Optics E-5730 Spring 2021

Wave Optics I

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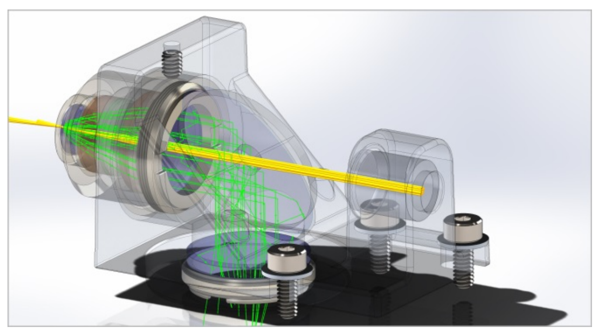
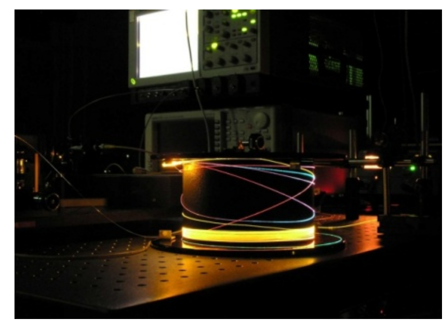
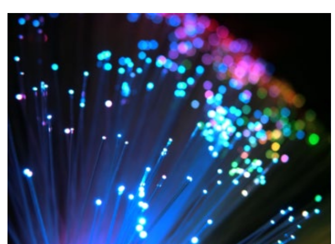
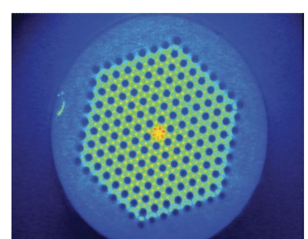
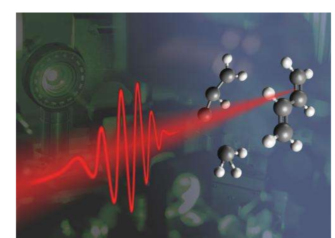










Fundamentals of Optics, Spring 2021

ELEC E-5730 lectures online using Zoom at https://aalto.zoom.us/j/8453943170

exercises online using Zoom at https://aalto.zoom.us/j/5703080612

week	day	date	time	topic	
2	Mon	11.1.2021	8-10	Lecture 1: Geometrical optics 1	
	Fri	15.1.2021	8-10	Lecture 2: Geometrical optics 2	
3	Mon	18.1.2021	8-10	Lecture 3: Wave optics 1	
	Mon	18.1.2021	10-12	Exercise 1	
	Fri	22.1.2021	8-10	Lecture 4: Wave optics 2	
4	Mon	25.1.2021	8-10	Lecture 5: Coherence 1	
	Mon	25.1.2021	10-12	Exercise 2	
	Fri	29.1.2021	8-10	Lecture 6: Coherence 2	
5	Mon	1.2.2021	8-10	Lecture 7: Radiometry	
	Mon	1.2.2021	10-12	Exercise 3	
	Fri	5.2.2021	8-10	Lecture 8: Interferometry + 30 mins mid-term exam	
6	Mon	8.2.2021	8-10	Lecture 9: Fibre optics + Optical telecom	LAB WORKS PERIOD
	Mon	8.2.2021	10-12	Exercise 4	LAB WORKS PERIOD
	Fri	12.2.2021	8-10	Lecture 10: Diffraction 1	LAB WORKS PERIOD
7	Mon	15.2.2021	8-10	Lecture 11: Diffraction 2	LAB WORKS PERIOD
	Mon	15.2.2021	10-12	Exercise 5	LAB WORKS PERIOD
	Fri	19.2.2021	8-10	NO LECTURE	LAB WORKS PERIOD
8	Mon	22.2.2021	8-10	NO LECTURE	
	Mon	22.2.2021	10-12	Exercise 6	
	Fri	26.2.2021		Examination	

Last Lecture – Geometrical Optics II

- Lens maker’s formula and thin lens equation
- Basics of ray tracing in optical systems
- Different types of lenses, magnification, numerical aperture, f-number
- Non-ideal lenses - aberrations
- Matrix formalism for ray tracing
- Reduction of an optical system ’into a thin lens’: principal planes

Wave Optics I

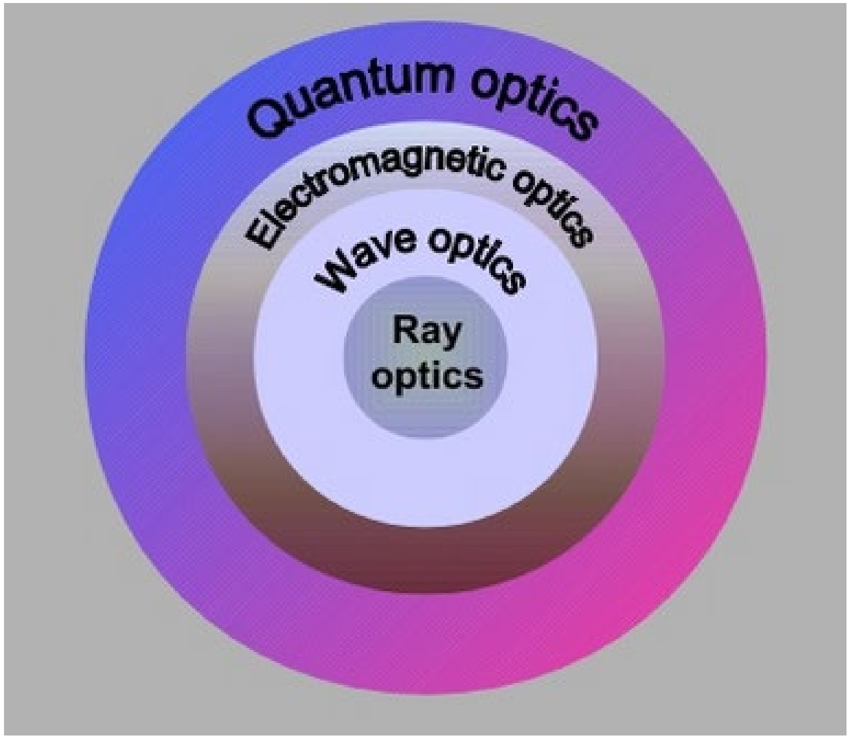
Recap
- Wave motion
- Electric and magnetic fields: Maxwell’s equations
- Wave equation and speed of light

- Polarisation of light

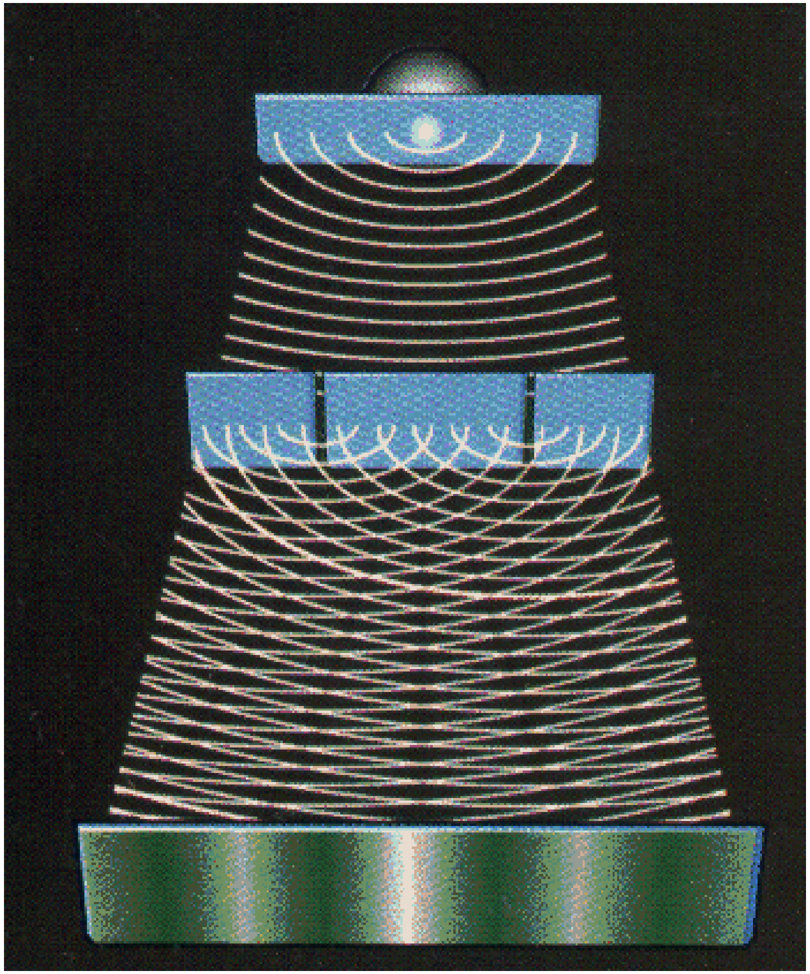
Wave Optics II

- polarising optical components: ’polarisers’
- dichroism and birefringence
- waveplate components: quarter-wave plate and half-wave plate
- reflection and refraction coefficients for E field amplitude and intensity
- Brewster’s angle
- anti-reflection (AR) coating
- interference

