Optics E-5730 Spring 2021

Wave Optics II

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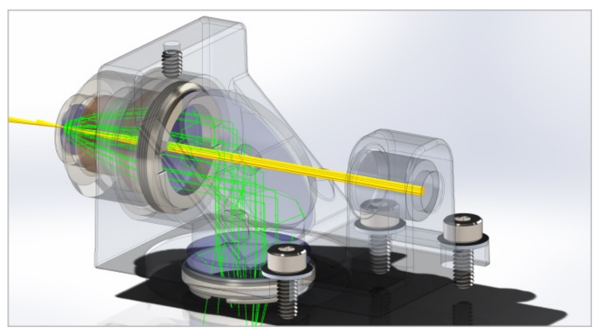
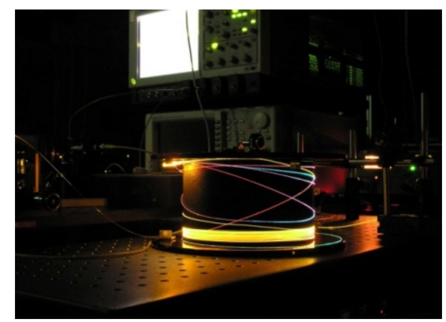
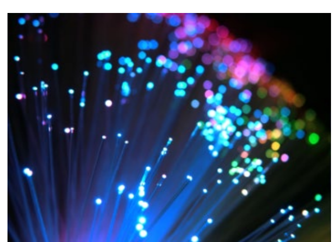
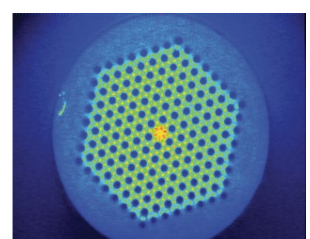
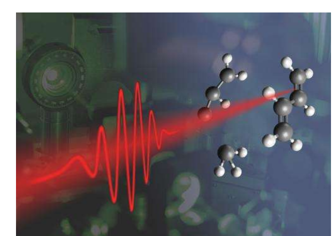



Fundamentals of Optics, Spring 2021

ELEC E-5730

lectures online using Zoom at https://aalto.zoom.us/j/8453943170

exercises online using Zoom at https://aalto.zoom.us/j/5703080612

week	day	date	time	topic	
2	Mon	11.1.2021	8-10	Lecture 1: Geometrical optics 1	
	Fri	15.1.2021	8-10	Lecture 2: Geometrical optics 2	
3	Mon	18.1.2021	8-10	Lecture 3: Wave optics 1	
	Mon	18.1.2021	10-12	Exercise 1	
	Fri	22.1.2021	8-10	Lecture 4: Wave optics 2	
4	Mon	25.1.2021	8-10	Lecture 5: Coherence 1	
	Mon	25.1.2021	10-12	Exercise 2	
	Fri	29.1.2021	8-10	Lecture 6: Coherence 2	
5	Mon	1.2.2021	8-10	Lecture 7: Radiometry	
	Mon	1.2.2021	10-12	Exercise 3	
	Fri	5.2.2021	8-10	Lecture 8: Interferometry + 30 mins mid-term exam	
6	Mon	8.2.2021	8-10	Lecture 9: Fibre optics + Optical telecom	LAB WORKS PERIOD
	Mon	8.2.2021	10-12	Exercise 4	LAB WORKS PERIOD
	Fri	12.2.2021	8-10	Lecture 10: Diffraction 1	LAB WORKS PERIOD
7	Mon	15.2.2021	8-10	Lecture 11: Diffraction 2	LAB WORKS PERIOD
	Mon	15.2.2021	10-12	Exercise 5	LAB WORKS PERIOD
	Fri	19.2.2021	8-10	NO LECTURE	LAB WORKS PERIOD
8	Mon	22.2.2021	8-10	NO LECTURE	
	Mon	22.2.2021	10-12	Exercise 6	
	Fri	26.2.2021		Examination	

Last Lecture – Wave Optics I

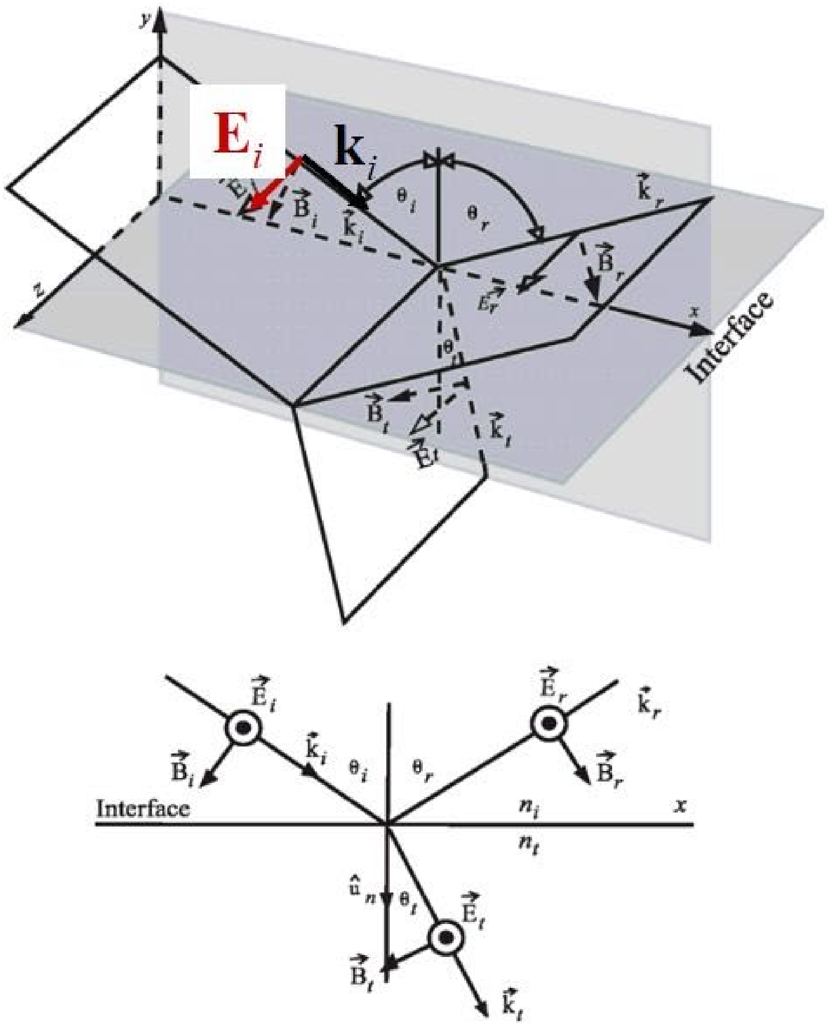
- wave motion
- electric and magnetic fields: Maxwell's equations
- wave equation and speed of light
- polarisation of light: linear and circular polarization

Today – Wave Optics II

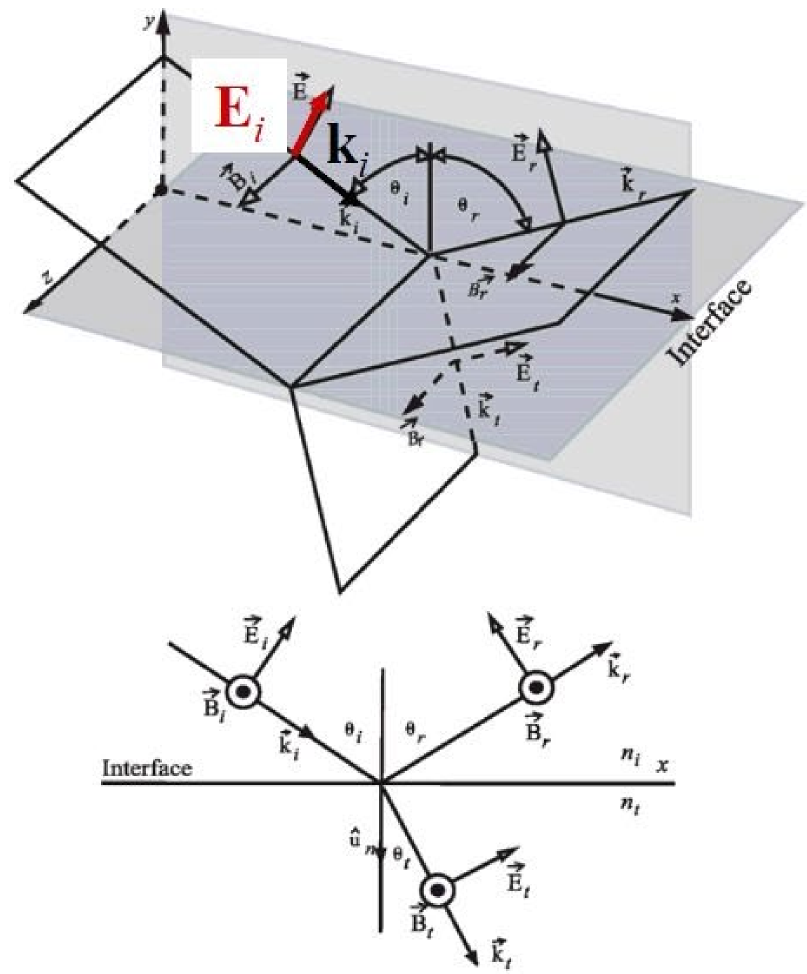
- reflection and refraction coefficients for E field amplitude and intensity
- Brewster's angle
- polarising optical components: 'polarisers'
- dichroism and birefringence
- waveplate components: quarter-wave plate and half-wave plate
- anti-reflection (AR) coating

Reflection and Refraction at an Interface

s-polarisation ('senkrecht')

