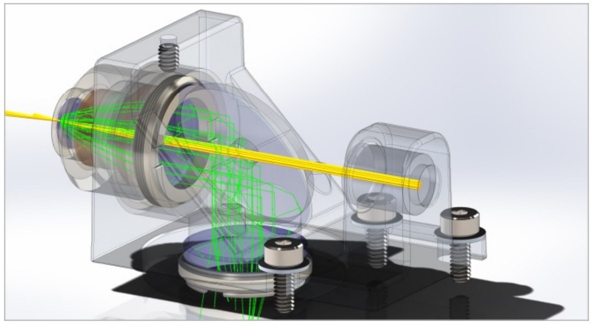
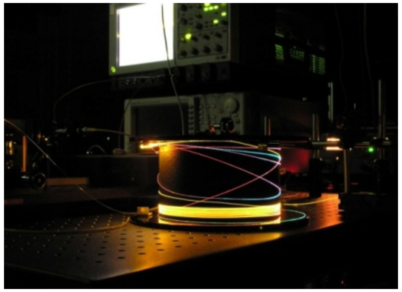
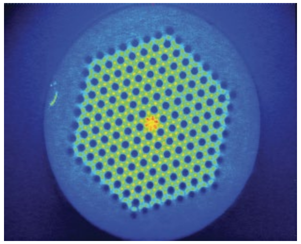
Optics

