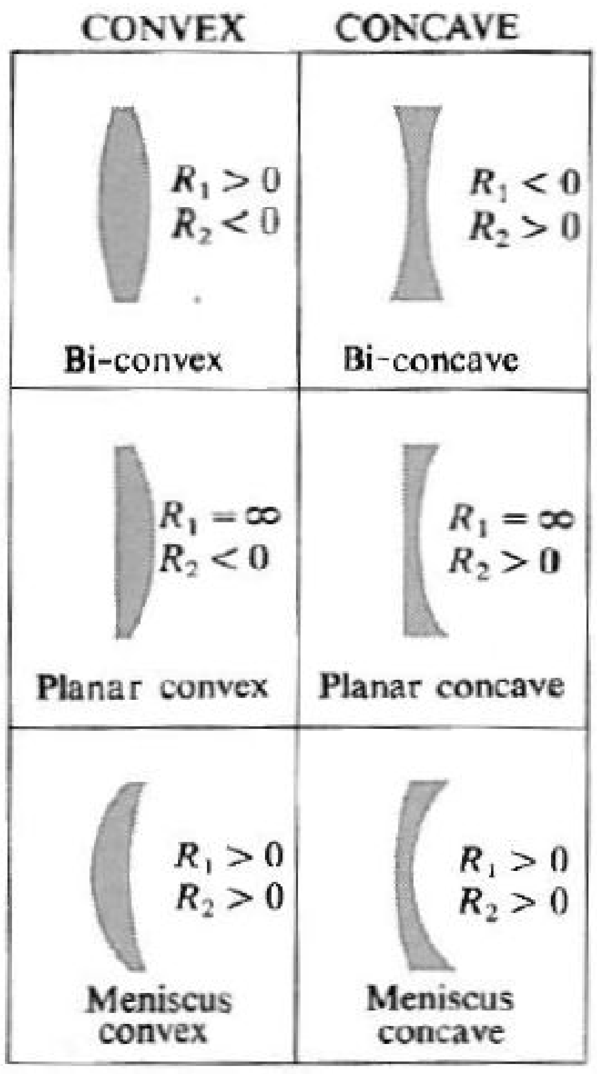


Refraction of a ray is proportional to the distance from the optical axis.

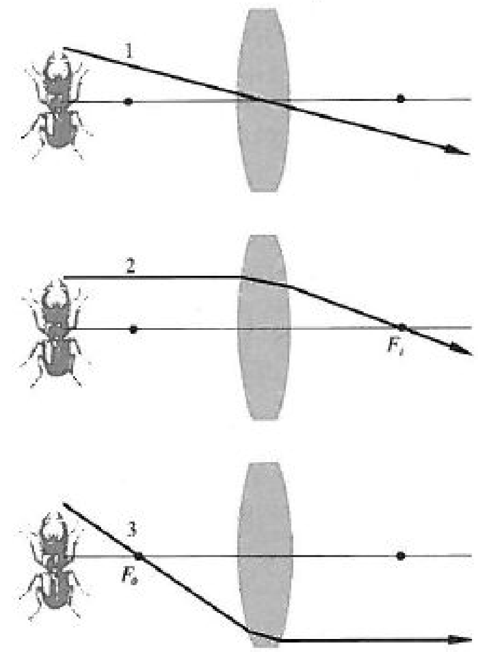
Sign Rules for Lenses

R_i is > 0 if the centre point of curvature is to the right of the surface

Also the terms ”positive lens” and ”negative lens” are used.



Principal Rays of a Lens (3 rays)

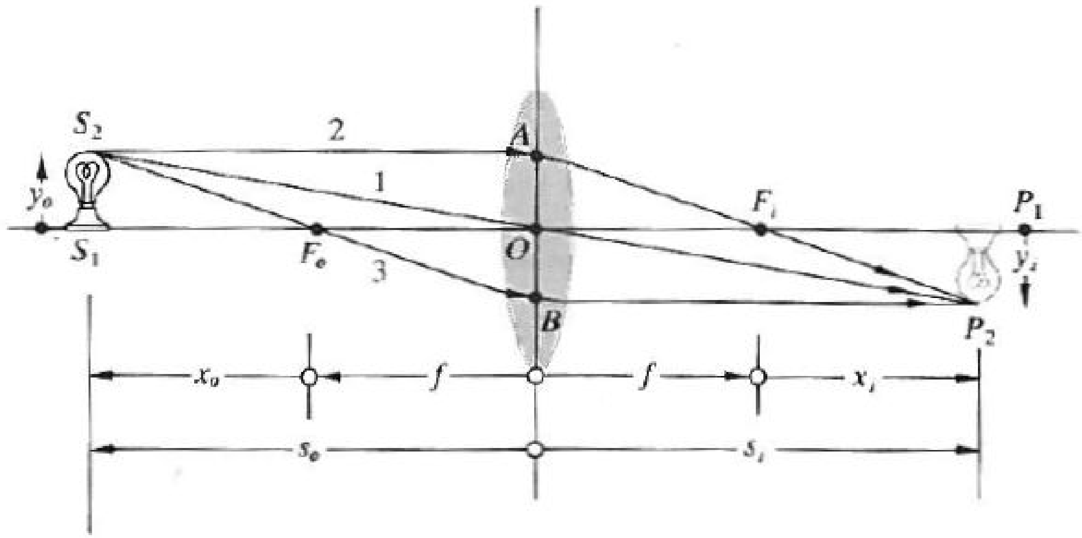


1. Ray propagates through the centre of the lens without changing its direction.

2. Collimated ray travels through the focal point behind the lens (virtual focal point in front of a negative lens)

3. Ray going through the front focal point will be collimated after the lens (back virtual focal for a negative lens).

Magnification M of a Lens



$$M \equiv y_i/y_o$$

From equivalent triangles S_1S_2O ja P_1P_2O it follows:

