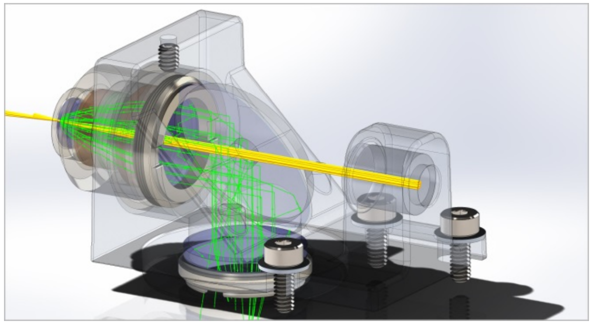
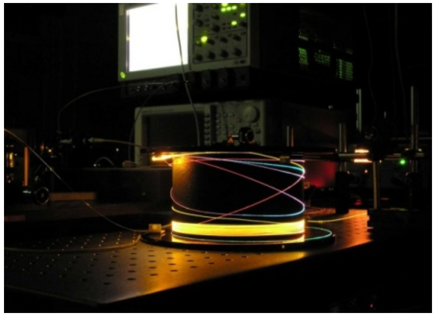
Optics E-5730 Spring 2021

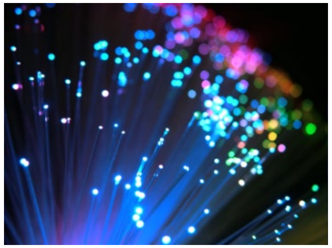
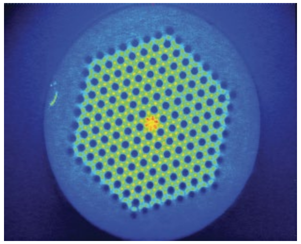
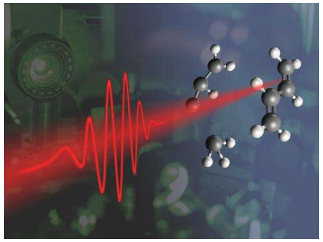
Coherence I

Lectures: Toni Laurila

Email: toni.k.laurila@aalto.fi

Tel. 050-358 3097

Fundamentals of Optics, Spring 2021

ELEC E-5730 lectures online using Zoom at https://aalto.zoom.us/j/8453943170

exercises online using Zoom at https://aalto.zoom.us/j/5703080612

week	day	date	time	topic	
2	Mon	11.1.2021	8-10	Lecture 1: Geometrical optics 1	
	Fri	15.1.2021	8-10	Lecture 2: Geometrical optics 2	
3	Mon	18.1.2021	8-10	Lecture 3: Wave optics 1	
	Mon	18.1.2021	10-12	Exercise 1	
	Fri	22.1.2021	8-10	Lecture 4: Wave optics 2	
4	Mon	25.1.2021	8-10	Lecture 5: Coherence 1	
	Mon	25.1.2021	10-12	Exercise 2	
	Fri	29.1.2021	8-10	Lecture 6: Coherence 2	
5	Mon	1.2.2021	8-10	Lecture 7: Radiometry	
	Mon	1.2.2021	10-12	Exercise 3	
	Fri	5.2.2021	8-10	Lecture 8: Interferometry + 30 mins mid-term exam	
6	Mon	8.2.2021	8-10	Lecture 9: Fibre optics + Optical telecom	LAB WORKS PERIOD
	Mon	8.2.2021	10-12	Exercise 4	LAB WORKS PERIOD
	Fri	12.2.2021	8-10	Lecture 10: Diffraction 1	LAB WORKS PERIOD
7	Mon	15.2.2021	8-10	Lecture 11: Diffraction 2	LAB WORKS PERIOD
	Mon	15.2.2021	10-12	Exercise 5	LAB WORKS PERIOD
	Fri	19.2.2021	8-10	NO LECTURE	LAB WORKS PERIOD
8	Mon	22.2.2021	8-10	NO LECTURE	
	Mon	22.2.2021	10-12	Exercise 6	
	Fri	26.2.2021		Examination	

Last Lecture – Wave Optics I

- reflection and refraction coefficients for E field amplitude and intensity
- Brewster's angle
- anti-reflection coating
- polariser components
- optically anisotropic materials:
    - dichroism
    - birefringence
- quarter-wave ($\lambda/4$) plate
- half-wave ($\lambda/2$) plate