ELEC E-5730 Spring 2020

Radiometry

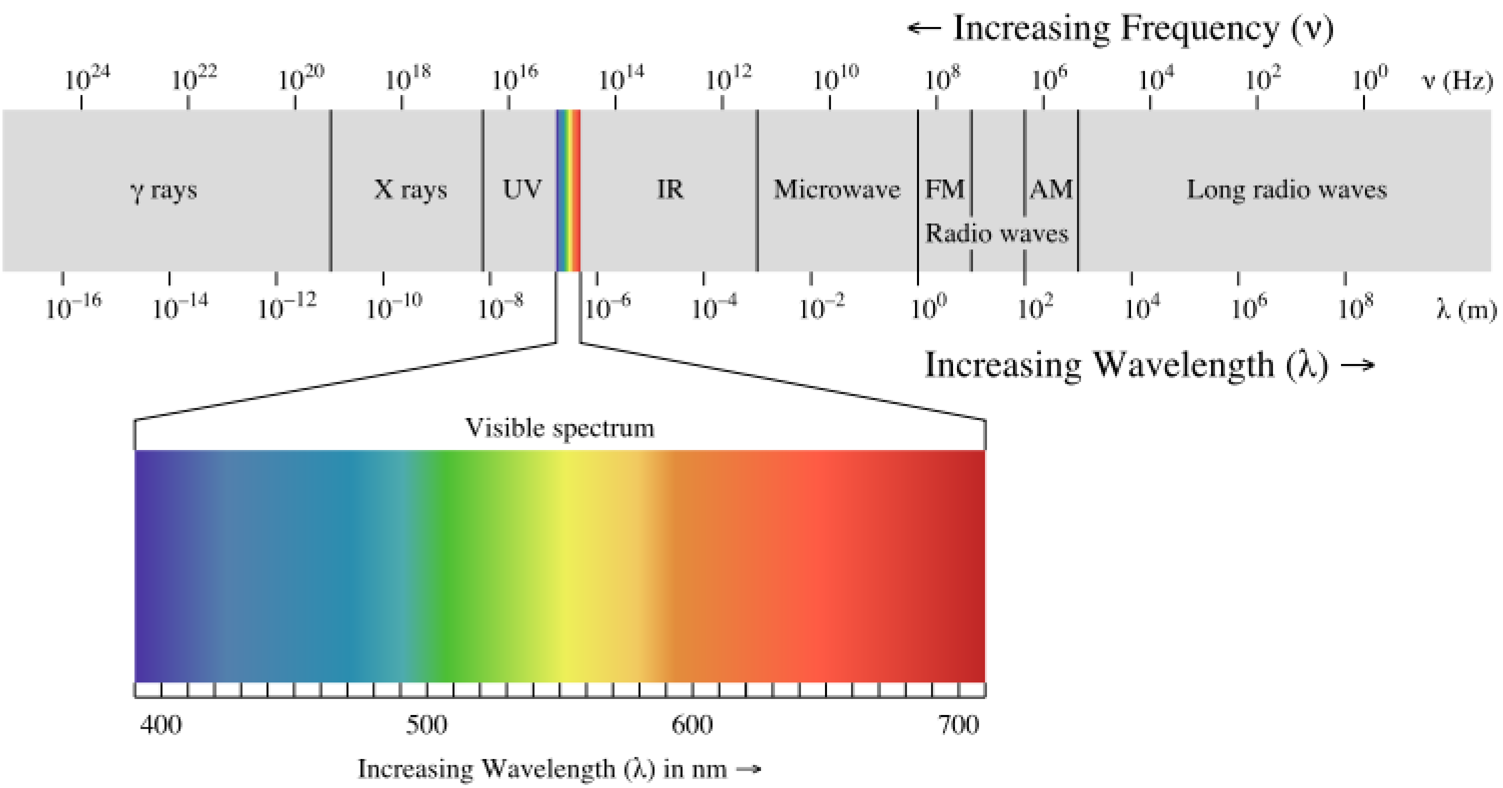




Radiometry

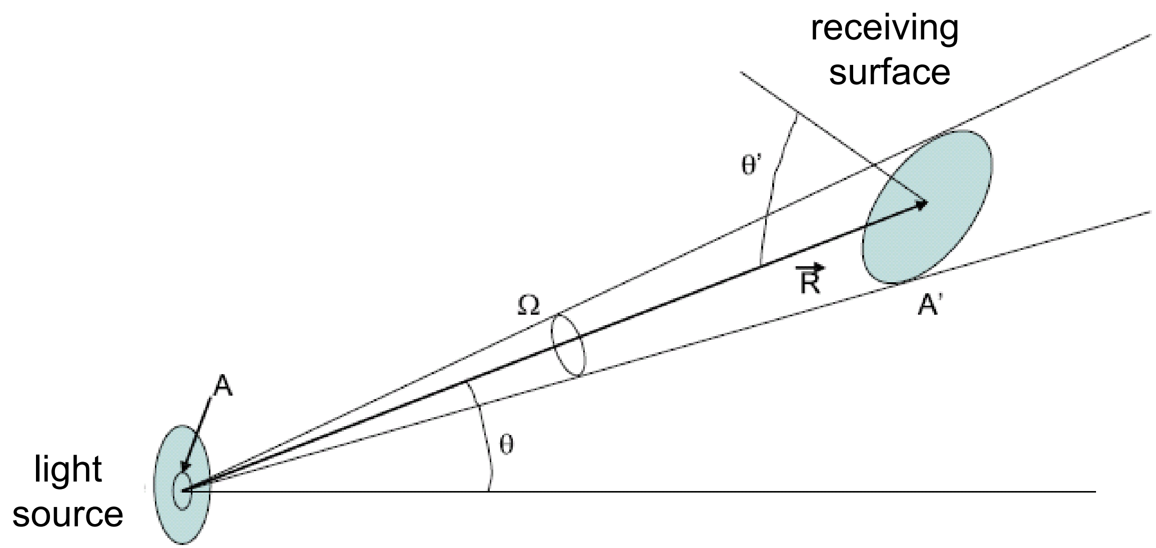
Radiometry is the science of measurement of radiant energy, including light, in terms of absolute power.

Electromagnetic radiation



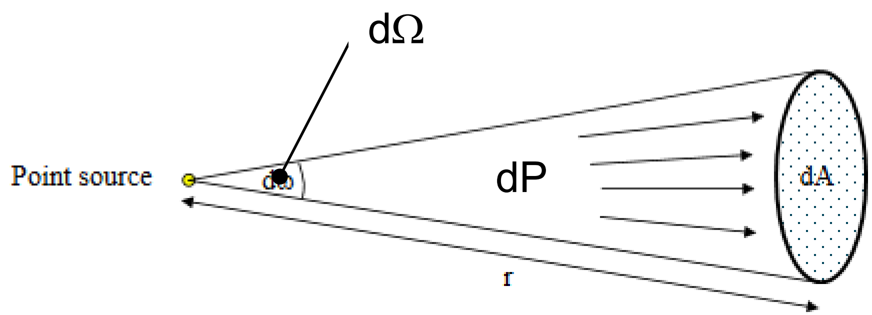
Fundamentals of radiometry

- Solid angle to direction **R** $\Omega = A' / R^2$
- Total optical power *P* or total radiant flux Φ_e
- Light source's radiant intensity $I_e = P / \Omega$
- Light source's radiance $L_e = P_A / (\Omega\ A \cos\theta) = I_e / (A \cos\theta)$
- Irradiance at the receiving location caused by a light source $E_e = P_A / A'$