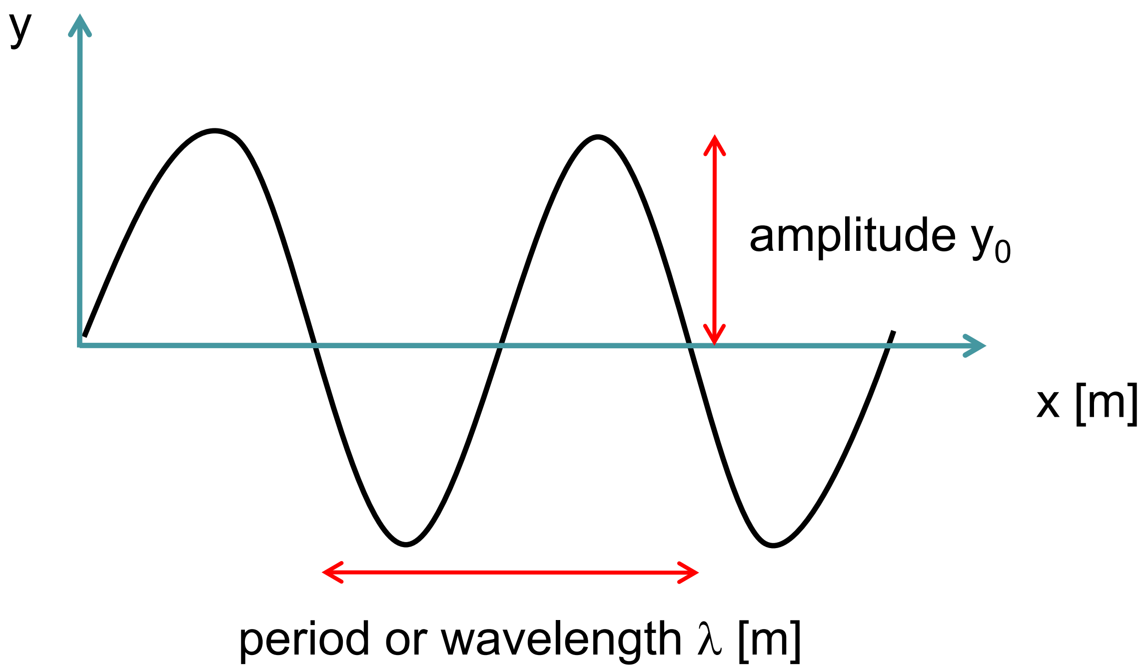
Geometrical Optics (Ray Optics) is the Starting Point



Wave Optics



Recap of Wave Motion (in Space/Spatial Coordinates)



$y = y_0 \sin(\theta)$

oscillating presentation **with respect to spatial coordinate x**:
$y = y_0 \sin(kx)$

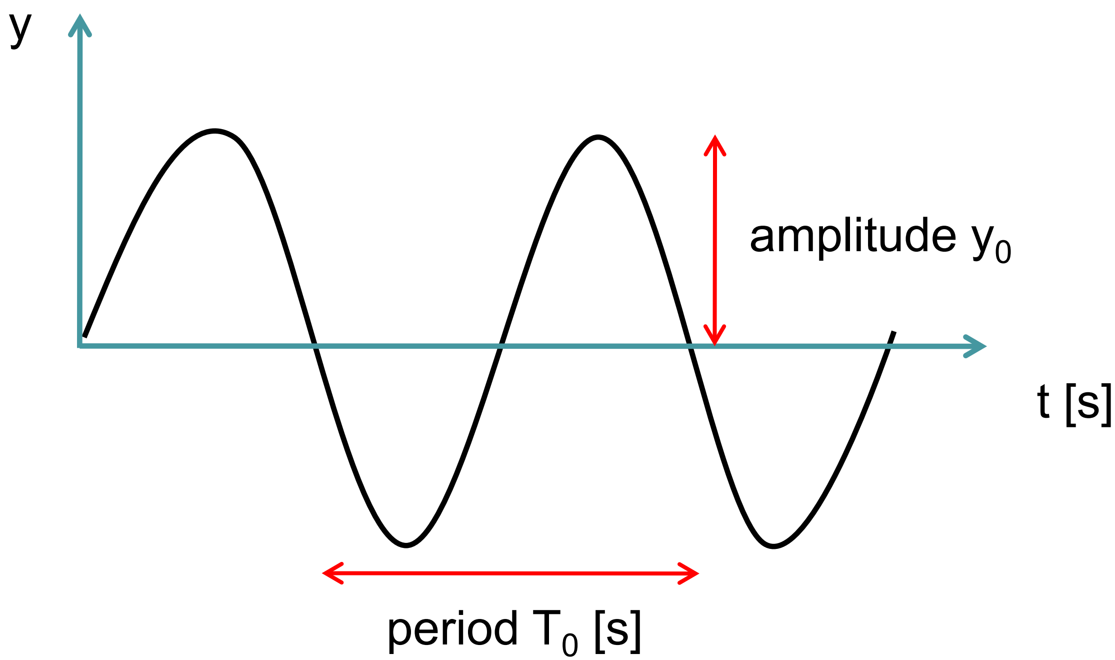
$\cos(kx) = \sin(kx + \pi/2)$

- unit for the argument of sine and cosine is radian
- k [?] x [m] = [rad] → unit for wavenumber k is [rad/m]
- period in the angle space is 2π and equivalently in space it is λ:

$$kx = \theta$$

$$k(x+\lambda) = \theta + 2\pi \quad \rightarrow \quad \boxed{k = 2\pi/\lambda}$$

Recap of Wave Motion (in Time/Temporal Coordinates)



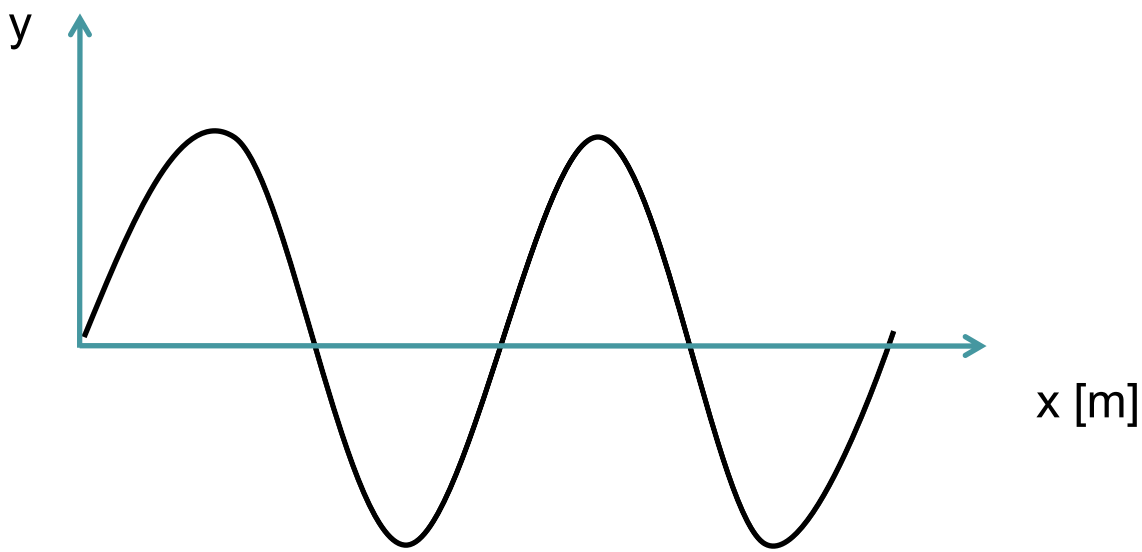
oscillating presentation **with respect to time**:
$y = y_0 \sin(\omega t)$

$\omega t = \theta$

$\omega(t+T_0) = \theta + 2\pi$ → $\boxed{\omega = 2\pi/T_0 = 2\pi\nu}$

where ν is frequency [s^{-1}]

