# E-5730 Optics
Exercise VI 22.2.2021

Question Q1. Consider the passage of light through a circular aperture of 1 mm radius. The wavelength of light is 550 nm (visible green). How large the distance z = L needs to be so that the Fraunhofer approximation is valid?

Question Q2. Two signals are launched into an optical fibre at wavelengths λ_1 and λ_2. Show that the difference in their transition times is

$$\Delta T_{12} = L \int_{\lambda_1}^{\lambda_2} D(\lambda) d\lambda$$

where D is the wavelength dependent fibre dispersion parameter. Using the dispersion data shown in the figure below, estimate the difference in the transition times when two signals at wavelengths 1.32 and 1.5 µm propagate through 50 km length of fibre.

Wavelength (µm)
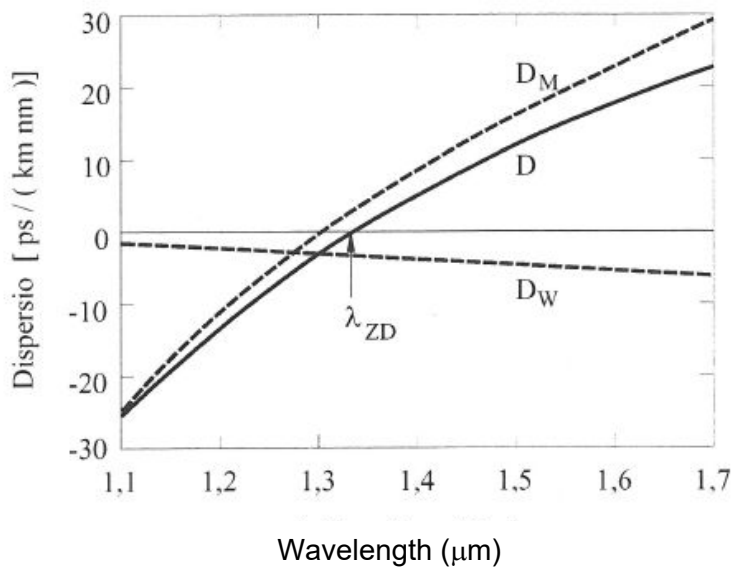
Question Q3. In a step-index optical fibre the refractive index of the cladding is 1.45 and the modal dispersion is less than 10 ns/km. What is the numerical aperture of the fibre and refractive index of the core? What is the maximum data transfer rate B over a 10 km long optical fibre in this case?

Homework Question HQ 1. (return by February 22) Ray A hits a transparent glass ball at right angle. The radius of curvature r of the ball is 20 mm and the refractive index is 1.5. Another ray of light (B) propagates in parallel with the ray A, but at height $h << r$ above it (see the figure on next page). At what distance behind the ball the two rays will cross each other?
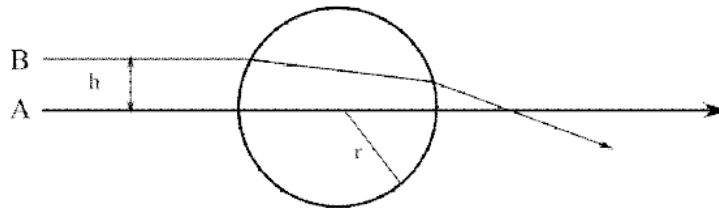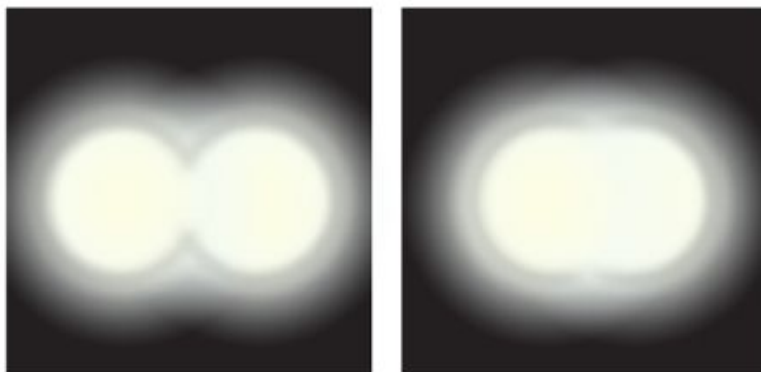