Irradience *E* caused by a point source having optical power P_0

$$E = \frac{dP}{dA}$$



$$dP = P_0 \frac{d\Omega}{4\pi} = P_0 \frac{1}{4\pi} \frac{dA}{r^2}$$

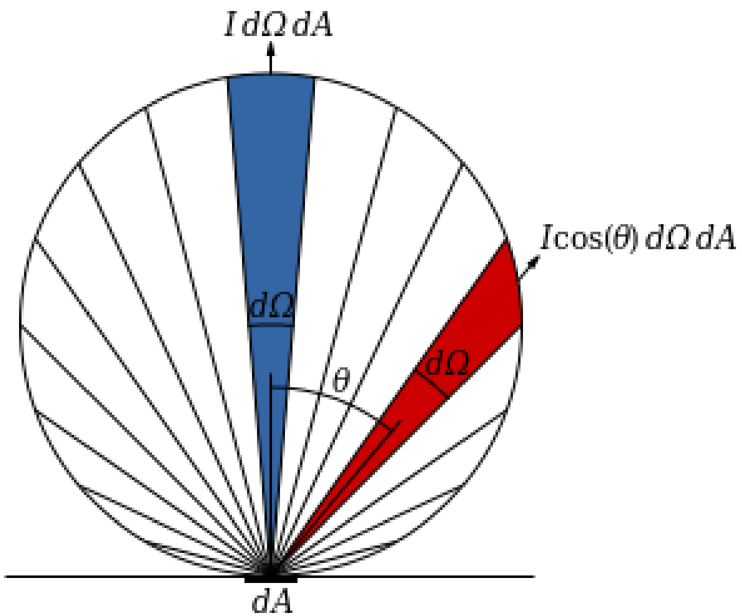
$$E = \frac{P_0}{4\pi r^2}$$

Corresponding spectral quantities are denoted by the subscript e or λ:

$$E_e = E_\lambda = \frac{dE}{d\lambda} \qquad \left[\frac{W}{nm\, m^2}\right]$$

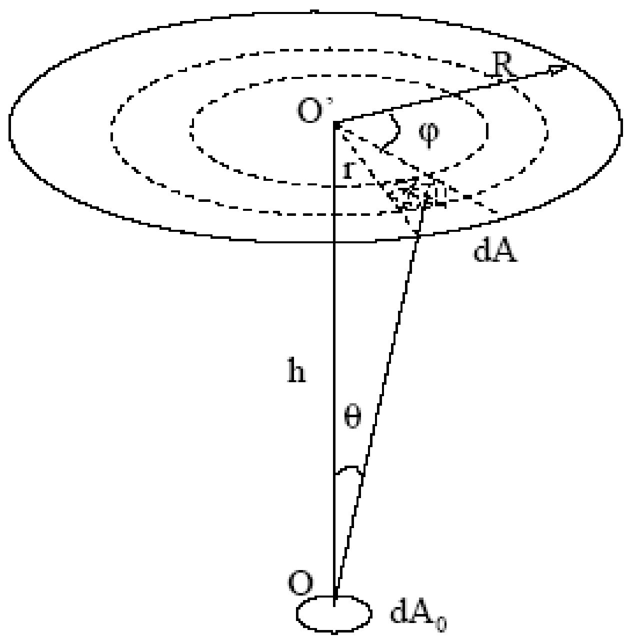
Fundamentals of radiometry

- **Inverse square law**: irradiance caused by a distant source decreases as the distance squared ($E = P_0 / 4\pi r^2$)
- **Lambertian source**: radiance is constant irrespective of the observation angle or Lambertian source's optical power to direction θ is $P_e = P_e(0) \cos \theta$.



Example in the Exercises

Calculate the irradiance at O due to a disk source at O’.



Photometry

Photometry is the measurement of power or energy of electromagnetic radiation, including light, weighted by the human eye's response.