Propagating Wave Motion

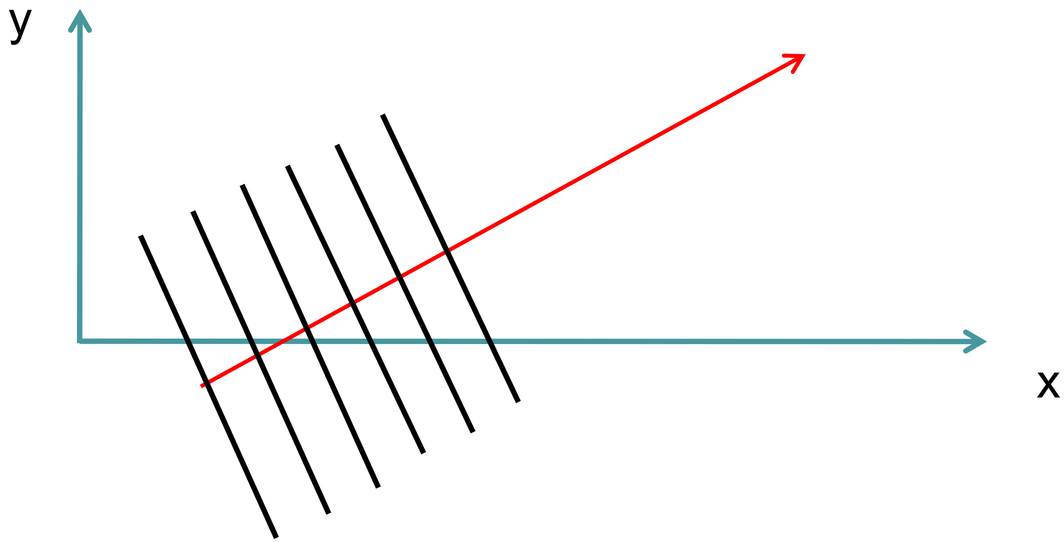


oscillating presentation
for a propagting wave:
$y = y_0 \sin(kx - \omega t)$

Argument $(kx-\omega t)$, and thus, amplitude *y* remains constant if kx increases proportionally to ωt. Therefore the wave described by the function y propagates along the positive x axis.

On the other hand, a wave propagating to the negative direction of the x axis has a form $f = f(kx+\omega t)$.

Propagating Wave Motion in 3D



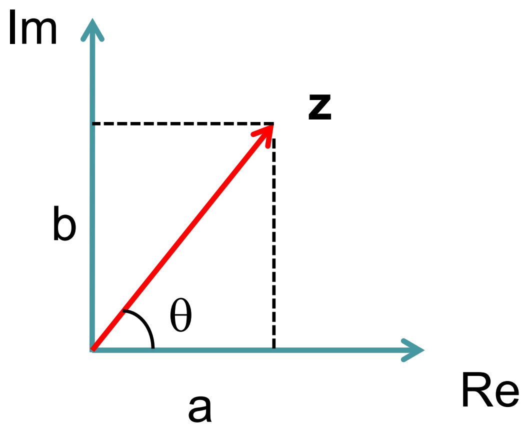
wave vector **k** is perpendicular to the direction of the wave front

$$\mathbf{k} = k_x\mathbf{e_x} + k_y\mathbf{e_y} + k_z\mathbf{e_z}$$

wave fronts of a plane wave
(wave front = plane where the wave has constant phase)

$$k = ||\mathbf{k}|| = \mathrm{sqrt}(\mathbf{k}\cdot\mathbf{k}) = \mathrm{sqrt}(k_x^2 + k_y^2 + k_z^2)$$
$$= 2\pi/\lambda$$

Complex Numbers – Quick Recap



z = a + ib, in addition r = ||**z**|| = sqrt(a^2+b^2)
and complex conjugate (c.c.) **z***= = a - ib

sin θ = b/r
cos θ = a/r

z = a + ib = r(cos θ + i sin θ)

z = r $e^{i\theta}$ ← $\equiv e^{i\theta}$ (Euler's formula)

- propagating electric field can thus be expressed as **E = E**$_0$ $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$
- <u>real part of the electric field</u> can always be found with the help of the c.c.: Re(**E**) = 0.5 (**E**+**E***) = **E**$_0$ cos θ
- by using complex valued fields the math becomes easier to follow:

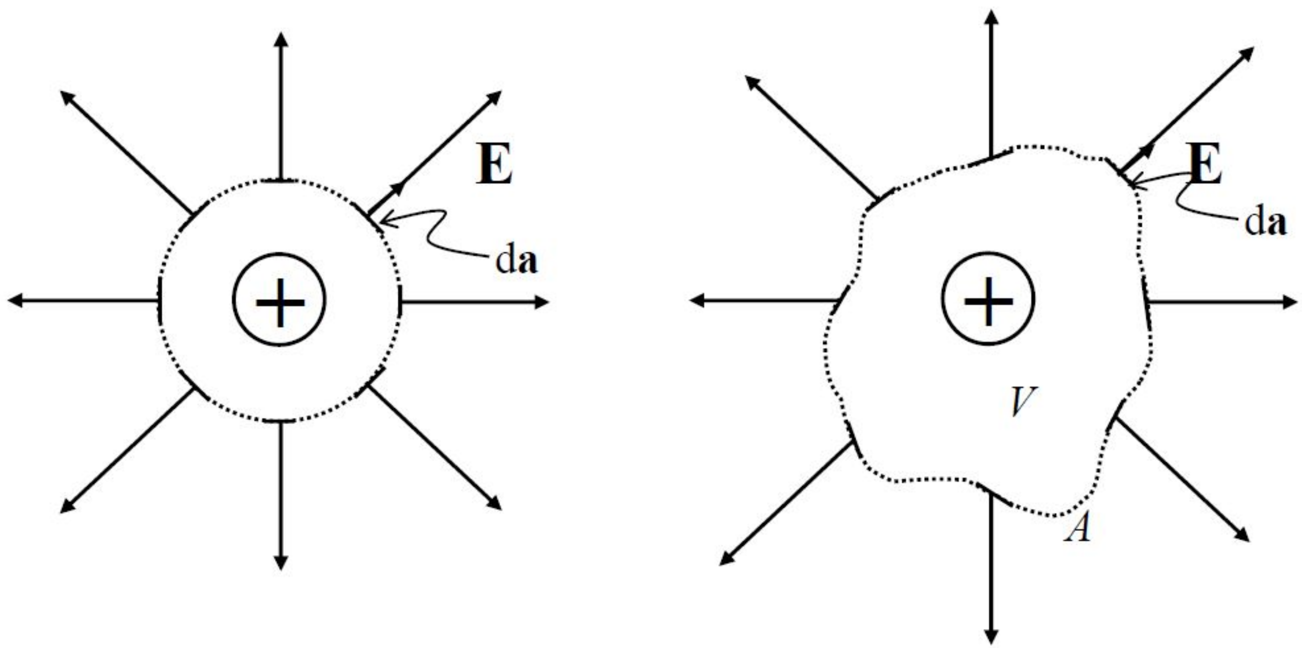
$$E_1 = E_{01}e^{i\theta_1} \qquad E_2 = E_{02}e^{i\theta_2}$$

$$E_1E_2 = E_{01}E_{02}e^{i(\theta_1+\theta_2)}$$

$$\frac{E_1}{E_2} = \frac{E_{01}}{E_{02}}e^{i(\theta_1-\theta_2)}$$

Maxwell's Equations (1/4)

Gauss's Law for the Electric Field



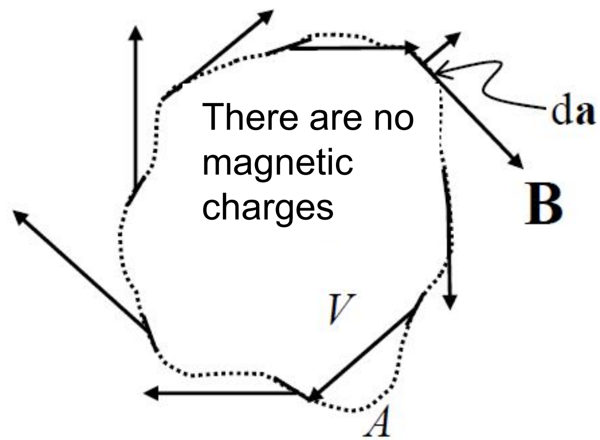
$$\oint\!\!\oint_A \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\varepsilon_0} \iiint_V \rho \, dV \quad \Longleftrightarrow \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

divergens theorem

density of free charges

Maxwell's equations (2/4)

Gauss's Law for the Magnetic Field



$$\oint\!\!\oint_A \mathbf{B} \cdot d\mathbf{a} = 0$$

divergens theorem

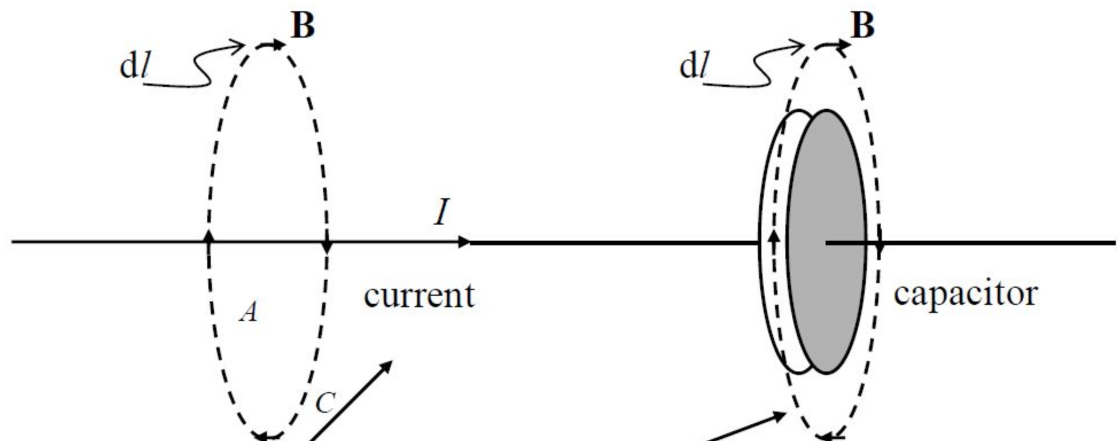
$\Longleftrightarrow$

$$\nabla \cdot \mathbf{B} = 0$$

density of "magnetic charges"

