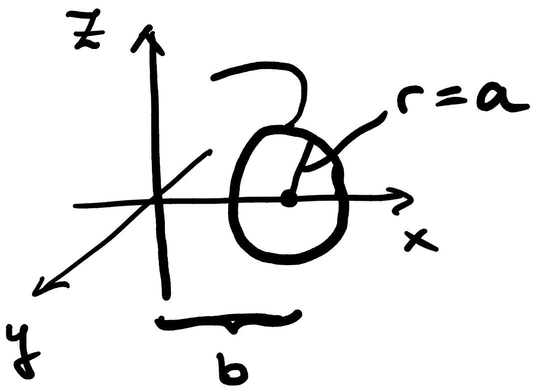
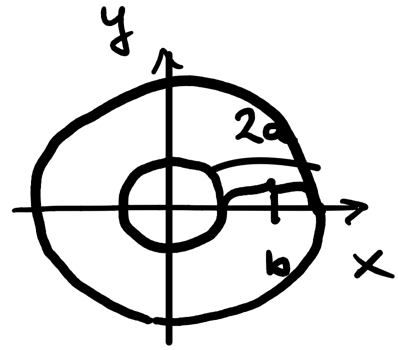
Parametrisoidut pinnat : TORUS

xz - tason ympyrä : kp $(b, 0)$; $r = a$

$$\begin{cases} x = a\cos\theta + b \\ z = a\sin\theta \end{cases}, \quad \theta \in [0, 2\pi]$$





Kierretään xz - tasoa z - akselin ympäri :

Formaalisti : $\begin{cases} x \leftarrow x\cos\varphi \\ y \leftarrow x\sin\varphi \end{cases}, \quad \varphi \in [0, 2\pi]$

Pinnan parametrisointi :

$$\begin{cases} x = (a\cos\theta + b)\cos\varphi \\ y = (a\cos\theta + b)\sin\varphi \\ z = a\sin\theta \end{cases}, \quad \theta, \varphi \in [0, 2\pi]$$

Avaruuden suora

<u>Määritelmä</u> $\dfrac{x-x_0}{r} = \dfrac{y-y_0}{s} = \dfrac{z-z_0}{t}$

(suorakulmainen, oikeakätinen koordinaatisto)

Suuntavektori:

$$\begin{cases} \dfrac{x-x_0}{r} = \dfrac{y-y_0}{s} \\[2ex] \dfrac{y-y_0}{s} = \dfrac{z-z_0}{t} \end{cases} \quad \text{eli} \quad \begin{cases} sx - ry = sx_0 - ry_0 \\ ty - sz = ty_0 - sz_0 \end{cases}$$

$\rightarrow$ kaksi tasoa

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ s & -r & 0 \\ 0 & t & -s \end{vmatrix} = s\underbrace{(r\underline{i} + s\underline{j} + t\underline{k})}_{\text{suuntavektori}}$$

OSITTAISDERIVAATAT

Olkoon $D \subset \mathbb{R}^n$, $n \geq 2$, $f : D \to \mathbb{R}$

Muuttujia n kpl : x_j, $j = 1, \ldots, n$

$$f_j(\underline{x}) = \lim_{h \to 0} \frac{f(\underline{x} + h\underline{e}_j) - f(\underline{x})}{h}$$

$\longrightarrow$ osittaisderivaattaa laskettaessa muut kuin x_j ajatellaan vakioiksi

<u>Esim.</u> $f(x,y) = x^2 y$

$$f_1(x,y) = 2xy$$

$$f_2(x,y) = x^2$$

Merkintöjä: $\dfrac{\partial f}{\partial x_j} = \dfrac{\partial}{\partial x_j} f(x_1, \ldots, x_n)$

$$= f_j(x_1, \ldots, x_n) = D_j f(x_1, \ldots, x_n)$$

$n = 2$: $z = f(x,y)$; $f_1(x,y) = \dfrac{\partial z}{\partial x}$

$$f_2(x,y) = \frac{\partial z}{\partial y}$$

PINNAN TANGENTIT JA NORMAALIT

$$z = f(x,y)$$

Tarkastelupiste: (a,b)