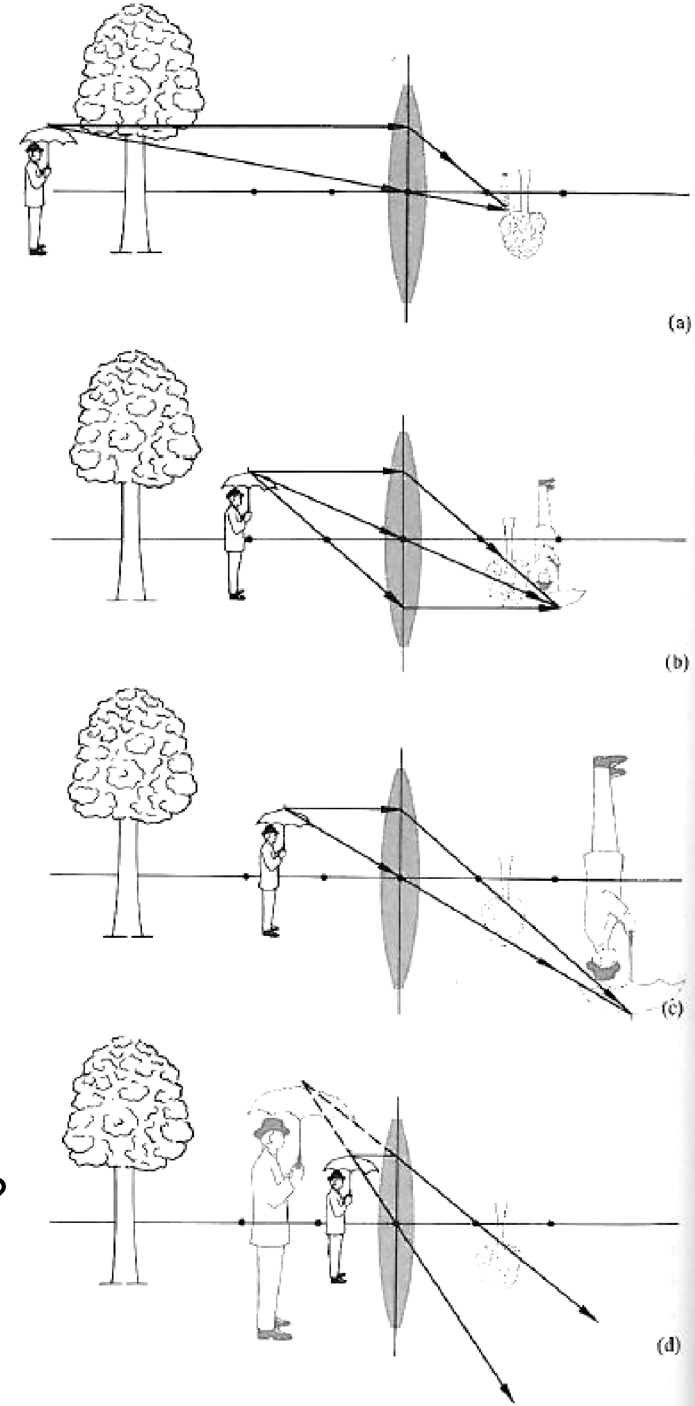
$$\boxed{M = y_i/y_o = s_i/s_o}$$

Magnification M

$M = y_i/y_o = s_i/s_o$

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

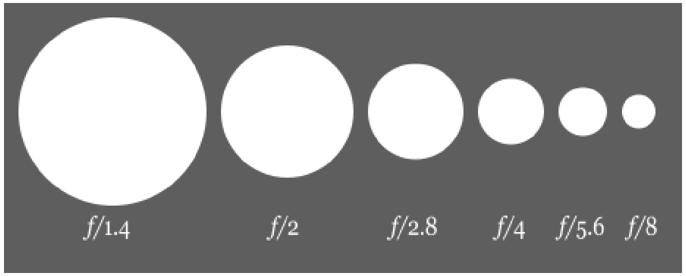
When the magnification is equal to 1?
Or where the object should be placed
to get an image having the original size?



f-number and Numerical Aperture N.A.

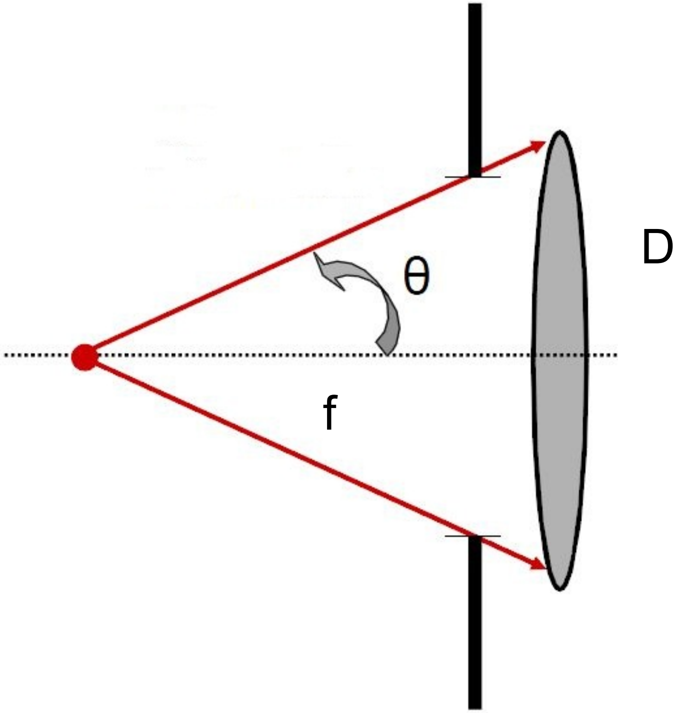
f-number (also known as f/#) = f/D

"f-number = 2" is thus the same as "f/2"

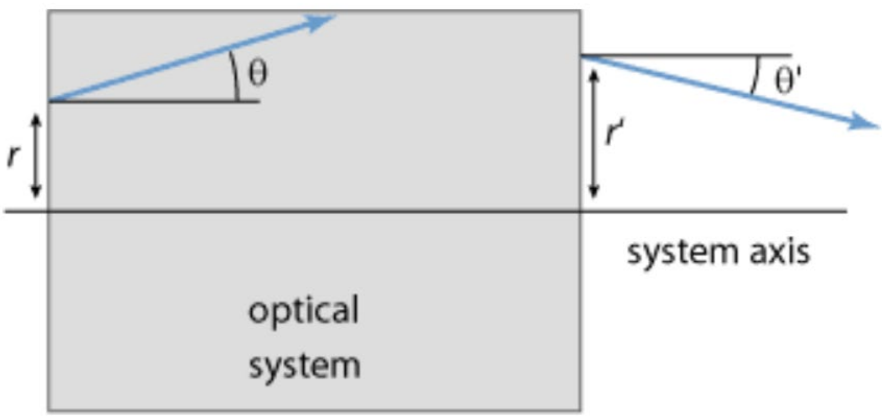


numerical aperture characterises the light gathering power of a lens

N.A. = n sin $\theta \approx$ n D/(2f) = n / (2 x f/#)

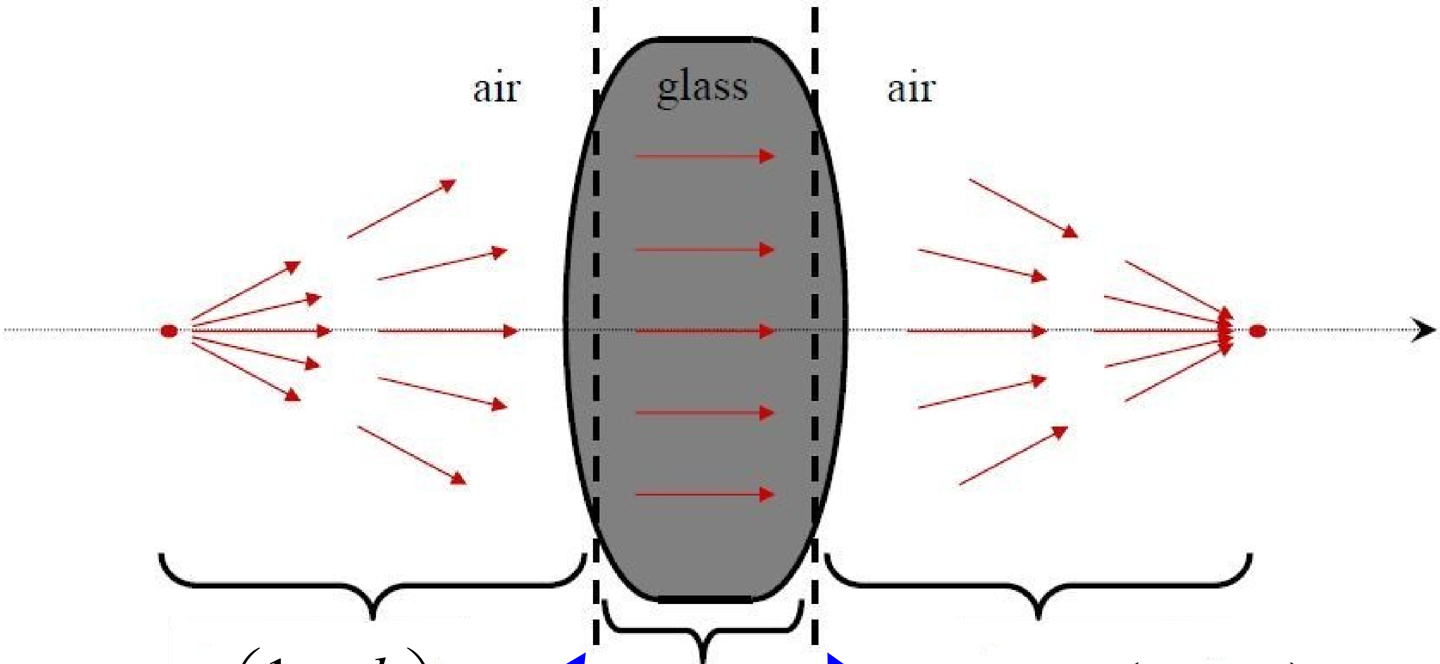


Matrix Formalism for Ray Tracing



$$\begin{pmatrix} r' \\ \theta' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix}$$

Matrix Formalism for Ray Tracing



$$T_1 = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}$$

$$T_3 = \begin{pmatrix} 1 & d_3 \\ 0 & 1 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{r_1} & \frac{1}{n_L} \end{pmatrix}$$

$$R_2 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{r_2} & \frac{n_L}{1} \end{pmatrix}$$

