

# Urban Economics

## Lecture 2: Monocentric city model

*Spring 2021*

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# Monocentric city model

- In this lecture, we analyze the **monocentric city model**
- **Origins in the work of Alonso (1964), Mills (1967), and Muth (1969)**
  - Alonso, W. (1964): *Location and land use*. Harvard University Press.
  - Mills, E. (1967): An Aggregative Model of Resource Allocation in a Metropolitan Area. *American Economic Review* 57(2), 197–210.
  - Muth, R. (1969): *Cities and housing*. Chicago: University of Chicago Press.

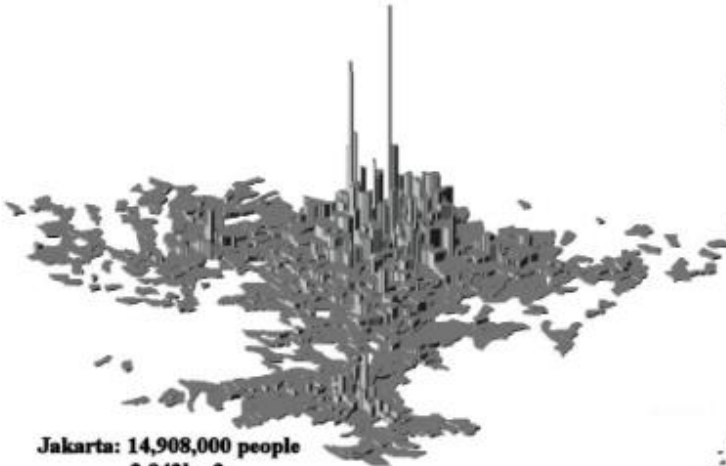
# Monocentric city model

- **Main goal of the model is to explain the empirical regularities that we observe in real-life cities**
- **Main mechanism is the relationship between commuting costs, housing prices, and housing consumption**
- **Another key ingredient is identical utility levels and developer profits across space, i.e. spatial equilibrium within the city**

# Outline

- **Empirical regularities of real-life cities**
- **Monocentric city model assumptions**
- **Consumer analysis**
- **Producer analysis**
- **Empirical example**
- **This lecture will follow Brueckner's Chapter 2**

# Population density in 7 major cities



**Jakarta: 14,908,000 people  
2,942km<sup>2</sup>**



**Paris: 7,877,000 people  
893 km<sup>2</sup>**



**Moscow: 8,543,000 people  
470 km<sup>2</sup>**



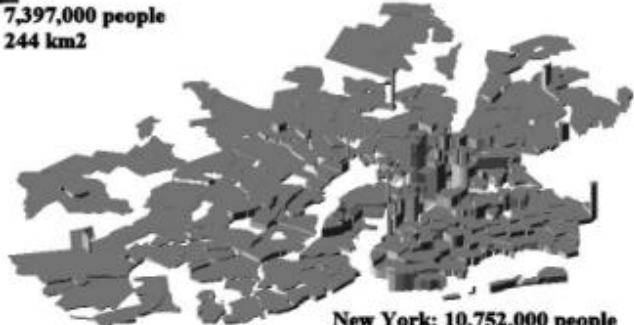
**Shanghai: 7,397,000 people  
244 km<sup>2</sup>**



**Berlin: 4,212,000 people  
1,176 km<sup>2</sup>**



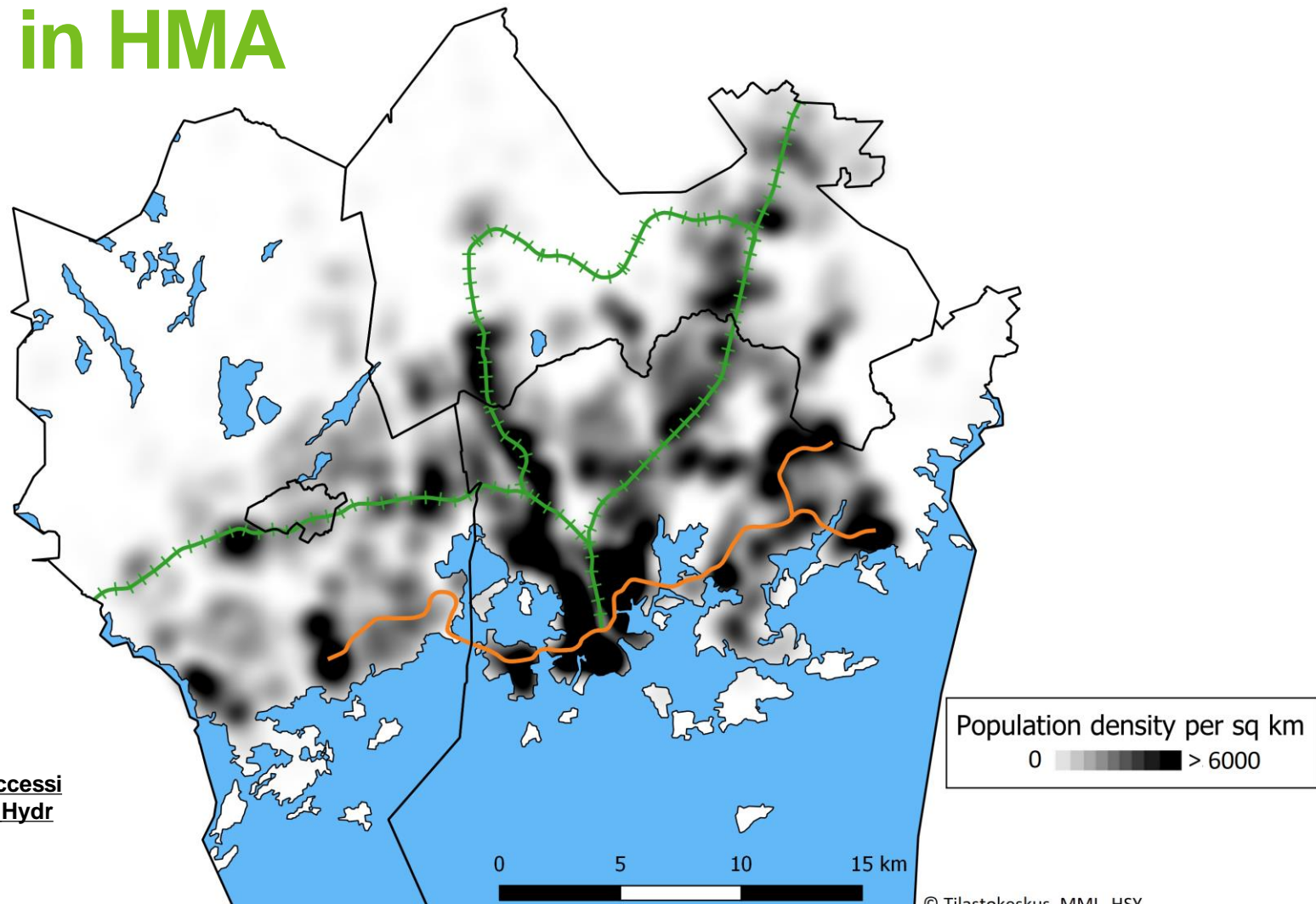
**London: 6,626,000 people  
1,062 km<sup>2</sup>**



