Optics E-5730 Spring 2021

Coherence II

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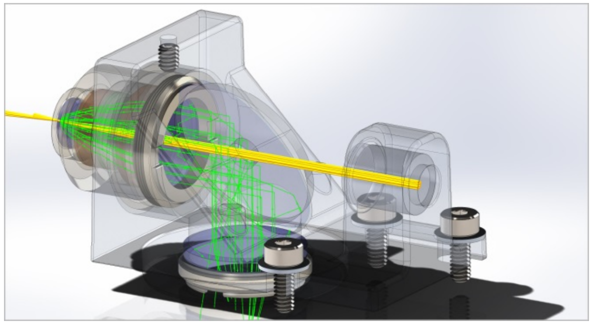
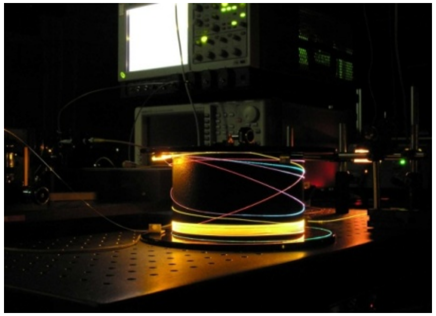
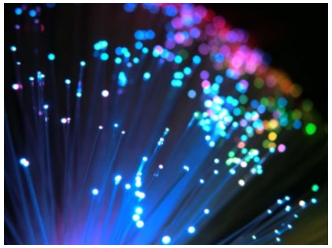
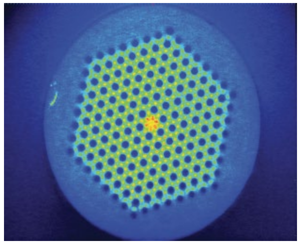
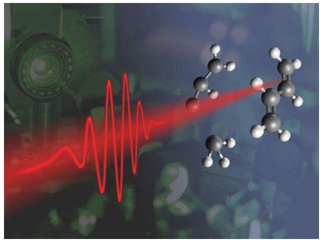



NOTE: Monday, Feb 1 lecture at https://aalto.zoom.us/j/8059841110

Fundamentals of Optics, Spring 2021

ELEC E-5730

lectures online using Zoom at https://aalto.zoom.us/j/8453943170

exercises online using Zoom at https://aalto.zoom.us/j/5703080612

week	day	date	time	topic	
2	Mon	11.1.2021	8-10	Lecture 1: Geometrical optics 1	
	Fri	15.1.2021	8-10	Lecture 2: Geometrical optics 2	
3	Mon	18.1.2021	8-10	Lecture 3: Wave optics 1	
	Mon	18.1.2021	10-12	Exercise 1	
	Fri	22.1.2021	8-10	Lecture 4: Wave optics 2	
4	Mon	25.1.2021	8-10	Lecture 5: Coherence 1	
	Mon	25.1.2021	10-12	Exercise 2	
	Fri	29.1.2021	8-10	Lecture 6: Coherence 2	
5	Mon	1.2.2021	8-10	Lecture 7: Radiometry	
	Mon	1.2.2021	10-12	Exercise 3	
	Fri	5.2.2021	8-10	Lecture 8: Interferometry + 30 mins mid-term exam	
6	Mon	8.2.2021	8-10	Lecture 9: Fibre optics + Optical telecom	LAB WORKS PERIOD
	Mon	8.2.2021	10-12	Exercise 4	LAB WORKS PERIOD
	Fri	12.2.2021	8-10	Lecture 10: Diffraction 1	LAB WORKS PERIOD
7	Mon	15.2.2021	8-10	Lecture 11: Diffraction 2	LAB WORKS PERIOD
	Mon	15.2.2021	10-12	Exercise 5	LAB WORKS PERIOD
	Fri	19.2.2021	8-10	NO LECTURE	LAB WORKS PERIOD
8	Mon	22.2.2021	8-10	NO LECTURE	
	Mon	22.2.2021	10-12	Exercise 6	
	Fri	26.2.2021		Examination	

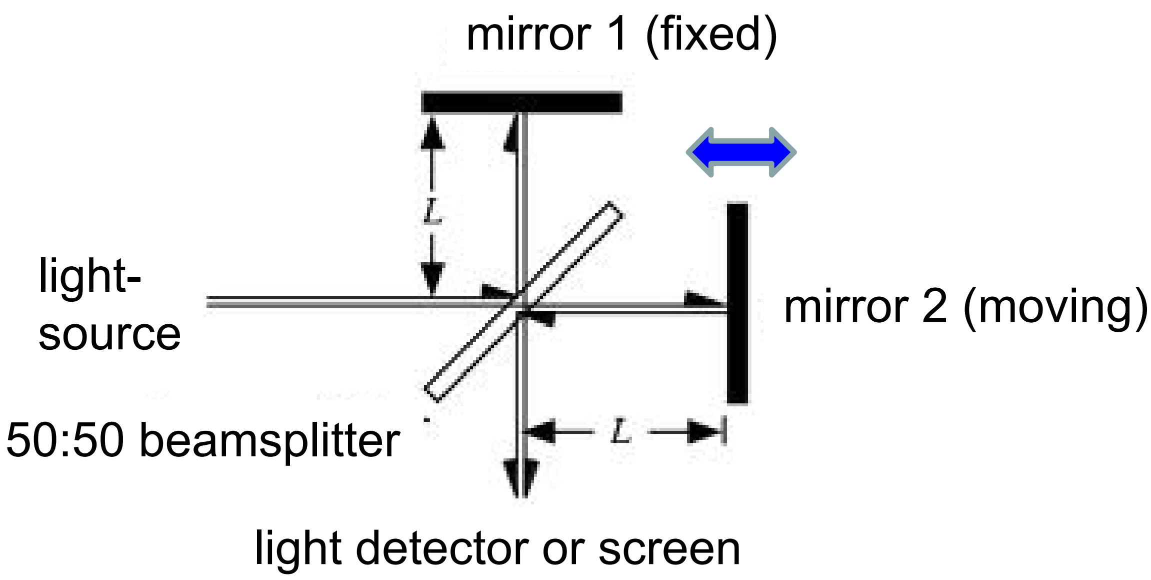
Last Lecture – Coherence I

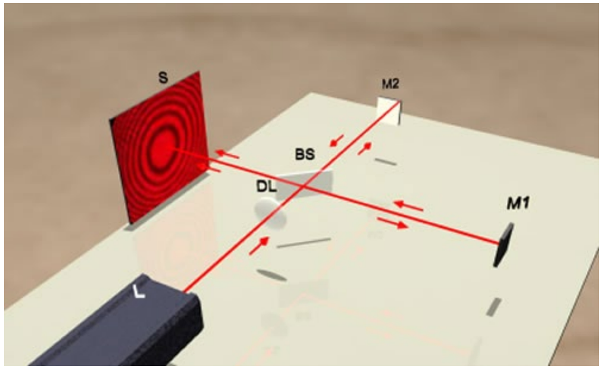
- Interference
- Michelson interferometer

This Lecture – Coherence II

- Fourier transform (FT) spectroscopy
- FTIR spectrometer
- Coherence length and time
- Young’s double-split experiment

Michelson Interferometer





Michelson Interferometer

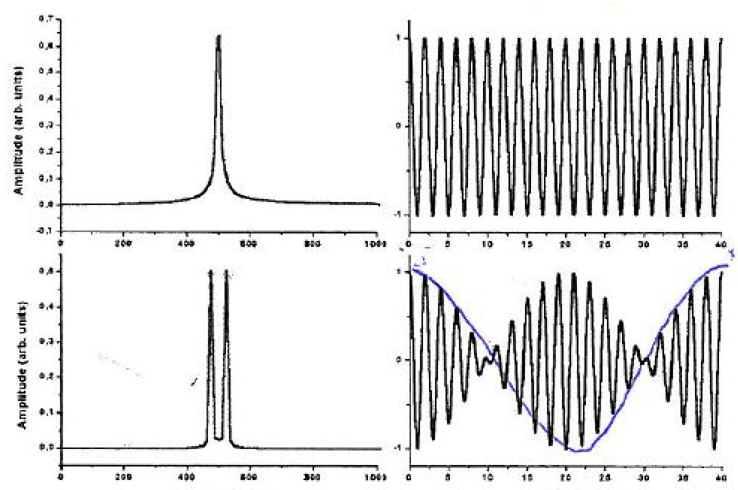
Monochromatic light produces interferogram signal:

$$I_{TOT}(\Delta x) = I_1\left[1 + \cos\left(\frac{2\pi\Delta x}{\lambda}\right)\right]$$

Bichromatic light produces interferogram signal:

$$I_{TOT}(\Delta x) = I_1(1 + \cos(k_1\Delta x)) + I_2(1 + \cos(k_2\Delta x))$$

frequency space   Δx or time space



Michelson Interferometer & Spectrally Broadband Light

The interferogram for bichromatic light can be extended for light containing multiple discrete frequencies:

$$I_{TOT}(\Delta x) = \sum_j I_j \left[1 + \cos\left(k_j \Delta x\right)\right]$$

The above can be generalised for continuous spektrum *I*(k):

$$I(\Delta x) = \int_0^\infty I(k)\left[1 + \cos(k\Delta x)\right] dk$$

$$= I_0 + \boxed{\int_0^\infty I(k)\cos(k\Delta x)\, dk}$$

which is the cosine transformation of function I(k)

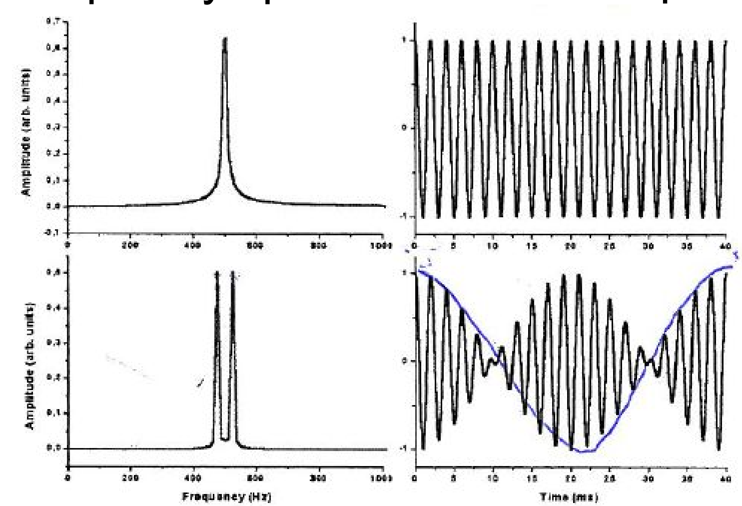
Michelson Interferometer & Fourier Transform (FT) Spectroscopy

Next it will be shown that from the time-dependent light detector signal, which varies as a function of the mirror movement (interferogram), the spectrum of input light can be obtained by Fourier transformation.