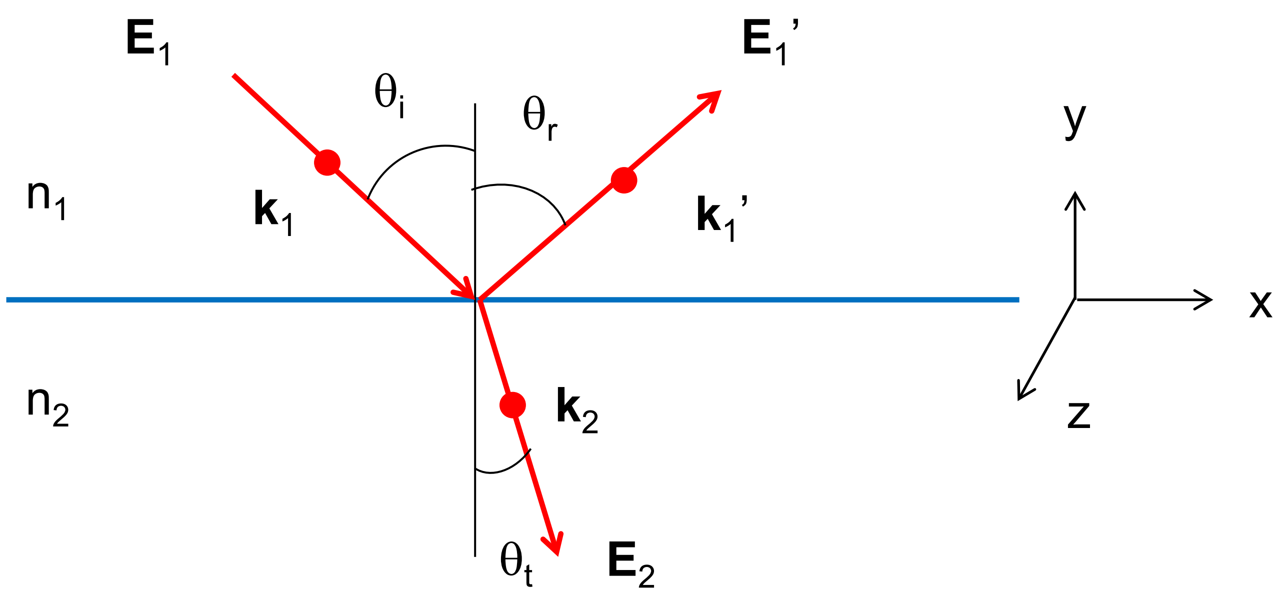


p-polarisation ('parallel')

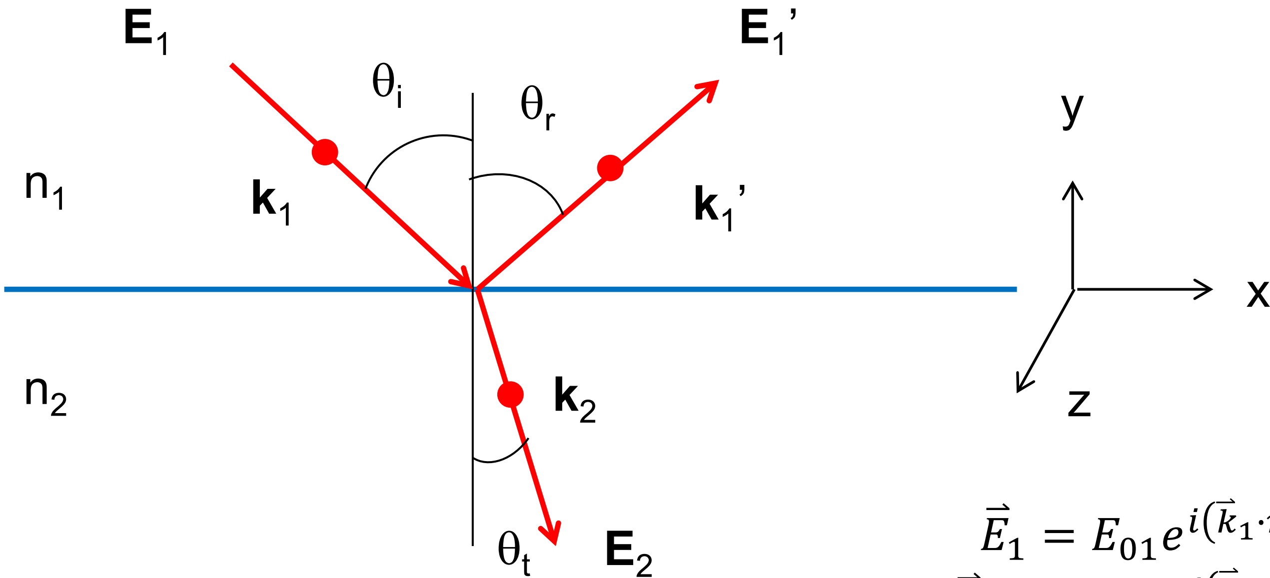


Reflection and Refraction Coefficients for E-field over an Interface between Two Media

Boundary conditions limit the number of Maxwell equations' solutions of EM fields. One such condition is that E and B field components along the interface must be continuous. Let us investigate how the S-polarised E field behaves upon crossing the interface.



$$k_{1z} = k'_{1z} = k_{2z} = 0$$ why?



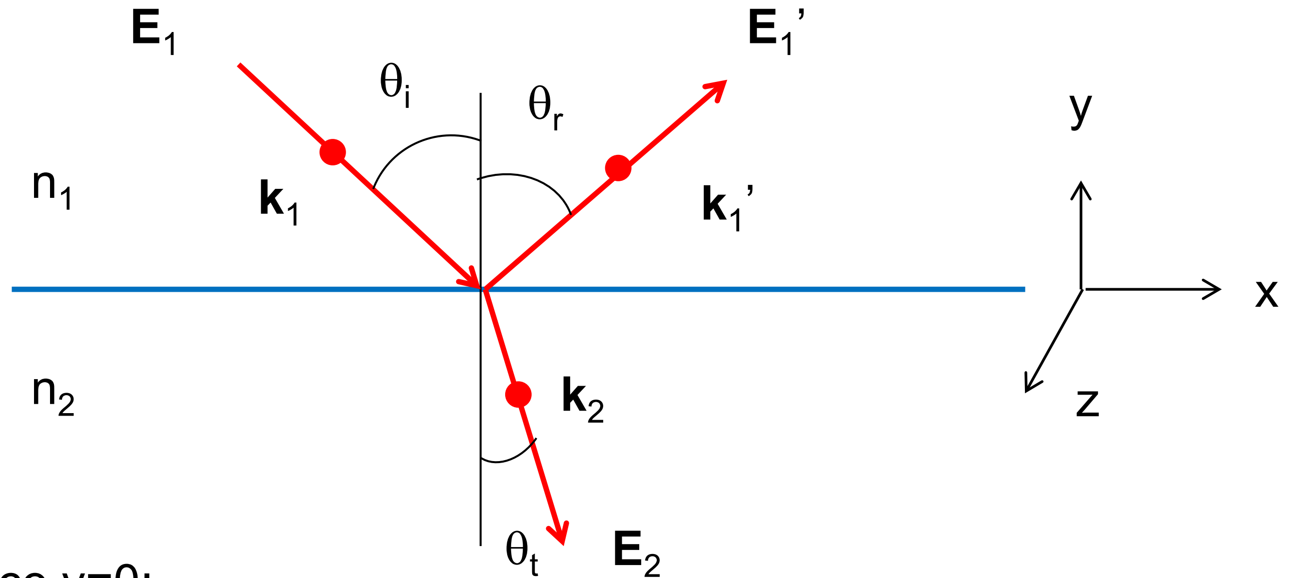
$$\vec{E}_1 = E_{01} e^{i(\vec{k}_1 \cdot \vec{r} - \omega_1 t)}$$
$$\vec{E'}_1 = E'_{01} e^{i(\vec{k'}_1 \cdot \vec{r} - \omega'_1 t)}$$
$$\vec{E}_2 = E_{02} e^{i(\vec{k}_2 \cdot \vec{r} - \omega_2 t)}$$

Let us investigate the continuity of the tangential components of E.

$$sin\theta_i = \frac{k_{1x}}{k_1} \quad cos\theta_i = \frac{k_{1y}}{k_1} \quad sin\theta_r = \frac{k'_{1x}}{k'_1} \quad cos\theta_r = \frac{k'_{1y}}{k'_1} \quad sin\theta_t = \frac{k_{2x}}{k_2} \quad cos\theta_r = \frac{k_{2y}}{k_2}$$

$$\vec{k}_1 \cdot \vec{r} = k_{1x} \cdot x + k_{1y} \cdot y + 0$$

$$E_{01} e^{i(k_1 \cdot sin\theta_i x + k_1 \cdot cos\theta_i y - \omega_1 t)} + E'_{01} e^{i(k'_1 \cdot sin\theta_r x + k'_1 \cdot cos\theta_r y - \omega'_1 t)}$$
$$= E_{02} e^{i(k_2 \cdot sin\theta_t x + k_2 cos\theta_t (-y) - \omega_2 t)}$$



At the interface y=0:

$$E_{01}e^{i(k_1 \cdot sin\theta_i x - \omega_1 t)} + E'_{01}e^{i(k'_1 \cdot sin\theta_r x - \omega'_1 t)} = E_{02}e^{i(k_2 \cdot sin\theta_t x - \omega_2 t)}$$

The continuity must be valid over the whole interface, that is, at any value of x or t. Therefore, the exponential terms must be equal (and can be devided out of the equation). E_{01}, E'_{01} and E_{02} do not depend on x or t.