Typical Transformation Matrices

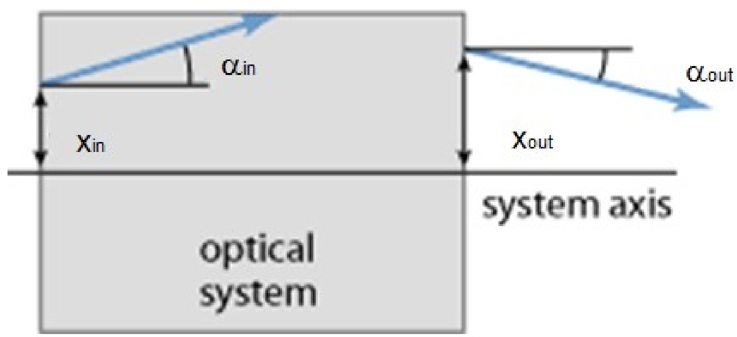
Table of ray transfer matrices

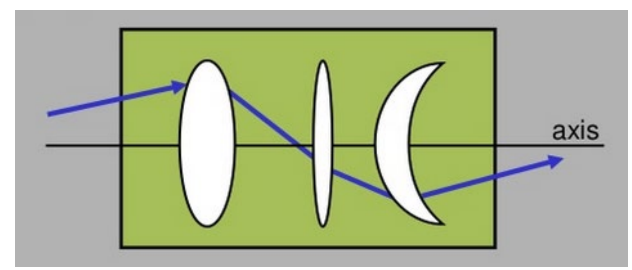
for simple optical components

Element	Matrix	Remarks
Propagation in free space or in a medium of constant refractive index	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	d = distance
Refraction at a flat interface	$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$	n_1 = initial refractive index n_2 = final refractive index.
Refraction at a curved interface	$\begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{R\cdot n_2} & \frac{n_1}{n_2} \end{pmatrix}$	R = radius of curvature, $R > 0$ for convex (centre of curvature after interface) n_1 = initial refractive index n_2 = final refractive index.
Reflection from a flat mirror	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	
Reflection from a curved mirror	$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$	R = radius of curvature, R > 0 for concave
Thin lens	$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	f = focal length of lens where $f > 0$ for convex/positive (converging) lens. Only valid if the focal length is much greater than the thickness of the lens.

Matrix Formalism of Ray Tracing
”ABCD matrix method”

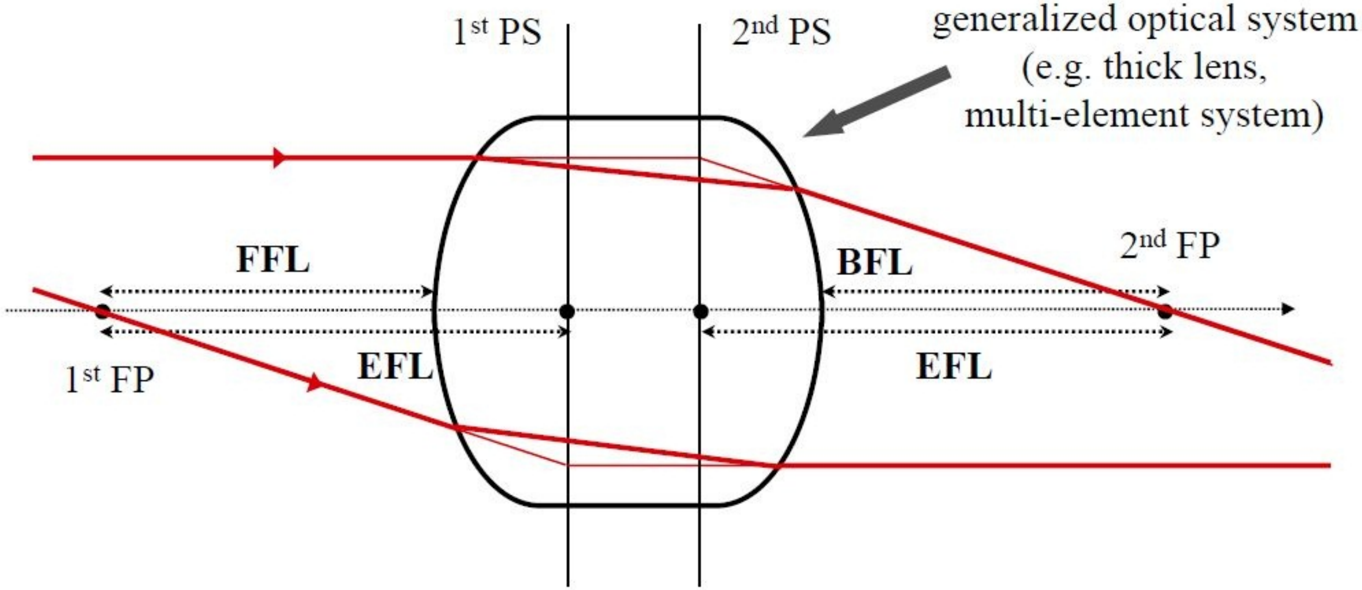
For many optical unit transformations we can define 2 x 2 ray matrices or ABCD matrices. An element’s effect on a ray is found by multiplying the ray with the element’s ABCD matrix.





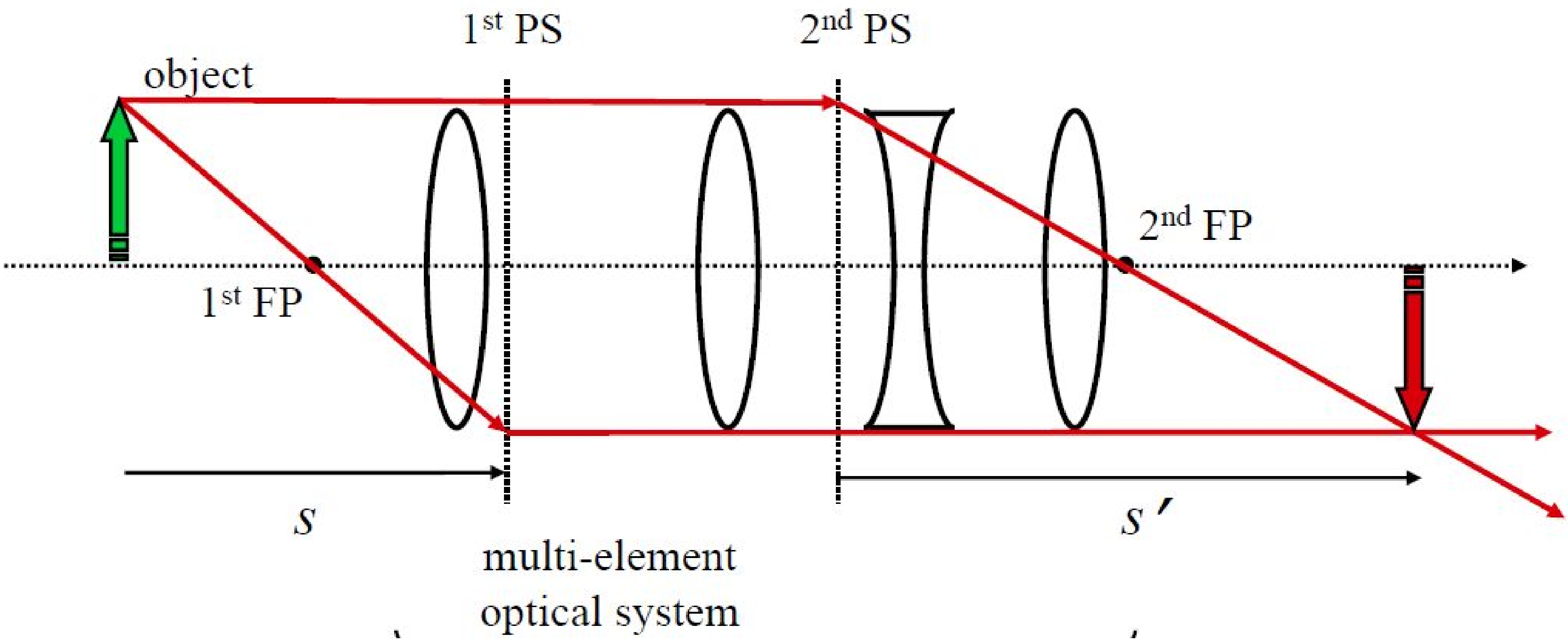
$$\begin{bmatrix} x_{in} \\ \alpha_{in} \end{bmatrix} \longrightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \begin{bmatrix} x_{out} \\ \alpha_{out} \end{bmatrix}$$