Basic concepts:

- spectrum = quantity expressed as a function of frequency or energy
- for example: intensity spectrum, absorption spectrum, transmission spectrum
- intensity I can be expressed: $I(\nu)$, $I(\lambda)$, $I(\bar{\nu})$, $I(\omega)$

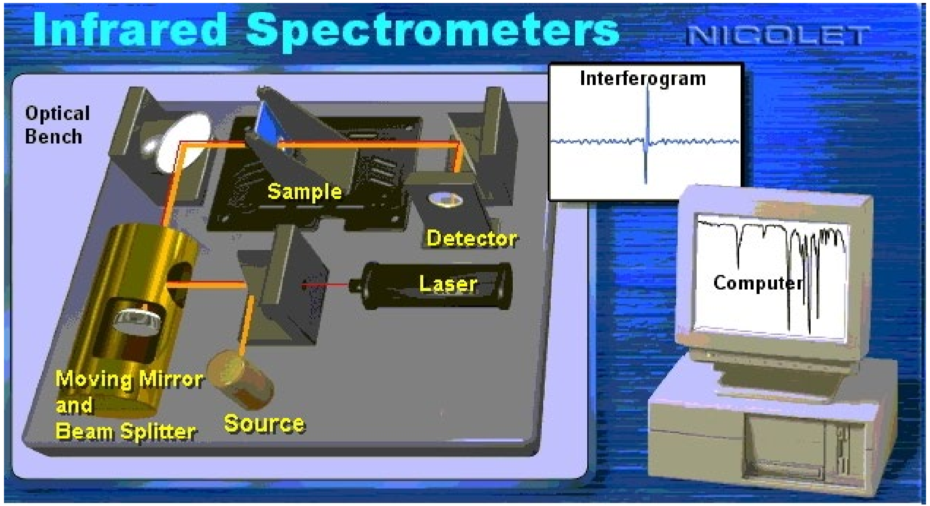
frequency space Δx or time space



Michelson Interferometer & Fourier Transform (FT) Spectroscopy

FTIR Spectrometer

$$I_{TOT}(\Delta x) = I_1\left[1+\cos\left(\frac{2\pi\Delta x}{\lambda}\right)\right]$$



FTIR spectrometer operation for monohromatic input spectrum:

- interferogram is of the cosine type
- from the time domain signal, first, the monochromatic frequency ν and, secondly, corresponding wavelenght λ can be determined (v_0 is mirror speed (m/s))

$$\lambda = \frac{v_0}{\nu}$$

- mirror movement must be "interferometrically" smooth which is difficult to realise over several cm: The solution is to compensate for the velocity variations by using a reference laser, e.g, frequency stabilised 633 nm helium-neon (HeNe) laser.

FTIR Spectrometer

- wavelength of the reference laser λ_R is constant and the corresponding frequency ν_R can be obtained by detecting the rising zero crossings of cosine:

$$\lambda = \lambda_R \frac{\nu_R}{\nu}$$

- Nyquist theorem: the sampling frequency must be twice the maximum frequency of the signal so that at minimum $\nu_R / \nu = 2$, and thus, the shortest wavelength that can be measured is $2\lambda_R$ = 1266 nm. This is why FT spectrometer operates typically only in the IR (infrared) and it is generally known as the **FTIR spectrometer**

FTIR Instrument Characteristics

- resolution (smallest measurable difference in wavenumbers):

$$\Delta\bar{\nu}\left[\mathrm{cm}^{-1}\right] = \frac{1}{\text{optical path difference}} = \frac{1}{\Delta \mathrm{x}}$$

- one measurement (mirror scan) yields full frequency spectrum (Felgett advantage)
- most of light (even 50%) can reach the detector (Jacquinot advantage)

We will return to these latter two advantages when we study the grating spectrometer.

Interference

The General Case

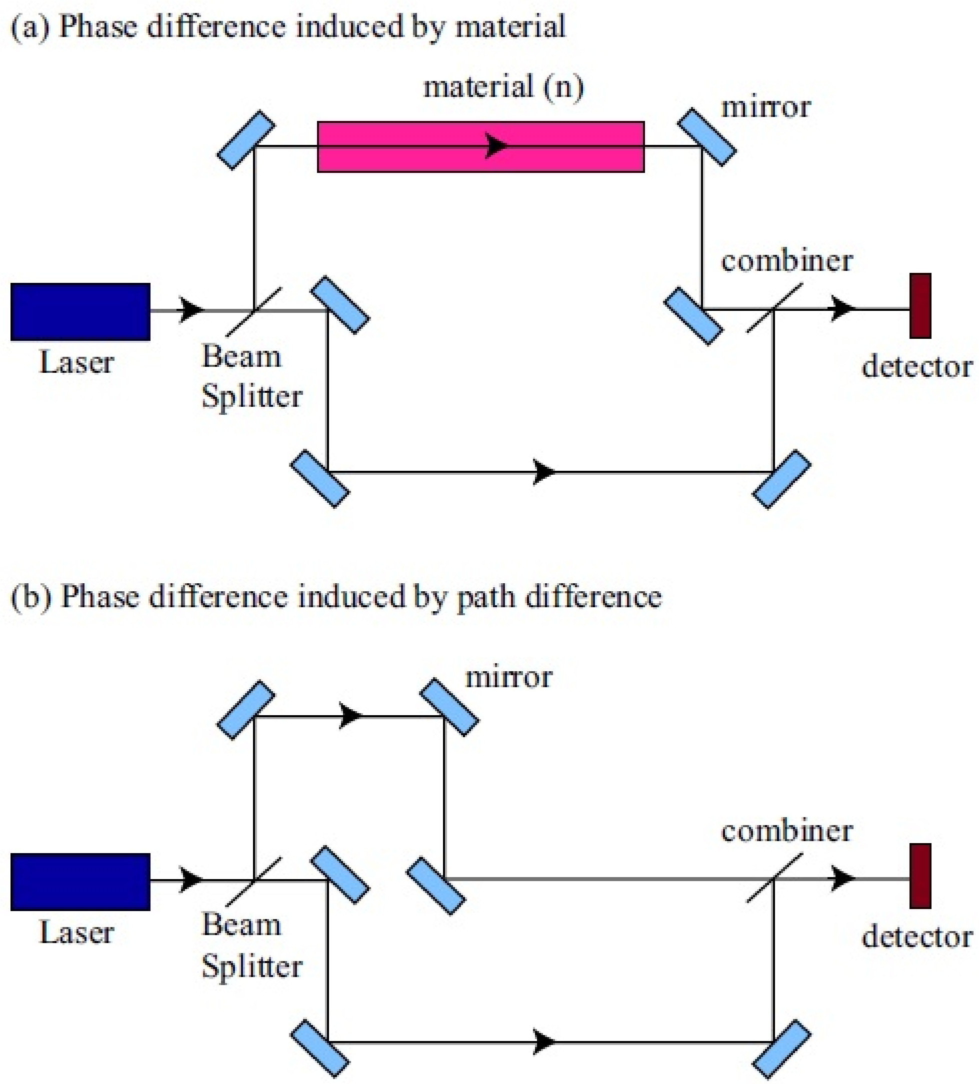
In Exercise IV it will be shown that in the most general case the intensity due to the interference between two plane waves can be expressed as

$$I_{TOT} = 2I_1\left[1 + \cos\left(\Delta\bar{k}\cdot\bar{r} - \Delta\omega t + \Delta\phi\right)\right]$$

The phase difference causing interference can thus be produced by

- different frequencies of the field
- path difference
- initial difference in the phases of the EM fields
- sum of the above contributions

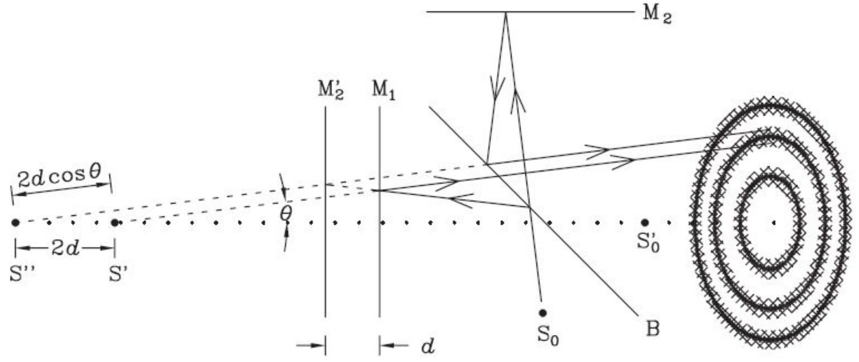
Interference



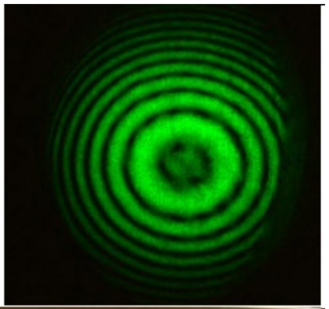
Interference Due to Path Lenght Difference in Michelson Interferometer