**New York: 10,752,000 people  
2,674 km<sup>2</sup>**

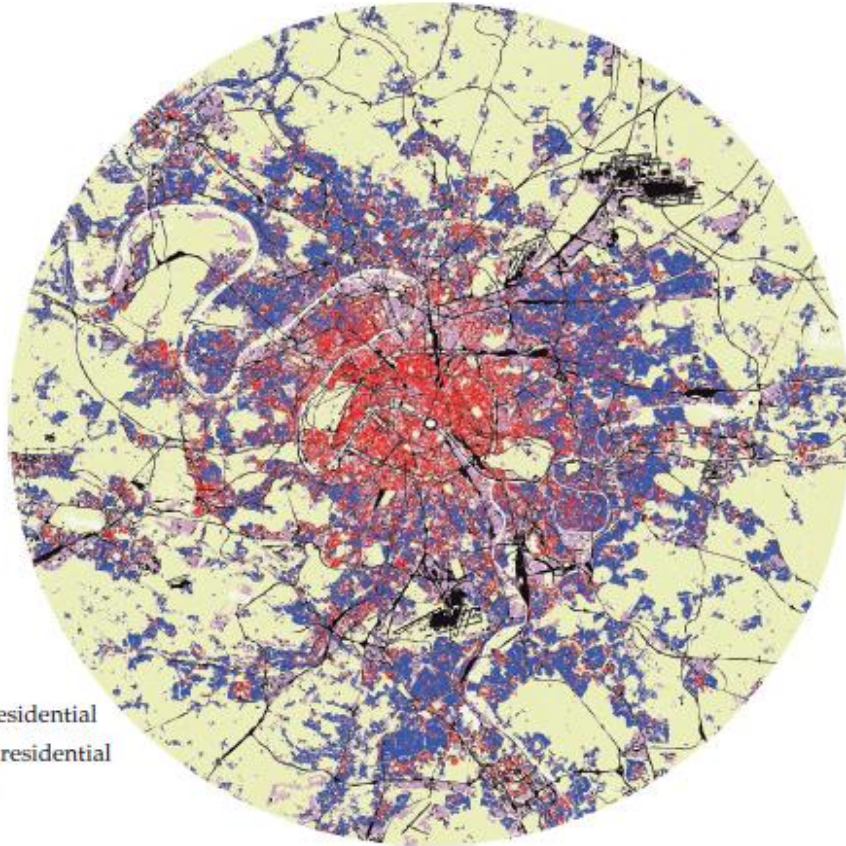


# Population density in HMA

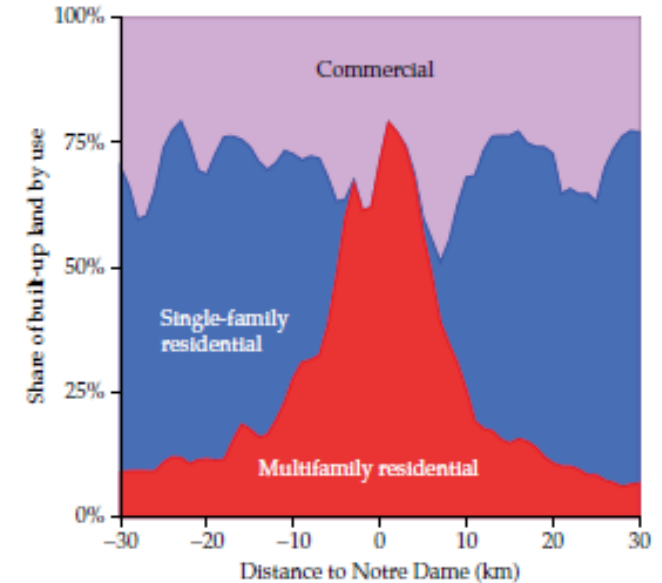
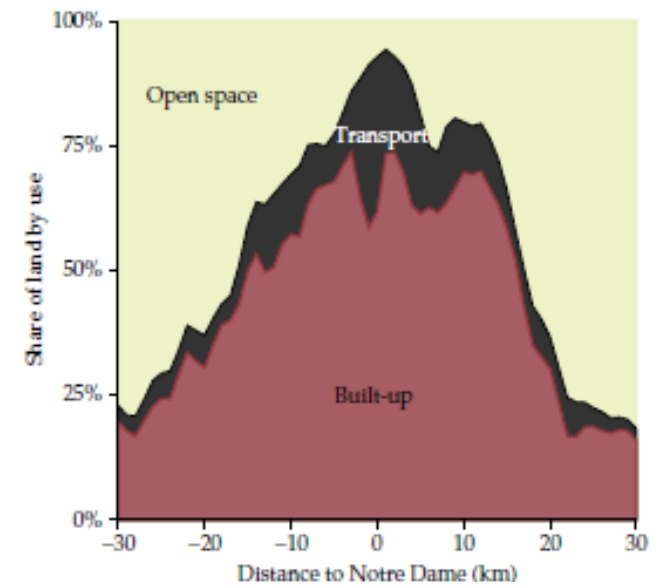


[https://blogs.helsinki.fi/accessibility/files/2018/10/HSPA\\_Hydro\\_pulse50ms.gif](https://blogs.helsinki.fi/accessibility/files/2018/10/HSPA_Hydro_pulse50ms.gif)

