

E-5730 Optics

Exercise III Feb 1 2021

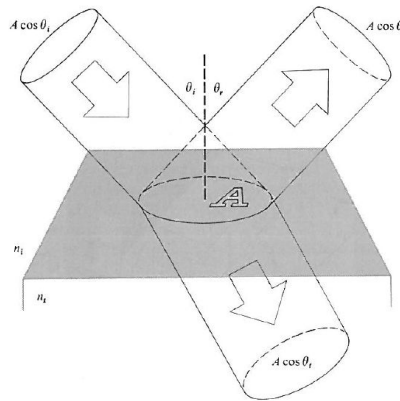
Question Q1. Show that the reflectance R and transmittance T for light intensity hitting the boundary between two materials (refractive indexes n_1 and n_2) at normal incidence ($\theta_i=0$) are

$$R = R_{\parallel} = R_{\perp} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

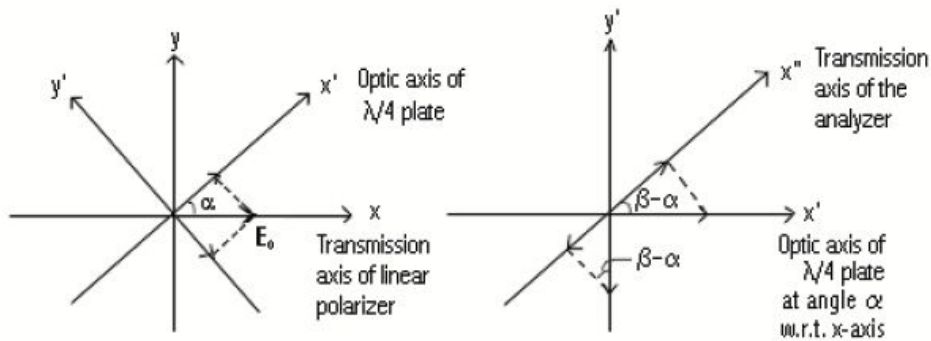
$$T = T_{\parallel} = T_{\perp} = \frac{4n_1n_2}{(n_1 + n_2)^2}$$

Show also that $R+T=1$.

Question Q2. Show that if the changes in the cross-sectional area of a light beam are taken into account, the total intensity is conserved upon oblique crossing of the material boundary (see figure).



Question Q3. Derive the intensity transmission function for a system consisting of a linear polarizer, a quarter wave-plate and second linear polarizer. The angle between the transmission axis of the first polarizer and the optical axis of the quarter wave-plate is α and the angle between the transmission axes of the two polarizers is β (see figure below).



Question Q4. Intensity I of a plane wave in homogenous dielectric medium is defined as

$$I = \frac{1}{2} c_0 \epsilon_0 \langle E^2 \rangle$$

where the brackets denote temporal averaging. Assuming sinusoidal electric field $E = E_0 \sin(\omega t)$, express the intensity I as a function of E_0 , the refractive index of the medium n and the vacuum speed of light c_0 . Show that the energy density u of the field is

$$u = \frac{n}{c_0} I$$

The Poynting vector represents the energy flux (energy per unit time and per unit area (W/m^2)). Poynting's vector \mathbf{S} is defined as

$$\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B}$$

Show that in free space the energy flux is in the same direction as the wave vector \mathbf{k} . Show that for sinusoidal fields the average of the Poynting vector $\langle \mathbf{S} \rangle$ becomes equal to the expression for intensity I .

Homework Question HQ 1. (return by Feb 1) Simple ray matrix calculations using Matlab.