1. $\omega_1 = \omega'_1 = \omega_2$ <u>frequency remains the same upon crossing the interface</u>

1. $\omega_1 = \omega'_1 = \omega_2$ frequency remains the same upon crossing the interface

2. wavelength and propagation speed do change upon crossing the interface:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{c_1}{\nu}}{\frac{c_2}{\nu}} = \frac{\frac{c_0}{n_1 \nu}}{\frac{c_0}{n_2 \nu}} = \frac{n_2}{n_1}$$

3. $k_1 sin\theta_i = k_2 sin\theta_t$

$$\frac{\omega}{c_1} sin\theta_i = \frac{\omega}{c_2} sin\theta_t$$

$$\omega_1 = \omega'_1 = \omega_2$$

$$k_i = \frac{2\pi}{\lambda_i} = \frac{2\pi\nu}{c_i} = \frac{\omega}{c_i}$$

$$\frac{\omega n_1}{c_0} sin\theta_i = \frac{\omega n_2}{c_0} sin\theta_t \Rightarrow n_1 sin\theta_i = n_2 sin\theta_t$$

law of refraction

4. both k_1 and k'_1 are in the same medium:

$$k_1 = k'_1$$

$$\Rightarrow sin\theta_i = sin\theta_r$$

$$\Leftrightarrow \theta_i = \theta_r$$

law of reflection

5. $$E_{01} + E'_{01} = E_{02} \qquad (1)$$

The normal component of E is discontinuous. For B field the normal component is continuos as is also the tangential component of B/μ:

$$-\frac{B_{01}}{\mu_1}cos\theta_i + \frac{B'_{01}}{\mu_1}cos\theta_r = -\frac{B_{02}}{\mu_t}cos\theta_t$$

also $$B_{01} = \frac{E_{01}}{c_1} \qquad B'_{01} = \frac{E'_{01}}{c_1} \qquad B_{02} = \frac{E_{02}}{c_2}$$

for transparent dielectric materials $\mu_i \approx \mu_t \approx \mu_0$

$$\Longrightarrow \quad n_1(E_{01} - E'_{01})cos\theta_i = n_2 E_{02} cos\theta_t \qquad (2)$$

combining equations 1 and 2:

$$r_{12\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = \frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t}$$

$$t_{12\perp} = \left(\frac{E_{02}}{E_{01}}\right)_{\perp} = \frac{2n_1 cos\theta_i}{n_1 cos\theta_i + n_2 cos\theta_t}$$

Fresnell amplitude reflection and transmission factors for s polarisation

In the same way the amplitude reflection and transmission factors can be derived for the p polarised component:

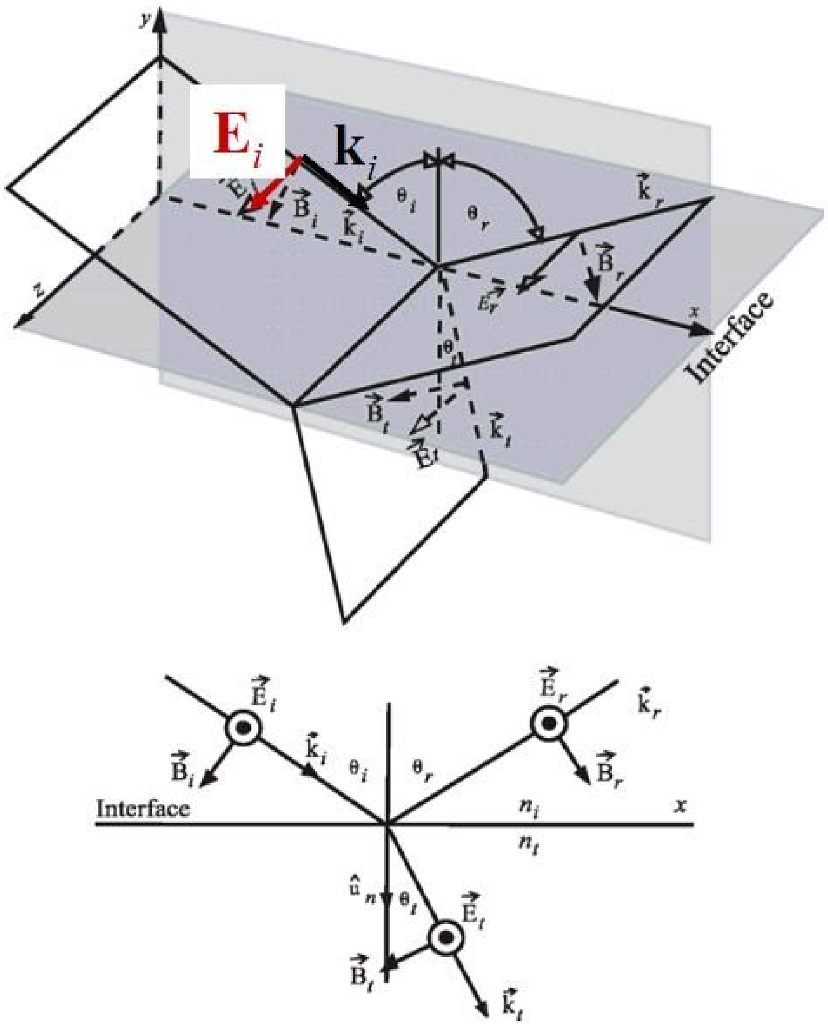
$$r_{12\parallel} = \left(\frac{E'_{01}}{E_{01}}\right)_{\parallel} = \frac{n_2 cos\theta_i - n_1 cos\theta_t}{n_2 cos\theta_i + n_1 cos\theta_t}$$

$$t_{12\parallel} = \left(\frac{E_{02}}{E_{01}}\right)_{\parallel} = \frac{2n_1 cos\theta_i}{n_2 cos\theta_i + n_1 cos\theta_t}$$

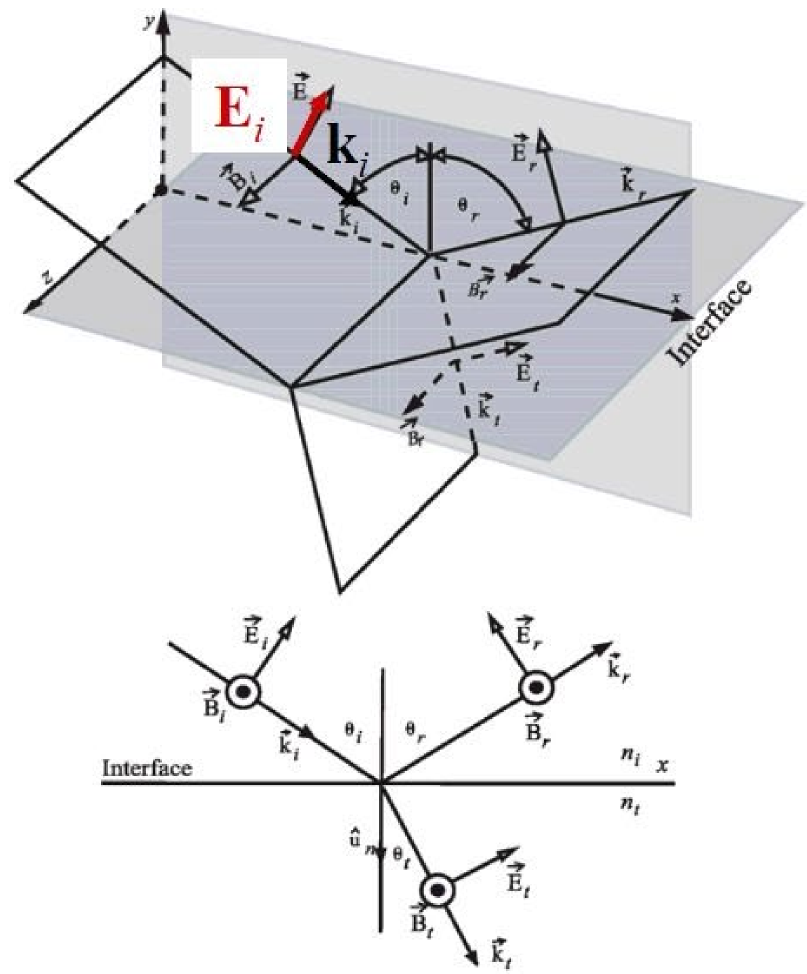
Fresnell amplitude reflection and transmission factors for p polarisation

Reflection and Refraction at an Interface

s-polarisation ('senkrecht')



p-polarisation ('parallel')



Fresnell reflection and transmission factors

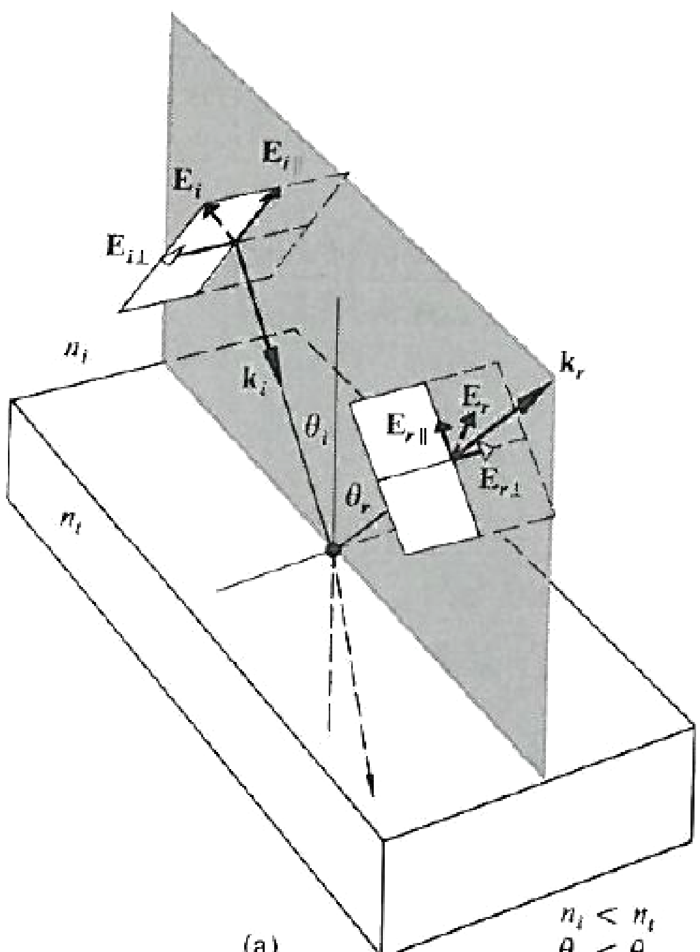
$$r_{12\perp} = \left(\frac{E'_{01}}{E_{01}}\right)_{\perp} = \frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t}$$

$$t_{12\perp} = \left(\frac{E_{02}}{E_{01}}\right)_{\perp} = \frac{2n_1 cos\theta_i}{n_1 cos\theta_i + n_2 cos\theta_t}$$

$$r_{12\parallel} = \left(\frac{E'_{01}}{E_{01}}\right)_{\parallel} = \frac{n_2 cos\theta_i - n_1 cos\theta_t}{n_2 cos\theta_i + n_1 cos\theta_t}$$