# Land use in Paris

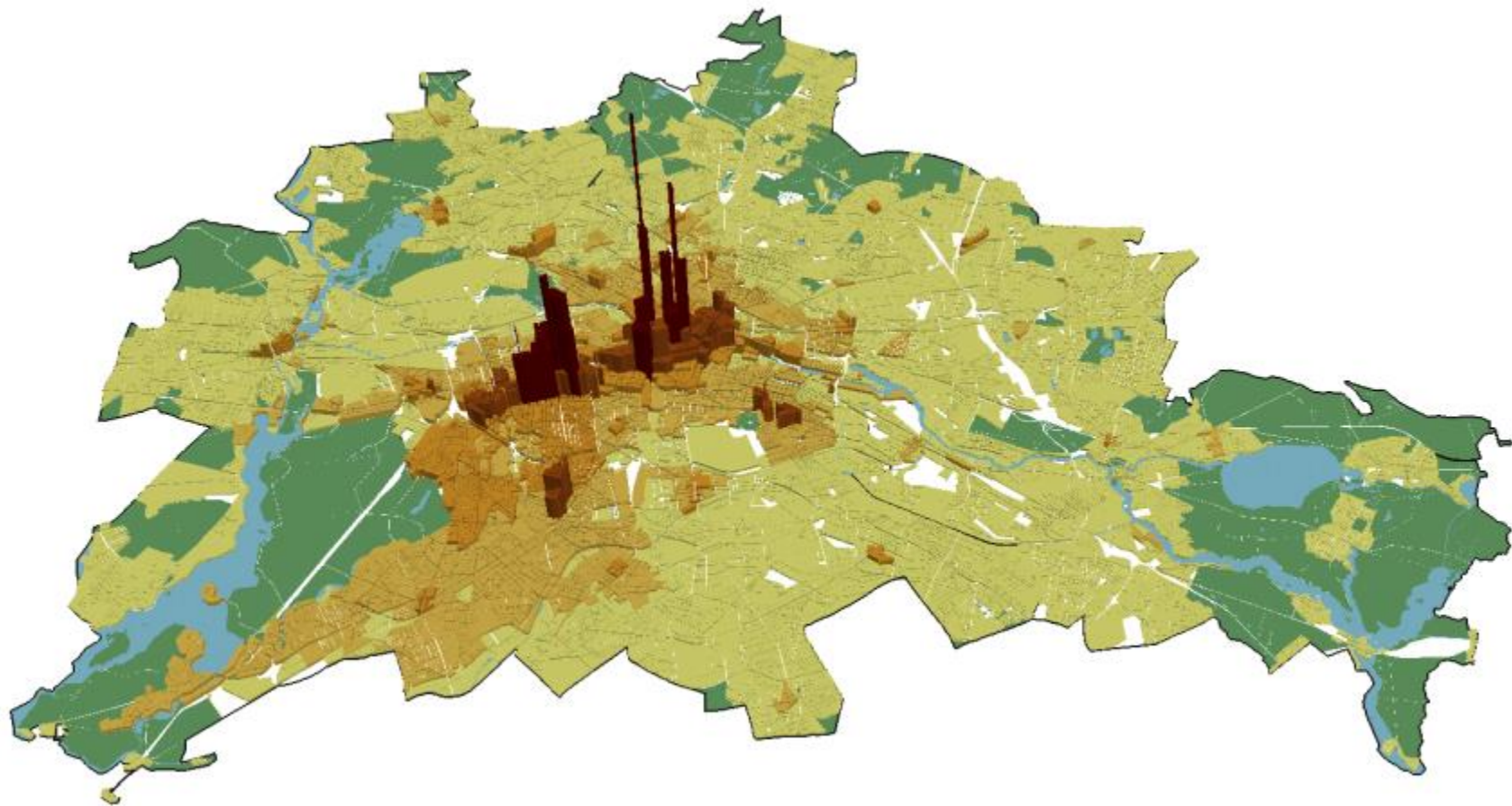


- Multifamily residential
- Single-family residential
- Commercial
- Transport
- Open space



Duranton, G. & D. Puga. 2015. Urban Land Use. In G. Duranton, J.V. Henderson, W.C. Strange (ed.), *Handbook of Regional and Urban Economics*, Vol 5, 467-560