Principal Planes of an Optical System

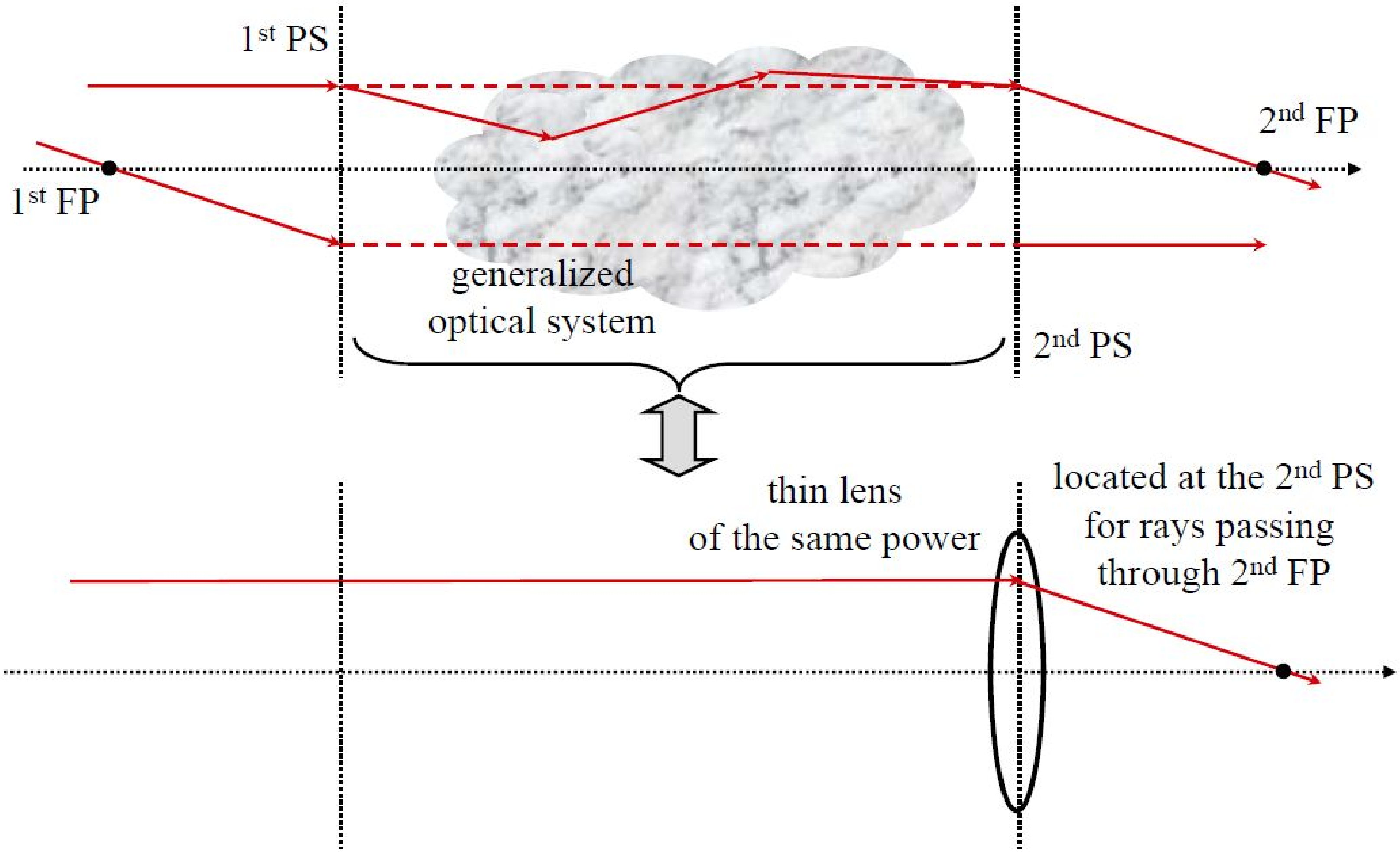


EFL: Effective Focal Length (or simply "focal length")
FFL: Front Focal Length
BFL: Back Focal Length
FP: Focal Point/Plane **PS**: Principal Surface/Plane