Today – Coherence I

- interference
- Michelson interferometer

Coherence

Part 1

Interference

Let us start by looking at the total electric field of two monochromatic plane waves. We assume that the frequencies and propagation directions are the same:

$$E^{(1)} = E_1 e^{i(kx - \omega t + \phi_1)}$$

$$E^{(2)} = E_2 e^{i(kx - \omega t + \phi_2)}$$

Total intensity is (ignoring the ½ n c_o ε_o and time-dependent $e^{-i\omega t}$ term):

$$I_{TOT} = \left|E_{TOT}(x,t)\right|^2 = \left|E_1 e^{i\phi_1} + E_2 e^{i\phi_2}\right|^2$$

$$= \left|E_1\right|^2 + \left|E_2\right|^2 + E_1 E_2^* e^{i(\phi_1 - \phi_2)} + E_1^* E_2 e^{-i(\phi_1 - \phi_2)}$$

Interference

$$I_{TOT} = \left|E_{TOT}(x,t)\right|^2 = \left|E_1 e^{i\phi_1} + E_2 e^{i\phi_2}\right|^2$$

$$= \left|E_1\right|^2 + \left|E_2\right|^2 + E_1 E_2^* e^{i(\phi_1 - \phi_2)} + E_1^* E_2 e^{-i(\phi_1 - \phi_2)}$$

notation $\quad E_1 = \sqrt{I_1} \quad \text{ja} \quad E_2 = \sqrt{I_2}$

$$I_{TOT} = \left(I_1 + I_2\right) + 2\,\mathrm{Re}\left\{E_1 E_2^* e^{i\Delta\phi}\right\}$$

$$= \left(I_1 + I_2\right) + 2\sqrt{I_1 I_2}\,\mathrm{Re}\left\{e^{i\Delta\phi}\right\}$$

$$= \left(I_1 + I_2\right) + 2\sqrt{I_1 I_2}\cos\left(\Delta\phi\right)$$

$$= \left(I_1 + I_2\right)\left[1 + \frac{2\sqrt{I_1 I_2}}{\left(I_1 + I_2\right)}\cos\left(\Delta\phi\right)\right]$$

Interference term

Interference

If $I_1 = I_2$ the total intensity becomes:

$$I_{TOT} = 2I_1\left(1+\cos\{\Delta\phi\}\right)$$

If two light sources are mutually <u>completely coherent</u> (same phase), then $\Delta\phi=0$ and total intensity is $4I_1$.

A light detector (also the human eye) has a certain response time T_m (time constant) which is the minimum time that the measurement event takes. During the response or measurement time the signal gets 'averaged'. If $\Delta\phi$ changes more rapidly than the response time then an average is observed:

$$\left\langle\cos(\Delta\phi)\right\rangle = \frac{1}{T_m}\int_{-\frac{T_m}{2}}^{\frac{T_m}{2}}\cos(\Delta\phi)\,dt$$

Interference

$$I_{TOT} = 2I_1\left[1 + \langle\cos(\Delta\phi)\rangle\right] \qquad \langle\cos(\Delta\phi)\rangle = \frac{1}{T_m}\int_{-\frac{T_m}{2}}^{\frac{T_m}{2}}\cos(\Delta\phi)\,dt$$

In the case of classical (thermal) light, e.g, two light bulbs, the phases are completely random so the averaged phase term is zero and interference term disappears. Total intensity is thus ’classically’ the sum of the two input intensities.

Two wave sources in physics are generally speaking perfectly coherent if they have a constant phase difference, the same frequency and the same waveform.

For light sources showing some coherence or stability in their phase, e.g., lasers the interference is commonly observed because the coherence time is typically long (ms or more for stable lasers). Between the coherent and incoherent waves are the partially coherent waves (and light sources).

