# E-5730 Optics
# Exercise I January 18 2021

The exercises will be held on Mondays at 10-12 am. There will be six exercise sessions in total. The exercises take place at https://aalto.zoom.us/j/5703080612 and supervisor is Mikhail Korpusenko.

It is recommended that the students try to solve all the questions in advance. The demonstration questions (Q) will be discussed and solved together during the exercises.

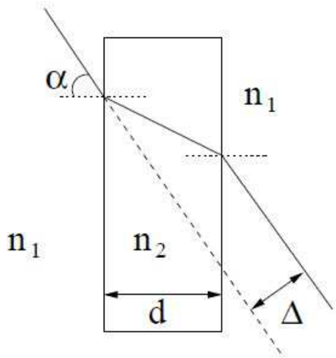
Each question sheet also includes three Homework Questions (HQ) which the participating students are expected to solve in advance and return the written answers to the exercise supervisor before the beginning of the exercise session. The supervisor will mark the returned HQ solutions. By solving the Homework Questions one can earn maximum 9 extra points contributing to the total mark of the course. There will also be a 30-minute mid-term exam where it is possible to earn maximum 6 points.

The total mark of the course (max 45 points) is based on the following:

- written online exam (max 30 points)
- homework (HQ) questions (max 9 points)
- mid-term online exam (max 6 points)

**Question Q1.** A ray of light hits the surface normal of a uniform glass plate at $\theta \ll 1$ rad. Show that the parallel shift caused by the glass plate is

$$\Delta \approx \alpha d \frac{n_2 - n_1}{n_2}$$




where $d$ is the thickness of the glass plate, $n_2$ refractive index of the glass and ja $n_1$ refractive index of the surrounding medium (see schematic on the right).

**Question Q2.** Show that the transformation matrix $M$ ($M = R_2 T_{12} R_1$, see Eq. 1.22 in the book) for a simple lens is

$$M = \begin{pmatrix} 1 - \frac{P_1 t_c}{n_2} & \frac{n_1 t_c}{n_2} \\ \frac{P_1 P_2 t_c}{n_2 n_1} - \frac{P_1}{n_1} - \frac{P_2}{n_1} & 1 - \frac{P_2 t_c}{n_2} \end{pmatrix}$$

where $t_c$ is the lens thickness, $n_2$ refractive index of the lens, $n_1$ refractive index of the surrounding medium and $P_i$ is the power of the surface i=1,2 (Eq. 1.15).

**Question Q3.** Starting from the transformation matrix of Question 2, show that for a thin lens ($t_c = 0$) the matrix becomes

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

where $f = (n_2/n_1 - 1)^{-1} r_1 r_2/(r_2 - r_1)$.

**Homework Question HQ1.** (Return by January 18 2021) Show that the laws of reflection and refraction (Snell's law) can be derived from the Fermat principle.

{Hint: Calculate the time for a light ray to travel from point A to B. For refraction this means crossing an interface between two media. The distances from the interface are $h_A$ and $h_B$, respectively. Find the minimum of the travel time as a function of the coordinate $x$}.

**Homework Question HQ2.** (Return by January 18 2021) Using the transformation matrices for a thin lens and for free space propagation, show that all paraxial rays ($\alpha_1$=0) will focus at a distance $f$ behind the lens.

**Homework Question HQ3.** (Return by January 18 2021) You have got an industrial production process where liquid ethanol is produced at +20 deg C temperature. Sometimes in the process, however, water appears as a contaminant. Your task is to design an optical online sensor to detect if there is water in the ethanol flow. The sensor is based on the measurement of the refractive index of the process liquid.

The process pipeline has 100 mm internal diameter and you can have high quality optical windows (refractive index 1.5) installed on both sides of the pipeline. The windows can be either flat 5 mm thick windows or they can be prism/semi-circular shaped glass blocks so your light source can enter the glass window at right angle from air. **a)** Sketch at least one design showing how you can determine the refractive index of the liquid flowing in the pipeline. Explain the principle of your measurement.

Assuming that the refractive index of the process liquid is a linear combination of the constituent refractive indices

$$n_{liquid} = X_{ethanol} n_{ethanol} + X_{water} n_{water}$$

where $X_i$ are the volume fractions (0-1) of ethanol and water. At 589 nm wavelength and at +20 deg C temperature the refractive indices for water and ethanol are $n_{water}$ = 1.3330 and $n_{ethanol}$ = 1.3615, respectively. **b)** Estimate what is the minimum amount of water ('limit of detection') expressed in vol-% you expect to detect with your sensor design?

{Hint: Regarding part b) and some potential solutions it may be useful to know that there exist motorised rotation stages with 5 arcmin angular resolution or, on the other hand, linear one-dimensional CCD (charge-coupled device) sensor arrays exist where one light detector element ("pixel") has a size of 10 µm * 10 µm. The total number of pixels in such 1D CCD arrays is typically 500-2000.}