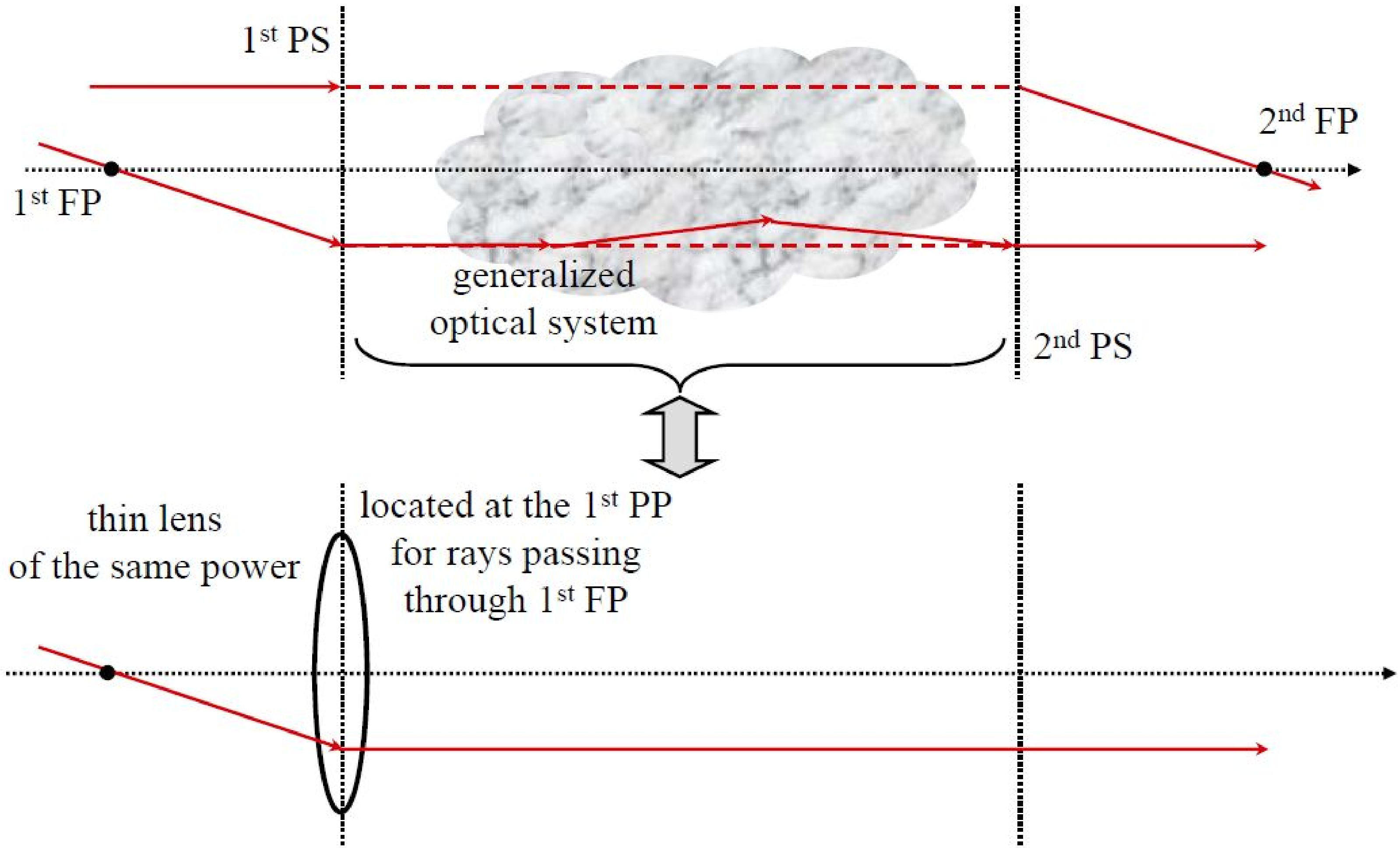
Principal Planes of an Optical System



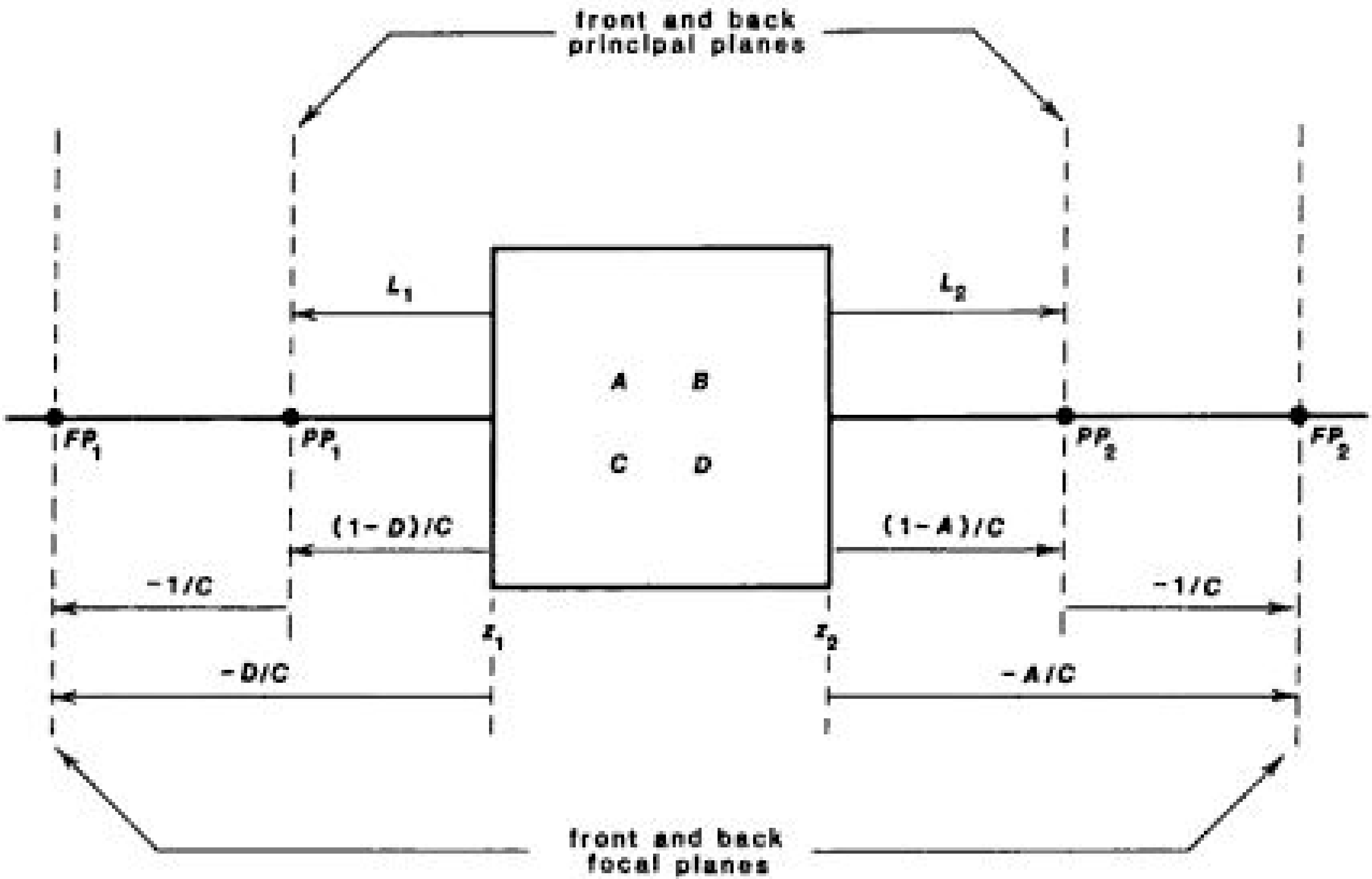
Principal Planes of an Optical System



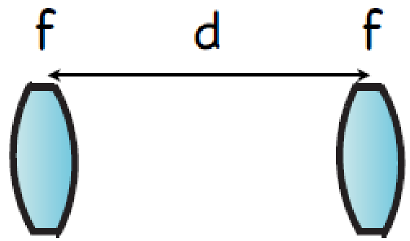
Principal Planes of an Optical System



Principal Planes of an Optical System



Example: Find the Back Focal Length (b.f.l.) of the Following System



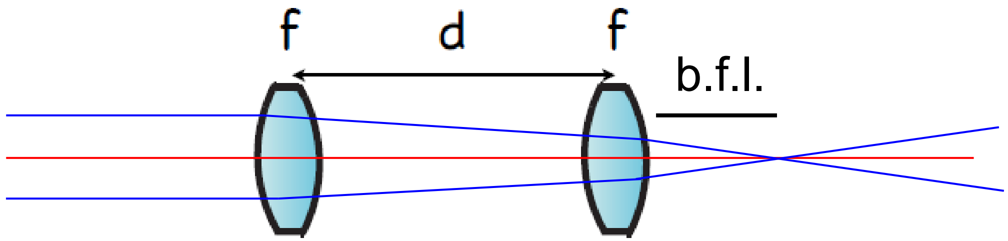
Step 1/3. Find the system's transformation matrix

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 - \dfrac{d}{f} & d \\ -1/f & 1 \end{bmatrix}$$

$$M_{total} = \begin{bmatrix} 1 - \dfrac{d}{f} & d \\ -\dfrac{2}{f} + \dfrac{d}{f^2} & 1 - \dfrac{d}{f} \end{bmatrix}$$

Example: Find the Back Focal Length (b.f.l.)



Step 2/3. Require input rays parallel to the optical axis to pass through the optical axis after the lens system and travel an additional distance equal to b.f.l.

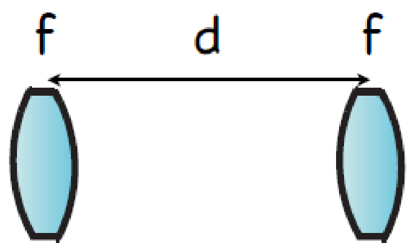
$$\begin{bmatrix} 0 \\ \alpha_{out} \end{bmatrix} = \begin{bmatrix} 1 & b.f.l. \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 - \frac{d}{f} & d \\ -\frac{2}{f} + \frac{d}{f^2} & 1 - \frac{d}{f} \end{bmatrix} \begin{bmatrix} x_{in} \\ 0 \end{bmatrix}$$

Step 3/3. Solve for b.f.l.

$$0 = (1 - \frac{d}{f} + b.f.l.\left(-\frac{2}{f} + \frac{d}{f^2}\right))x_{in}$$

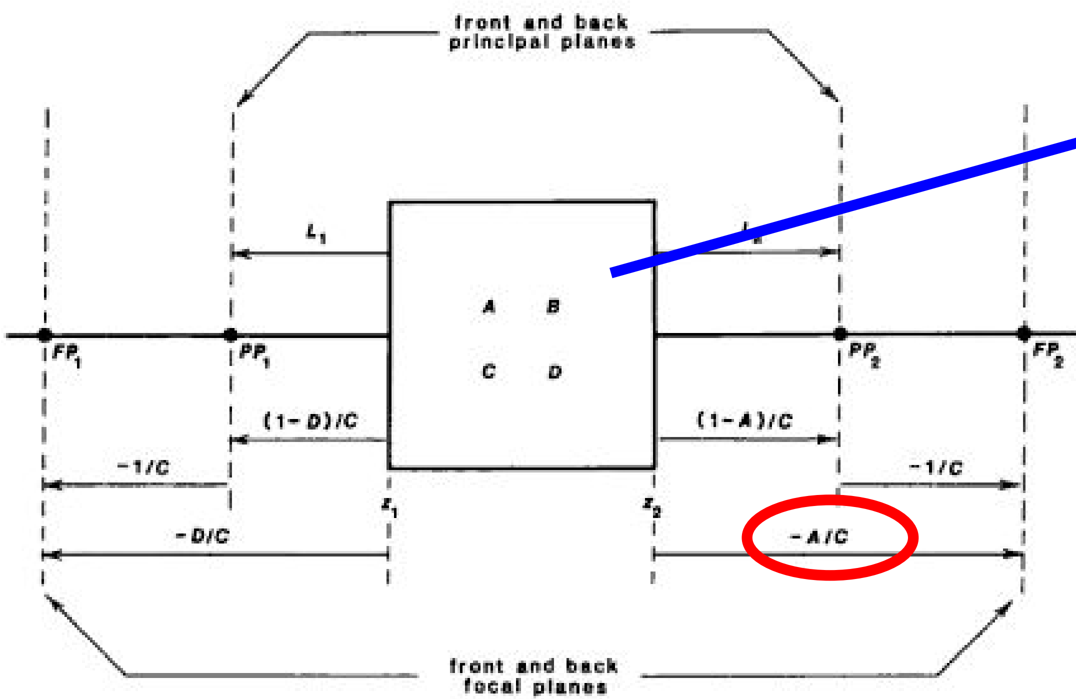
$$\text{b.}f.l. = \frac{\frac{d}{f} - 1}{-\frac{2}{f} + \frac{d}{f^2}} = \frac{df - f^2}{d - 2f}$$

Example: Find the Back Focal Length (b.f.l.)