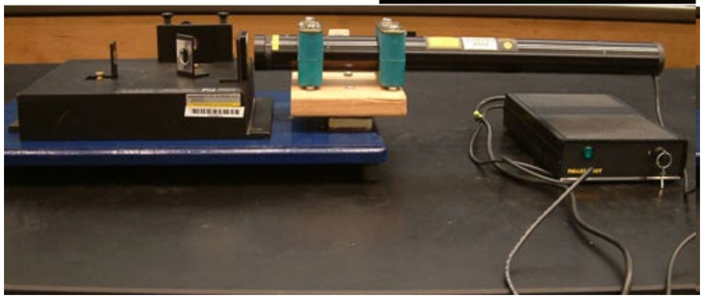
Interference Due to Path Lenght Difference in Michelson Interferometer

The rays of a divergent beam cause a circular diffraction pattern



diverging beam





$$I_{TOT} = 2I_1\left[1 + \cos\left(\Delta\bar{k}\cdot\bar{r} - \Delta\omega t + \Delta\phi\right)\right]$$

Destructive Interference

Where Does the Light Go?

https://www.youtube.com/watch?v=RRi4dv9KgCg

Longitudinal Coherence Length and Coherence Time

- For FTIR we studied the intereference as a function of the mirror movement Δx.
- For real light sources the amplitude of the interferogram starts to decrease as Δx gets large.

- <u>Coherence length l_c</u> of a light source is defined as the path length difference where the amplitude of the interference has dropped to about 1/3 (1/e) from the maximum value at $\Delta x=0$.

$$I_{TOT} = \left(I_1 + I_2\right)\left[1 + \frac{2\sqrt{I_1 I_2}}{\left(I_1 + I_2\right)}\cos\left(\Delta\phi\right)\right]$$

- <u>Coherence time</u> $t_c = l_c/c$ is the time corresponding to the coherence length
- We will see that coherence time is inversely proportional to the width of the light source's frequency spectrum $t_c \propto 1/\Delta\nu$

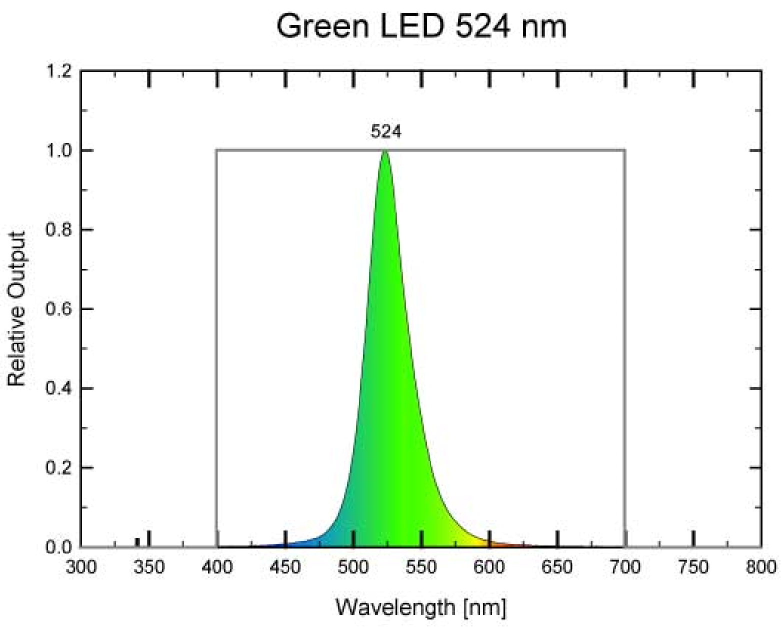
Examples of Coherence Lengths and Times

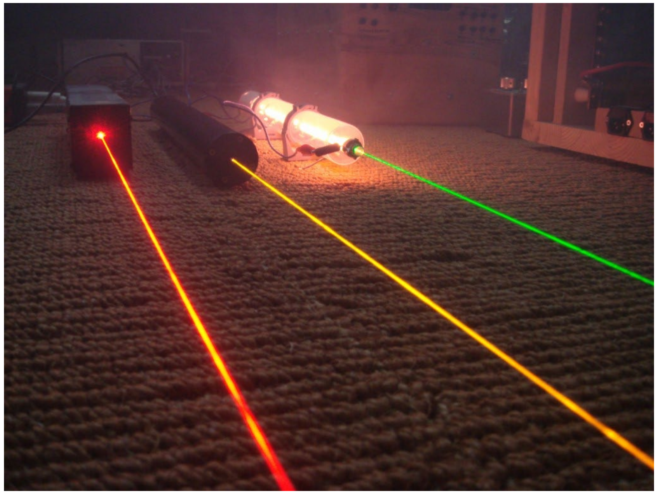
a) LED's $\Delta\lambda_{FWHM}$ is around 50 nm and coherence length l_c is in the µm range

b) multi-mode laser's l_c is around 20 cm (t_c = 0,7 ns)

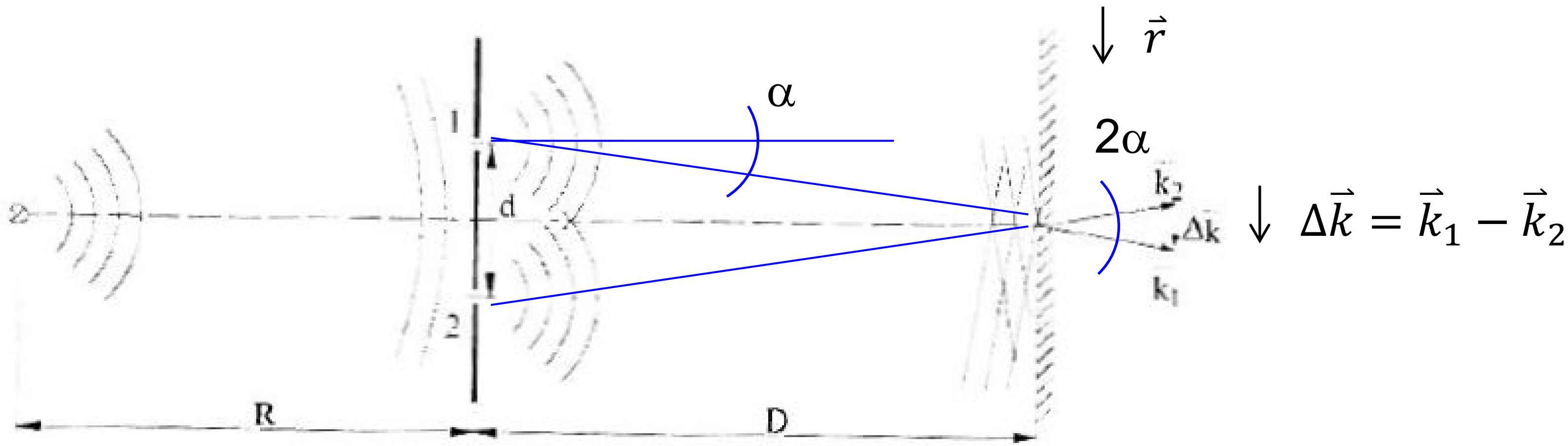
c) single-mode HeNe laser's l_c is around 100 m (t_c = 0,3 µs)

d) Well frequency-stabilised laser has a spectral width $\Delta\nu_{FWHM}$ in the kHz range and l_c 100 km (t_c = 0,3 ms)





Young's Experiment and Transverse Coherence Length



$$I_{TOT} = 2I_1\left[1+\cos\left(\Delta\bar{k}\cdot\bar{r} - \Delta\omega t + \Delta\phi\right)\right]$$

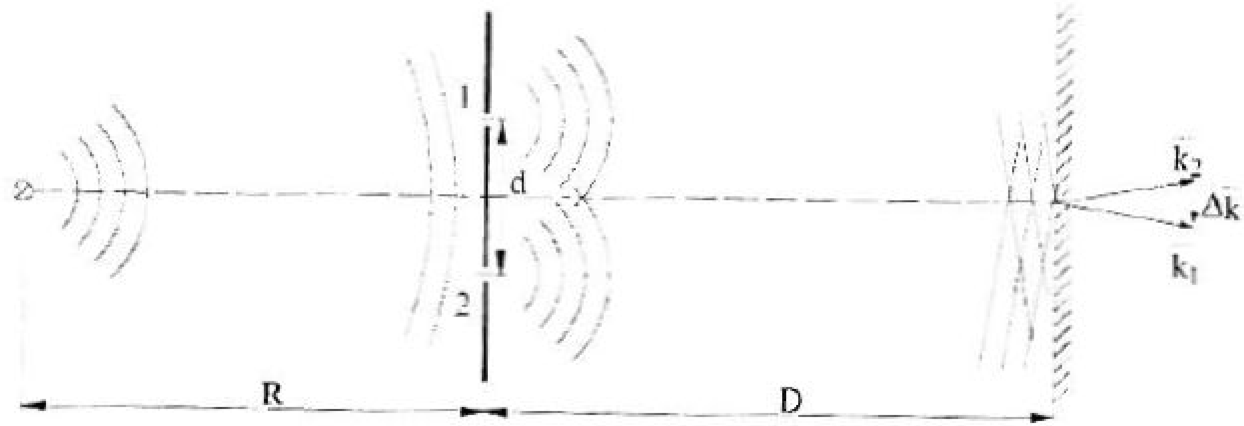
now $\Delta\omega = 0$ ja $I_1 = I_2$

$$\Delta\bar{k}\cdot\bar{r} = \left\|\Delta\bar{k}\right\|\left\|\bar{r}\right\|\cos\theta$$