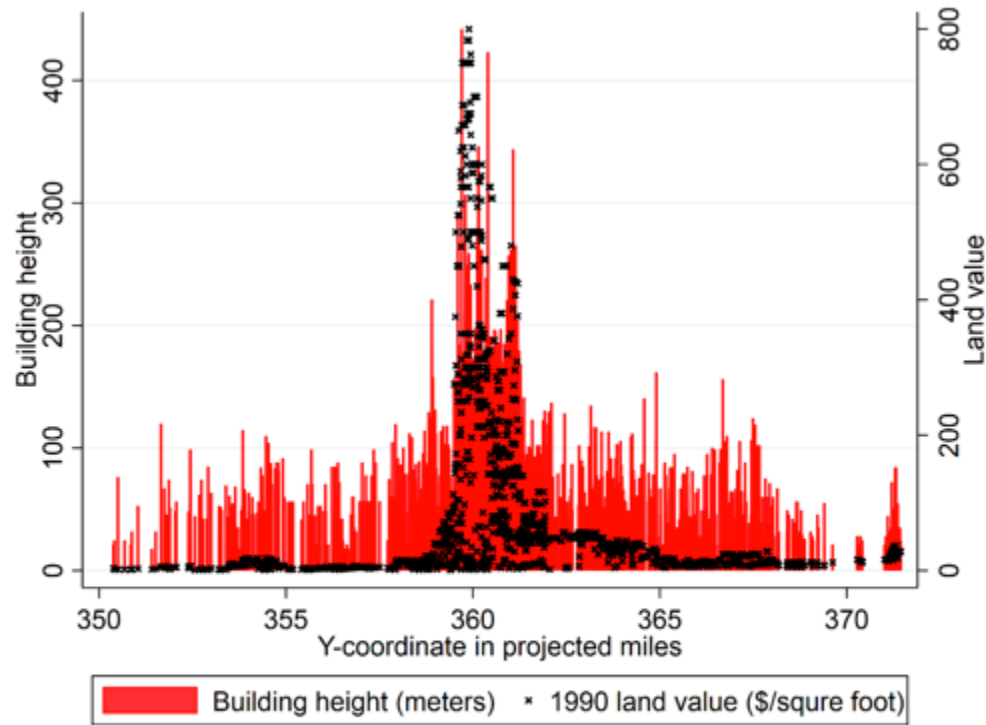
# Land prices in Berlin



Source: Ahlfed et al. (2015), *Econometrica*



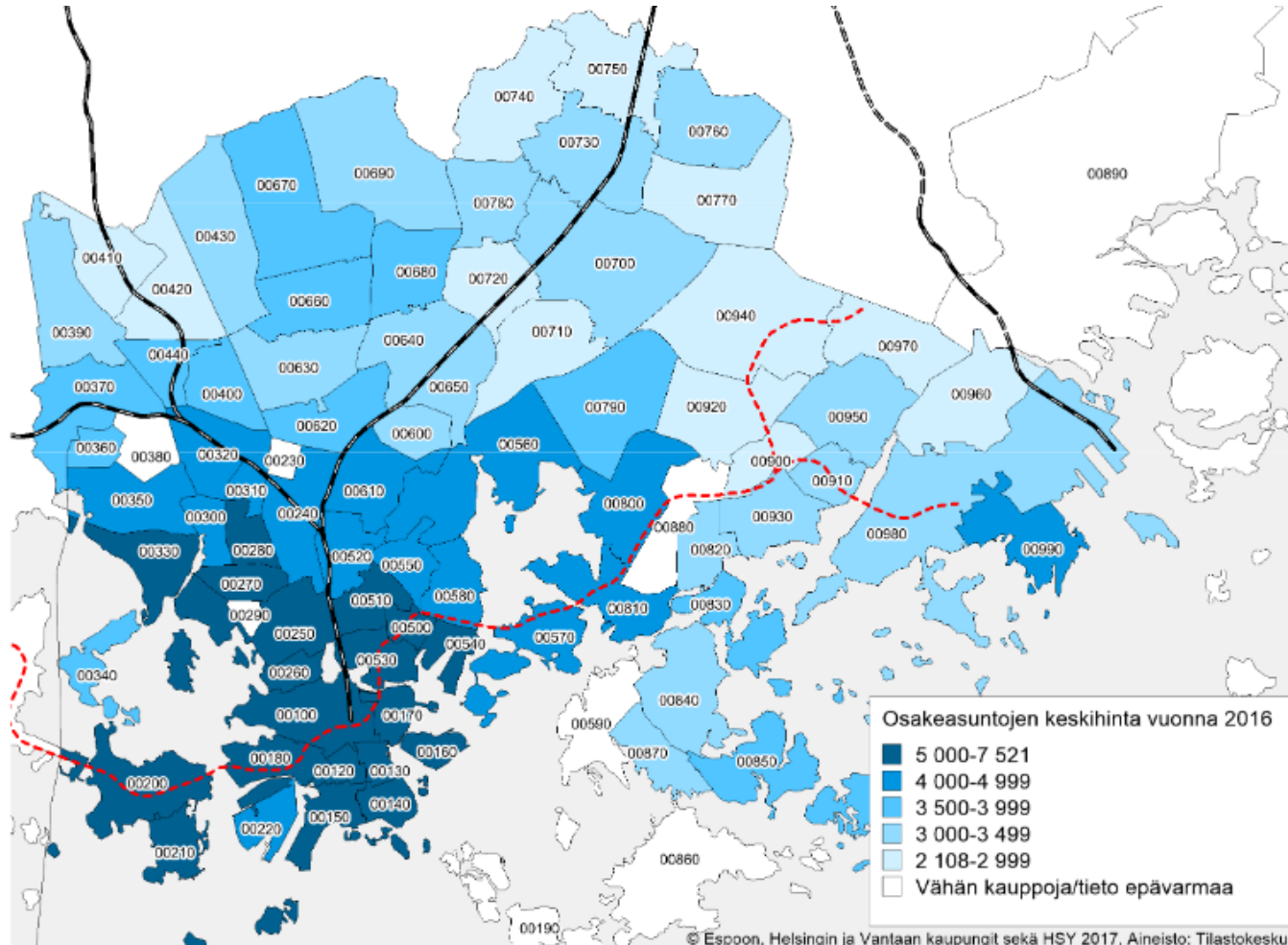
# Building height and land prices in Chicago



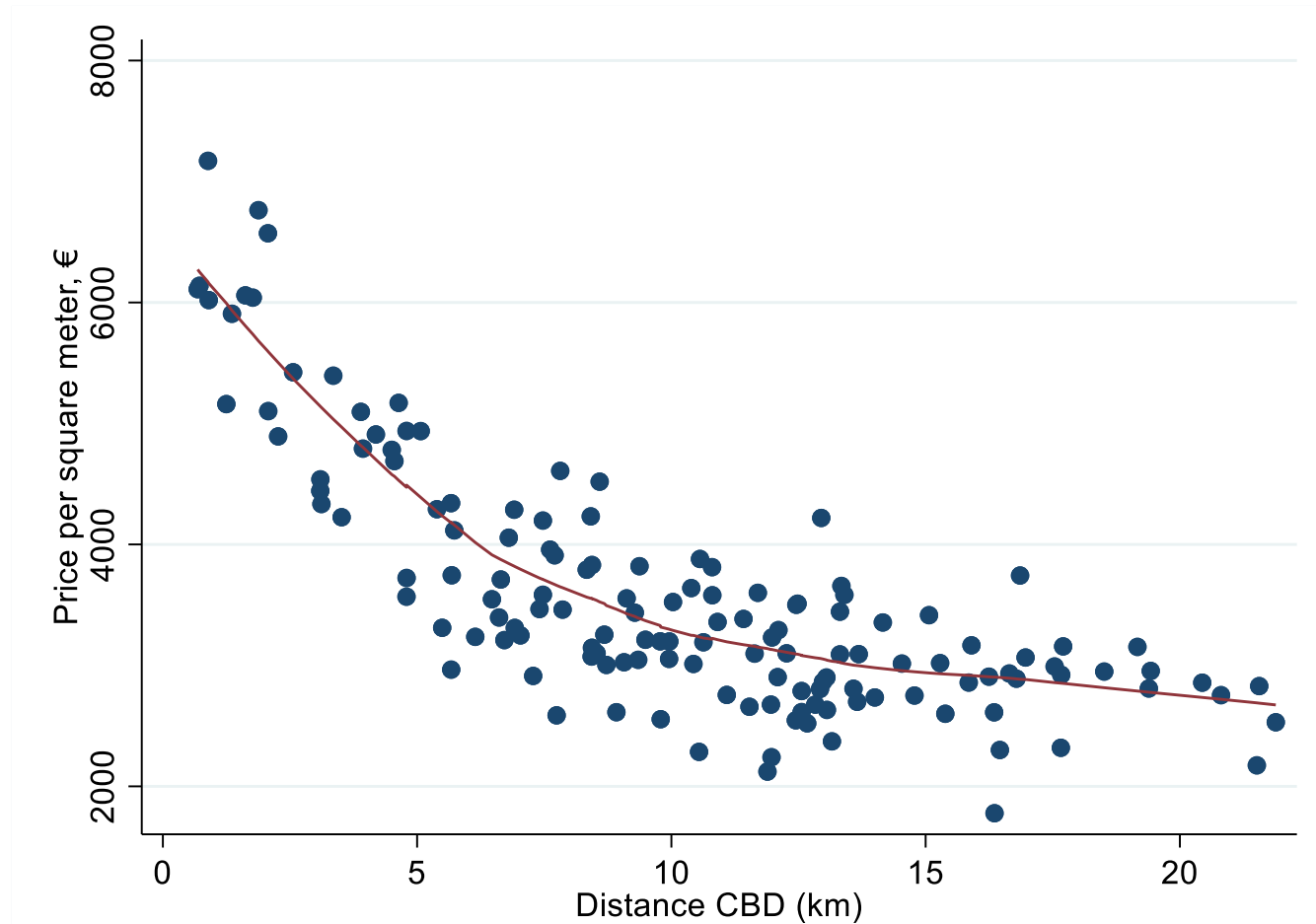
Source: Ahlfed and McMillen (2018), *Review of Economics and Statistics*.