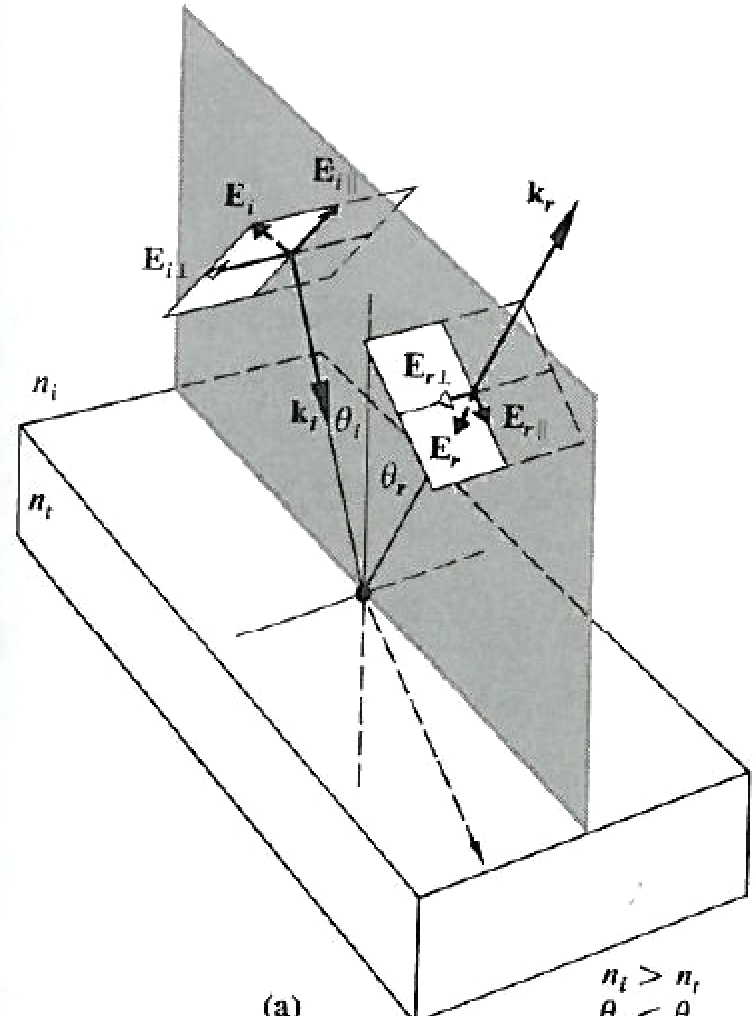
$$t_{12\parallel} = \left(\frac{E_{02}}{E_{01}}\right)_{\parallel} = \frac{2n_1 cos\theta_i}{n_2 cos\theta_i + n_1 cos\theta_t}$$

Phase upon Reflection



(a)

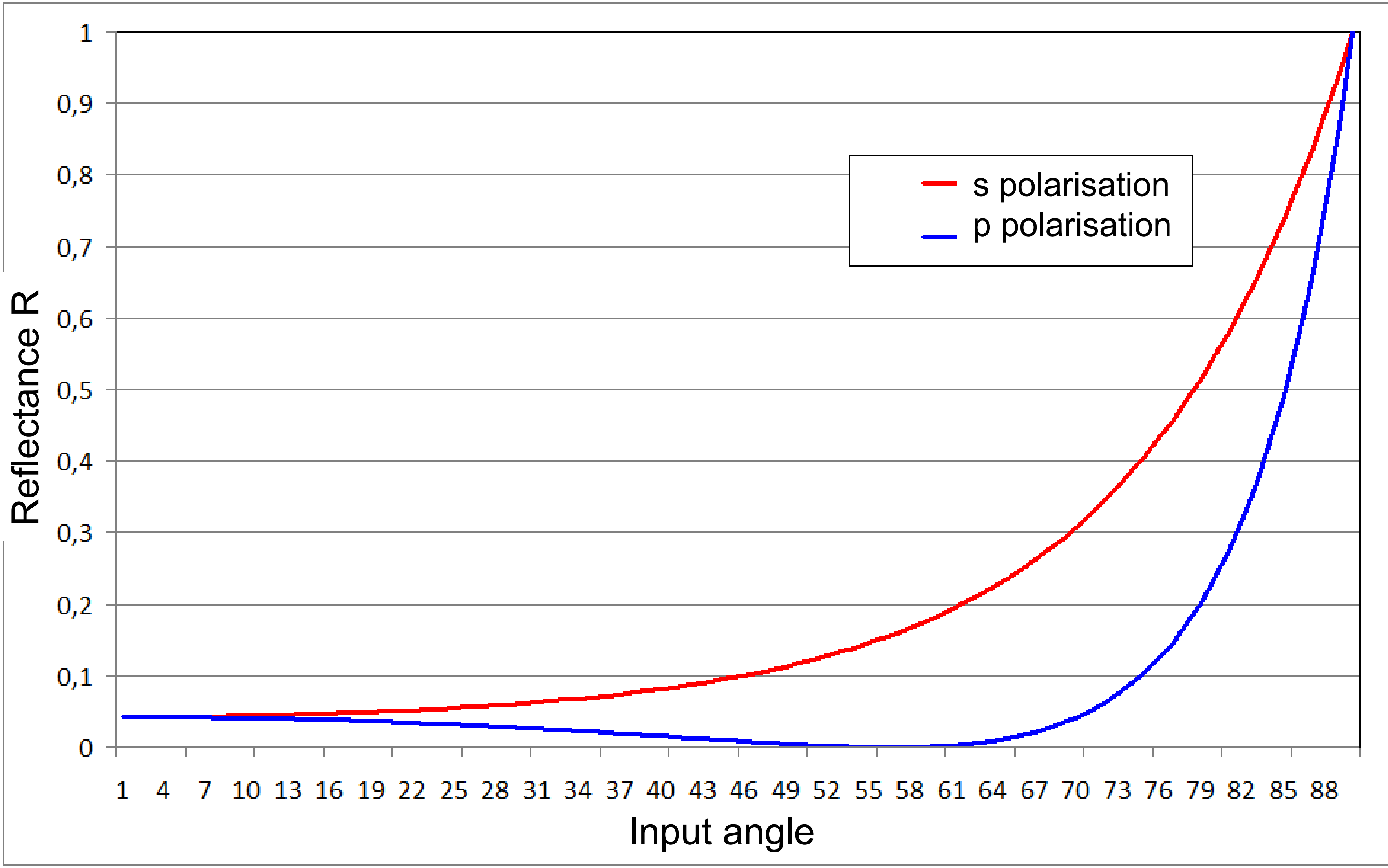
$n_i < n_t$
$\theta_i < \theta_p$



(a)

$n_i > n_t$
$\theta_i < \theta_p$

Reflectance R (Reflection of Intensity)



Brewster's Angle

$$r_p \propto n_2 \cos\theta_1 - n_1 \cos\theta_2 = 0$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