Interference

If $I_1 = I_2$ the total intensity becomes:

$$I_{TOT} = 2I_1\left(1 + \cos\{\Delta\phi\}\right)$$

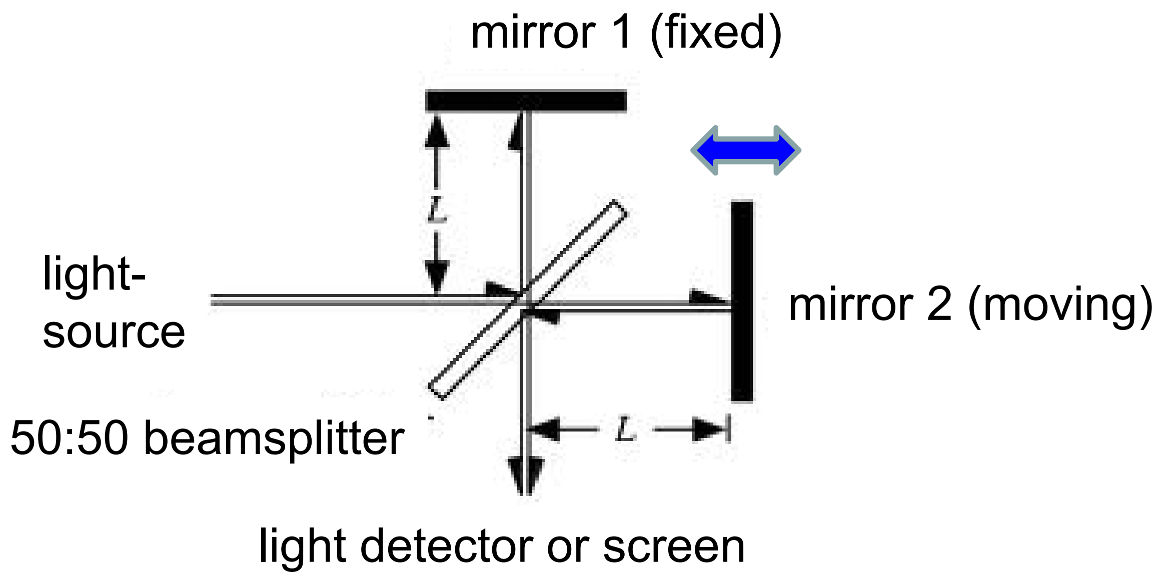
If two light sources are mutually <u>completely coherent</u> (same phase), then $\Delta\phi$=0 and total intensity is $4I_1$.

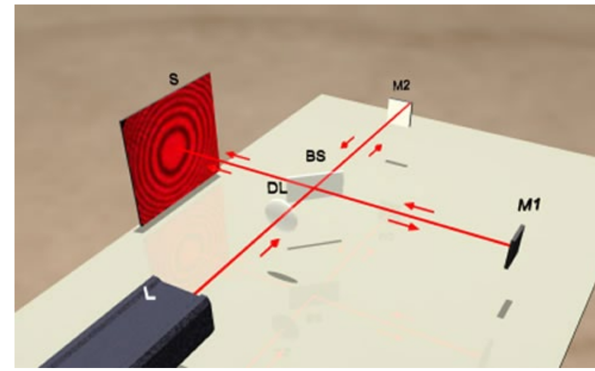
Mathematically the difference between ’classical’ and ’interfering’ cases is in how we add the two fields together:

$$A_{TOT} = \|A_1\|^2 + \|A_2\|^2$$

$$B_{TOT} = \|B_1 + B_2\|^2$$

Michelson Interferometer





If the light source is a fully coherent laser (intensity I) and the mirror distances are the same (L) the intensity at the detector is at maximum 2I:

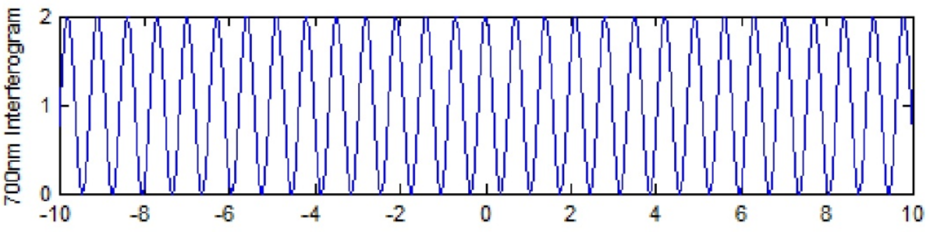
$$I_{TOT} = I_1[1 + \cos(\Delta\phi)]$$

If we move mirror 2 by a distance $\Delta x/2$ the phase difference due to path difference is:

$$\Delta\phi = k \cdot \Delta L = k2\frac{\Delta x}{2} = k\Delta x = \frac{2\pi\Delta x}{\lambda}$$