$$\alpha \approx \frac{\frac{d}{2}}{D} \qquad 2\alpha \approx \frac{\Delta k}{k_1} \qquad \Rightarrow \qquad \frac{d}{D} = \frac{\Delta k}{k_1} \quad \text{or} \quad \Delta k = k_1\frac{d}{D}$$

$$\Rightarrow \qquad \Delta\bar{k}\cdot\bar{r} \approx k_1\frac{d}{D}r$$

Young's Experiment and Transverse Coherence Length

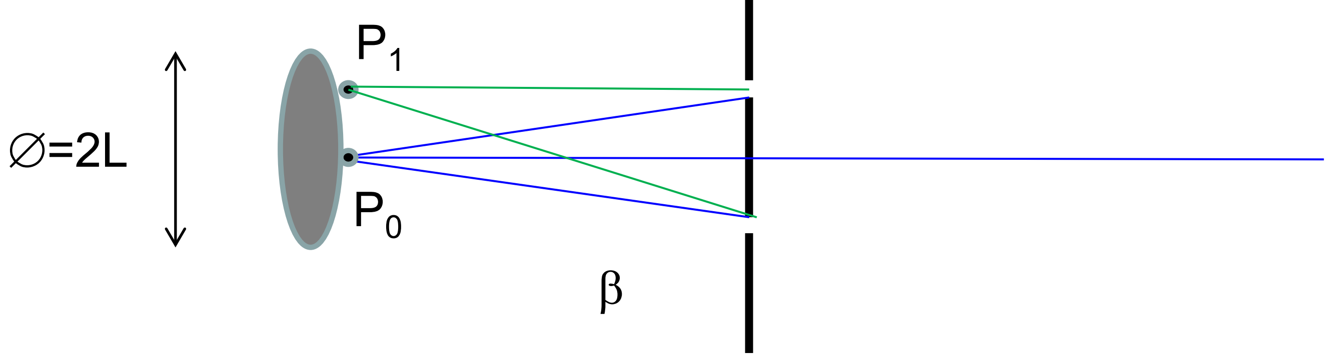


$$I_{TOT}(r) = 2I_1\left[1+\cos\left(2\pi\frac{d}{\lambda D}r+\Delta\phi\right)\right]$$

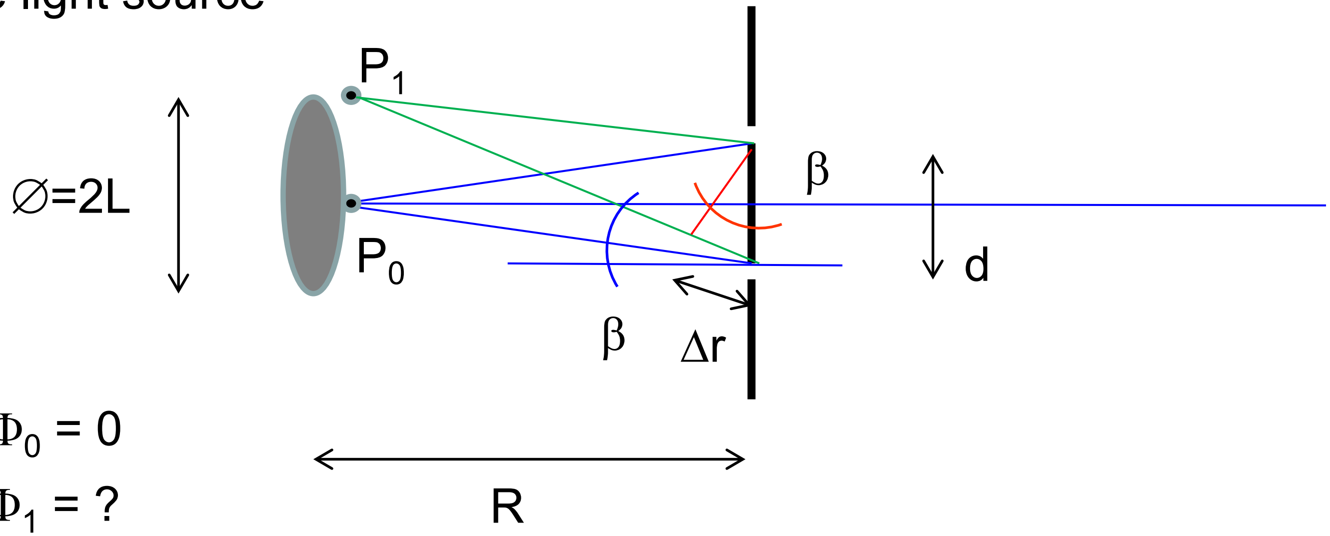
period and position of the maximum intensity:

$$\frac{\lambda D}{d} \text{ and } r_0 = -\frac{\lambda D \Delta\Phi}{2\pi d}$$

disk-like light source



disk-like light source



P_0: $\Delta\Phi_0 = 0$

P_1: $\Delta\Phi_1 = ?$

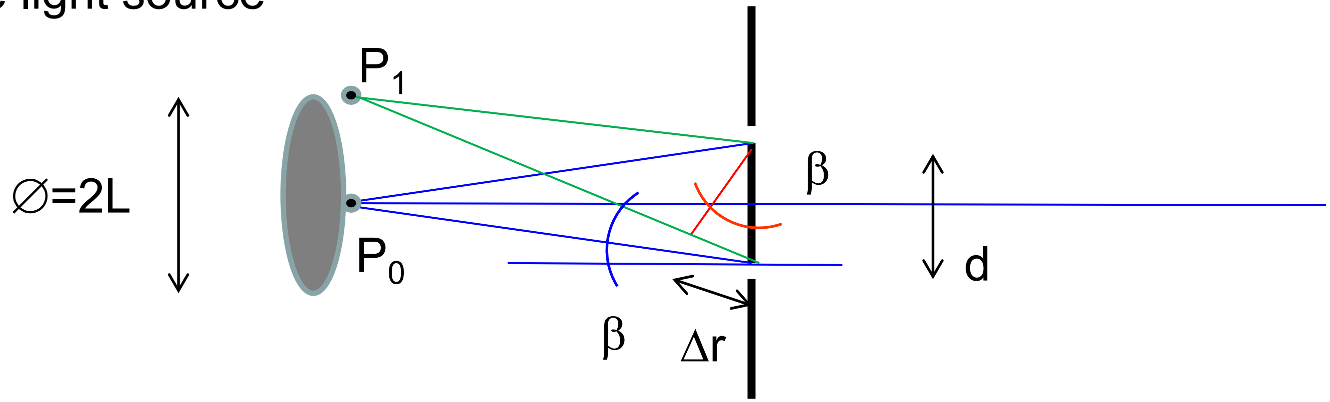
$\Delta\beta \approx \frac{\Delta r}{d}$ on the other hand $\beta \approx \frac{\frac{d}{2} + L}{R}$

when $\Delta r \rightarrow 0$ then $L \rightarrow 0$: $\beta_0 = \frac{d}{2R}$ $\beta = \beta_0 + \Delta\beta = \frac{d}{2R} + \frac{L}{R}$

Let us make the two Δβ's equal: $\frac{\Delta r}{d} = \frac{L}{R}$ or $\Delta r = \frac{Ld}{R}$

$$\Delta\Phi_1 = \bar{k} \cdot \Delta\bar{r} \approx \frac{2\pi}{\lambda}\frac{Ld}{R}$$

disk-like light source



P_0: $\Delta\Phi_0 = 0$

P_1: $\Delta\Phi_1 = ?$

$$\Delta\Phi_1 = \bar{k}\cdot\Delta\bar{r} \approx \frac{2\pi}{\lambda}\frac{Ld}{R}$$

Rays emerging from points P_0 and P_1 both create their own interference patterns on the screen whose centre points are located at

$$r_{0j} = -\frac{\lambda D \Delta\Phi_j}{2\pi d}, \text{ j} = 0,1$$

If the spatial shift on the screen for light emerging from point P_1

$$\frac{\lambda D \Delta\Phi_1}{2\pi d} = \frac{\lambda D}{2\pi d}\frac{2\pi}{\lambda}\frac{dL}{R} = \frac{LD}{R}$$

equals half of the interference pattern's period $\dfrac{\lambda D}{d}$

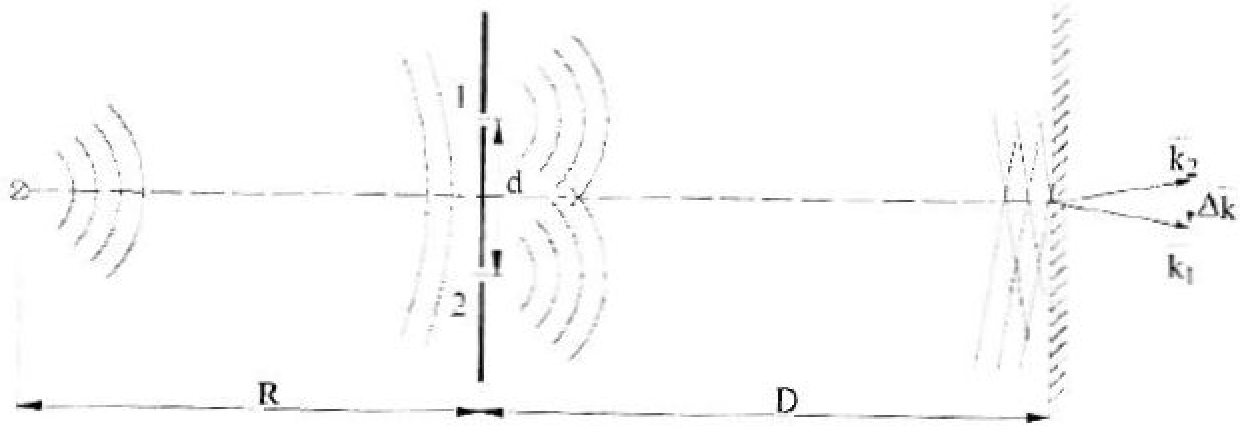
that is $$\frac{DL}{R} = \frac{1}{2}\frac{\lambda D}{d}$$