Notes: The building heights in 2014 are from Emporis.com. The 1990 land values are from Olcott's blue books. The y-coordinate is the vertical Cartesian coordinate in the State Plane Coordinate System (Illinois East).

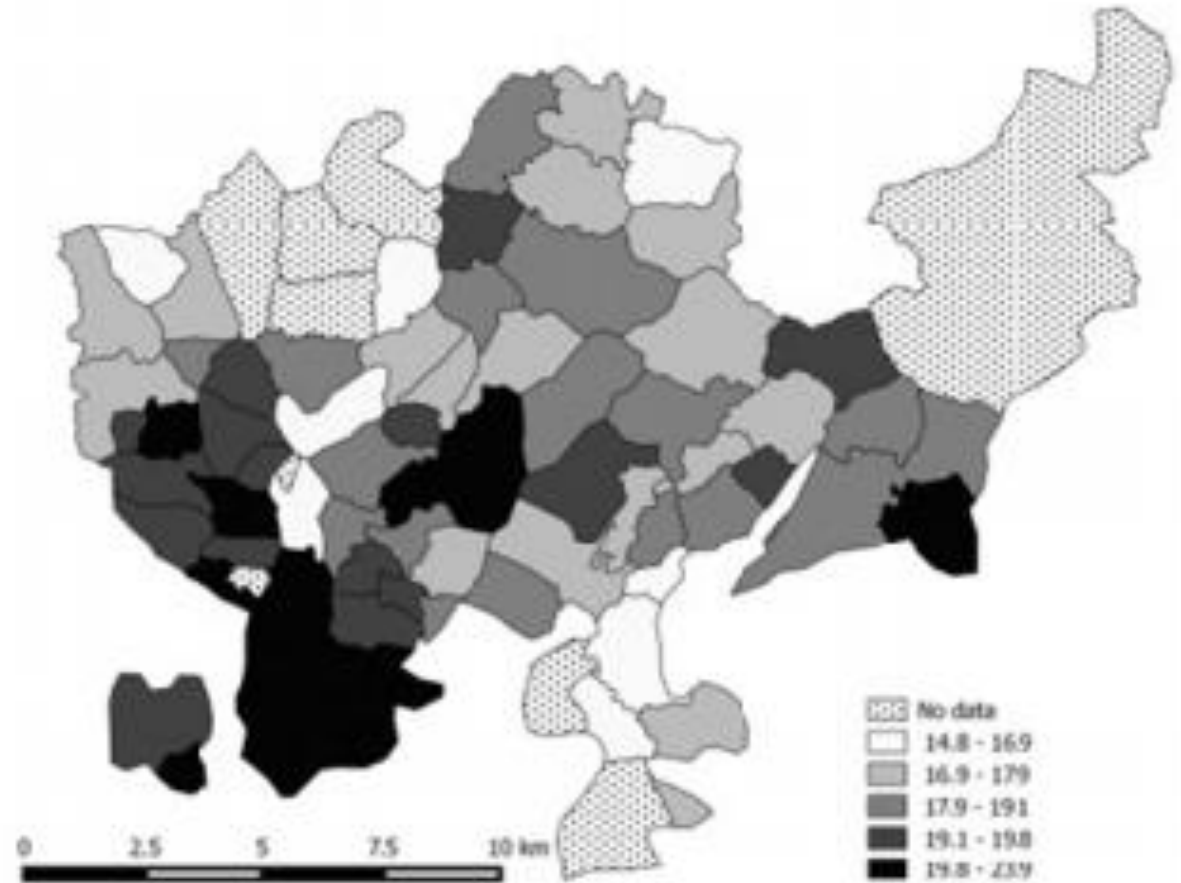
# House prices (€/m<sup>2</sup>) in Helsinki postcodes



# House prices (€/m<sup>2</sup>) in HMA postcodes



# Monthly rents per square meter in Helsinki



Source: Eerola and Saarimaa (2018),  
*Journal of Housing Economics*.

# Patterns

- **We generally see a pattern of declining density radiating from one center, or sometimes multiple centers**
- **Tall multi-family buildings tend to be located near the city center, while single-family houses are at the fringe**
- **Land and housing prices per square meter/foot tend to be high near the city center and lower farther away**
  - Think about the spatial equilibrium condition!
- **Of course, these patterns are not purely market driven as land use planning has played a major in role**
- **Next, we try to explain these patterns through a simple model**

# Monocentric city model

# Useful concepts – Land

- **Land rent** is the price for using one unit of land, say a hectare, for one unit of time, say a year
- **Land value** is the price of buying one unit of land, again say a hectare
- Land is an asset; like any asset its **price (= value) is the present value of the benefits (= net rent) from owning it**

# Determination of land value

Value of a  
Land Parcel

Net Rent of  
Land in Year 2



$$V_L = \frac{R_{L1}}{(1+i)} + \frac{R_{L2}}{(1+i)^2} + \frac{R_{L3}}{(1+i)^3} + \frac{R_{L4}}{(1+i)^4} + \dots$$



Discount Rate

$$= \sum_{t=1}^{\infty} \frac{R_{Lt}}{(1+i)^t} \approx \frac{R_L}{i}$$



Assumes  $R_L$  is constant over time



# Important lesson: cheap land => cheap housing?

- The price of land is high in some locations because people are willing to pay a lot for housing or commercial activities at that location
- It is not correct to say that the price of housing is high because land is expensive!
- **Policy question:** will we get cheap housing if the municipality sells land to developers at a discount?