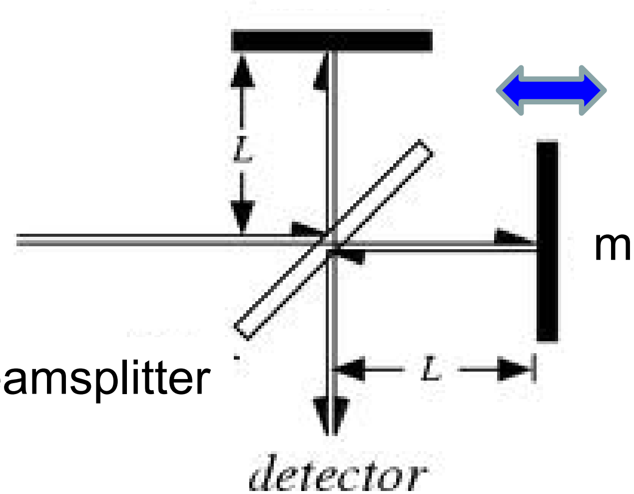
Michelson Interferometer's Interferogram for Monochromatic Light

$$I_{TOT} = I_1\left[1+\cos\left(\frac{2\pi\Delta x}{\lambda}\right)\right]$$

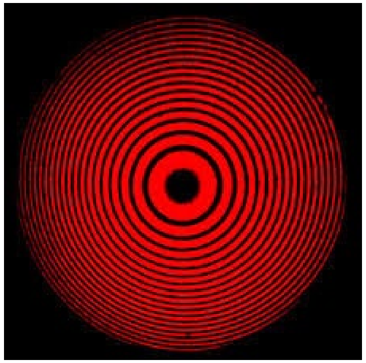


Mirror 2 relative position

When the mirror 2 is moving a cosine like intensity variation between 0 and $2I_1$ is observed. This is the <u>interferogram</u> of the monochromatic light source.



https://www.youtube.com/watch?v=j-u3IEgcTiQ



Interferogram of a red diverging laser beam on screen

Michelson Interferometer's Interferogram for Bichromatic Light

$$E(x,t) = 2E_1 e^{i(k_1 x - \omega_1 t + \phi_1)} + 2E_2 e^{i(k_2 x - \omega_2 t + \phi_2)}$$

We assume that the two waves 1 and 2 are mutually completely incoherent when averaged over the detector's response time:

$$\langle \cos(\phi_1 - \phi_2) \rangle = \langle \sin(\phi_1 - \phi_2) \rangle = 0$$

Waves reaching the detector after being reflected from mirrors 1 and 2:

$$E_{mirror\_1} = E_1 e^{i(k_1 x - \omega_1 t + \phi_1)} + E_2 e^{i(k_2 x - \omega_2 t + \phi_2)}$$

$$E_{mirror\_2} = E_1 e^{i(k_1 [x + \Delta x] - \omega_1 t + \phi_1)} + E_2 e^{i(k_2 [x + \Delta x] - \omega_2 t + \phi_2)}$$

Electric field and intensity at the detector are:

$$E_{TOT} = E_1 e^{i(k_1 x - \omega_1 t + \phi_1)} \left(1 + e^{ik_1 \Delta x}\right) + E_2 e^{i(k_2 x - \omega_2 t + \phi_2)} \left(1 + e^{ik_2 \Delta x}\right)$$

$$I_{TOT} = \left|E_{TOT}\right|^2 = \left|E_1 \left(1 + e^{ik_1 \Delta x}\right)\right|^2 + \left|E_2 \left(1 + e^{ik_2 \Delta x}\right)\right|^2$$

$$+ \left\langle 2\,\mathrm{Re}\left\{E_1 E_2^* e^{i(\Delta k x - \Delta \omega t + \Delta \phi)} \left(1 + e^{ik_1 \Delta x}\right)\left(1 + e^{-ik\,2\Delta x}\right)\right\}\right\rangle$$