then there is destructive interference and we get the expression for the transverse coherence length d_c:

$$\frac{DL}{R} = \frac{1}{2}\frac{\lambda D}{d_c}$$

$$d_c = \frac{\lambda R}{2L} = \frac{\lambda R}{\text{light source diameter}}$$

Young's Experiment and Transverse Coherence Length



$$I_{TOT}(r) = 2I\left[1 + \cos\left(2\pi \frac{d}{\lambda D} r + \Delta\phi\right)\right]$$

$$d_C = \frac{\lambda R}{2L}$$

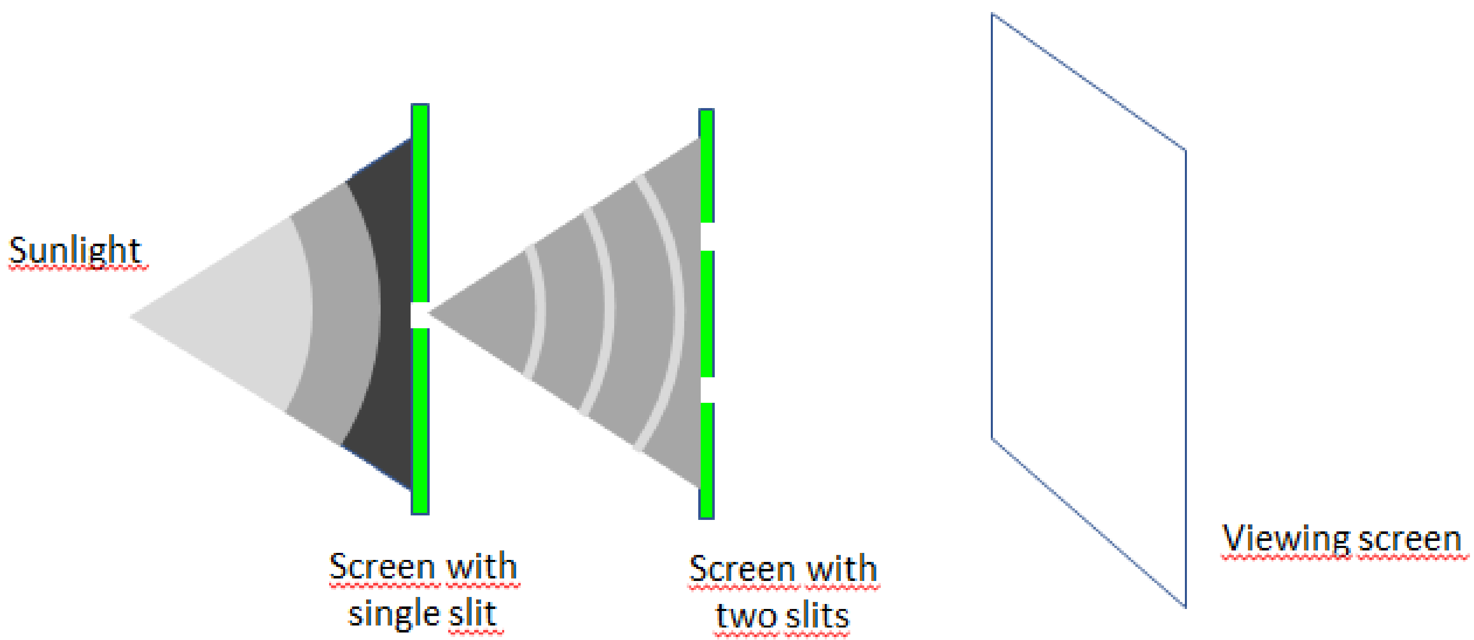
Young’s Double-slit Experiment

In 1803 Thomas Young performed an extraordinary optical experiment which would later serve as evidence for the wave model of light. He proposed that light exhibits properties of a wave which did not agree with the views of many other physicists during the early 19th century who believed that light consisted of only particles.

Thomas Young 1773-1829

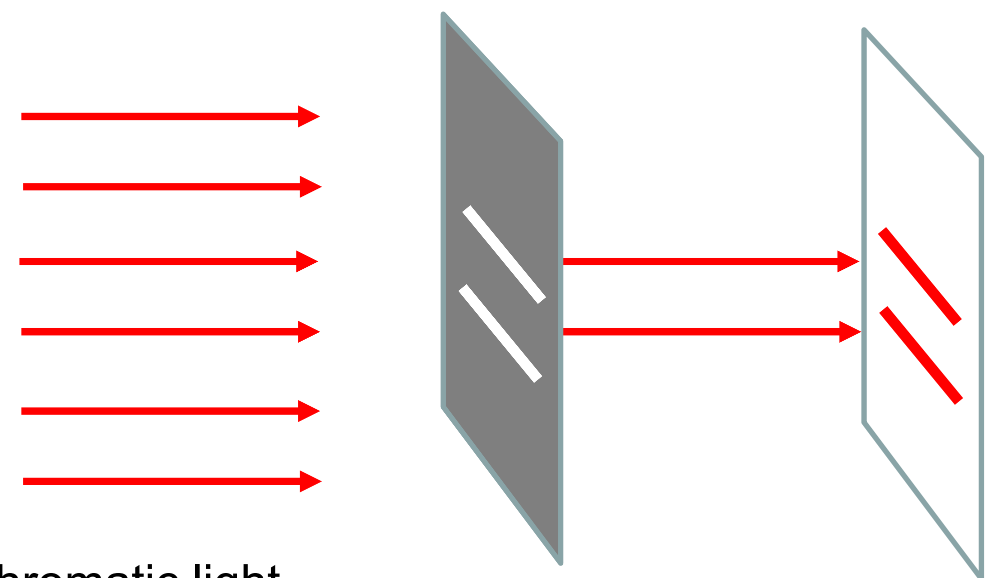
Young's Double-slit Experiment

Two narrow slits were illuminated with a light source which would project an image onto a screen behind the slits.



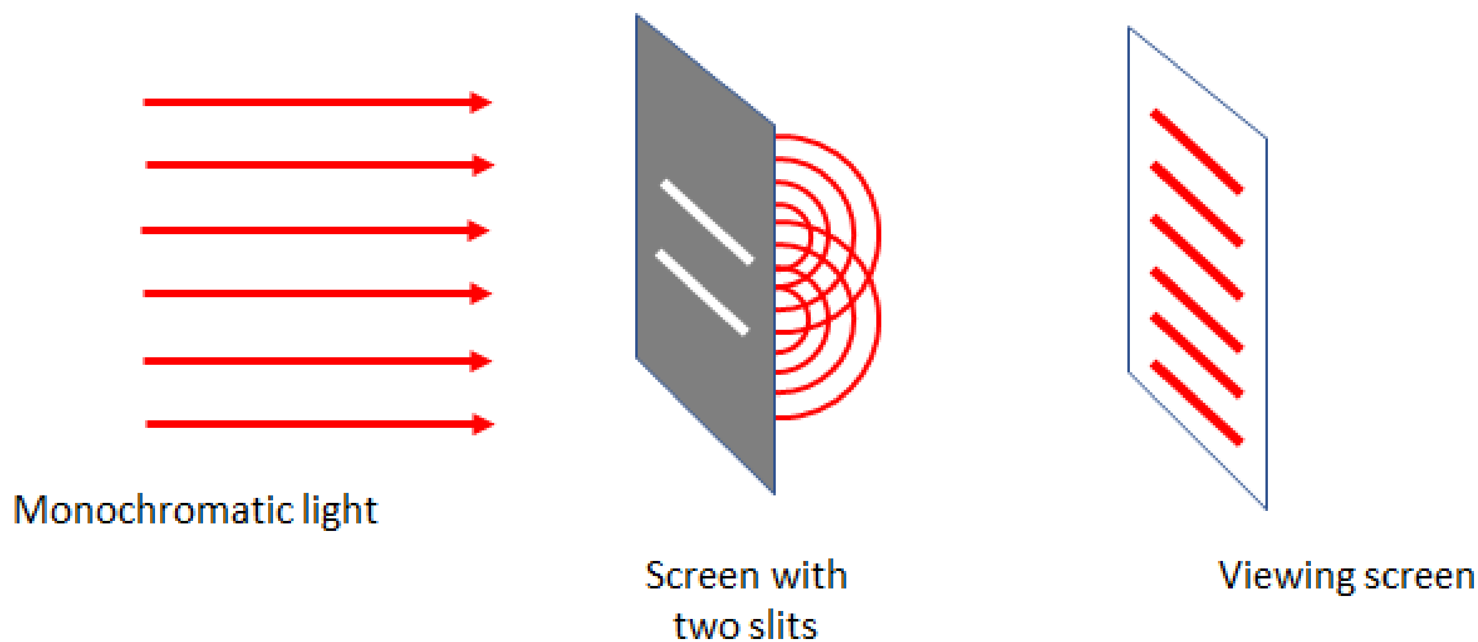
What if Light Were Only Particles?

If light were particles then we would see nothing but two intensity maximas corresponding to rays hitting directly the two slits.