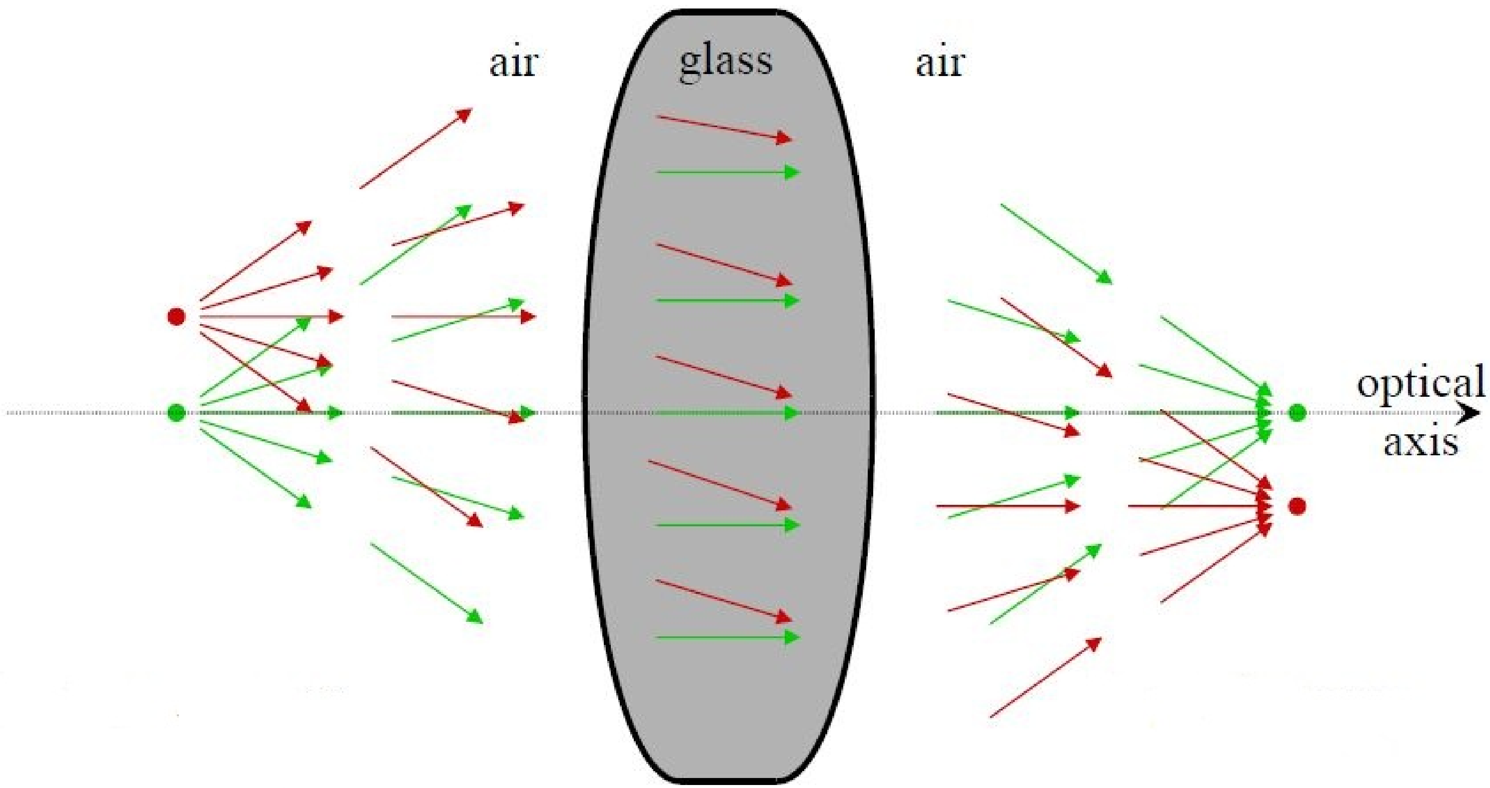
Alternative approach: using matrix tables

$$M_{total} = \begin{bmatrix} 1 - \frac{d}{f} & d \\ -\frac{2}{f} + \frac{d}{f^2} & 1 - \frac{d}{f} \end{bmatrix}$$



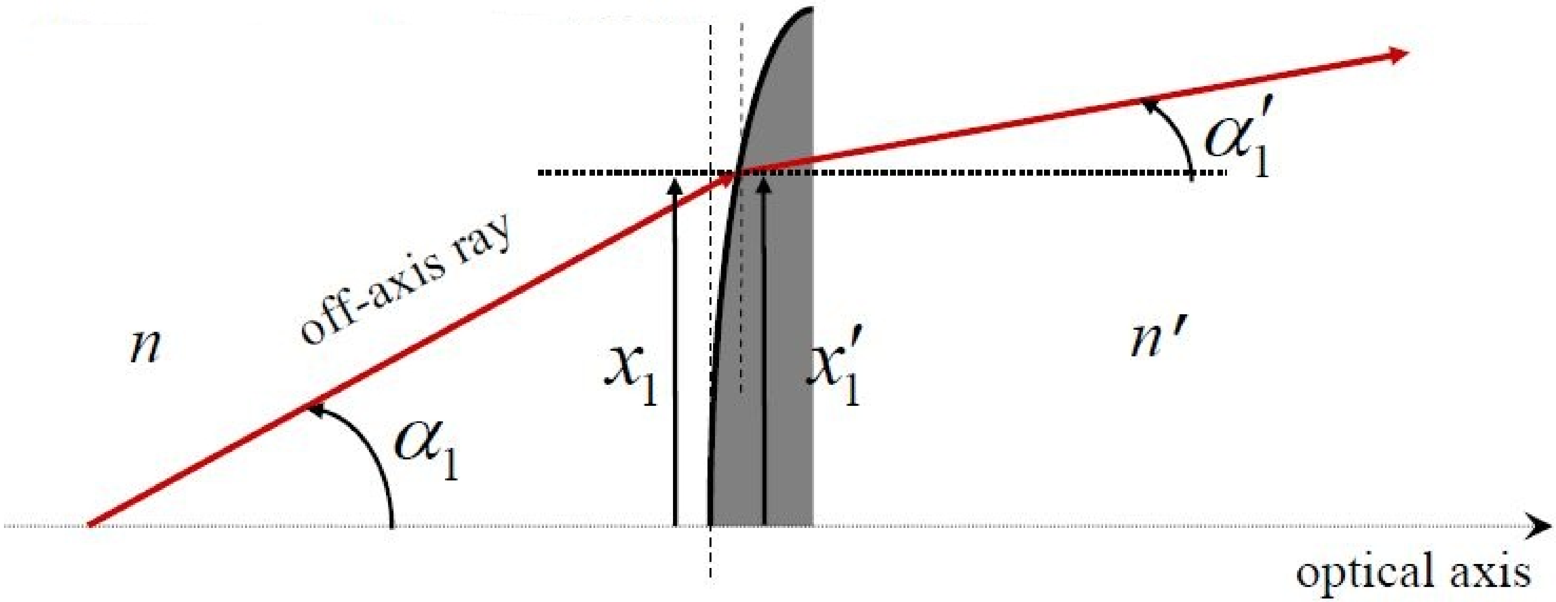
$$-\frac{A}{C} = \frac{\frac{d}{f} - 1}{-\frac{2}{f} + \frac{d}{f^2}} = \frac{df - f^2}{d - 2f}$$

Ideal Lens



Validity of the Paraxial Approximation

Further away from the optical axis the paraxial approximation is not good enough anymore.



Aberrations = Imperfections of Lenses

Ignoring the third and higher order terms in the series expansion of sine and cosine functions causes aberrations. Typical aberrations for a monochromatic system are:

- spherical aberration
- astigmatism
- coma
- field curvature
- distortion.