If you are new to Matlab please watch the following three short tutorials on the basics of Matlab on YouTube (the videos are currently available only in Finnish, for videos in English please search YouTube for "MIT open course introduction to Matlab", for example):

- Osa 1, peruslaskutoimitukset/Basic maths (5:52 min), <http://youtu.be/fQFZD-DlejI>
- Osa 2, vektorit ja matriisit/Vectors and matrices (7:24 min), http://youtu.be/dq_3OZ1D3KQ
- Osa 3, m-tiedostojen käyttö/Use of m-files (7:18 min), <http://youtu.be/-1ji2qal10s>

Open Matlab, then go to File → New → Script. Type in the following code (you will create a function called "simplelens"):

```
function [detS, r1]=simplelens(r0,d0,f,z);
% Function for output ray vector of
% a single lens system
Md=[1, d0;0,1];
Mf=[1,0;-(1/f),1];
Mz=[1,z;0,1];
S=Mz*Mf*Md;
% see question a)
detS=det(S);
% see question a)
r1=S*r0;
```

After typing in, save the text file as “simplelens.m” Next call this function by typing in the Matlab’s command prompt:

```
>> [detS,r1]=simplelens([0;0.2], 15, 10, 30)
```

Output from Matlab will be:

```
detS = 1  
r1 =  
0  
-0.1000
```

Note that “simplelens.m” must be located in the active Matlab folder for you to be able to run the code.

- a) On your own words, please explain what is the meaning of the numbers you typed in the command prompt, that is, ([0;0.2], 15, 10, 30) and what is the meaning of the answer?

The example script below shows how to use FOR loop structure in Matlab. If you want to test it, in Matlab go File → New → Script and type in the below code and save it as looptest.m. Then run this script either by typing looptest in the command prompt or press F5 while in the Matlab editor.

```
clear all  
  
for j=1:1:20  
  
    x(j)=j;  
  
    y(j)=j.^2;  
  
end  
  
figure(1)  
plot(x,y)
```

- b) Build a Matlab script for finding the image position. Accomplish this by using the above simplelens function inside a FOR loop to scan image position z from 1 cm to 100 cm at 0.1 cm steps. Using the plot Matlab command plot a figure that shows the image height coordinate as a function of the image position for input parameters r0=[0;1], d0=27.5cm, f=20. As your answer please return a paper copy of your Matlab code and a copy of the figure. Indicate the image position in the figure.

Hint 1: r1(1,1) gives the height and r1(2,1) the angle of the image ray vector r1.

Hint 2: some ideas for one possible solution

```
clear all
```

```
for j=10:1:1000 % FOR loop index must be an integer
```

```

x(j)=j;

[detS,r1]=simplelens( ... ;

y(j)= ... ;

```

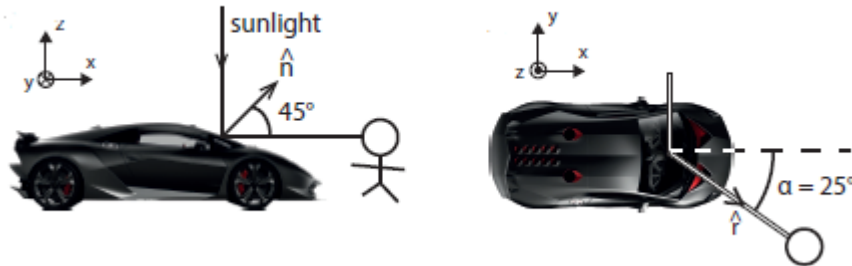
end

```

figure(1)
plot(x(10:end),y(10:end))

```

Homework Question HQ 2. (return by Feb 1) A police standing beside the road who wants to study the use of seatbelts in cars has good use for a pair of polarising sunglasses. How much is the reflection from the windscreen reduced when he wears them if he is positioned according to the figures below?



Homework Question HQ 3. (return by Feb 1) Anti-reflection (AR) coating has thickness $d = \lambda_2/4$. Show that the reflectance R (Eq. 2.46) for a light beam at normal incidence ($\theta_i=0$)

$$R = \left| \frac{E_T}{E_0} \right|^2 = \frac{r_{12}^2 + r_{23}^2 + 2r_{12}r_{23} \cos \beta}{1 + r_{12}^2 r_{23}^2 + 2r_{12}r_{23} \cos \beta}$$

becomes

$$R = \left(\frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2} \right)^2$$

Alice wants to measure the power of a near-infrared laser operating at 1575 nm vacuum wavelength. She plans to direct the laser on a semiconductor photodiode (refractive index 3.4) at normal incidence and she wants to minimise the reflection losses by applying an AR coating on the photodiode. What is optimal refractive index for the coating material and how thick layer of this material she should grow on the photodiode surface?