# Useful concepts – Housing

- **Housing is measured in units of housing services =  $q$** 
  - $q$  = quality-adjusted square meters
  - Depends on housing characteristics
  - For now, we **assume that floor space is the only characteristic**
- **$p$  = the price (rent) per unit of  $q$  per year or month (e.g. 20 €/m<sup>2</sup>/month)**
- **$r$  = rent for a housing unit =  $pq$  (e.g. 20\*50 = 1000 €/month)**
  - If the unit is a rental apartment,  $r$  = contract rent
  - If the unit is owner-occupied,  $r$  is not observed

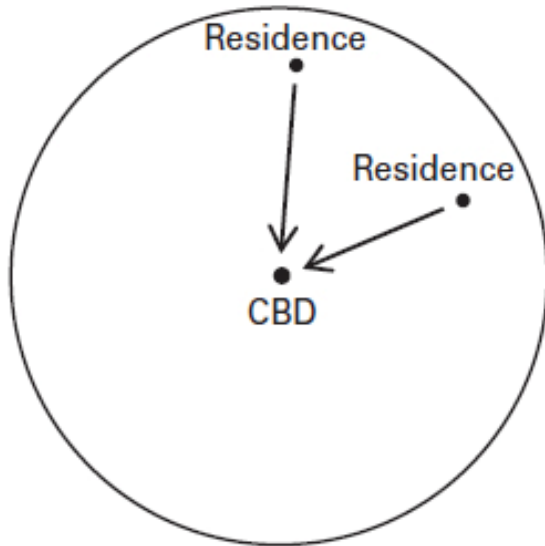
# Determination of house value

- **$V$  = the value of a housing unit = the present value of the rental flow**
- **So, with a long lifetime,  $T$ , for housing:**

$$V = \sum_{t=1}^T \frac{p_t q_t}{(1+i)^t} \approx \frac{pq}{i}$$

# Monocentric city model – assumptions

1. All jobs are in the city center (**central business district, CBD**)
  - Jobs do not take up space
2. The city has a dense network of radial roads



# Monocentric city model – assumptions

1. All jobs are in the city center (CBD)
2. The city has a dense network of radial roads
3. The city contains **identical households** or consumers or workers
  - Same income/wage ( $y$ ) and preferences (will be relaxed later)
4. The residents consume (get utility from) only two goods: **housing ( $q$ )** and a **composite good**, say bread ( $c$ )
  - The price of the composite good is the same everywhere (equal to 1)
  - Land and the housing that sits on it are allocated competitively to the highest bidder at each location

# Commuting costs

- The per-kilometer **cost of commuting** is  $t$ , so a resident living at distance  $x$  from the CBD incurs a commuting cost  $tx$ 
  - Commuting has only a **monetary cost**
  - Later we will introduce the opportunity cost of time used in commuting
  - Also, everyone uses the **same commuting mode** so that  $t$  is the same for everyone
- This leaves  $y - tx$  for expenditure on housing and the composite good (= disposable income)
- **Disposable income** decreases as  $x$  increases

# Housing consumption and budget constraint

- **A housing unit or a dwelling has variety of characteristics**
  - Floor space, yard size, construction quality, age, amenities
- **Here we simplify things and assume that dwellings differ only in size**
  - I.e.  $q$  represents square meters and  $p$  is measured as rental price per square meter
- **The consumer's budget constraint is  $y - tx = pq + c$** 
  - It states that the expenditure on bread and housing is equal to disposable income (income after commuting costs)

# Consumer analysis



# Consumer analysis

- **Consumers want to maximize the utility (welfare) they get from consuming housing and bread, while taking into account their budget constraint**
- **That is, the consumer chooses the  $c$  and  $q$  to maximize utility  $U(c, q)$  subject to the budget constraint at each distance  $x$**
- **Location “choice” enters the problem only through commuting costs**
  - We assumed that dwellings differ only with respect to size, not with respect neighborhood amenities

# Price of housing

- One of the empirical regularities that we saw earlier was that **price per square meter of housing falls as distance to the CBD increases** ( $p$  falls as  $x$  increases)
- Can this simple model predict this regularity?
- Yes! And there are several ways to demonstrate this

# Locational or spatial equilibrium

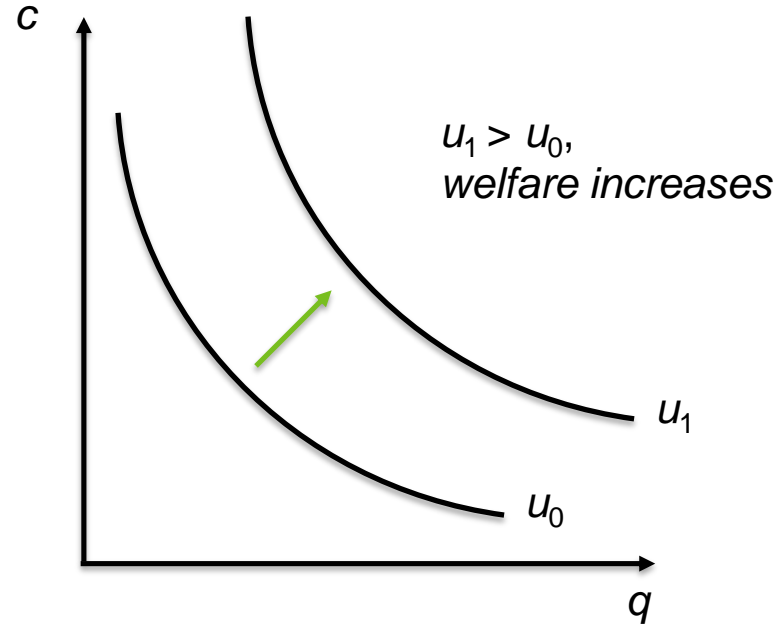
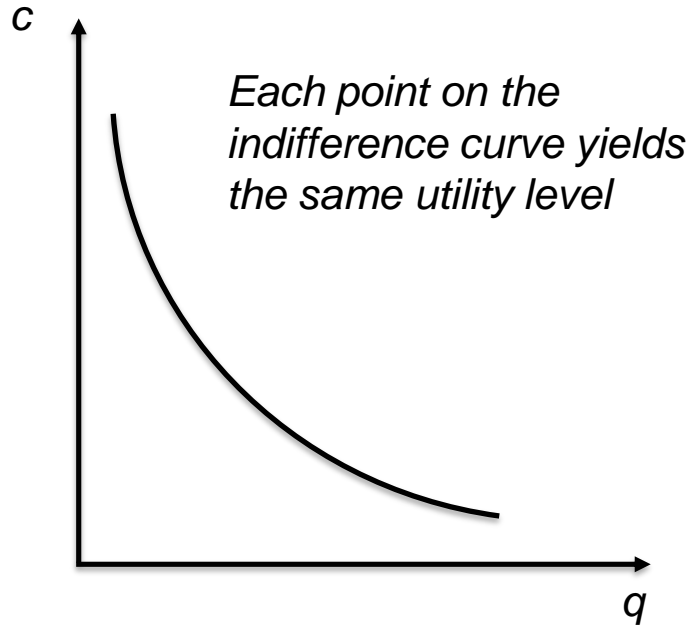
- **Everyone would want live right next to the CBD, but everyone cannot live in the same location**
- **But as consumers are identical, they must be equally well-off regardless of where they live in the city**
  - If this condition did not hold, then consumers in a low-utility area could gain by moving into a high-utility area (not an equilibrium)
- **This equilibrium can hold only if price of housing per square meter falls as distance increases**
  - Since higher commuting costs mean that disposable income falls as  $x$  increases, some offsetting benefit must be present to keep utility from falling