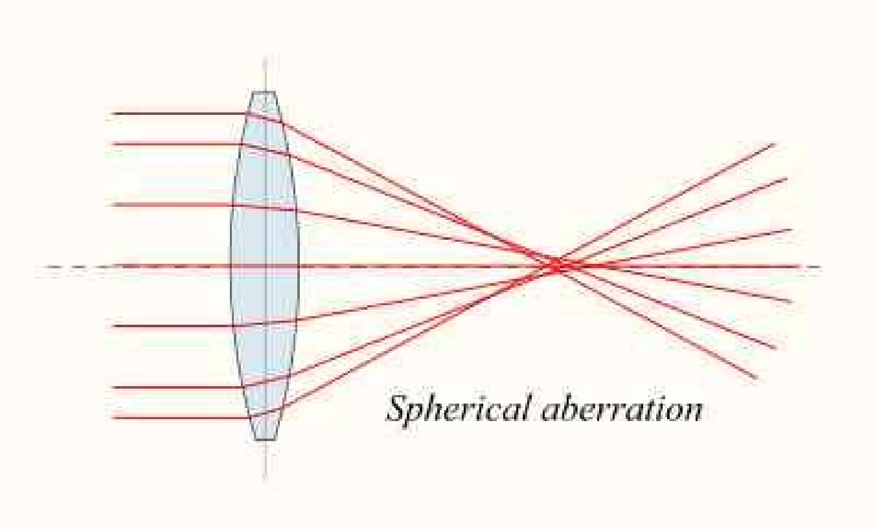
For light containing several colours there are additional aberrations:

- chromatic aberration
- lateral colour

Spherical Aberration

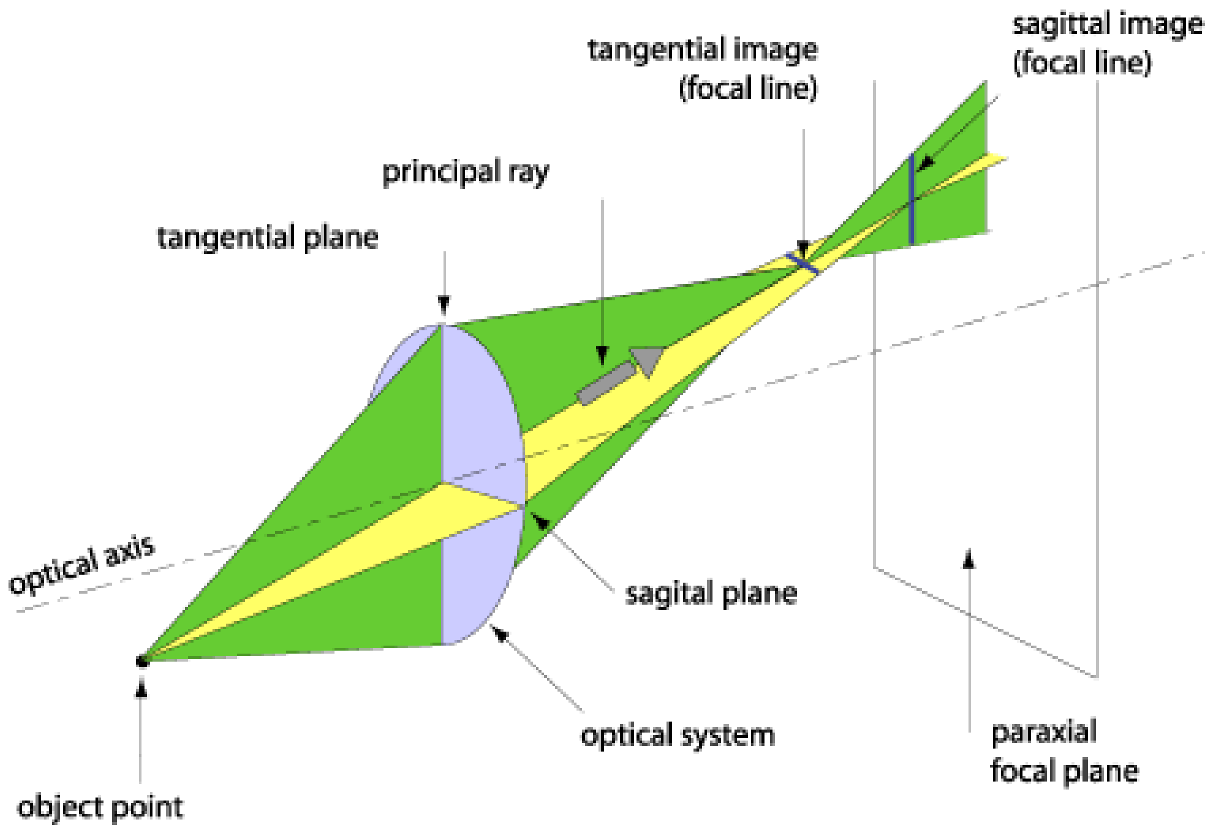
Ideal lens shape is not a spherical surface. However, a spherical surface is the easiest and most affordable to manufacture. In a bi-convex lens, for example, the rays of light propagating far away from the optical distance will refract closer to the lens than paraxial rays.

With modern manufacturing systems aspherical lenses can already be manufactured at a reasonable cost by pressing lenses out of melt glass or plastic using a mold. An aspheric surface can compensate for the spherical aberration.



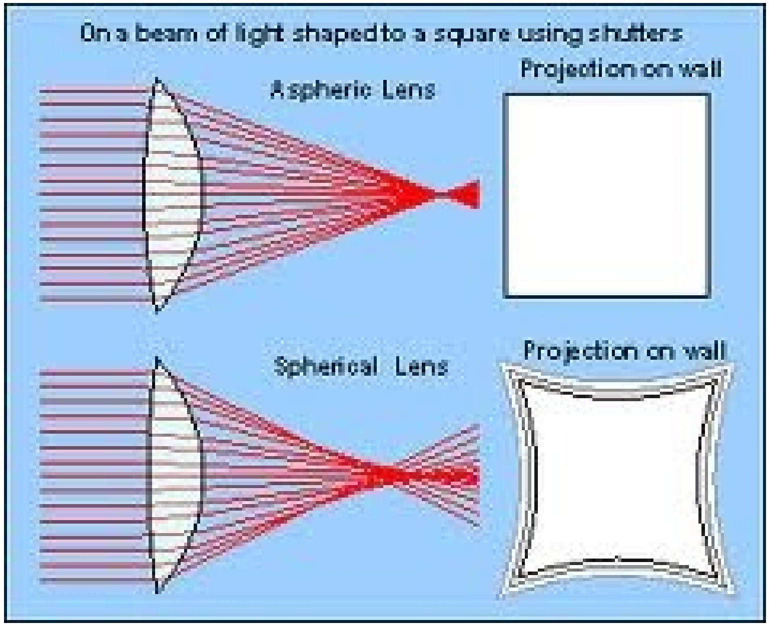
Astigmatism

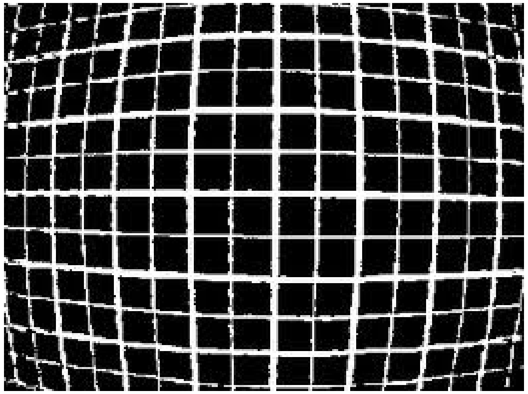
A non-paraxial ray has different focal points in the horizontal and vertical directions.



Field Distortion

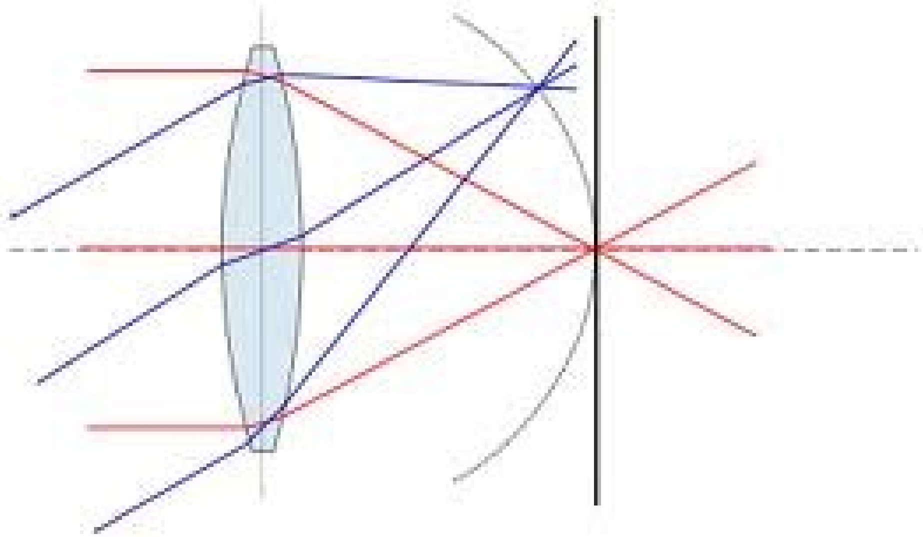
Lens can distort the image in many ways even though the image is sharp or the focusing properties are good.