Maxwell’s equations (3/4)

Ampére’s Circuital Law



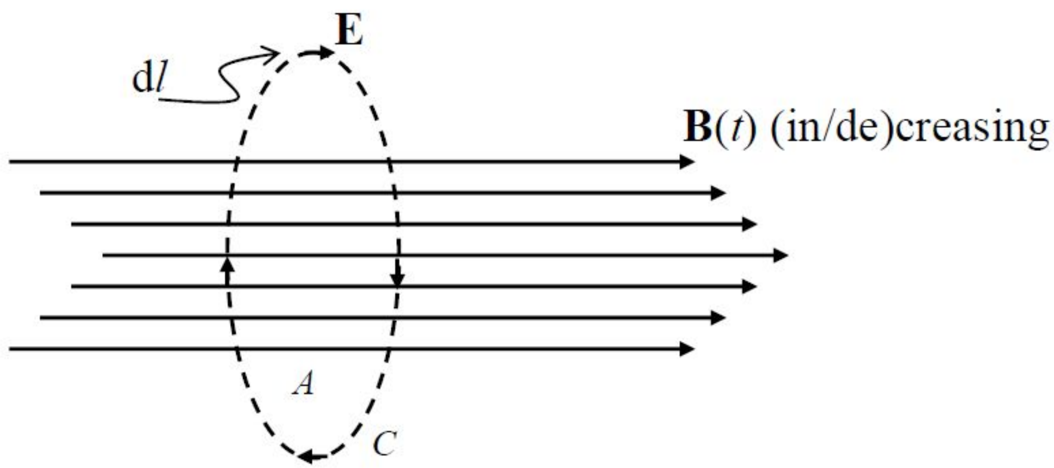
$$\oint_C \mathbf{B} \cdot \mathrm{d}l = \mu_0 \left(\iint_A \mathbf{J} \cdot \mathrm{d}\mathbf{a} + \iint_A \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathrm{d}\mathbf{a} \right) \quad \overset{\text{Stokes theorem}}{\Longleftrightarrow} \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

current density

Maxwell’s extension, *Displacement current*

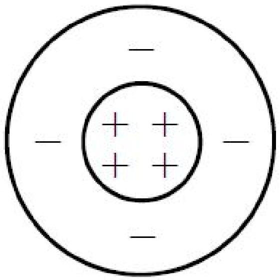
Maxwell's equations (4/4)

Faraday's Law of Induction

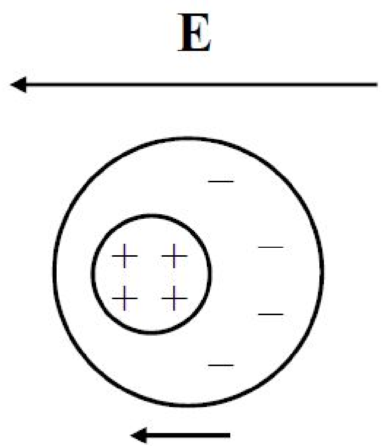


$$\oint_C \mathbf{E} \cdot \mathrm{d}l = -\frac{\mathrm{d}}{\mathrm{d}t} \int_A \mathbf{B} \cdot \mathrm{d}\mathbf{a} \quad \overset{\text{Stokes theorem}}{\Longleftrightarrow} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Constitutive Relations ("Material Equations")



free atom,
total charge is zero
(neutral atom)



Atom in external E field:

- total charge remains zero
- charge distribution is unsymmetric

$$\mathbf{p} = q_+\mathbf{r}_+ - q_-\mathbf{r}_-$$

dipole moment

$$\mathbf{P} = \sum \mathbf{p}$$

polarisation

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

E: external electric field
P: polarisation of the medium
D: displacement field
ε_0: permittivity of free space (=vacuum)
χ_e: electric suscebtibility

for linear,isotropic medium:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \varepsilon_0 (1 + \chi_e)\vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$$

How about non-linear, non-isotropic medium?

Constitutive Relations ("Material Equations")

similarly for magnetic fields

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

H: external magnetising field
M: magnetisation of the medium
B: (total) magnetic field
μ_0: permeability of free space (=vacuum)
χ_m: magnetic suscebtibility

for linear, isotropic medium: $\vec{M} = \varepsilon_0 \chi_m \vec{H}$

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H}$$

Maxwell's equations in differential form for linear isotropic isolating medium (dielectric)

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \varepsilon\mu \frac{\partial \vec{E}}{\partial t}$$

material equations

$$\vec{D} = \varepsilon_0 (1 + \chi_e)\vec{E} = \varepsilon\vec{E}$$

$$\vec{B} = \mu_0 (1 + \chi_m)\vec{H} = \mu\vec{H}$$

Wave Equation and Speed of Propagation

1. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Big\| \; \nabla \times$

2. $\nabla \times \vec{B} = \varepsilon\mu \frac{\partial \vec{E}}{\partial t} \quad \Big\| \; \frac{\partial}{\partial t}$

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla \times \frac{\partial \vec{B}}{\partial t} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial^2 t}$$

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\varepsilon\mu \frac{\partial^2 \vec{E}}{\partial^2 t}$$

Wave Equation and Speed of Propagation

$$\nabla \times (\nabla \times \vec{E}) = -\varepsilon\mu \frac{\partial^2 \vec{E}}{\partial^2 t}$$

vector calculus gives

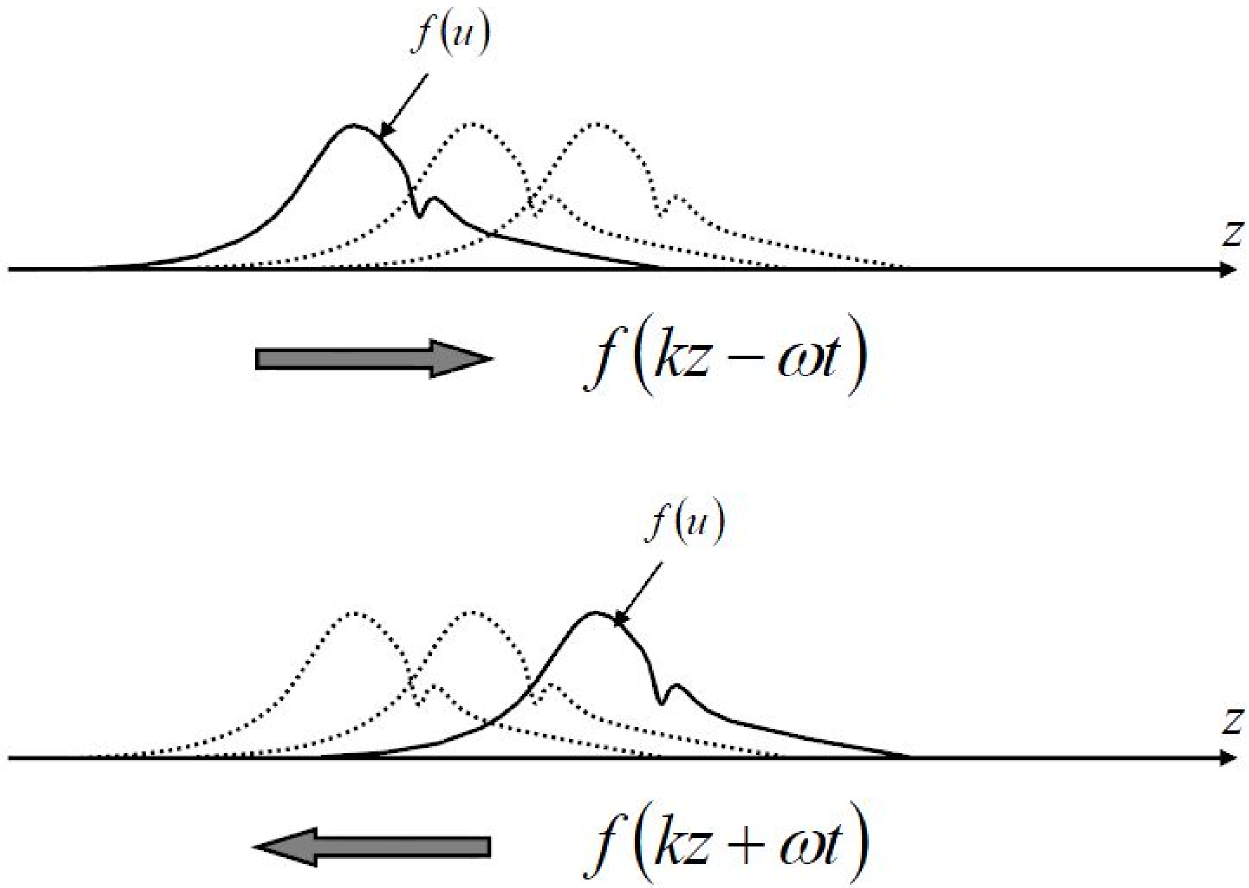
$$\nabla \times (\nabla \times \vec{E}) = -\nabla^2 \vec{E} + \nabla(\nabla \cdot \vec{E})$$