Tangenttivektorit: $\underline{T}_1 = \underline{i} + f_1(a,b)\,\underline{k}$

$$\underline{T}_2 = \underline{j} + f_2(a,b)\,\underline{k}$$

Pinnan normaalivektori $\underline{n} = \underline{n}(a,b)$

$$\underline{n} = \underline{T}_2 \times \underline{T}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & f_2(a,b) \\ 1 & 0 & f_1(a,b) \end{vmatrix}$$

$$= f_1(a,b)\,\underline{i} + f_2(a,b)\,\underline{j} - \underline{k}$$

Tangenttitaso: $\underline{n} \cdot (\underline{r} - \underline{r}_0) = 0$

$$\underline{r}_0 = (a, b, f(a,b))$$

$$\Rightarrow \boxed{z = f(a,b) + f_1(a,b)(x-a) + f_2(a,b)(y-b)}$$

Normaalisuora: $z = f(x,y)$; $\underline{r}_0 = (a, b, f(a,b))$

$$\underline{n} = f_1(a,b)\,\underline{i} + f_2(a,b)\,\underline{j} - \underline{k}$$

Parametrisointi: $s = \{\underline{r}_0 + t\underline{n} \mid t \in \mathbb{R}\}$

Jos $f_1(a,b) \neq 0$, $f_2(a,b) \neq 0$, niin

$$\frac{x-a}{f_1(a,b)} = \frac{y-b}{f_2(a,b)} = \frac{z-f(a,b)}{-1}$$

Esimerkki Pinta $z = f(x,y) = \dfrac{a^3}{xy}$ $(a > 0)$

Tutki pinnan pisteeseen asetetun tangenttitason ja koordinaattitasojen määräämän tetraedrin tilavuutta.

Piste: $(r, s, f(r,s))$

Tangenttitaso: $z = f(r,s) + f_x(r,s)(x-r)$

$$+ f_y(r,s)(y-s)$$

$$= \ldots = f(r,s)\left(3 - \frac{x}{r} - \frac{y}{s}\right)$$

$$\Rightarrow \quad \frac{x}{r} + \frac{y}{s} + \frac{z}{f(r,s)} = 3$$

Leikkaa

(a) x-akselin : $y = z = 0 \quad \Rightarrow \quad x = 3r$

(b) y-akselin : $x = z = 0 \quad \Rightarrow \quad y = 3s$

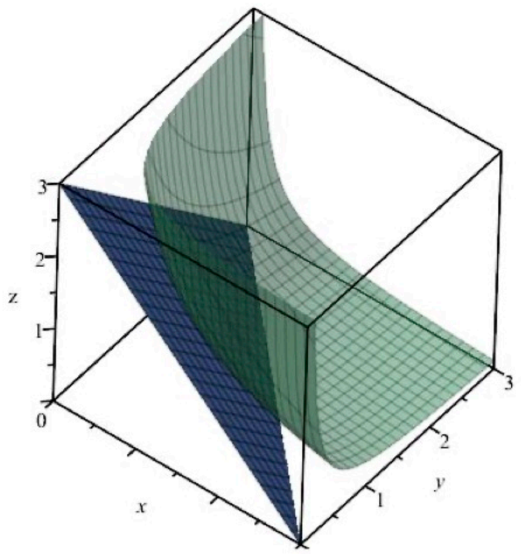
(c) z-akselin : $x = y = 0 \quad \Rightarrow \quad z = 3f(r,s)$

$$= \frac{3a^3}{rs}$$

Pohja (xy-tasossa) :

$$V = \frac{1}{3}\,(\text{Pohjan pinta-ala}) \cdot \text{korkeus}$$

$$= \frac{1}{3}\left(\frac{3s \cdot 3r}{2}\right)\left(\frac{3a^3}{rs}\right) = \frac{9a^3}{2}$$


3
2
z
1
0
x
1
2
y
3

Korkeammat osittaisderivaatat

$z = f(x, y)$:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial z}{\partial x} = f_{11}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = f_{21}(x, y)$$