X

$$n_1 n_2 \sin\theta_1 \cos\theta_1 = n_1 n_2 \sin\theta_2 \cos\theta_2$$

$$2\sin\alpha\cos\alpha = \sin(2\alpha)$$

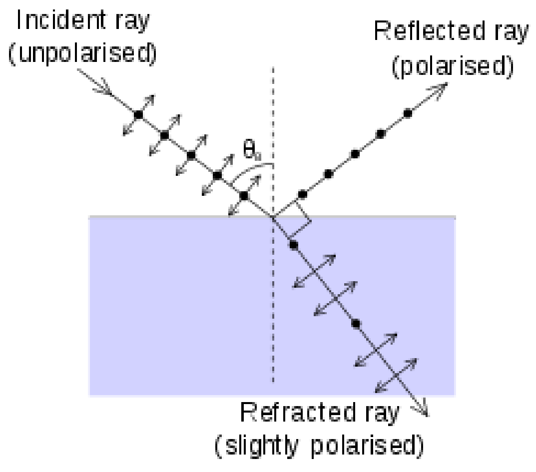
→

$$\sin(2\theta_1) = \sin(2\theta_2)$$

$$\theta_1 \neq \theta_2$$

$$\sin(\theta) = \sin(\pi - \theta)$$

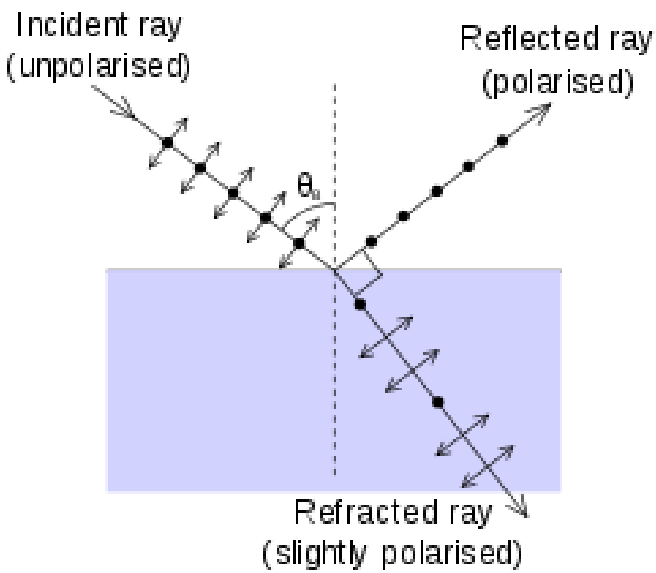
$$2\theta_1 = \pi - 2\theta_2$$

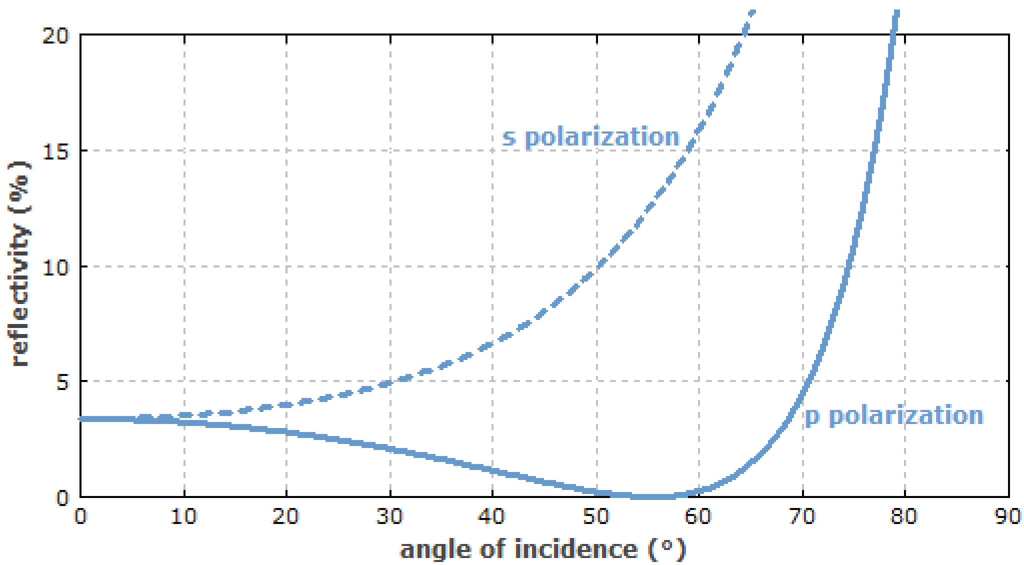


$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right)$$

Brewster's Angle

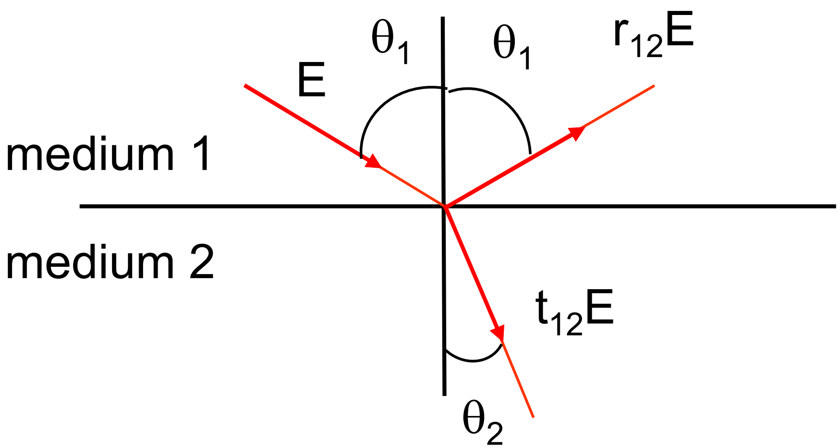




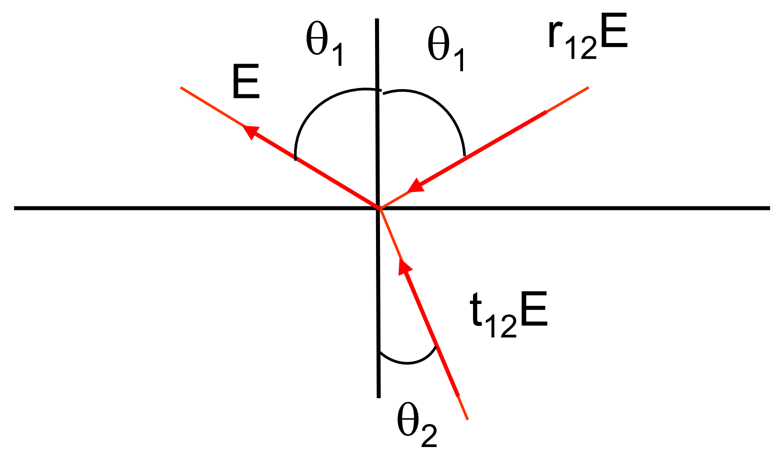
$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right)$$

Stokes' Relations

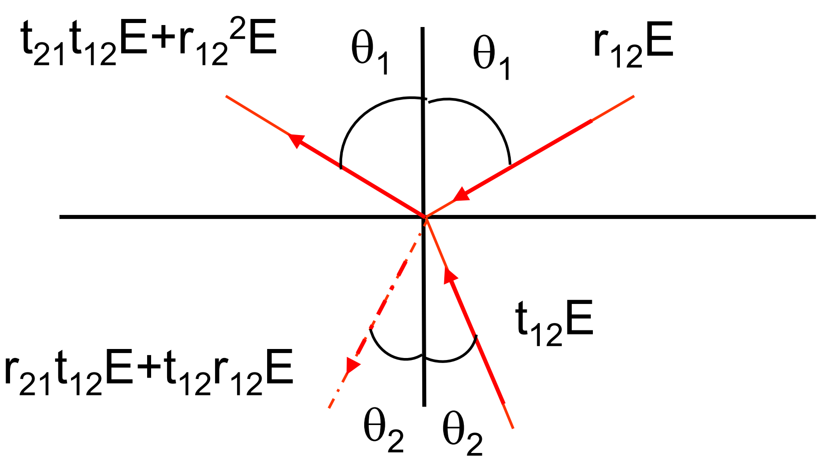
I



II "reversal of time"



III combining I and II



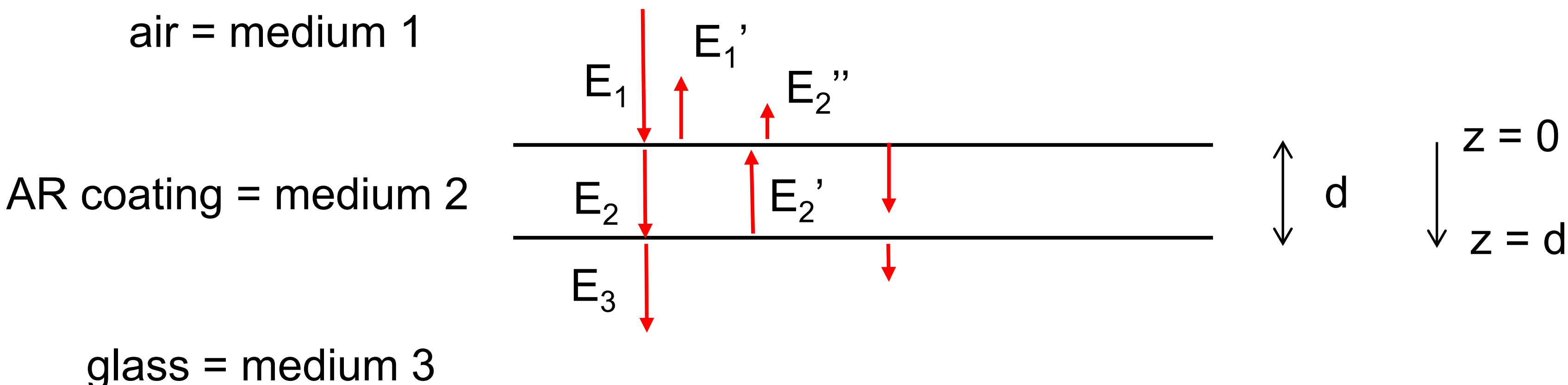
cases II and III must equal

$r_{21}t_{12}E+t_{12}r_{12}E = 0$ ⇨ $\mathbf{r_{21} = - r_{12}}$

$t_{21}t_{12}E+r_{12}^2E = E$ ⇨ $\mathbf{t_{21}t_{12} + r_{12}^2 = 1}$

Stokes' relations

Deriving Anti-reflection (AR) Coating Formula 1/4



$$E_1 = E_0 e^{i(k_1 z - \omega t)}$$
$$E_2 = t_{12} E_0 e^{i(k_2 z - \omega t)}$$
$$E_1' = r_{12} E_0 e^{i(-k_1 z - \omega t)}$$
$$\cdots$$

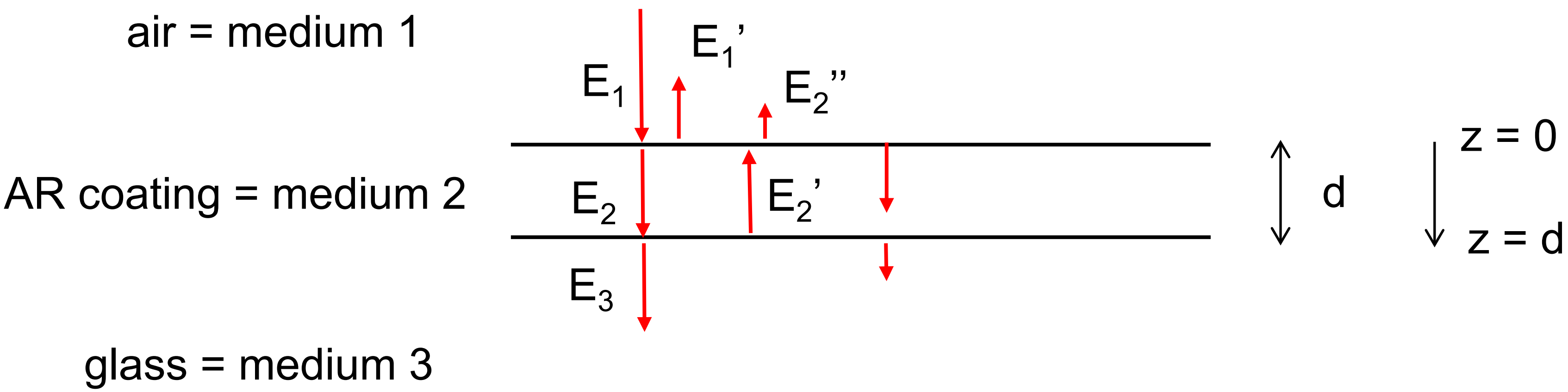
at surface z = d (ignoring temporal parts)

$$E_2' = r_{23} t_{12} E_0 e^{-ik_2 d}$$
$$\cdots$$

at surface z = 0 (ignoring temporal parts)

$$E_2'' = t_{21} r_{23} t_{12} E_0 e^{-i2k_2 d}$$
$$\cdots$$

Anti-reflection (AR) Coating 2/4



Total reflected field at surface z = 0

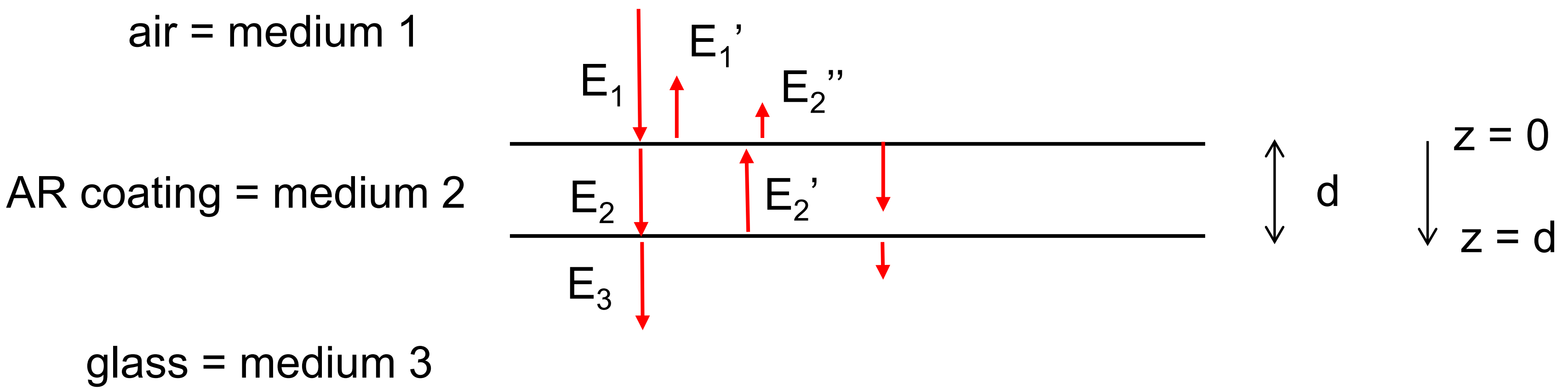
$$E_{TOT} = \left(r_{21}E_0 + t_{21}r_{23}t_{12}E_0e^{-i2k_2d} + t_{21}r_{23}r_{21}r_{23}t_{12}E_0e^{-i4k_2d} + \cdots\right)e^{-i\omega t}$$