= 0

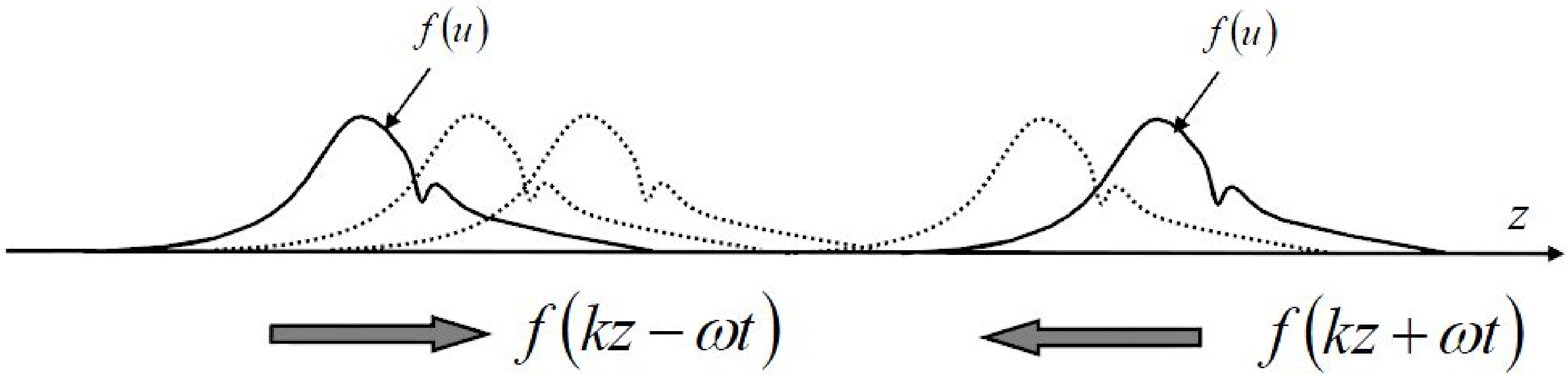
$$\nabla^2 \vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial^2 t} = 0$$

wave equation

Solutions to Wave Equation



Solutions to Wave Equation



If f(kz-ωt) and f(kz+ωt) are solutions to wave equation the also their sum is a solution, because the wave equation is linear. Thus f(kz-ωt) + f(kz+ωt) is also a solution.

Generally speaking, there can be multiple solutions but in most cases the solutions are limited by:

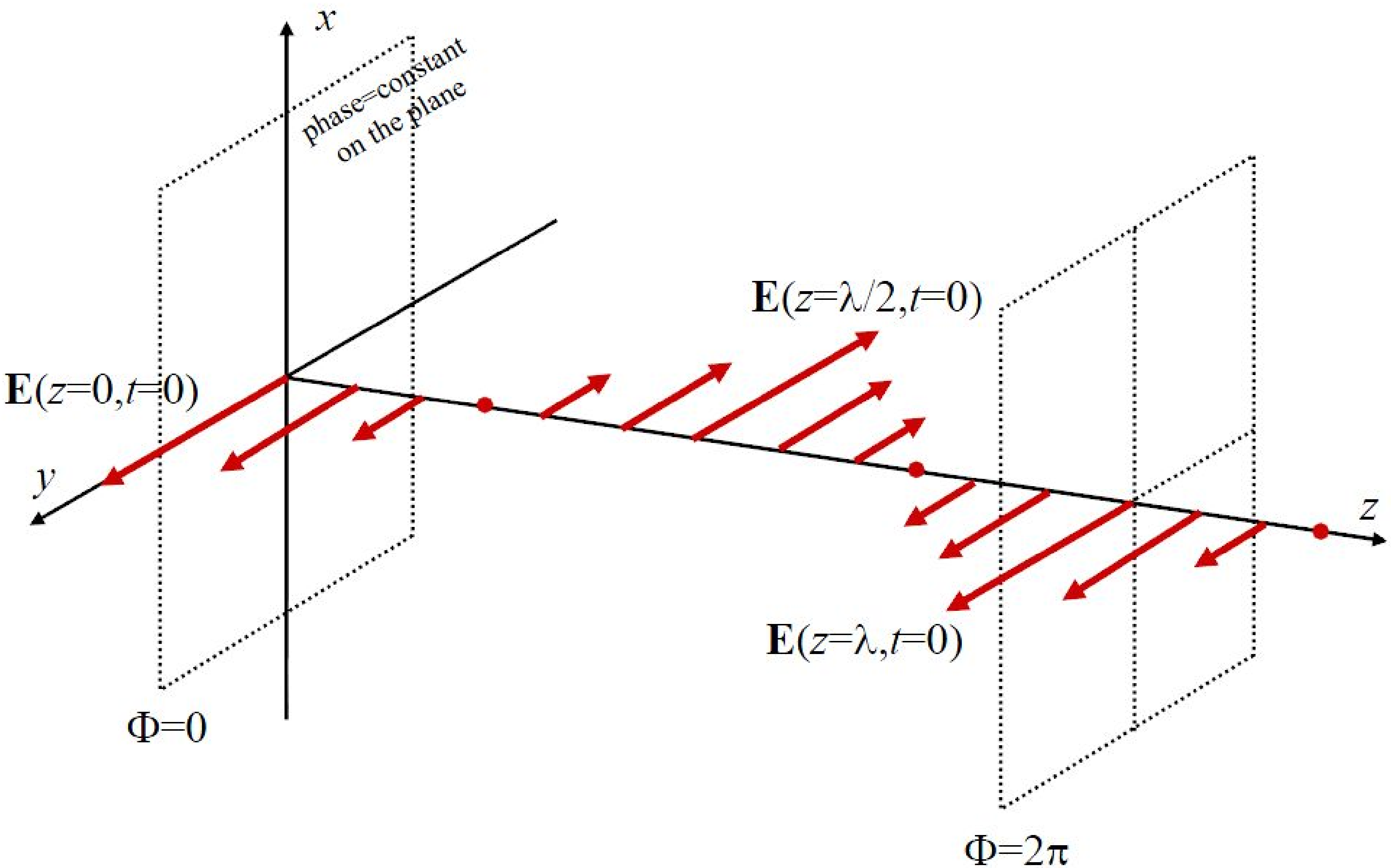
- initial values (e.g., laser light is coupled to an optical fibre so that we know the initial intensity and frequency of the laser light)
- boundary conditions (e.g., total internal reflection keeps light inside an optical fibre)