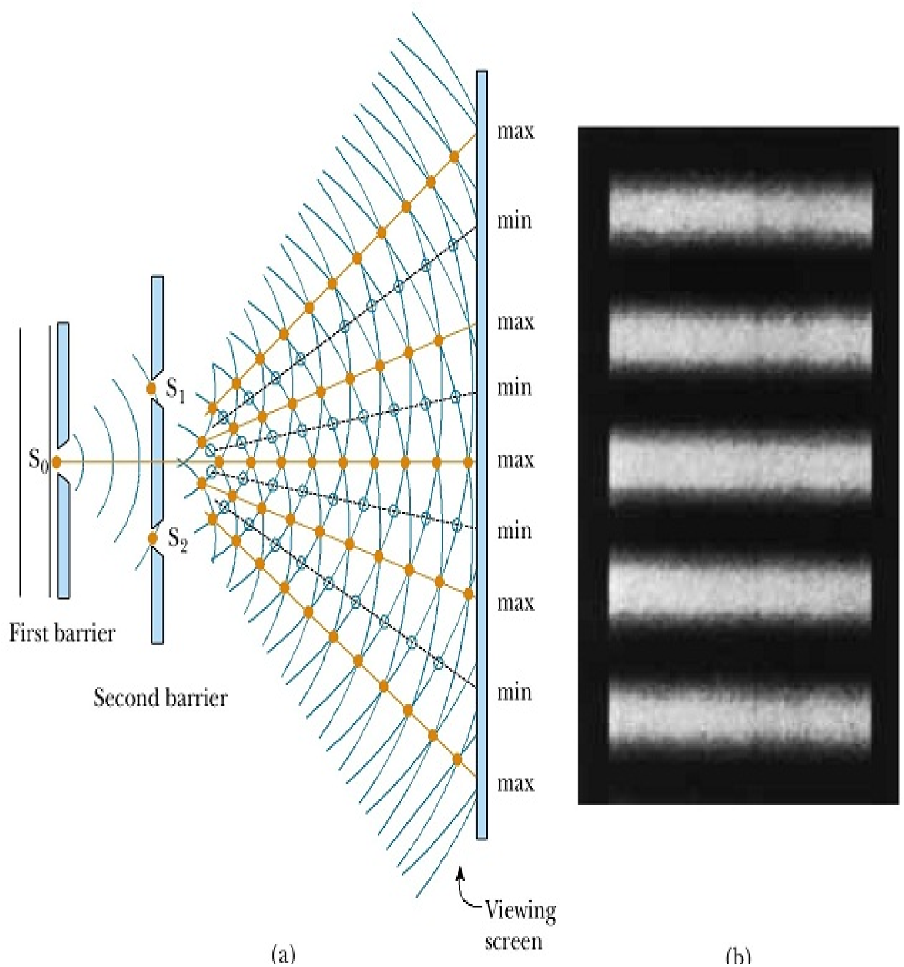


What if Light Had Wave Properties?

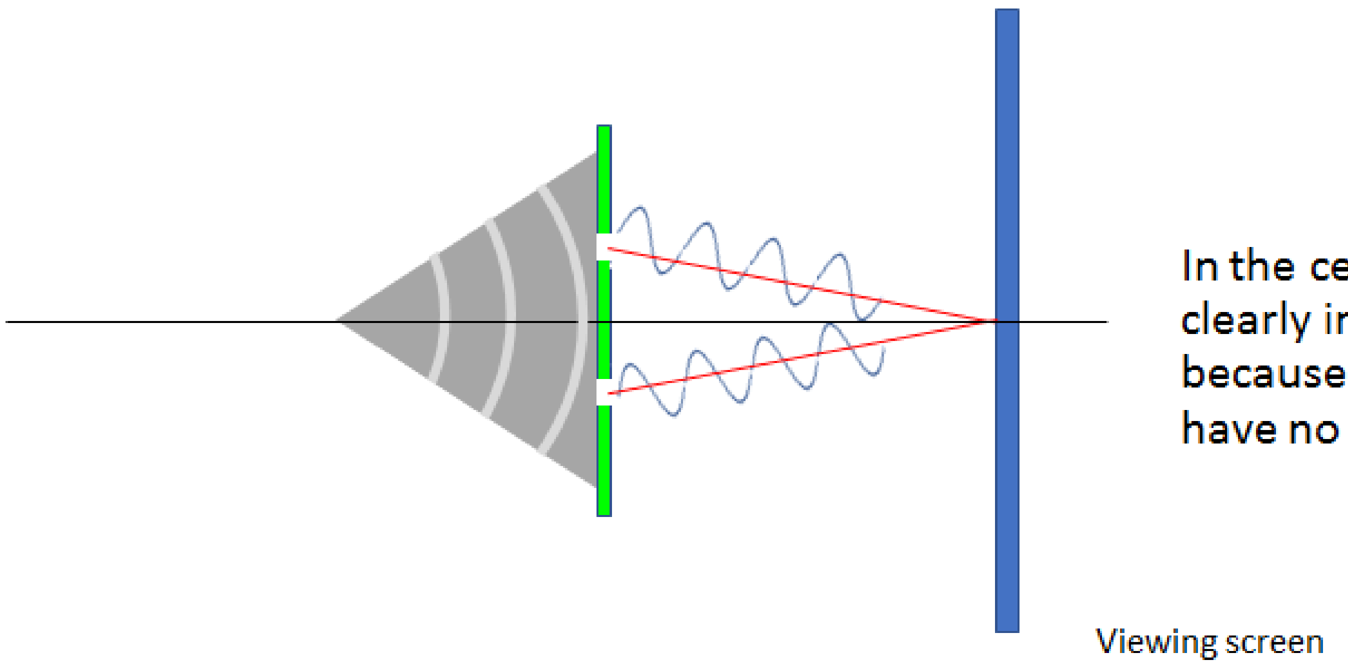
If light had wave properties then we would observe diffraction as we pass through the slits. We would then be able to see an interference pattern (constructive and desctructive) as the diffracted waves meet each other.



Young's Double-slit Experiment

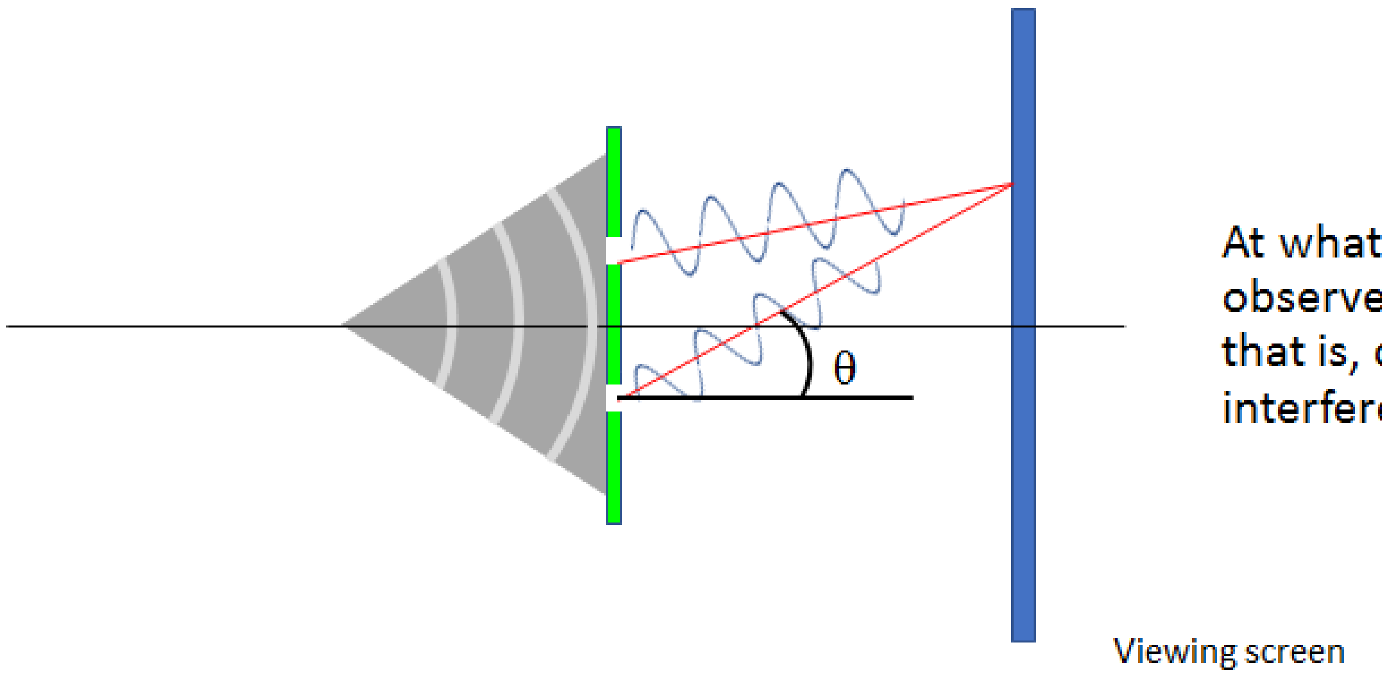


Mathematical Relationship



In the centre line there's clearly intensity maxima because the two waves have no phase difference.