Photometry

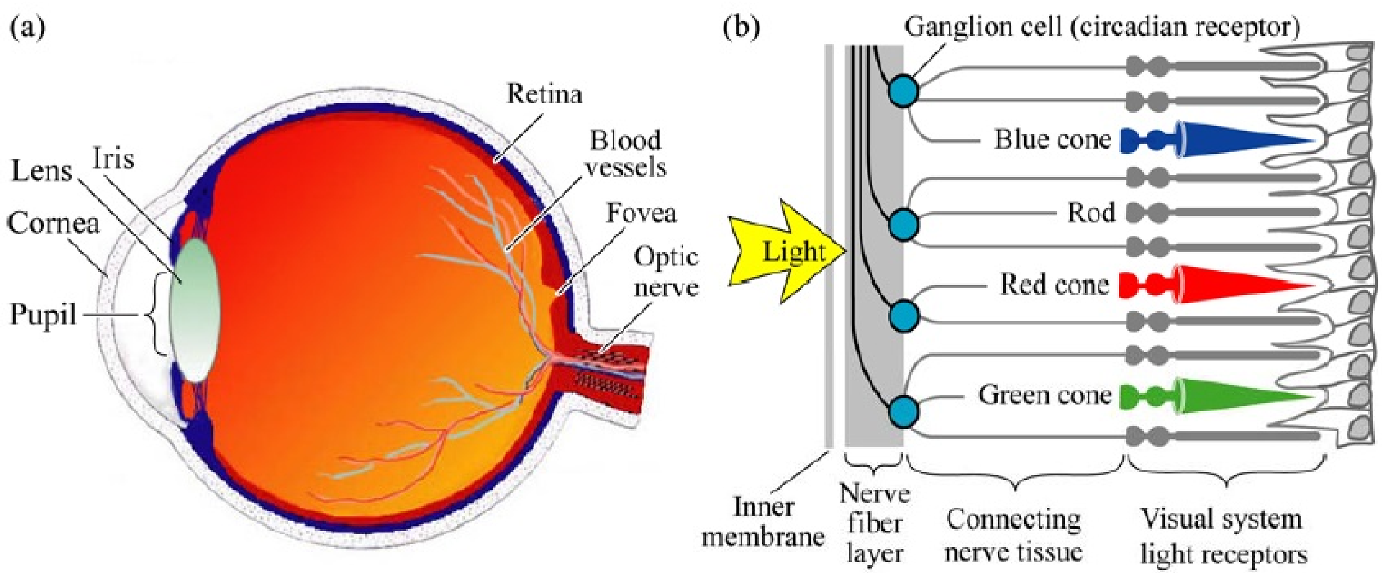
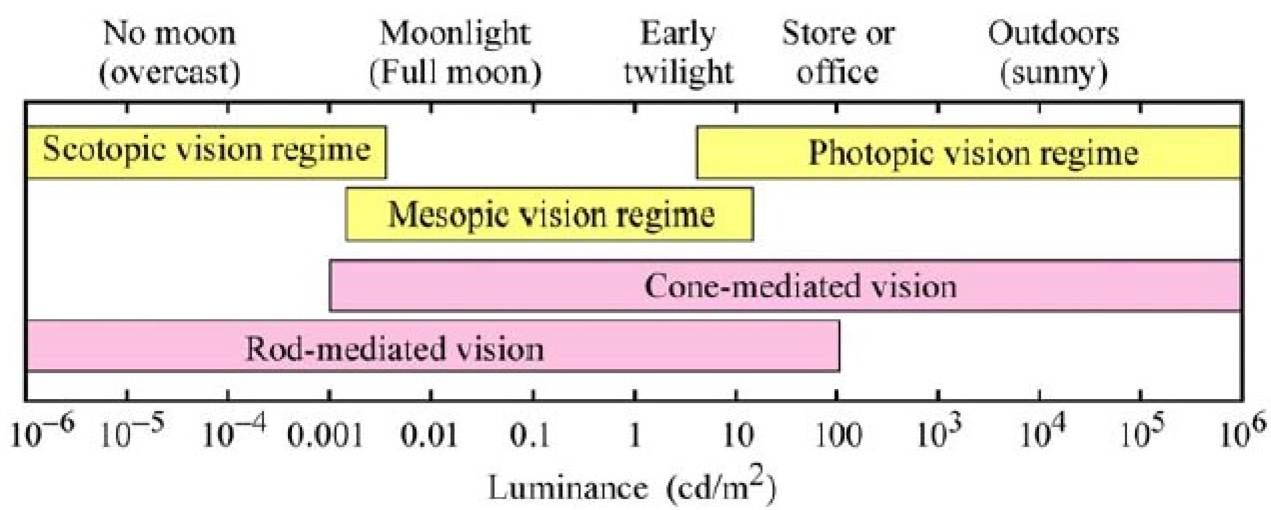
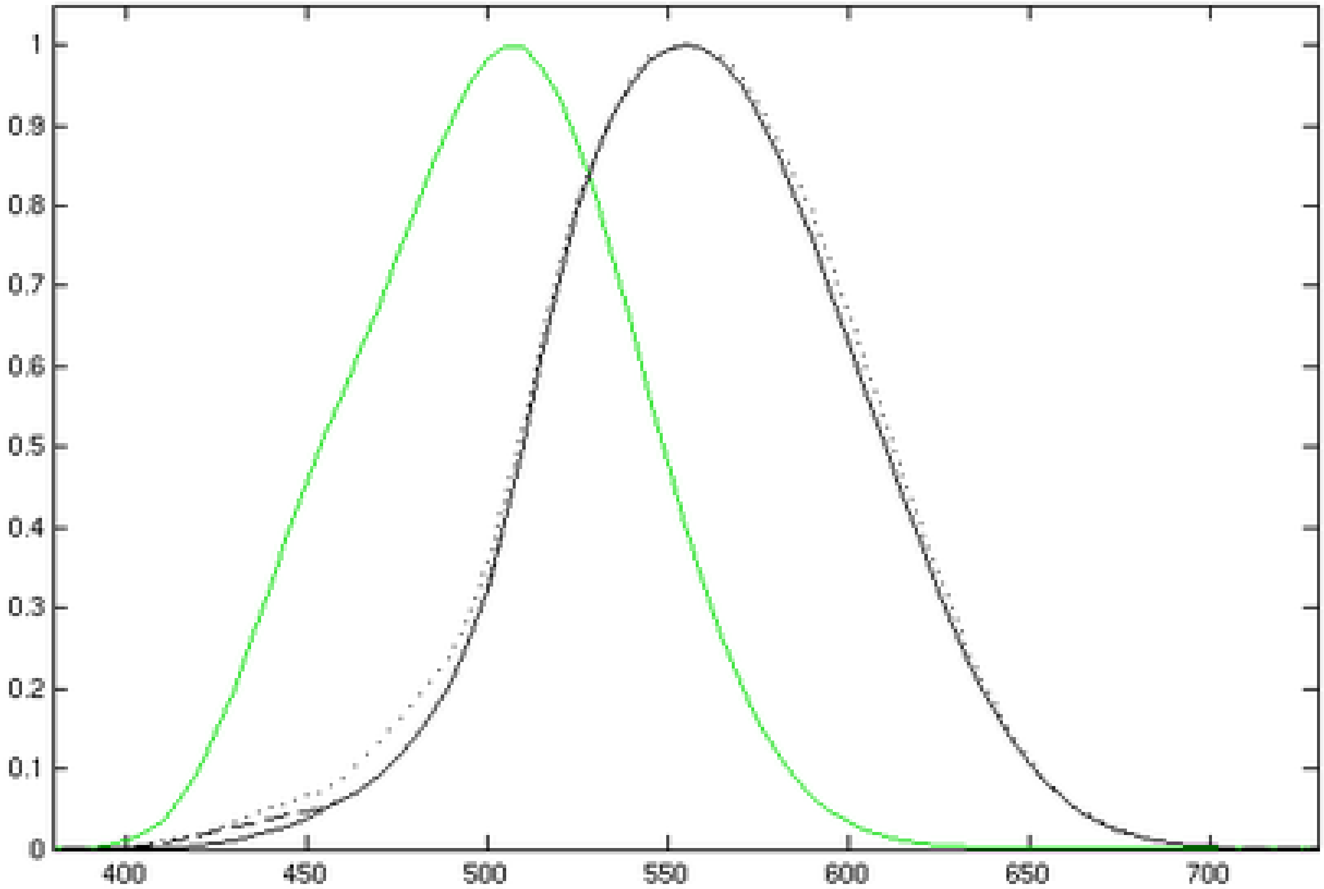