Wave Equation in Spherical Coordinates

Polarisation of Light

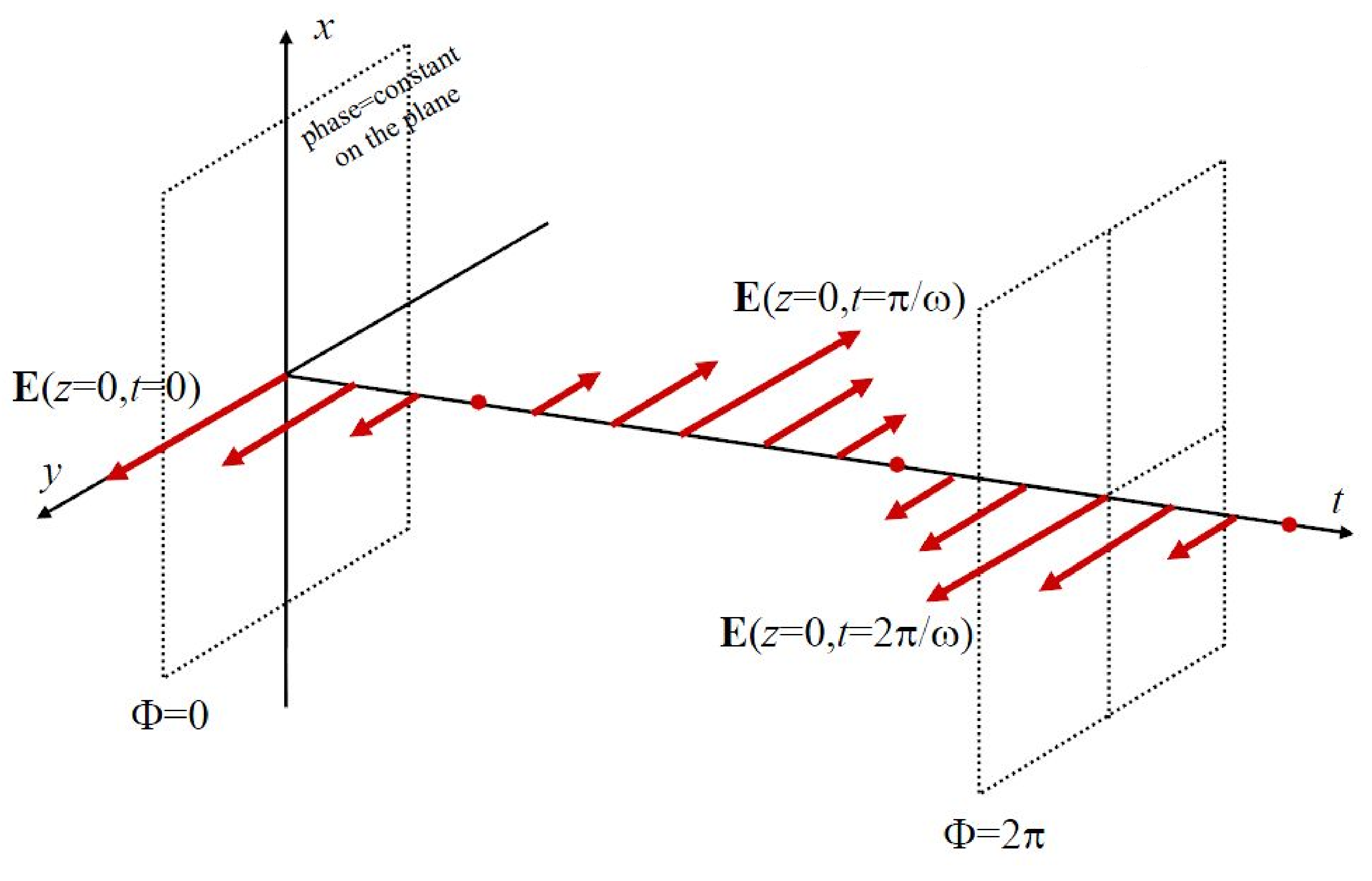
Linear Polarisation Along y Axis (E_{0y})

- plane wave f(kz-ωt) at the time t = 0



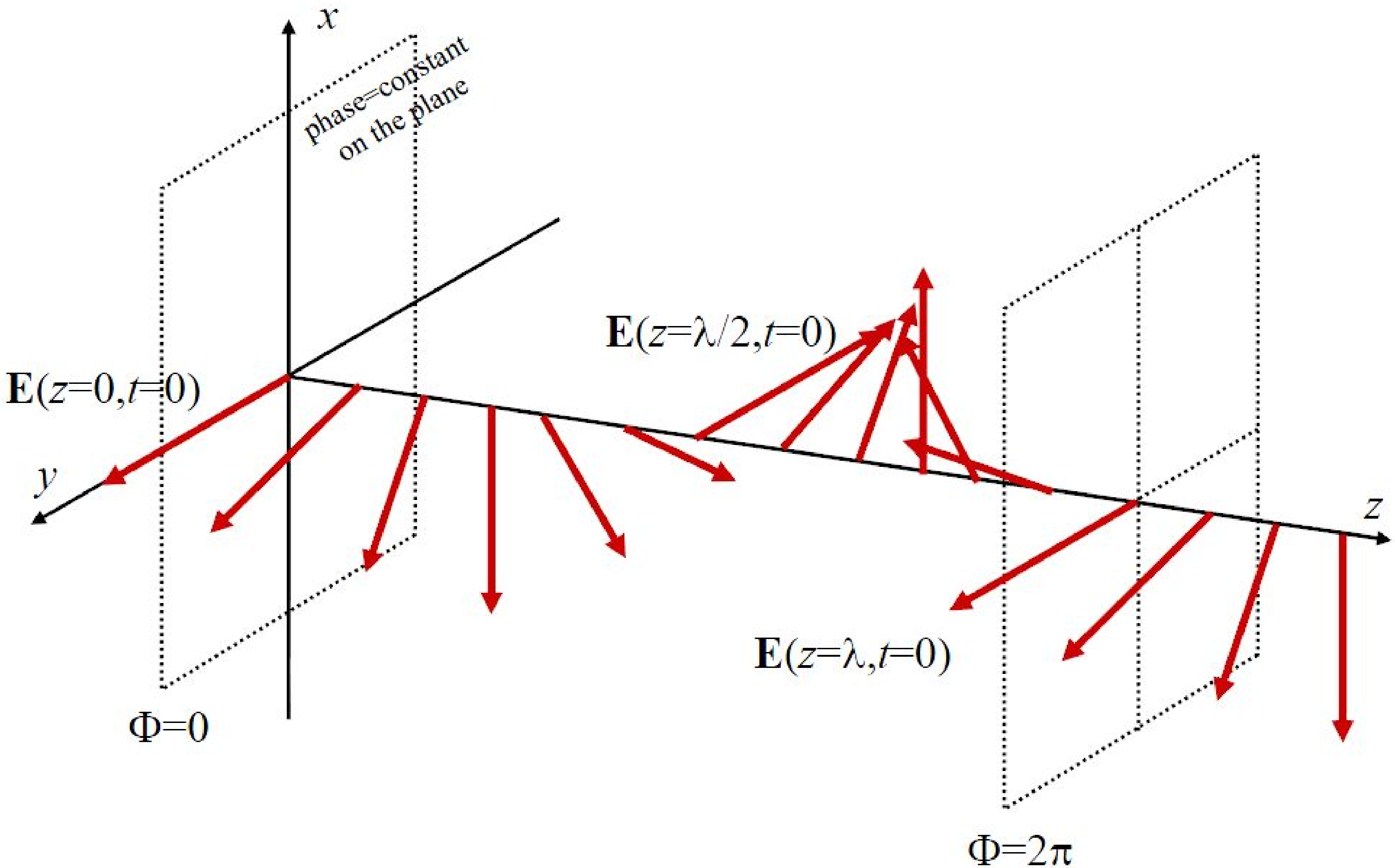
Linear Polarisation Along y Axis (E_{0y})

- plane wave f(kz-ωt) at the position z = 0



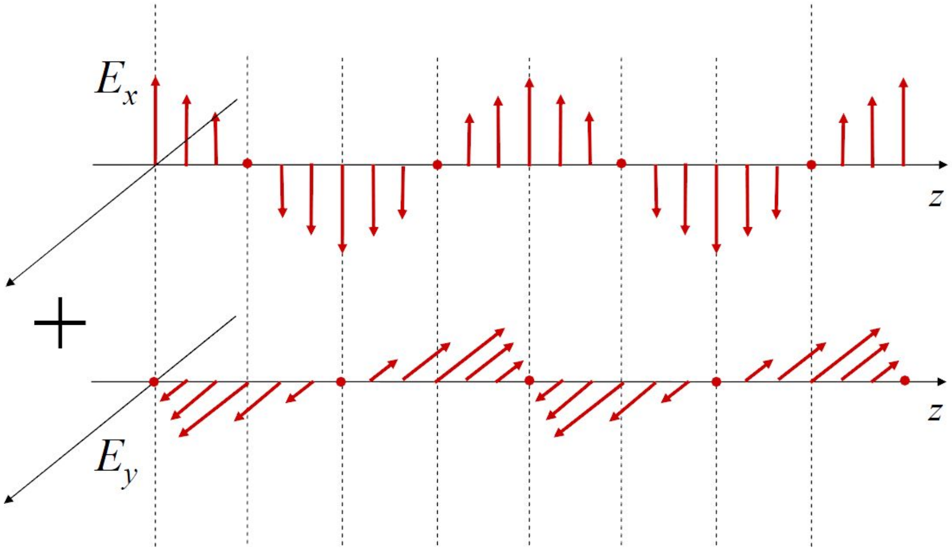
Circular Polarisation

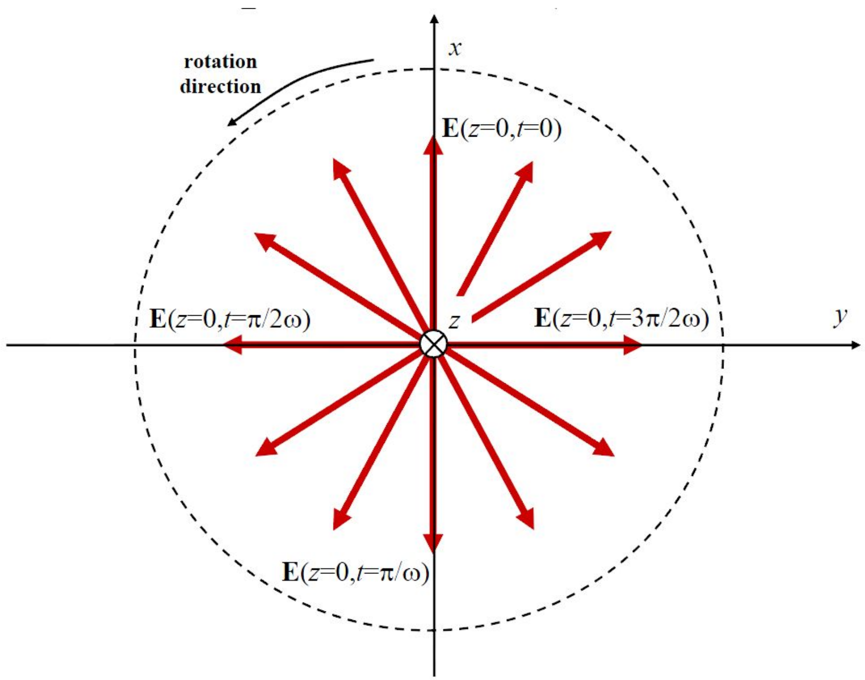
Circular Polarisation



Circular Polarisation

- can be considered as a sum of two linearly polarised plane waves, the phase difference of which is $\pi/2$