Starting from 2nd term: geometrical series

- ratio q = $r_{23}r_{21}e^{-i2k_2d}$

$$\sum_{j=0}^{\infty} aq^j = \frac{a}{1-q}$$

Anti-reflection (AR) Coating 3/4



Total reflected field at surface z = 0

$$E_{TOT} = \left(r_{21} E_0 + \frac{t_{21} r_{23} t_{12} E_0 e^{-i2k_2 d}}{1 - r_{23} r_{21} E_0 e^{-i2k_2 d}} \right) e^{-i\omega t}$$

For reflected intensity $$R = \left| \frac{E_{TOT}}{E_0} \right|^2 = \left| r_{21} + \frac{t_{21} r_{23} t_{12} e^{-i2k_2 d}}{1 - r_{21} r_{23} e^{-i2k_2 d}} \right|^2$$

Anti-reflection (AR) Coating 4/4

$t_{21}t_{12} + r_{12}^2 = 1$

$\Rightarrow$ $t_{21}r_{23}t_{12} = r_{23}(1 - r_{12}^2)$

$$R = \left|\frac{E_{TOT}}{E_0}\right|^2 = \left|r_{21} + \frac{t_{21}r_{23}t_{12}e^{-i2k_2d}}{1 - r_{21}r_{23}e^{-i2k_2d}}\right|^2$$

$\beta = 2k_d d$

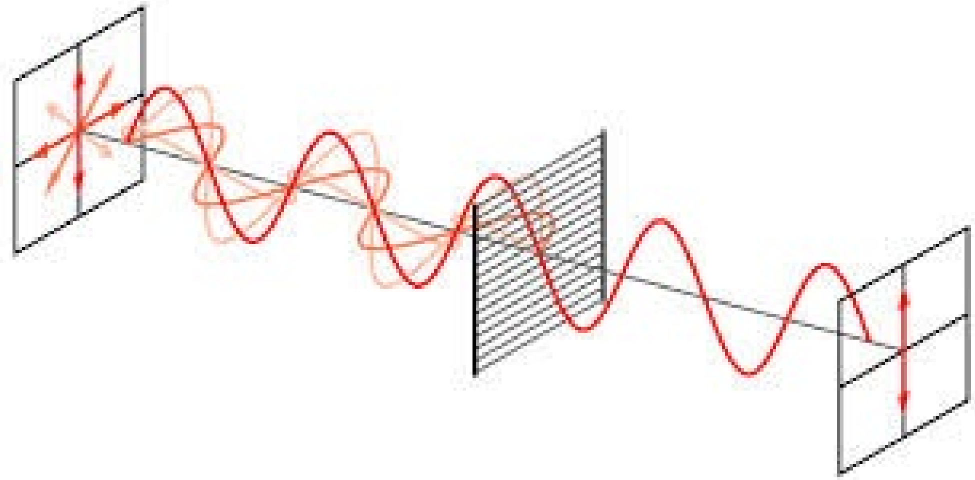
$r_{21} = - r_{12}$ $\Rightarrow$ $-r_{21} = + r_{12}$

$$\Rightarrow \quad R = \left|\frac{E_{TOT}}{E_0}\right|^2 = \frac{r_{12}^2 + r_{23}^2 + 2r_{12}r_{23}cos\beta}{1 + r_{12}^2 r_{23}^2 + 2r_{12}r_{23}cos\beta}$$

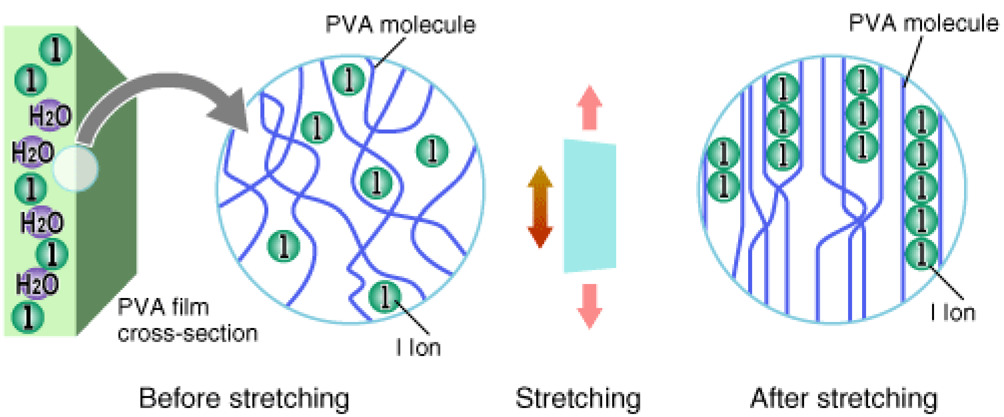
Exercise III: When d = $\lambda/4$ the above becomes

$$R = \left|\frac{E_{TOT}}{E_0}\right|^2 = \left(\frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}\right)^2$$

Polarisers



wire grid polariser
light gets absorbed (energy dissipation) in the direction of conduction

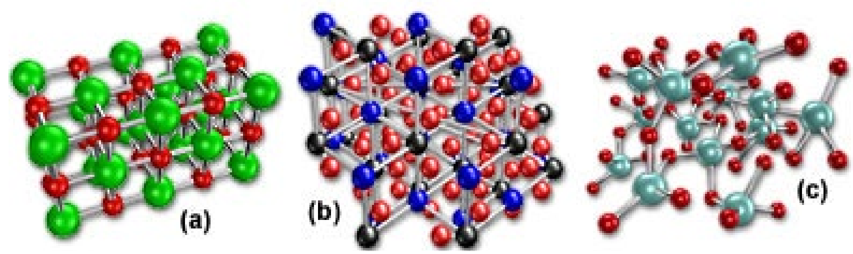


stretched conducting plastic film

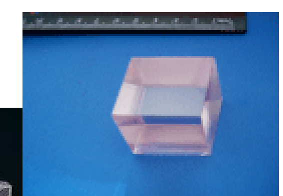
Anisotropic Optical Materials

Some optical materials have anisotropic structure, that is, structural geometry depends on the (observation) direction. This mean that macroscopic optical properties chracterising the material, such as absorption coefficient and refractive index, depend on the polarisation direction of light. For such optical cystals specific directions called optical axes can be defined.

anisotropic material (crystals, ”stretched materials”) structures



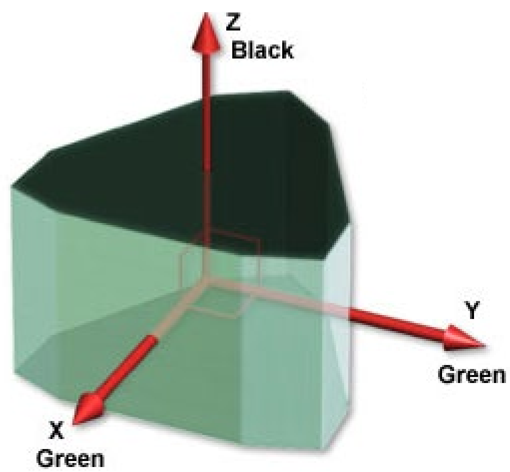
sodium chloride (NaCl)   calcium carbonite ($CaCO_3$)   amorfous polymer

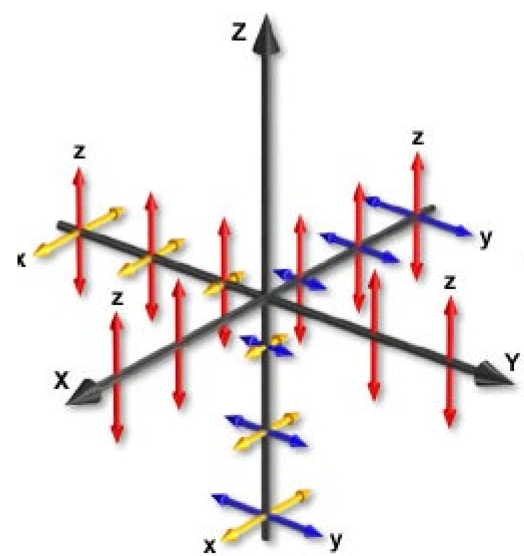


Dichroism

Some crystalline optical materials absorb more one polarisation direction than the other perpendicular polarisation. This anisotopy of absorption is called dichroism. When light propagates inside a such crystal, one polarisation gets attenuated and light becomes polarised.

example: tourmaline crystal



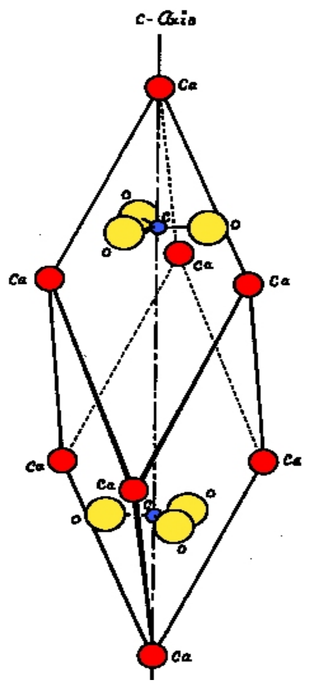


Birefringence

Some optical crystals do not have significant absorption but the refractive indices differ noticeably for different directions within the crystal. This means that the orthogonal polarisation components can refract into different directions so that the two polarisation components can be separated.