Fig. 16.1. (a) Cross section through a human eye. (b) Schematic view of the retina including rod and cone light receptors (adapted from Encyclopedia Britannica, 1994).



Photometry

Photopic (daytime-adapted, black curve) and scotopic (darkness-adapted, green curve) spectral luminosity functions V.



Photometry

It would be possible to use the same units for photometry and radiometry. For historical reasons, however, a coefficient K_m = 683 lm/W is used with photometric quantities to convert radiometric watt (W) to photometric lumen (lm).

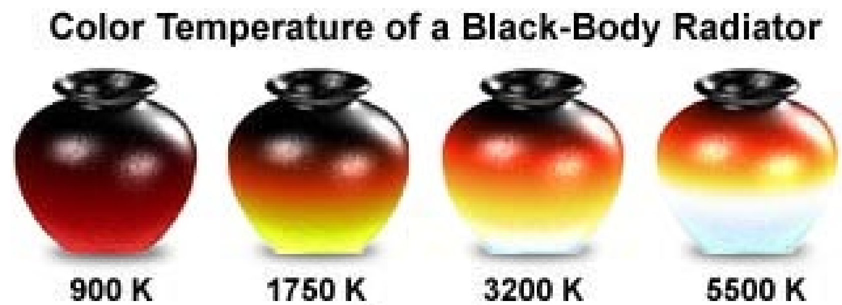
For example, the radiometric radiant intensity I_e can be converted into the corresponding photometric quantity, luminous intensity I_v in the form

$$I_v = K_m \int V(\lambda) I_e(\lambda) d\lambda$$

Radiometric and fotometric units

Spectroradiometry ($X_e(\lambda)$)	**Radiometry (X_e)** $X_e = \int_0^\infty X_e(\lambda) d\lambda$	**Photometry (X_v)**
Spectral intensity $I_e(\lambda)$ ($W \cdot sr^{-1} \cdot nm^{-1}$)	Radiant intensity I_e ($W \cdot sr^{-1}$)	Luminous intensity I_v (cd)
Spectral irradiance $E_e(\lambda)$ ($W \cdot m^{-2} \cdot nm^{-1}$)	Irradiance E_e ($W \cdot m^{-2}$)	Illuminance E_v (lx)
Spectral flux $\Phi_e(\lambda)$ ($W \cdot nm^{-1}$)	Radiant flux Φ_e (W)	Luminous flux Φ_v (lm)
Spectral radiance $L_e(\lambda)$ ($W \cdot sr^{-1} \cdot m^{-2} \cdot nm^{-1}$)	Radiance L_e ($W \cdot sr^{-1} \cdot m^{-2}$)	Luminance L_v (cd/m^2)
Spectral energy $Q_e(\lambda)$ ($J \cdot nm^{-1}$)	Radiant energy Q_e (J)	Luminous energy Q_v ($lm \cdot s$)