Michelson Interferometer's Interferogram for Monochromatic Light

$$I_{TOT} = \left|E_{TOT}\right|^2 = \left|E_1\left(1+e^{ik_1\Delta x}\right)\right|^2 + \left|E_2\left(1+e^{ik_2\Delta x}\right)\right|^2$$
$$+\left\langle 2\,\mathrm{Re}\left\{E_1E_2^* e^{i(\Delta kx-\Delta\omega t+\Delta\phi)}\left(1+e^{ik_1\Delta x}\right)\left(1+e^{-ik2\Delta x}\right)\right\}\right\rangle$$

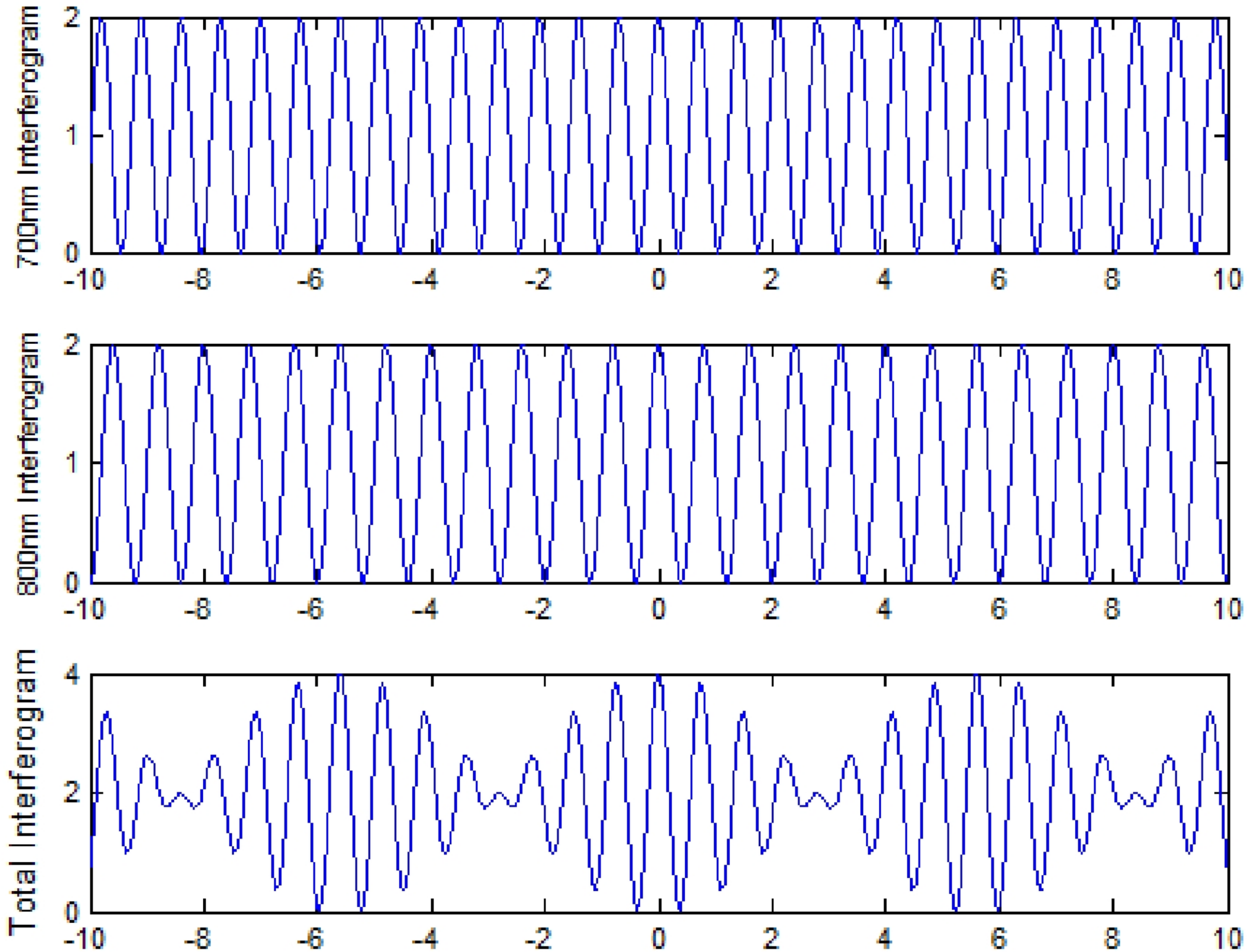
Because $\left\langle\cos(\phi_1-\phi_2)\right\rangle = \left\langle\sin(\phi_1-\phi_2)\right\rangle = 0$ the last term is zero.

