E-5730 Optics
Exercise V 15.2.2021

Question Q1. Show that for a Lorentzian spectrum

$$I(k) = \frac{I_0 \frac{\Gamma}{\pi}}{(k-k_0)^2 + \Gamma^2}$$

the interferogram of an FTIR spectrometer is

$$I(\Delta x) = I_0 + I_0 \exp^{-\Gamma|\Delta x|} \cos(k_0 \Delta x)$$

where the full-width at half-maximum (FWHM) 2Γ is much less than the centre wave number k_0. [Hint: for $\Delta x>0$

$$\int_{-\infty}^{\infty} \frac{e^{ik\Delta x}}{k-k_o-i\Gamma} dk = 2\pi i e^{-\Gamma\Delta x + ik_0\Delta x}$$ and the integral is zero for $\Delta x<0$].

Question Q2. Show that the intensity transmission of a Fabry-Perot interferometer for white light is

$$\langle T \rangle = \frac{1-r^2}{1+r^2}$$

[Hint: expand the expression for T (below) into a product of two geometric series.]

$$T = \frac{(1-r^2)^2}{\left|1-r^2 e^{i2kL}\right|^2}$$

Question Q3. By adding up the reflected fields, show that the reflectance of a Fabry-Perot interferometer is

$$R = \frac{F'\sin^2(kL)}{1+F'\sin^2(kL)}$$

where $F'=4r^2/(1-r^2)^2$. Solutions using "T = 1-R" are not acceptable. How the result relates to the reflectance R derived for a single layer anti-reflection coating?

Question Q4. Plane wave (amplitude E_0) hits a zero incident angle a rectangular aperture having dimensions of $2x_0$ and $2y_0$. Show that the Fraunhofer diffraction pattern behind the aperture is

$$I(\nu_x, \nu_y) = \frac{16x_0^2 y_0^2 E_0^2}{\lambda^2 z^2} \operatorname{sinc}^2(2\pi x_0 \nu_x) \operatorname{sinc}^2(2\pi y_0 \nu_y)$$

where sinc(u)=(sin(u)/u), $\nu_x = x/(\lambda z)$ and $\nu_y = y/(\lambda z)$.

Homework Question HQ 1. (return by February 15) The operating wavelength of a helium-neon laser is 630.0 nm and continuous wave (cw) power is 1 mW. The linewidth of the laser $\Delta\nu$ = 103 Hz.
a) Express the operating wavelength in frequency and wavenumber units and the linewidth in wavelength and wavenumber units.
b) How many photons are emitted per second?
c) If the output beam diameter is 1 mm, at what temperature would a blackbody radiator have to be to emit the same number of photons from an equal area and over the same frequency interval as the laser? [Ans: 3.7 x 1015 photons/s, 5.8 x 109 K]

Homework Question HQ 2. (return by February 15) a) At what angles do the orders m = 1,2 appear for light at λ = 630 nm incident on a reflection grating with 300 lines per millimetre at an angle θ_i = 15 degrees?
b) The grating is 50 mm wide. What is the resolution in the 2nd order if you illuminate the full grating?
c) If you would like to observe a spectrum from 300 nm to 630 nm, what is the degree of (angular) overlap between the 1st and 2nd order reflected light?
d) In a grating spectrometer the distance from the reflection grating (300 lines/mm) to the exit slit is 40 cm and the exit slit is 1 mm wide. The input slit is illuminated with collimated white light. What is the spectral bandwidth of light exiting the spectrometer if the grating is centred at 600 nm?

Homework Question HQ 3. (return by February 15) Below is the transmission spectrum of liquid ethanol in the 500 to 3800 cm^{-1} IR range. Your task is to repeat this measurement using an FTIR spectrometer based on a Michelson interferometer. To make sure that you can resolve the main four absorption peaks between 800 and 1500 cm^{-1} you decide that the resolution must be around 1/10 of the peaks' FWHM. What travelling distance of the moving mirror is required to achieve this resolution? At what speed the scanning mechanism must move the mirror for the photodetector to capture 100 intensity fringes per second? [Hint: one can assume a monochromatic light source].