# Locational or spatial equilibrium

- **Lower  $p$  at more distant locations serves as a compensating differential**
  - Compensating differentials arise in many economic settings
  - For example: dangerous or unpleasant jobs must pay higher wages than more appealing jobs with similar skill level requirements
  - Otherwise, no one would do the unpleasant jobs!
- **Note that here the price of the composite good is the same everywhere, and thus, cannot play a compensating role**
  - The prices of groceries and other non-housing goods are the same
  - May not be fully realistic, of course

# Indifference curve diagram

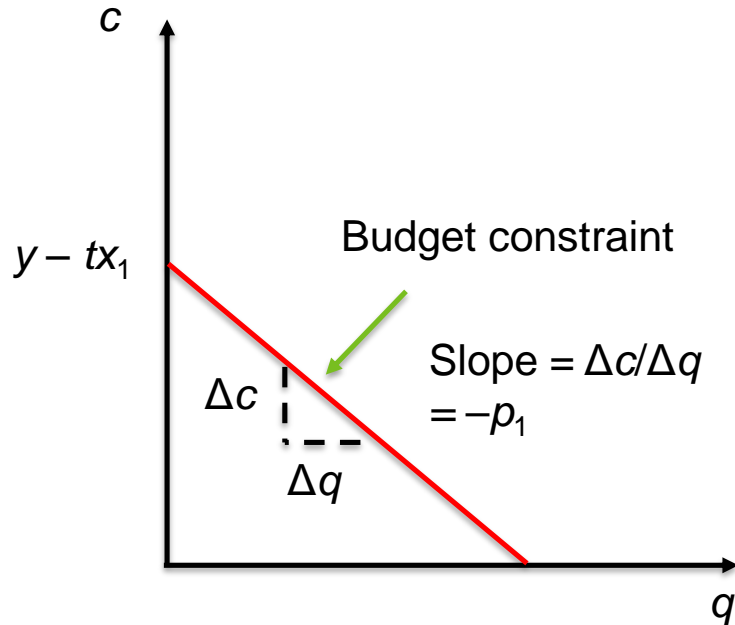
- A graphical way of deriving this result is using **indifference curves** and the budget constraint



# Budget constraint

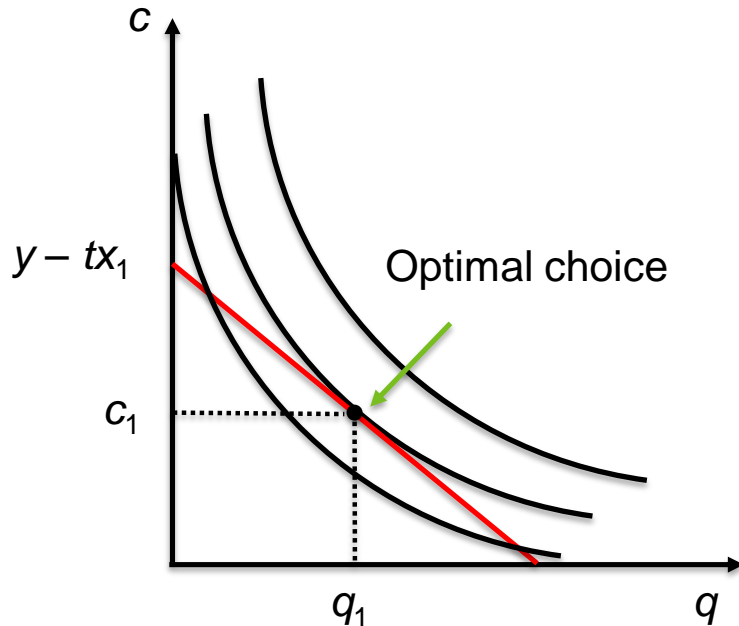
$$y - tx = pq + c$$

$$c = y - tx - pq$$



- The figure depicts the budget constraint/line for a consumer living at a distance of  $x_1$  away from CBD
- If housing consumption is zero, the consumer can consume  $y - tx_1$  worth of bread
- When the consumer starts to consume housing, it must give up on bread consumption
- The slope of the budget line for consumer living at  $x_1$  is  $-p_1$