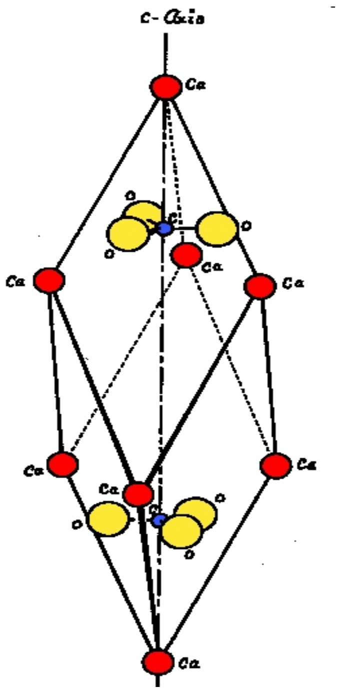
example: calcium carbonite ($CaCO_3$)

- Parallel carbonyl-groups create strong anisotrophy.
- **Optic axis** of a crystal is the direction where the crystal looks symmetric, i.e., in this direction there is no birefringence.



Calsium Carbonite ($CaCO_3$)

- Along optic axis (figure 4c) and along the perpendicular direction (4b) there are different refractive indices, yet in these two directions both polarisation still propagate together, because cystal looks symmetric (although the polarisations have different refractive indices, n_o and n_e). When light arrives at an angle with respect to the optic axis (figure 4a), the two polarisation components will separate from each other.
- ordinary (o) ray obeys the law of refraction, extra-ordinary (e) ray does not (e-ray obeys the rules of nonlinear optics)



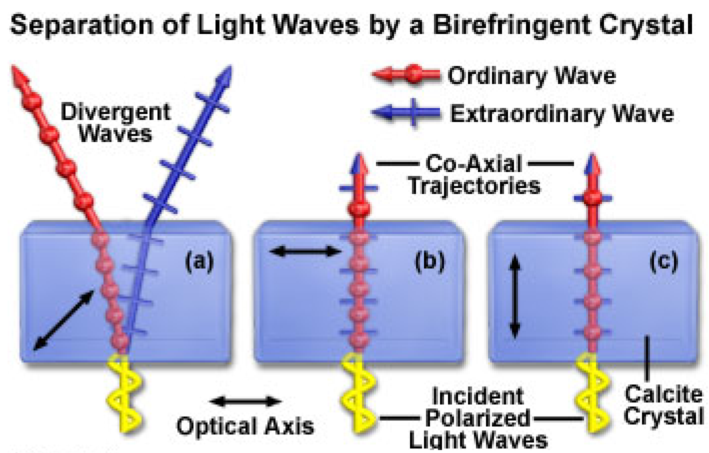
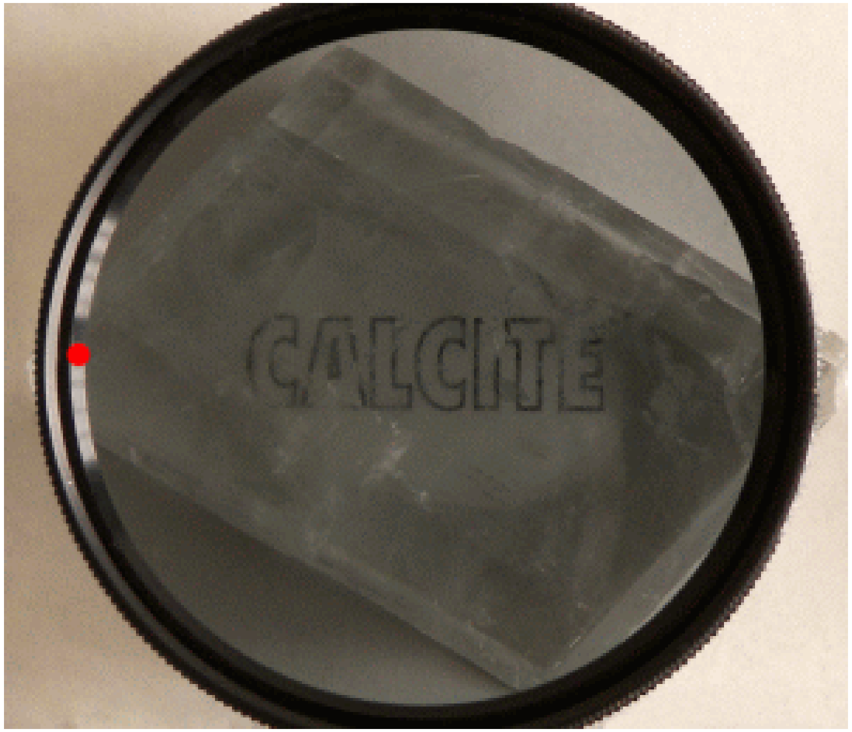


Figure 4

for CaCO3 at 590 nm wavelength: n_o = 1.658 ja n_e = 1.486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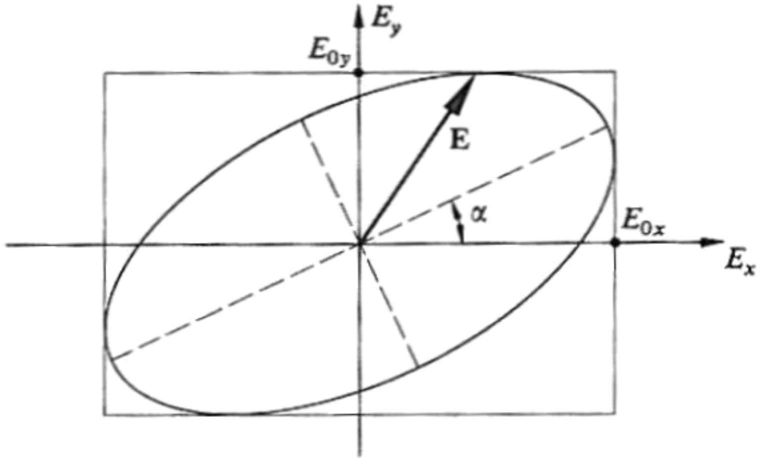
Polarisation, Continued

$$E_x = E_{ox}\cos(kz - \omega t)$$
$$E_y = E_{oy}\cos(kz - \omega t + \phi)$$

Polarisation pattern (drawn by $\mathbf{E}_{TOT}$) does not depend on position *z* and time, so the dependence on (kz-ωt) can be removed. Starting from E_y/E_{0y} it can be shown that in the most general case the polarisation is elliptical:

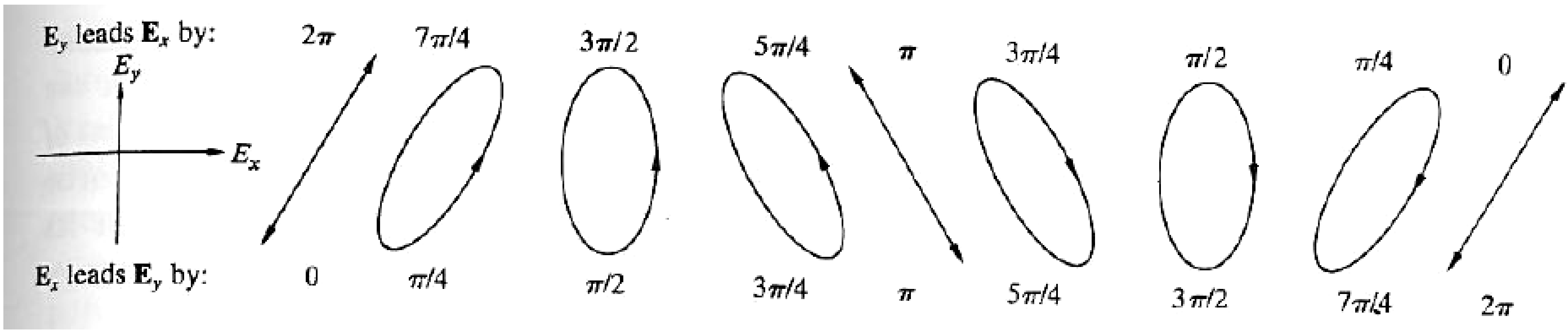
$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right)\cos\phi = \sin^2\phi$$

$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\phi}{E_{ox}^2 - E_{oy}^2}$$



Polarisation, Continued

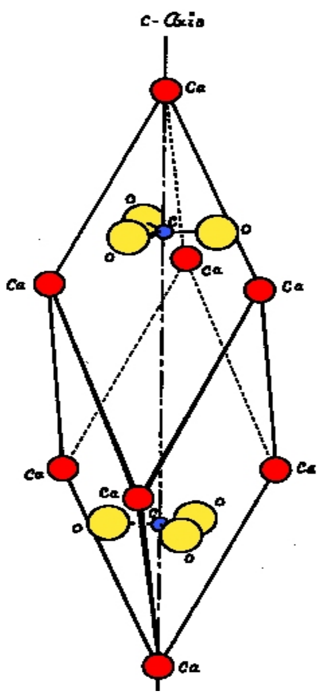
Depending on the phase difference various polarisation states can be achieved:

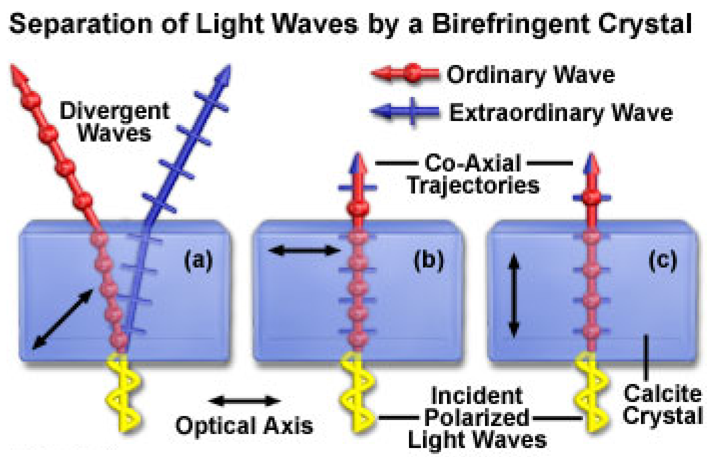


$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right)\cos\phi = \sin^2\phi$$

Phase Retardation

We saw that the polarisation along and at right angle to the optic axis propagate together but at different speeds $v_{||}$ and $v_{\perp}$, respectively. (figures b and c).