Field Curvature

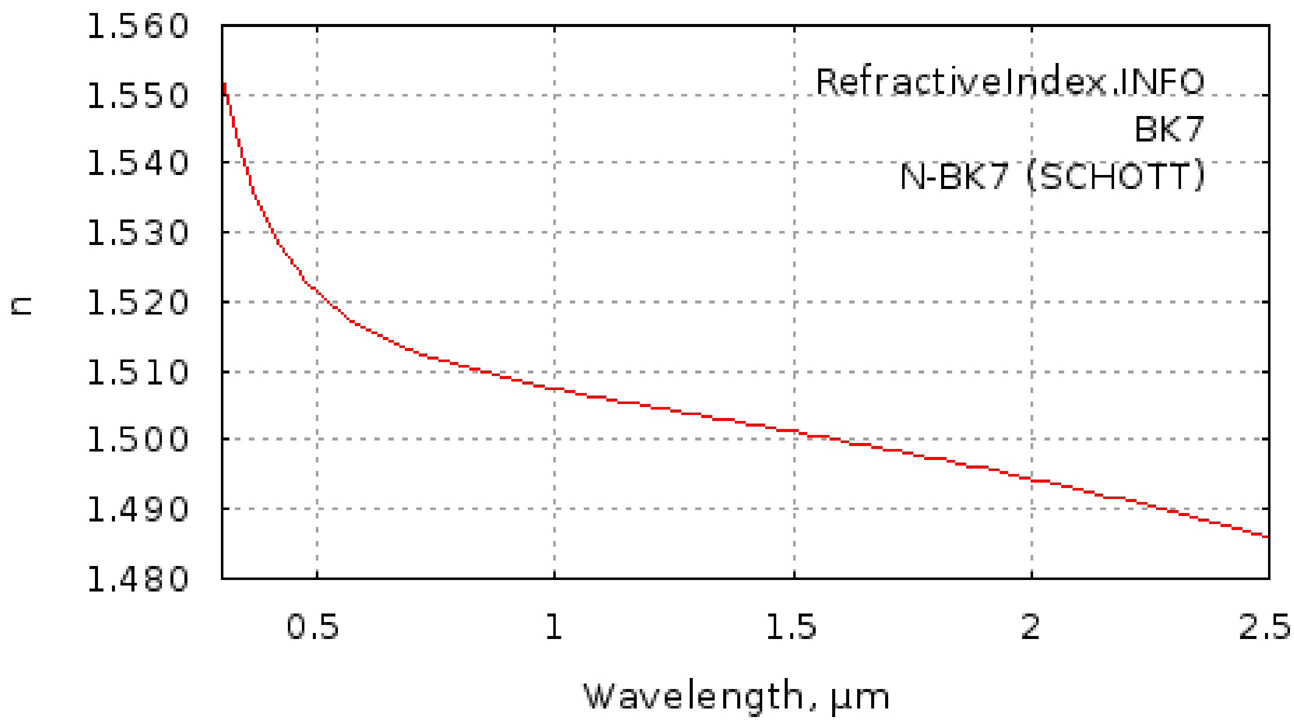
Non-paraksial rays do not focus on a place surface.



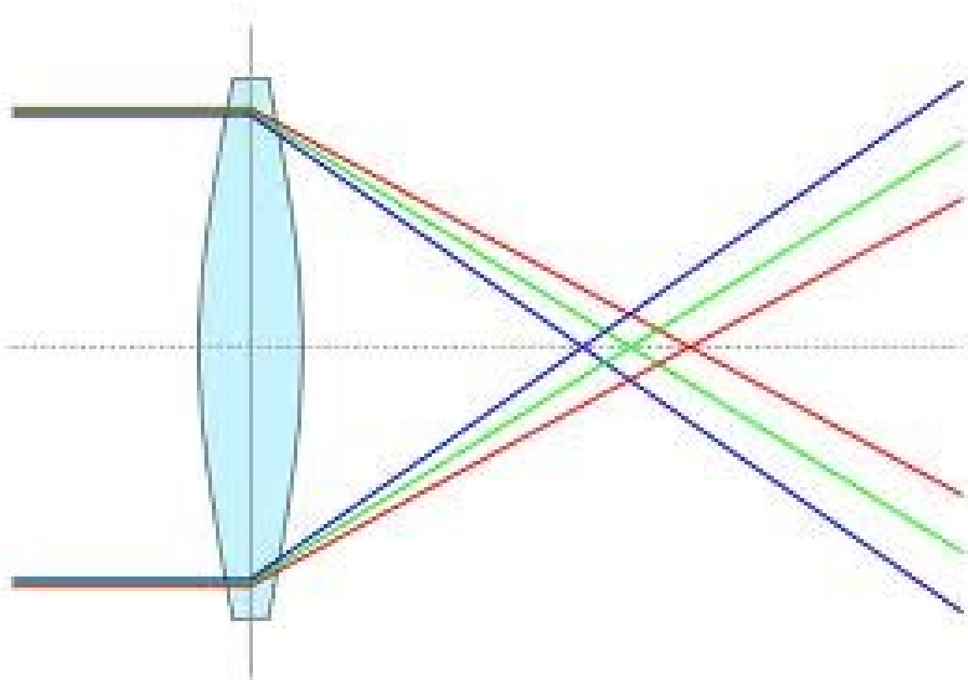
Chromatic Aberration

Is caused by the fact that the refractive index *n* is actually not a constant but depends on the wavelength/frequency of light = <u>material dispersion</u>.

$$n = n(\nu) = n(\lambda)$$



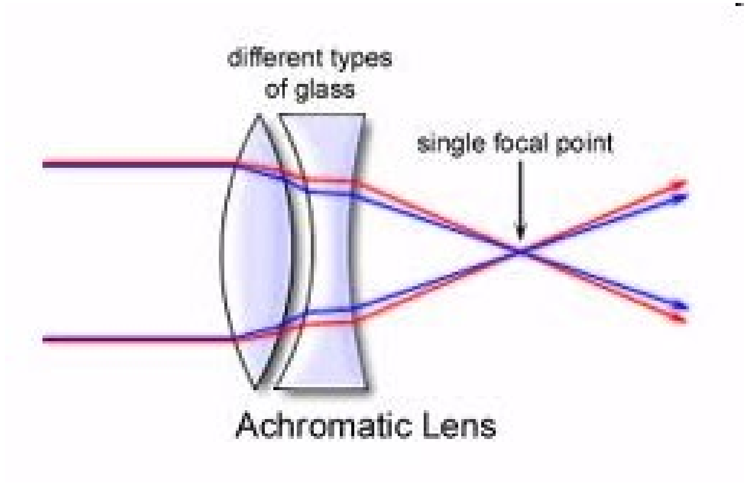
Chromatic Aberration in a Bi-convex Lens



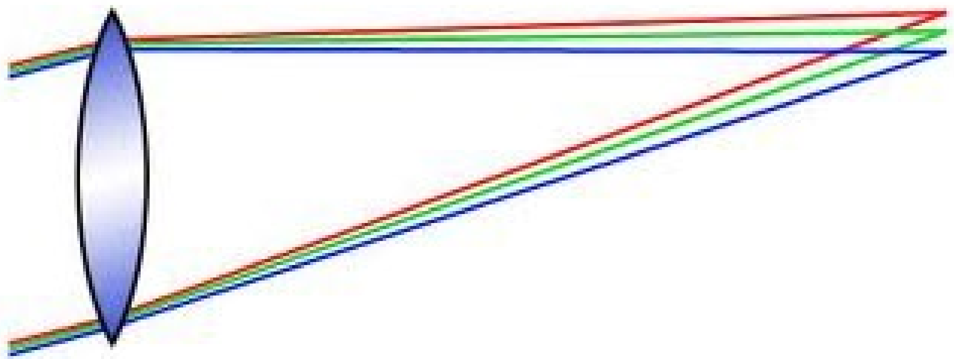
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Compensation of Chromatic Aberration

achromatic doublet lens



Lateral Colour



Summary

We have discussed the fundamental concepts of geometrical optics:

- **lens maker's formula** for a thin lens
- **thin lens equation**
- **ray tracing**:
    - principal rays of lenses
    - sign convention
- concepts: magnification, numerical aperture, f-number
- lens aberrations
- **matrix formalism for optical systems**
- reduction of an optical system into a thin lens: principal planes
- introduction to optical spectroscopy