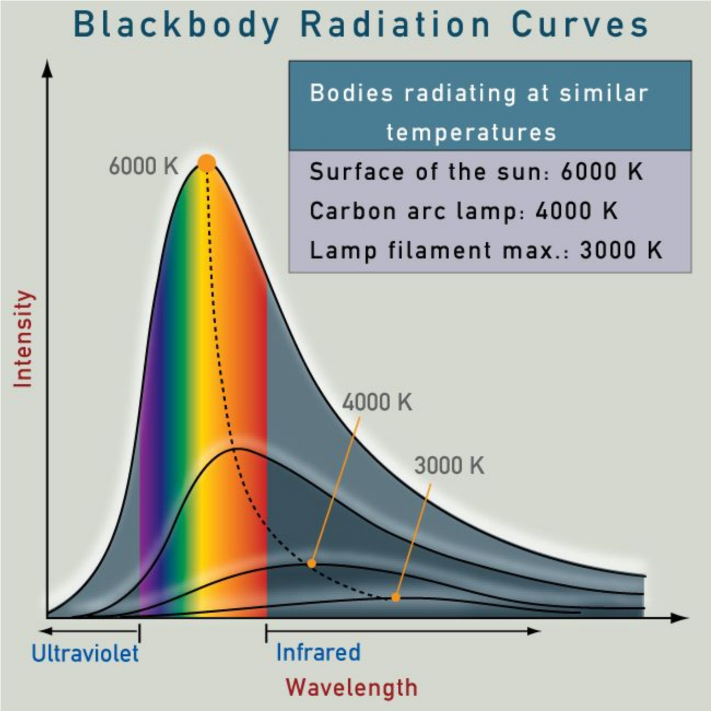
Blackbody radiation – continuous emission spectrum from solid objects



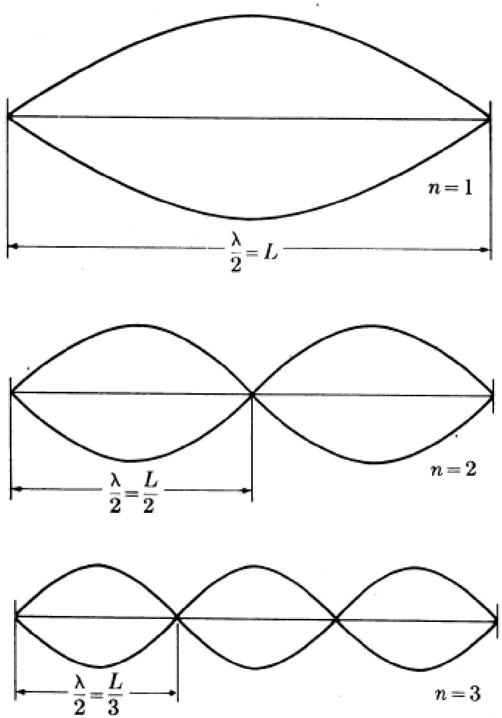
Blackbody radiation – Planck's distribution function

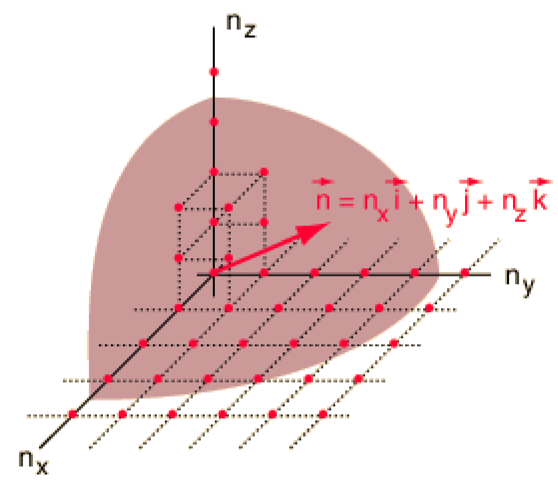
Max Planck's theory on blackbody radiation (year 1900) started the development of Quantum Theory and allowed the following important predictions:

- definition of Avogadro's number
- size of atoms
- charge of electrons
- mass of electrons



Derivation of Planck's energy density function 1/3





It can be shown that the density of electromagnetic modes per volume for frequencies from ν to dν is

$$\mathrm{d}N = \frac{8\pi\nu^2}{c^3}\,\mathrm{d}\nu$$

Derivation of Planck's energy density function 2/3

Energy density of radiation per volume and per unit frequency:

$$\mathrm{d}u = u(\nu)\,\mathrm{d}\nu = \frac{8\pi\nu^2}{c^3}\,\overline{E}\,\mathrm{d}\nu$$

The Rayleigh-Jeans Law

Classical physics $\overline{E} = kT$

$$u(\nu) = \frac{8\pi\nu^2 kT}{c^3}$$

The Ultraviolet Catastrophe

According to classical physics, the energy density of black-body radiation diverges at high frequencies

$$\int_0^\infty \frac{8\pi\nu^2 kT}{c^3}\,\mathrm{d}\nu \to \infty.$$

Derivation of Planck's energy density function 3/3

Energy density of radiation per volume and per unit frequency:

$$du = u(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\,\overline{E}\,d\nu$$