Mathematical Relationship

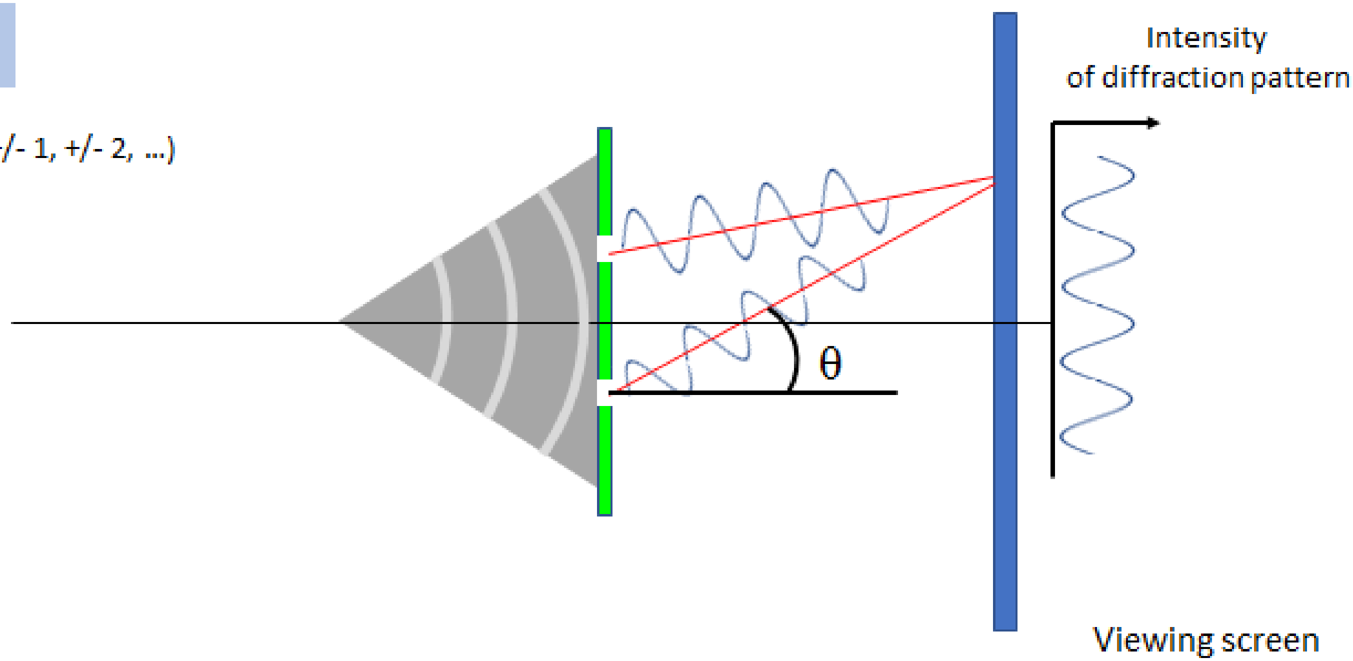


At what angles θ we observe intensity maxima, that is, construtive interference?

Mathematical Relationship

$$d\sin\theta = m\lambda$$

where m is the spectral order (+/- 1, +/- 2, ...)

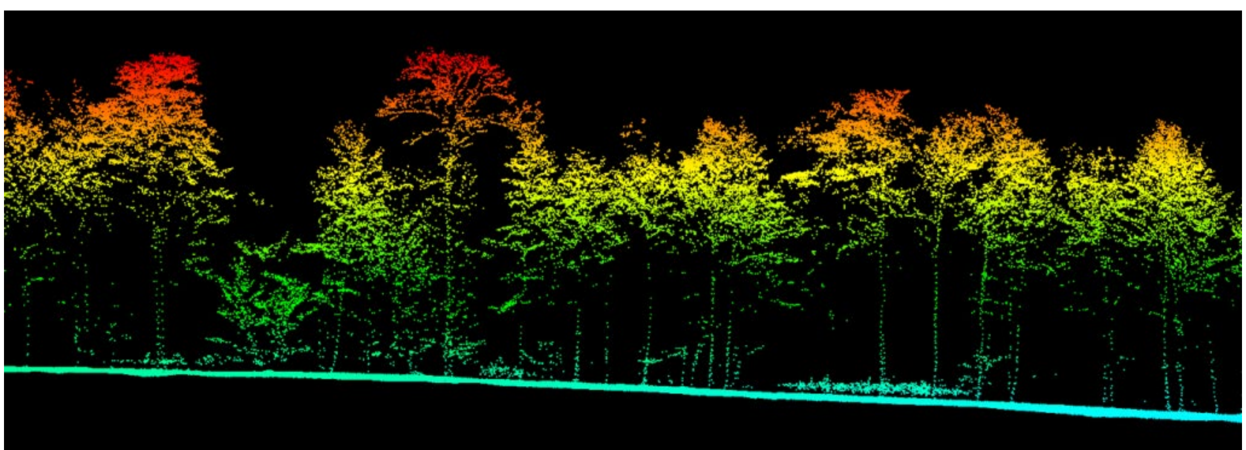


Young's Double-slit Experiment

Central Mystery of Quantum Mechanics

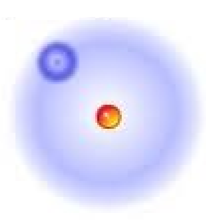
https://www.youtube.com/watch?v=A9tKncAdlHQ

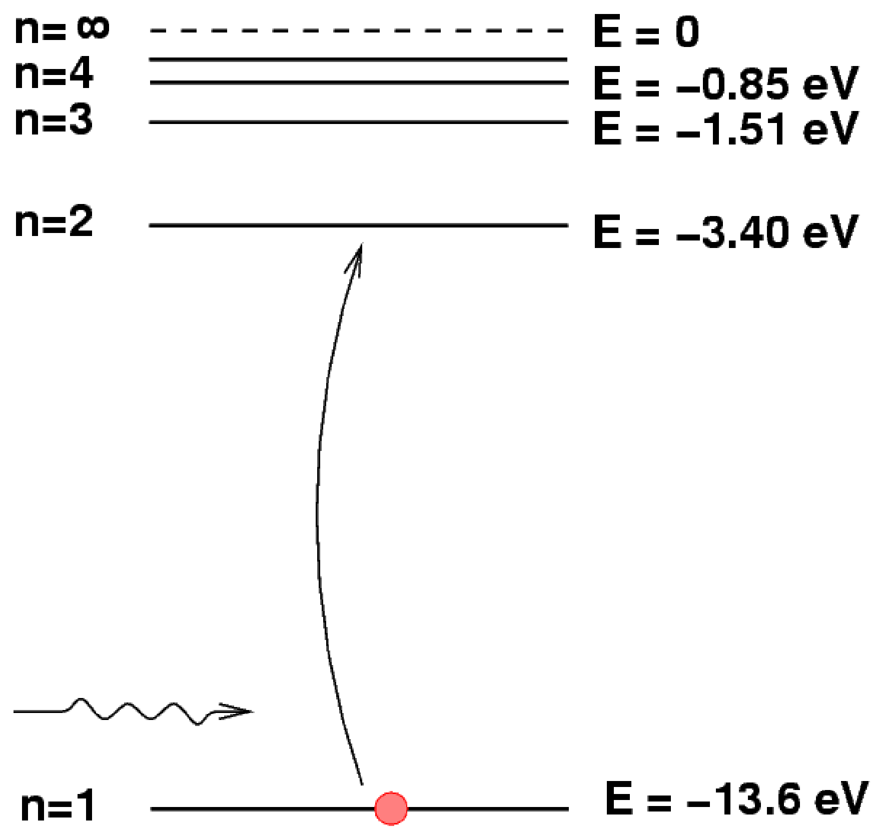
Optical Spectroscopy –
Studying Interaction between Light and Matter



Absorption of Ultraviolet/Visible Light in Atoms

When the energy (frequency) of photons matches the energy level difference of the atom's electrons, photons can interact with the particular atom.