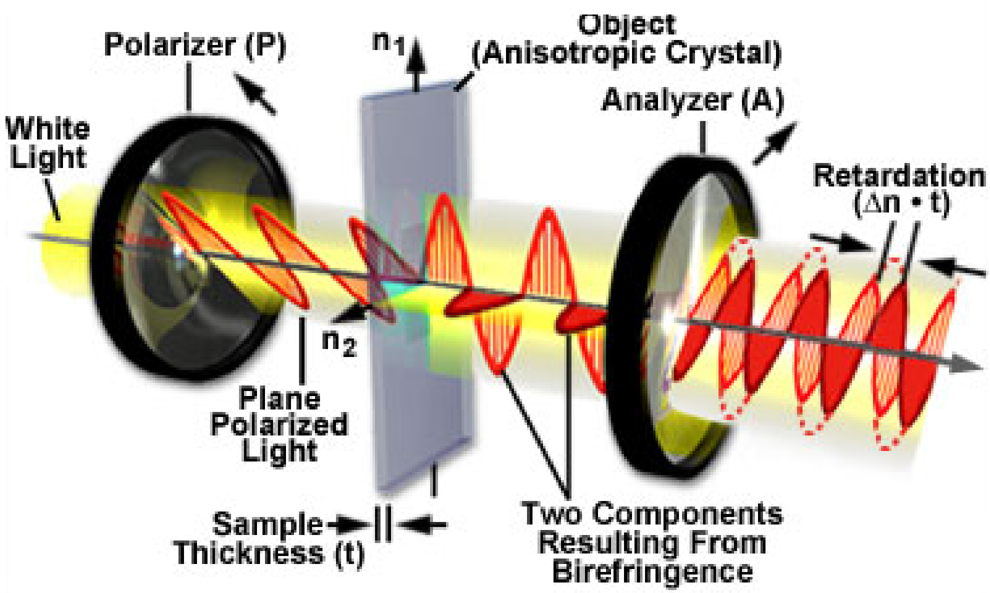
Different refrective indices (n_o and n_e) cause a phase difference between the two polarisations $\Delta\phi = k_o z - k_e z$, where z is pathlength is inside the crystal. For crystal having thickness d the phase difference is thus:

$$\Delta\phi = (k_o - k_e)d = \left(\frac{2\pi}{\lambda_o} - \frac{2\pi}{\lambda_e}\right)d \qquad \text{generally}: \lambda_1 = \frac{c_1}{\nu} = \frac{c_{\text{vacuum}}}{n_1 \nu} = \frac{\lambda_{\text{vac}}}{n_1}$$

$$= \frac{2\pi}{\lambda_{\text{vac}}}(n_o - n_e)d$$

Phase Retardation

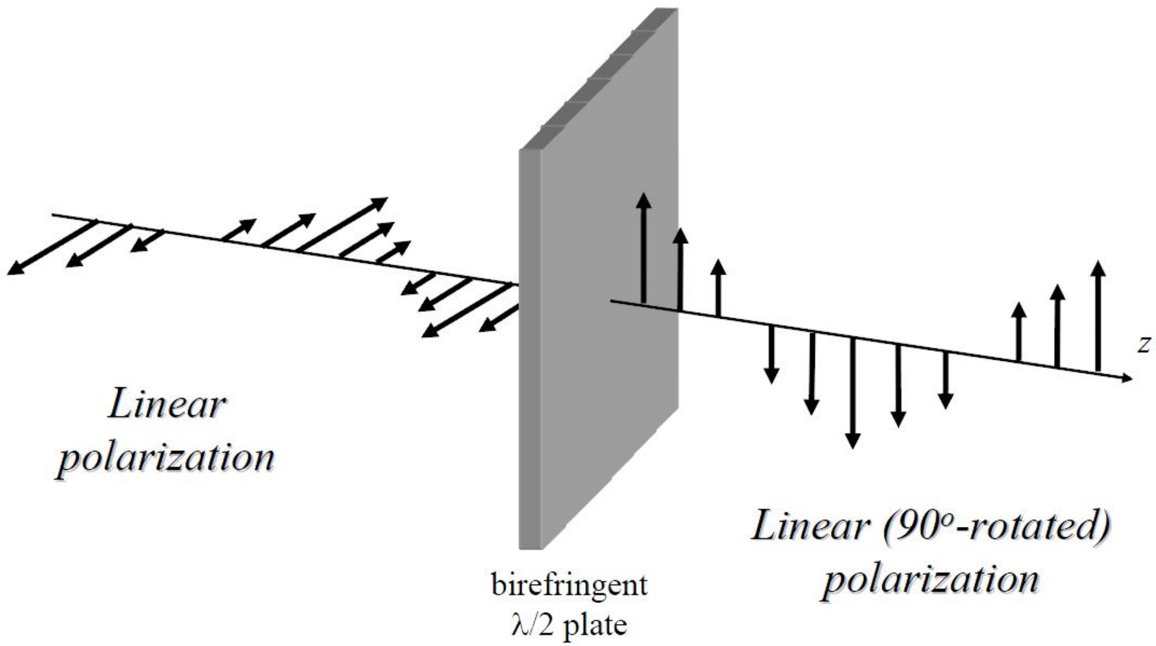
If the polarisation axis of a birefrigent crystal and the polarisation direction of light are not parallel, the polarisation state will change upon propagation though the crystal.



Wave Plates

Half-wave Plate ($\lambda/2$)

- causes π = 180° phase difference between the polarisation components
- rotates linear polarisation and changes the rotation direction of circular/elliptical polarisation

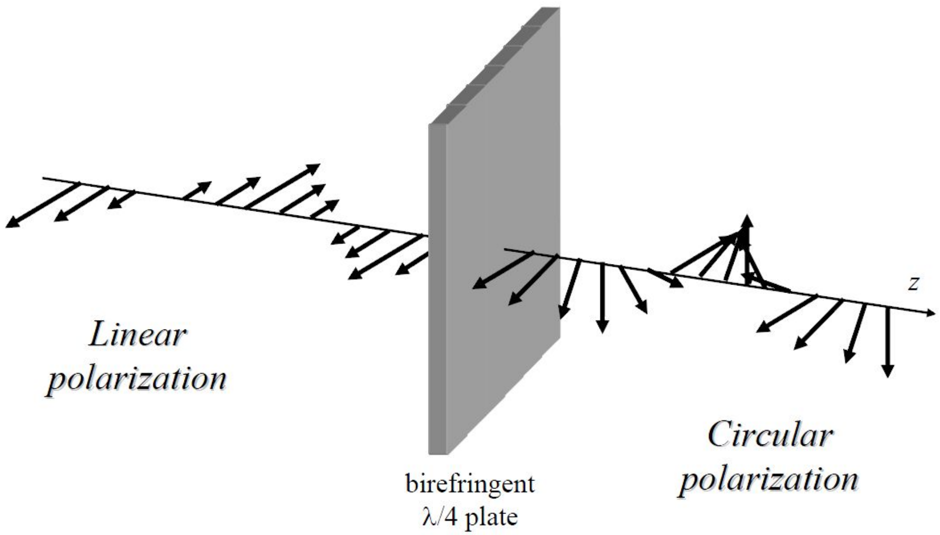


$$\Delta\phi = \pi = \frac{2\pi}{\lambda_{\text{vac}}}\left(n_o - n_e\right)d$$

$$d\Delta n = \frac{\lambda_{\text{vac}}}{2}$$

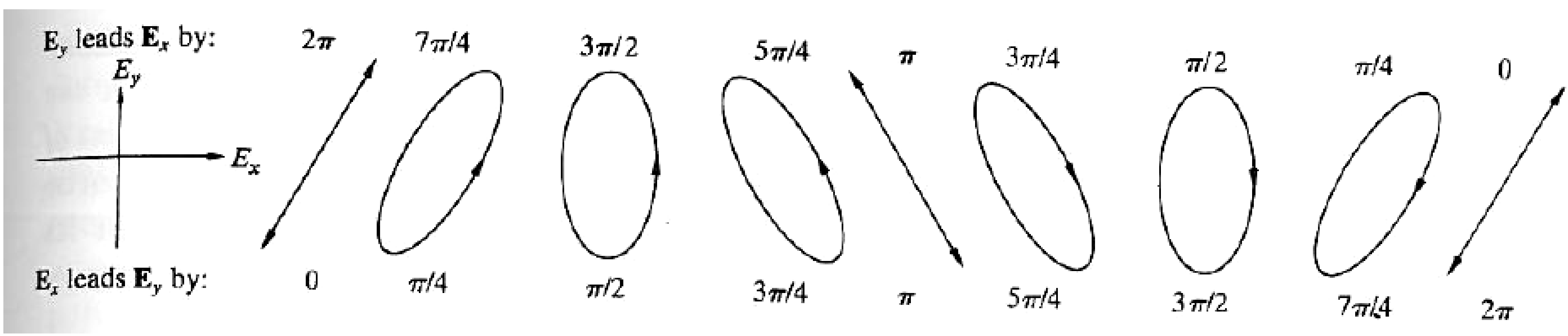
Quarter-wave Plate (λ/4)

- causes π/2 = 90° phase difference between the polarisation components
- changes linear polarisation into circular/elliptical polarisation and visa versa



$$\Delta\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda_{\text{vac}}}(n_o - n_e)d$$

$$d\Delta n = \frac{\lambda_{\text{vac}}}{4}$$



Video

Optics: Quarter-wave plate | MIT Video Demonstrations in Lasers and Optics

Electro-optic Modulator

- some optical crystals have electro-optic response, i.e. external electric field affects the optical properties of the crystal (small change in the position of electrons)
- a few kV voltage affects the polarisation rotation
- in an electro-optic modulator the intensity of light can be changes very quicly – at the scale of nanoseconds (”Q switch”)