Revolutionary proposal by Planck E = hν

$$\overline{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

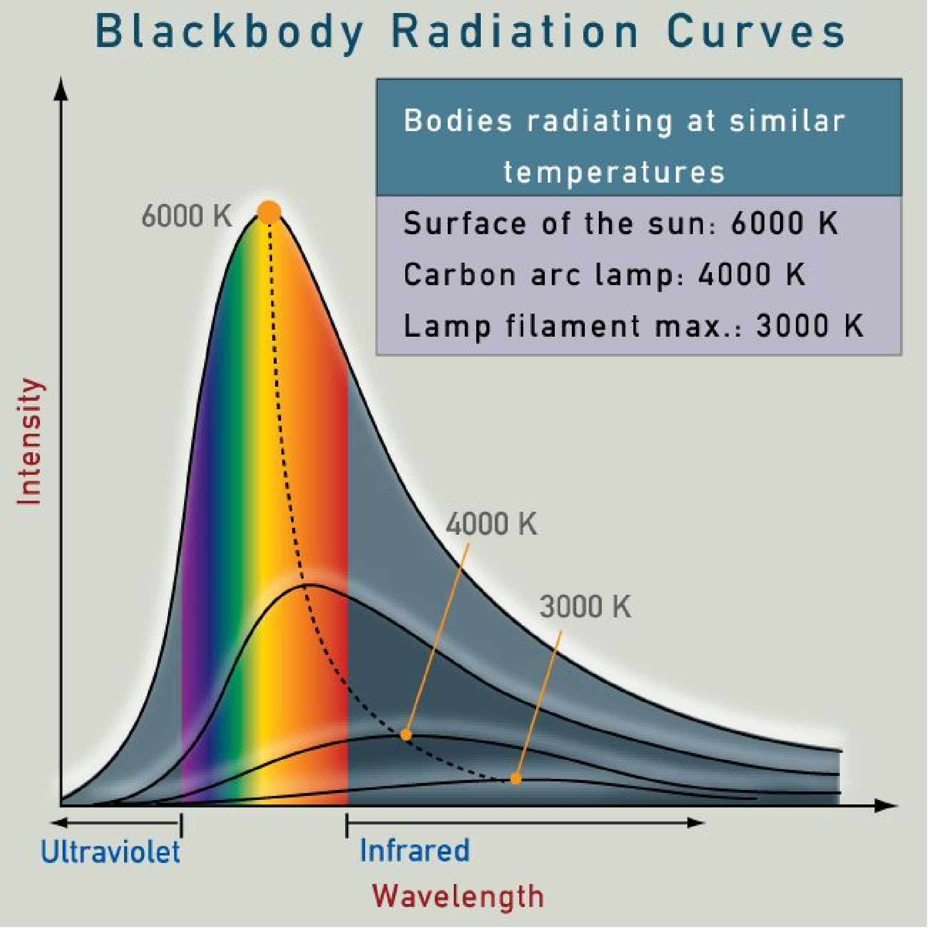
$$p(n) = \frac{\exp(-E_n/kT)}{\sum_{n=0}^{\infty}\exp(-E_n/kT)}$$

$$\overline{E_\nu} = \sum_{n=0}^{\infty} E_n\,p(n) = \frac{\sum_{n=0}^{\infty} E_n\,\exp(-E_n/kT)}{\sum_{n=0}^{\infty}\exp(-E_n/kT)}$$

$$= \frac{\sum_{n=0}^{\infty} nh\nu\,\exp(-nh\nu/kT)}{\sum_{n=0}^{\infty}\exp(-nh\nu/kT)}$$

Planck's spectral energy density function

$$u(\nu)\,\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(h\nu/kT\right)-1}\,\mathrm{d}\nu$$



Spectral radiance of a blackbody

It can be shown that the spectral radiance (W / (sr·m²·nm)) from a small opening in a blackbody cavity is:

$$L_e(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Radiance of a blackbody

$$L_e(\lambda) = \frac{2hc^2c}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Wien's displacement law $\lambda_{max}T \approx 3000\ \mu m \cdot K$

Stefan-Boltzmann law gives the total radiance $L = \int_0^\infty L_e(\lambda) = \frac{\sigma}{\pi}T^4$

where σ is Stefan-Boltzmann's constant ≈ 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

Applications of blackbody radiation measurement

Important applications include **remote optical temperature measurement and thermal imaging**:

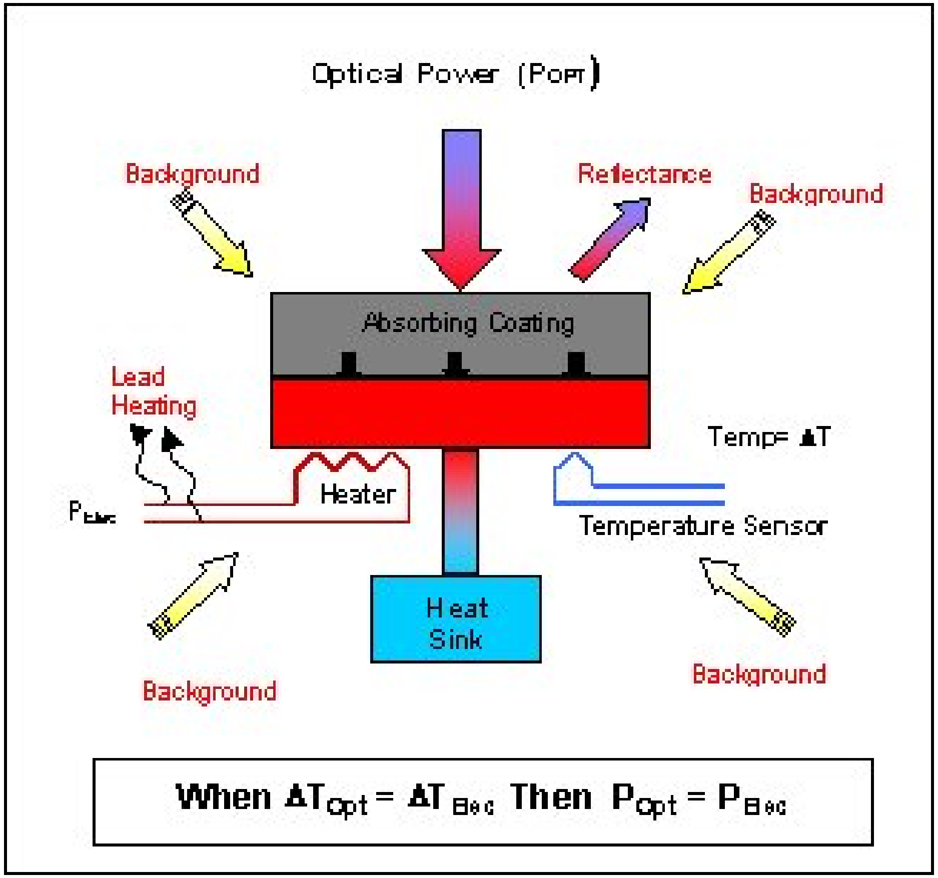
Primary standards in radiometry

Measurement of optical power can be either based on the known spectral radiance of the light source or the known spectral responsivity of the detector. The most significant primary standard in the history of radiometry is the **blackbody radiator** obeying Planck's spectral distribution function.

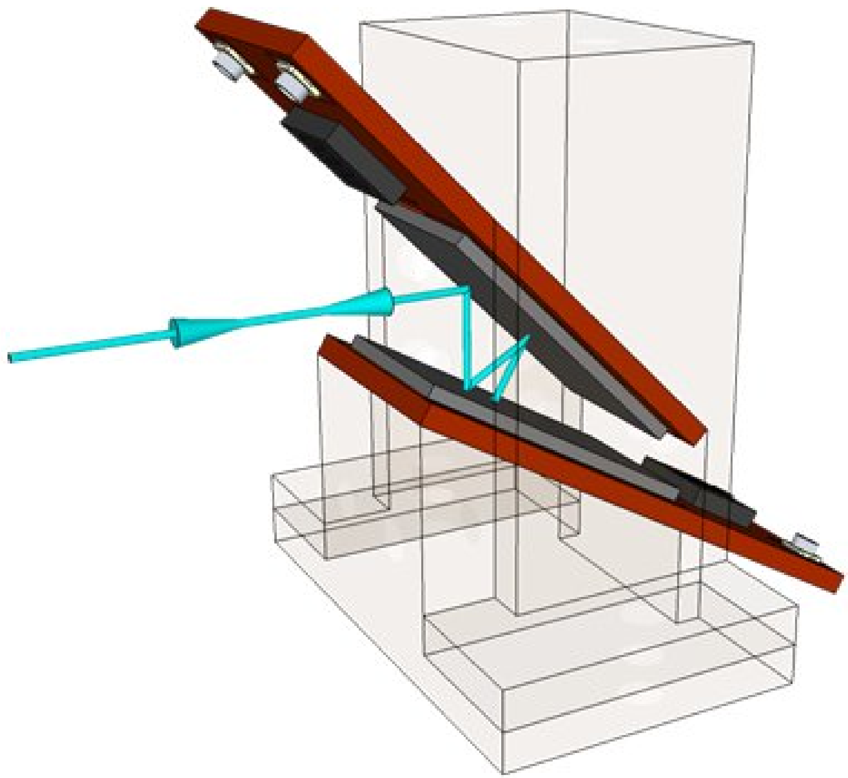
Newer detector-based primary standards include the **cryogenic radiometer** and **semiconductor detectors** with precisely known spectral responsivities.

Cryogenic radiometer

Based on the principal of electrical substitution radiometry (ESR), a thermometer is used to accurately measure the temperature rise of a thermal detector, relative to a constant temperature heat sink, during alternate optical and electrical heating cycles. By adjusting the electrical power so that the detector temperature rise is the same for both types of heating, the optical power can be considered equivalent to the electrical (heating) power.



Semiconductor trap-detector



Si photodiode's spectral sensitivity is about 30% less than for an ideal photodiode mainly due to reflections from the air-Si interface (n_{Si} = 4.3 @ 500 nm).

Using multiple photodiodes (so called trap-detector) in series, however, the amount of reflected light can be made very small. In the above figure with 7 reflections the relative amount of back-reflected light is $0.3^7 = 2.2 \times 10^{-4}$.