{Hint: Upon propagation from medium 1 to medium 2 the transformation matrix is

$$M = \begin{pmatrix} 1 & 0 \\ -\dfrac{P_2}{n_1} & \dfrac{n_1}{n_2} \end{pmatrix}$$

where P_2 is the power of the spherical surface. Here $P_1 = P_2 = P$.}

Homework Question HQ 2. (return by February 22) Diffraction limited performance.

a) The headlights of a car are 1.3 m apart. What is the maximum distance at which the eye can resolve these two headlights? Take the pupil diameter to be 0.40 cm and its refractive index 1.4. Hint: the effective focal length of a ball lens is determined in the HQ1 above.

b) What is the minimum diameter mirror on a telescope that would allow you to see details as small 5.00 km on the Moon some 384 000 km away? Assume an average wavelength of 550 nm for the light received. What other effects can be the limiting factors in this particular case, other than diffraction?

Homework Question HQ 3. (return by February 22) Matlab examples of Fourier filtering of an image. The following Matlab code *FourierImageFilteringQ.m* can be downloaded from the course's MyCourses (see "Materials") as well as test image *circles.mat*

Note that you need to upload the m-file and the test images into the same folder on your computer and update line 8 of the code accordingly.

In your answer to this question:
- a) Explain in your own words what happens in Sections A,B and C of the code. You don't have to explain all the programming details but rather the physical idea.
- b) Apply the code to *circles.mat* and run the code with different aperture parameters R=100, 50, 20 and 2 (line 36). Explain what is going on in the Figure 5 as R is decreased.
- c) On line 75, replace the filtering aperture *F* with *Finv* and run the code with R = 2,10,20,100. Explain what happens in the Figure 5 in this case as R is increased.

(You can also test *smilematrix.mat* if you want. Change the file name on lines 8 and 10).

```
%% Fourier filtering of an image file
% Toni Laurila
clear all
close all

%% load file image file; note that you need to edit the source location
load('C:\Users\Toni\Desktop\2014\circles.mat')

img=circles;
imgsize=size(img);
imgsize=imgsize(1);

figure(1)
imshow(img)
title('{\bf Original Image}')

%% Section A; Please explain what happens here?
img_shift = fftshift(img);

img_transform=fft2(img_shift);
img_fft_intensity = img_transform.*conj(img_transform);
img_fft = fftshift(img_fft_intensity)/max(img_fft_intensity(:));

% logarithmic image contrast enhancement helps to bring out
% details of the Fourier components
 img_fft = 1+log10(img_fft)/4;

figure(2)
imshow(img_fft)

% End of section A
```

A

```matlab
%% Create aperture function for filtering
% size of the filter radius R (expressed as pixels)
R=100;

imgsize_power2=log2(size(img_fft))
n = 2^imgsize_power2(1);
F = zeros(n);
I = 1:n;
x = I-n/2;
y = n/2-I;
[X,Y] = meshgrid(x,y);
A = (X.^2 + Y.^2 <= R^2);
F(A) = 1;

%% Section B: Explain what is F (Figure 3)? What about Finv?

F_ones=ones(imgsize,imgsize)
Finv=F_ones-F;

figure(3)
imshow(F)
%imshow(Finv)

% End of section B


%% Fraunhofer diffraction pattern
% Demonstration on how to calculate diffraction pattern through F

D1 = fft2(F);
diffracted_intensity = fftshift(D1);

figure(4)
imagesc(abs(diffracted_intensity))
colormap(hot)
title('{\bf Diffraction Pattern}')


%% Section C: Explain what is done here?

gF = fft2(img_shift);

gFfilt=fftshift(F).*real(gF)+i*imag(gF);

gBfilt = ifft2(gFfilt);
gB= fftshift(gBfilt);

gB=gB.*conj(gB);
gB=gB./max(gB(:));

figure(5)
imshow(gB)

% End of section C
```

B

C