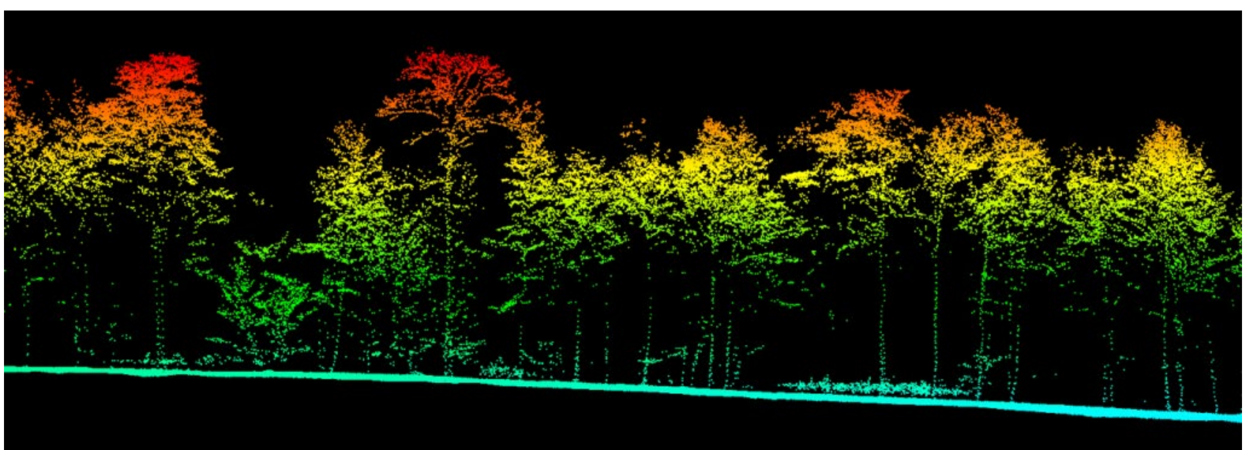




Circular Polarisation

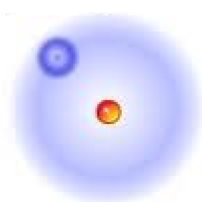
- https://www.youtube.com/watch?v=Fu-aYnRkUgg

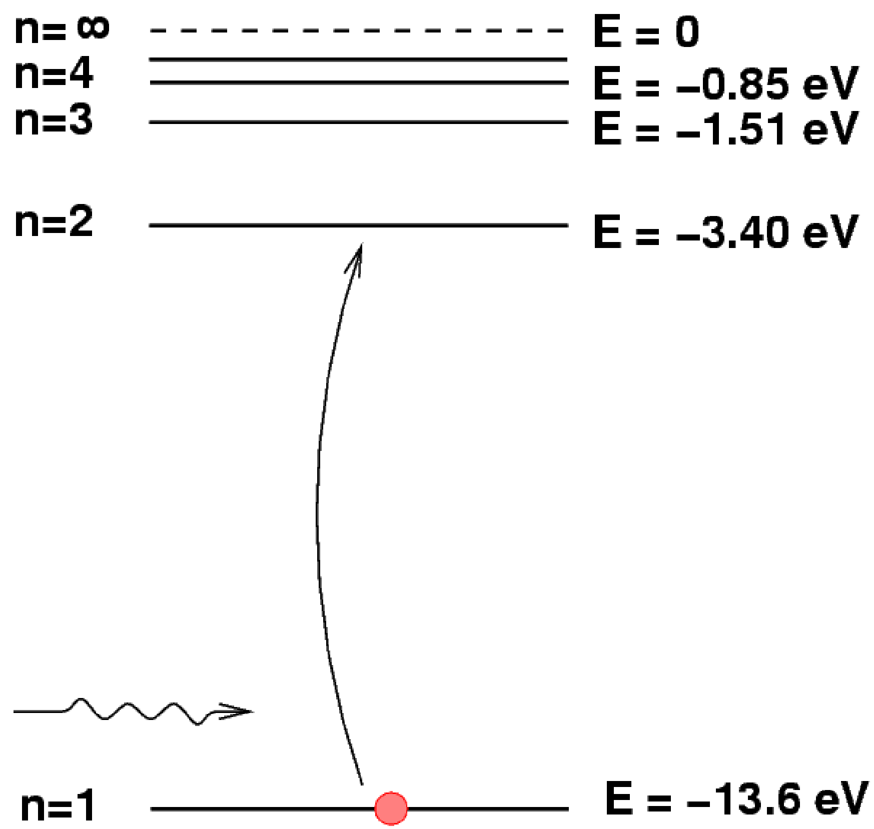
Introduction to the Concept of
Optical Spectroscopy –
Studying Interaction between Light and Matter



Absorption of Ultraviolet/Visible Light in Atoms

When the energy (frequency) of photons matches the energy level difference of the atom's electrons, photons can interact with the particular atom.





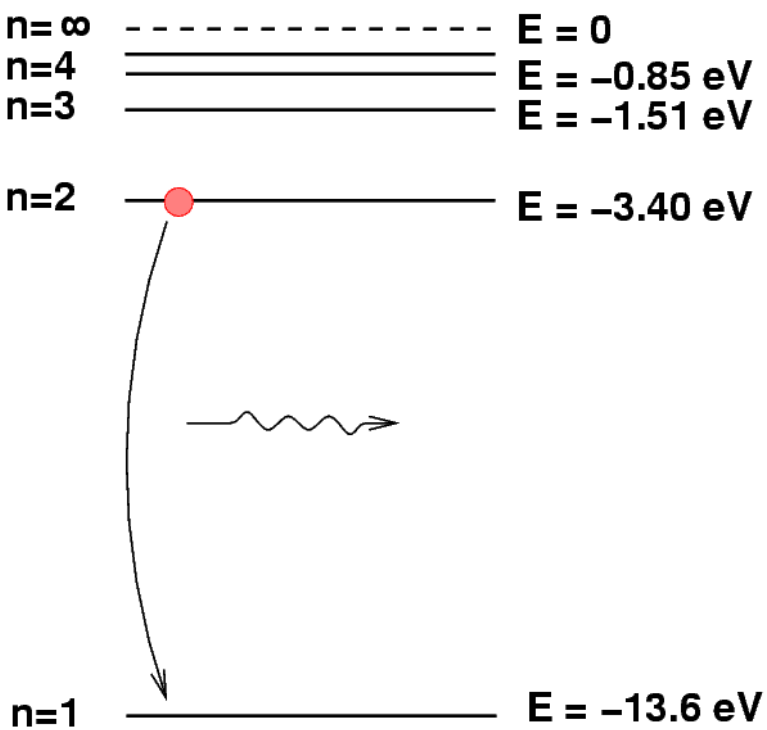
$E_p = hc/\lambda$

$E_p \sim 1.24$ eV μm / λ (μm)