$$I_{TOT} = I_1\left(1+\cos(k_1\Delta x)\right) + I_1\left(1+\cos(k_2\Delta x)\right) \text{if } \mathrm{I}_1 = \mathrm{I}_2$$

$$I_{TOT} = 2I_1 + I_1\left[\cos(k_1\Delta x) + \cos(k_2\Delta x)\right]$$
$$= 2I_1 + 2I_1\left[\cos\left(\frac{\{k_1+k_2\}\Delta x}{2}\right)\cos\left(\frac{\{k_1-k_2\}\Delta x}{2}\right)\right]$$

This is interferogram of bichromatic light. The interferogram oscillates at the average frequency $(k_1+k_2)/2$ and its envelope at difference frequency $(k_1-k_2)/2$.

Michelson Interferometer's Interferogram for Monochromatic Light



Mirror 2 relative position

Michelson Interferometer

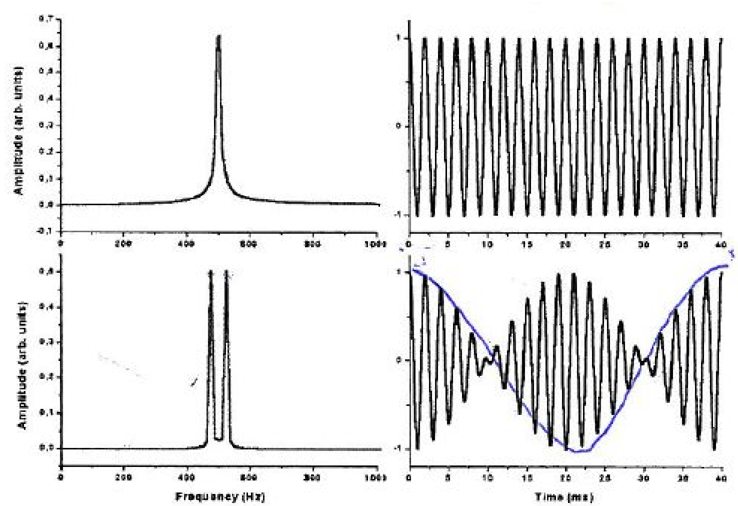
<u>Monochromatic light</u> produces the following interferogram signal:

$$I_{TOT}(\Delta x) = I_1\left[1 + \cos\left(\frac{2\pi\Delta x}{\lambda}\right)\right]$$

<u>Bichromatic light</u> produces the following interferogram signal:

$$I_{TOT}(\Delta x) = I_1\left(1 + \cos(k_1\Delta x)\right) + I_2\left(1 + \cos(k_2\Delta x)\right)$$

frequency space Δx or time space



Michelson Interferogram for Light Containing Multiple Frequencies

The interferogram for bichromatic light can be extended for light containing multiple frequencies:

$$I_{TOT}(\Delta x) = \sum_j I_j \left[1 + \cos\left(k_j \Delta x\right)\right]$$

The above can be generalised for continuous spektrum *I*(k):

$$I(\Delta x) = \int_0^\infty I(k)\left[1 + \cos(k\Delta x)\right] dk$$

$$= I_0 + \boxed{\int_0^\infty I(k)\cos(k\Delta x)\, dk}$$

which the cosine transformation of function I(k)

Michelson Interferometer vs. Fourier Transform (FT) Spectroscopy

Next it will be shown that from the time-dependent light detector signal, which varies as a function of the mirror movement (interferogram), the spectrum of input light can be obtained by Fourier transformation.

Basic concepts:

- spectrum = variable expressed as a function of frequency or energy
- for example: intensity spectrum, absorption spectrum, transmission spectrum
- intensity I can be expressed: $I(\nu)$, $I(\lambda)$, $I(\bar{\nu})$, $I(\omega)$

frequency space $\quad$ Δx or time space