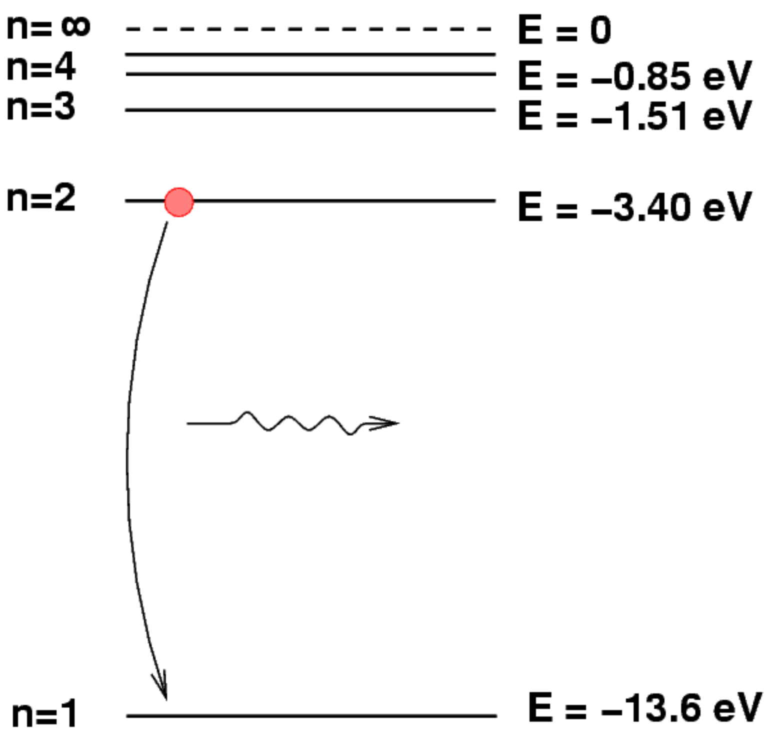
$E_p = hc/\lambda$

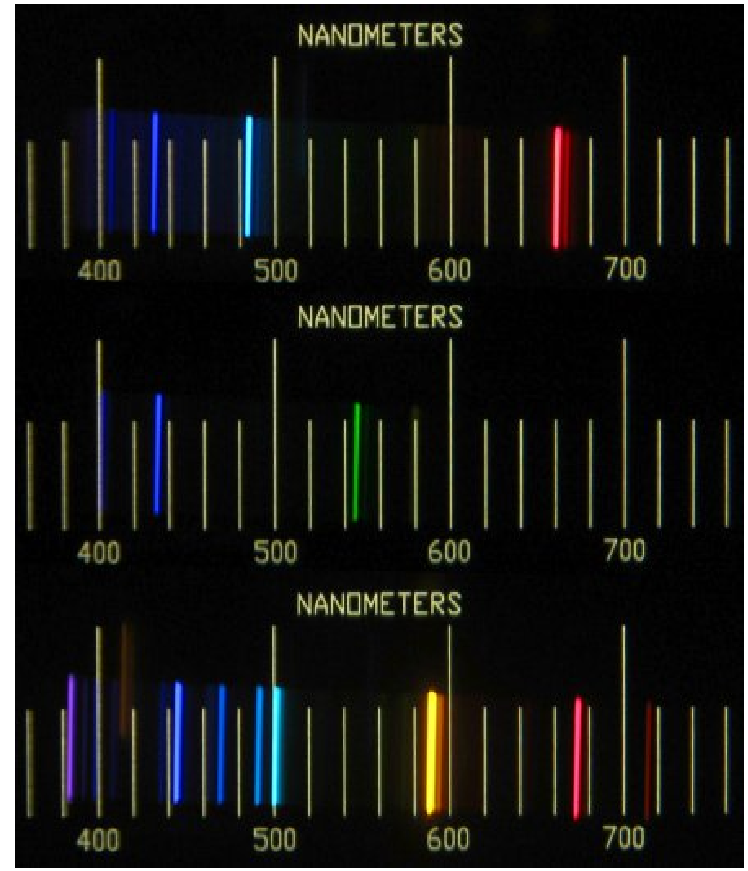
$E_p \sim 1.24$ eV μm / λ (μm)

$E_{1\text{-}2} = 10.2$ eV corresponds to $\lambda = 121$ nm

$E_{2\text{-}3} = 1.89$ eV corresponds to $\lambda = 656$ nm

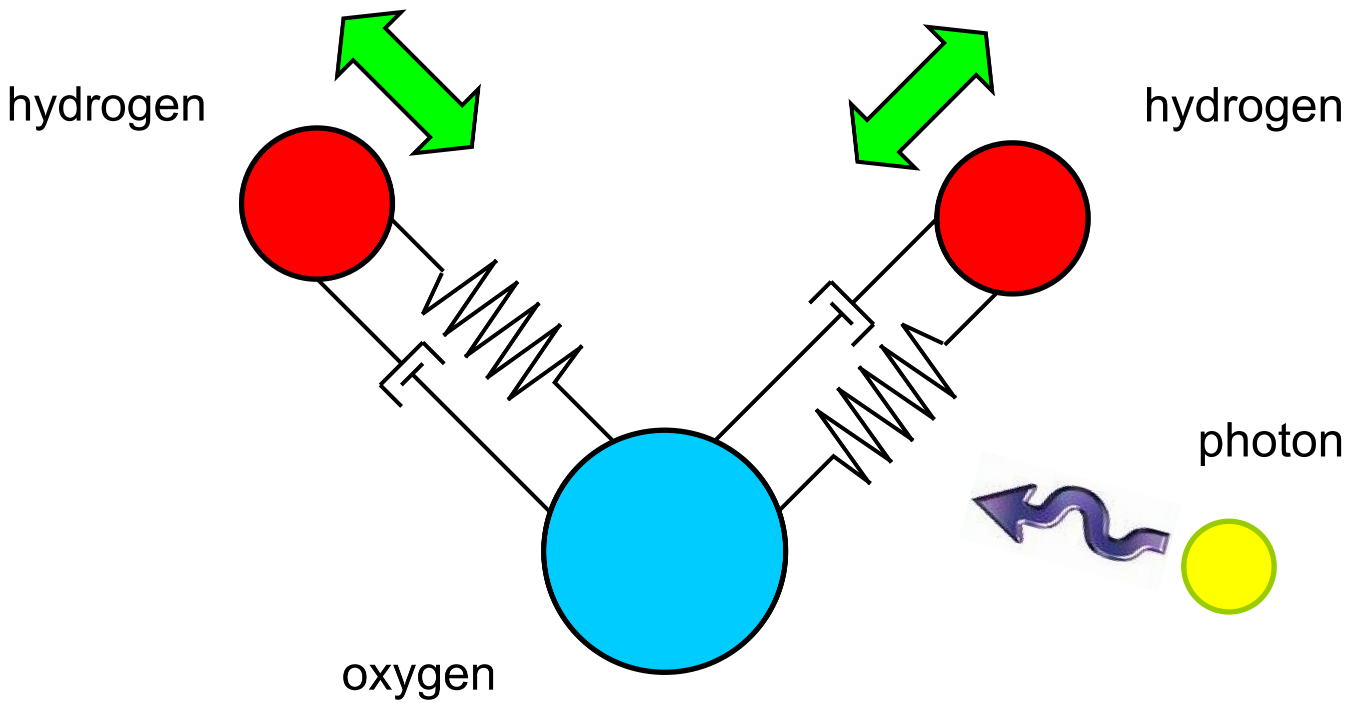
Emission of Light from Atoms





Absorption of Infrared Light in Molecules – Vibration and Rotation

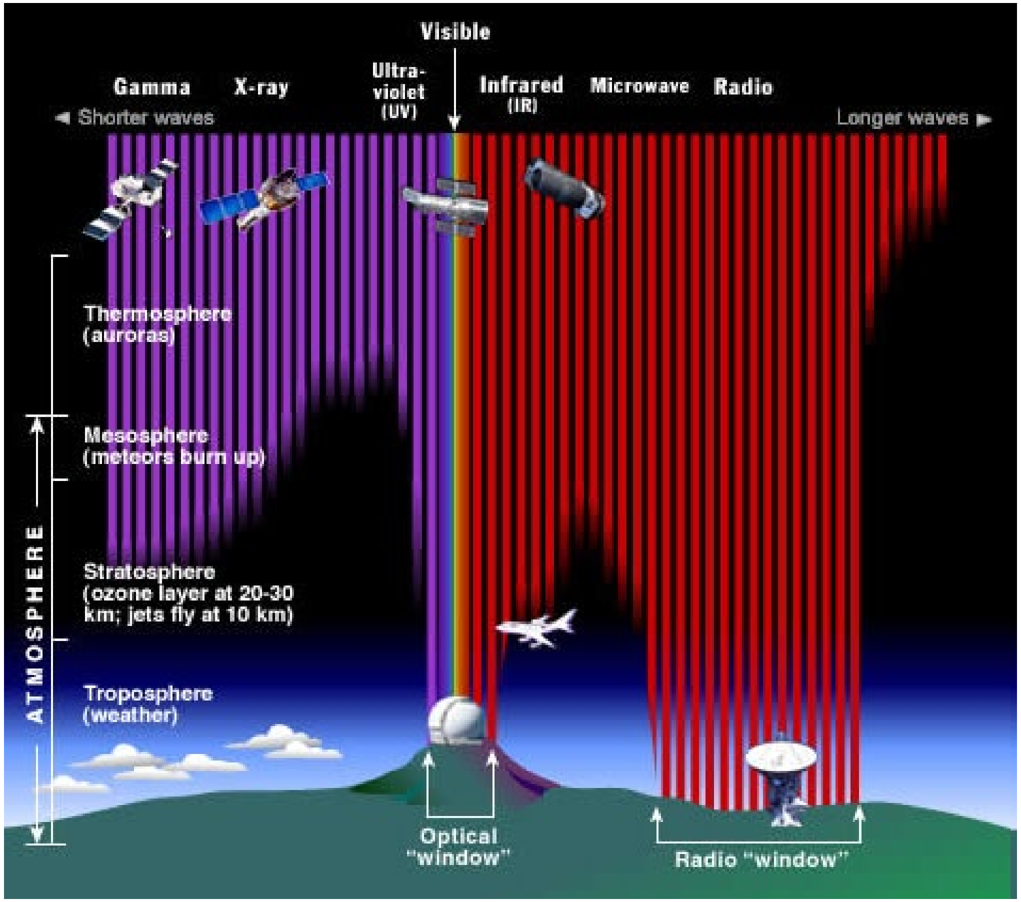
Mickey Mouse model of water molecule

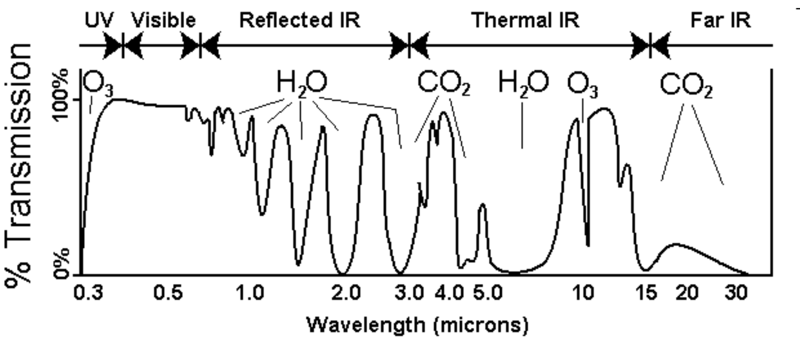


When the energy (frequency) of photons matches the quantised vibrational and/or rotational energy level differerences of the molecule, photon will interact with that molecule.

In absorption, the photon's energy (hv) get's converted into the molecules' electronic, vibrational or rotational energy.

Absorption of EM Radiation in the Atmosphere





For each molecule there are chracteristic energies/wavelengths that get absorbed – this is the foundation of optical spectroscopy.

Blackbody Radiation – Continuous Emission Spectrum

Max Planck's theory of blackbody radiation in the year 1900 started the development of quantum theory and allowed several fundamental predictions:

- definition of Avogadro's number
- size of atoms
- charge of electrons
- mass of electrons

Emission and Absorption of Light