$E_{1\text{-}2}$ = 10.2 eV corresponds to λ = 121 nm

$E_{2\text{-}3}$ = 1.89 eV corresponds to λ = 656 nm

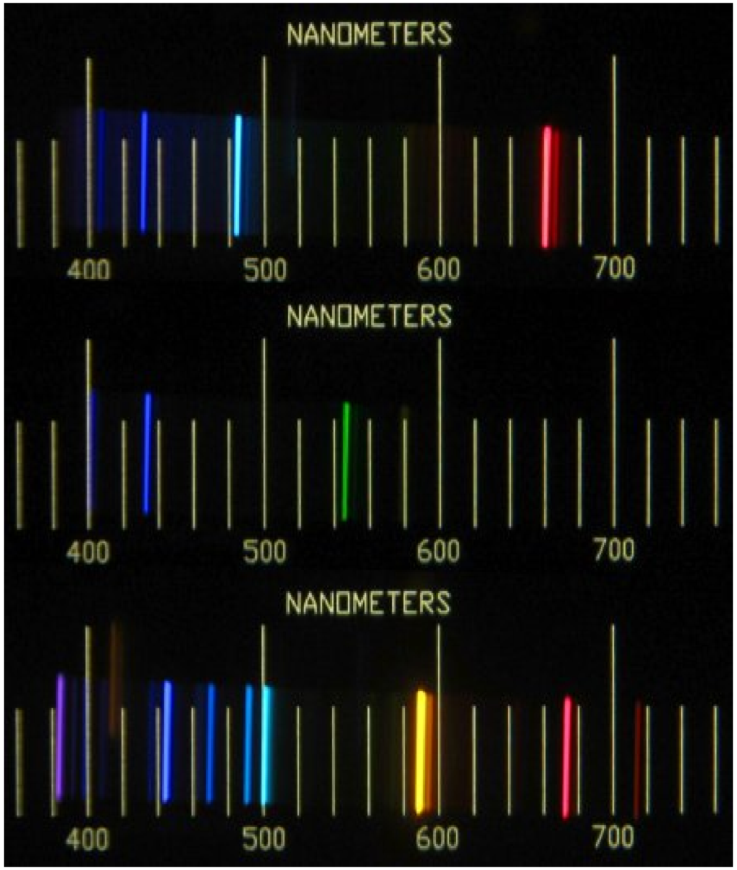
Emission of Light from Atoms



hydrogen

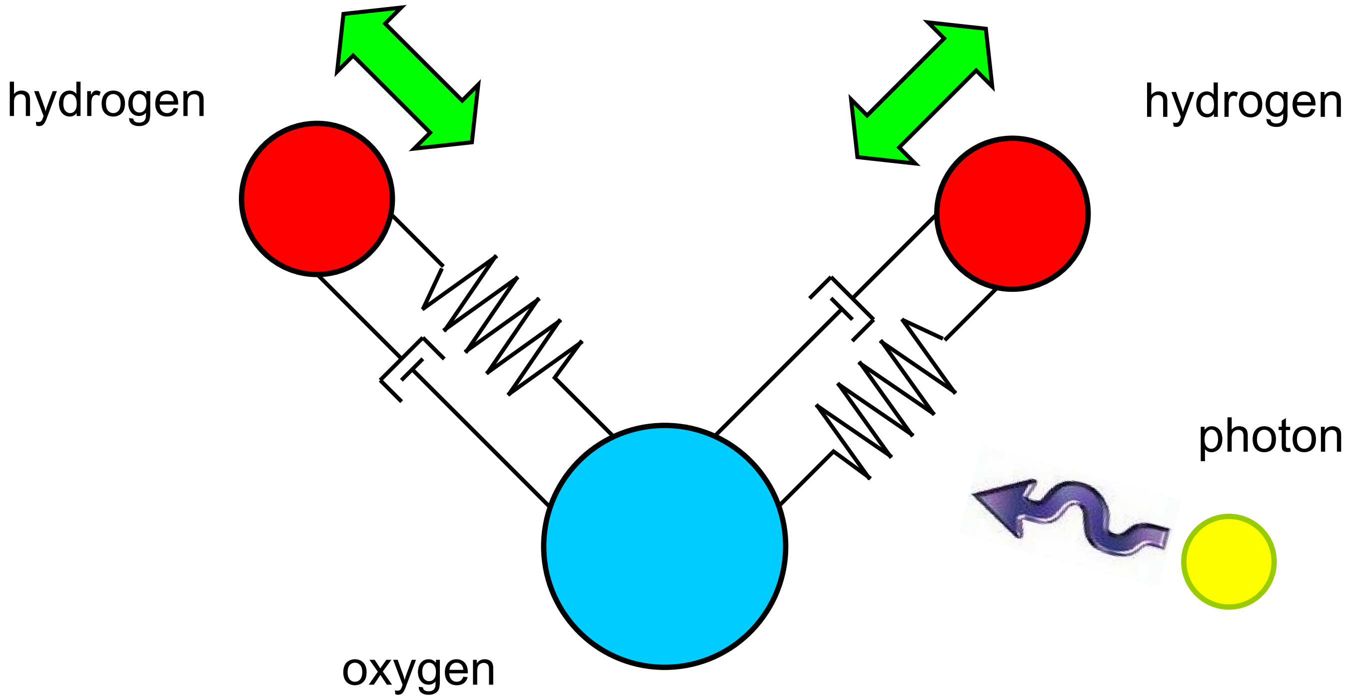
mercury

helium



Absorption of Infrared Light in Molecules – Vibration and Rotation

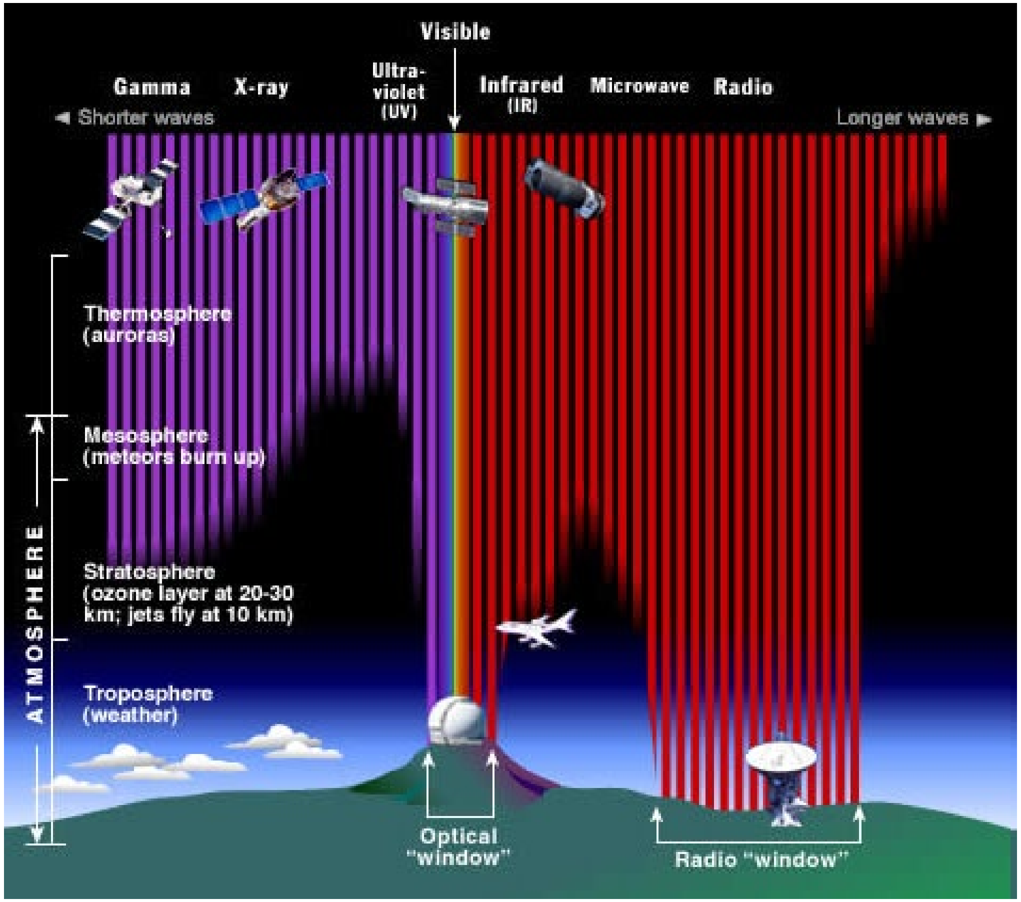
Mickey Mouse model of water molecule

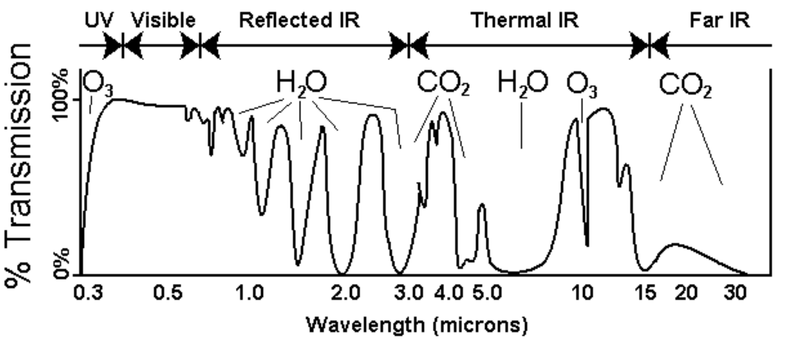


When the energy (frequency) of photons matches the quantised vibrational and/or rotational energy level differerences of the molecule, photon will interact with that molecule.

In absorption, the photon's energy (hv) get's converted into the molecules' electronic, vibrational or rotational energy.

Absorption of EM Radiation in the Atmosphere





For each molecule there are chracteristic energies/wavelengths that get absorbed – this is the foundation of optical spectroscopy.

Blackbody Radiation – Continuous Emission Spectrum

Max Planck’s theory of blackbody radiation in the year 1900 started the development of quantum theory and allowed several fundamental predictions:

- definition of Avogadro’s number
- size of atoms
- charge of electrons
- mass of electrons

Emission and Absorption of Light