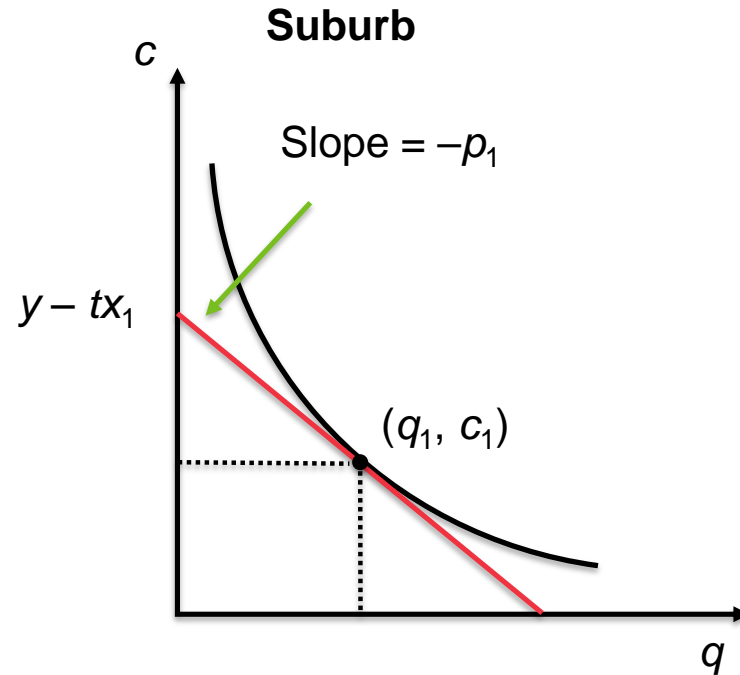
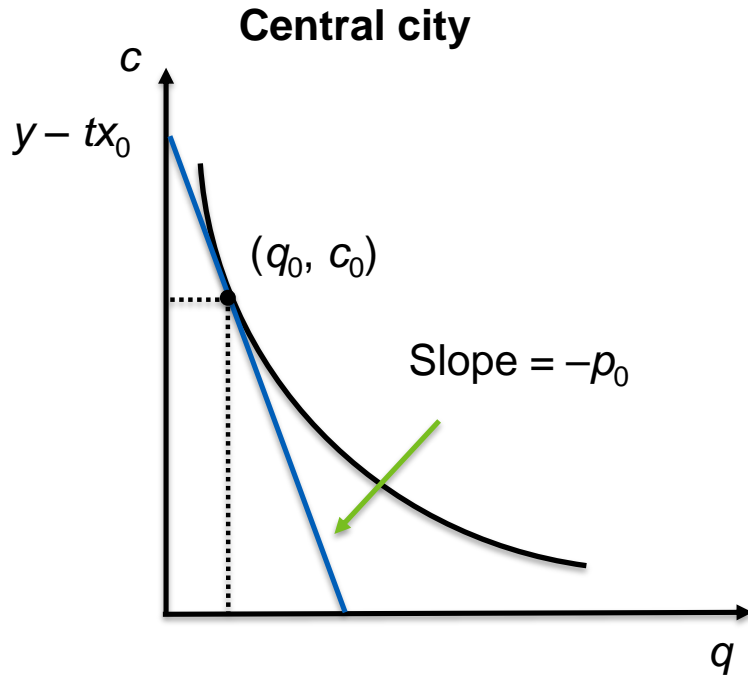
# Indifference curve diagram



- The consumer chooses the point where the indifference curve is tangent to the budget line  $(c_1, q_1)$
- This is the **highest possible indifference curve** that the consumer can reach within the budget constraint

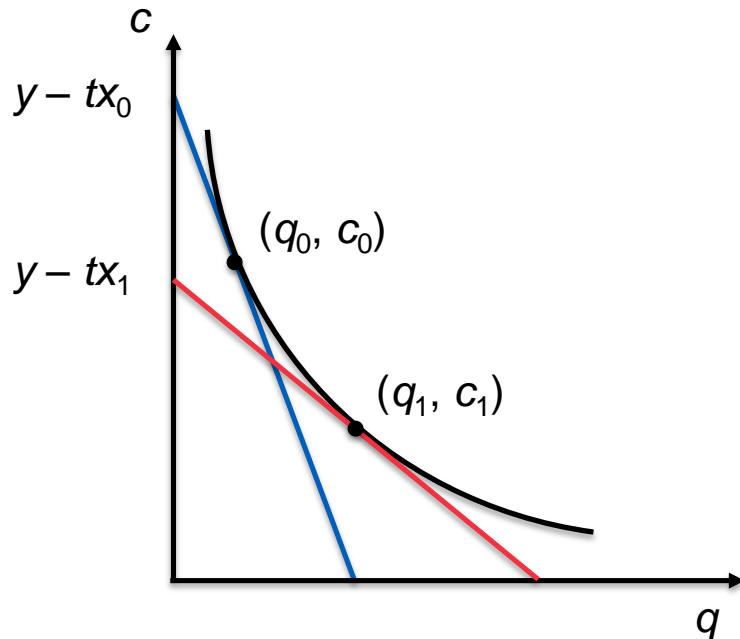
# Central-city and suburban consumer

- Consider now two consumers, one living central-city ( $x_0$ ) and the other in a suburban location ( $x_1$ ), so that  $x_1 > x_0$





# Central-city and suburban consumer



- What magnitude must the price of housing  $p_1$  be at distance  $x_1$  in order to ensure that the suburban consumer is just as well-off as the central-city consumer?
- The price must lead to a budget line that allows the suburban consumer to reach the **same indifference curve** as the central-city consumer
- That is, prices per square meter are higher in central-city,  $p_0 > p_1$

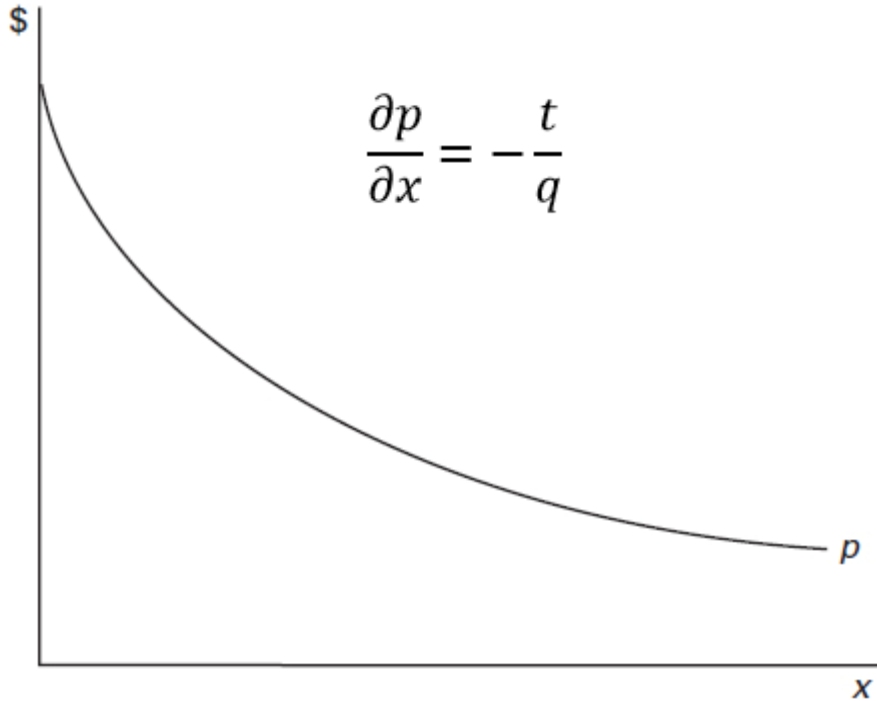
# Housing consumption in different parts of the city

- **The diagram reveals another important result of the model**
  - The suburban resident consumes more housing space ( $q_1 > q_0$ ) and less bread ( $c_1 < c_0$ ) than the central-city resident
  - This means that **dwelling size  $q$  increases as distance  $x$  from the CBD increases**
- **This substitution in favor of housing and away from bread is the consumer's response to the decline in the relative price of housing as  $x$  increases**
  - Remember that the price of bread is the same everywhere in the city

# Model predictions

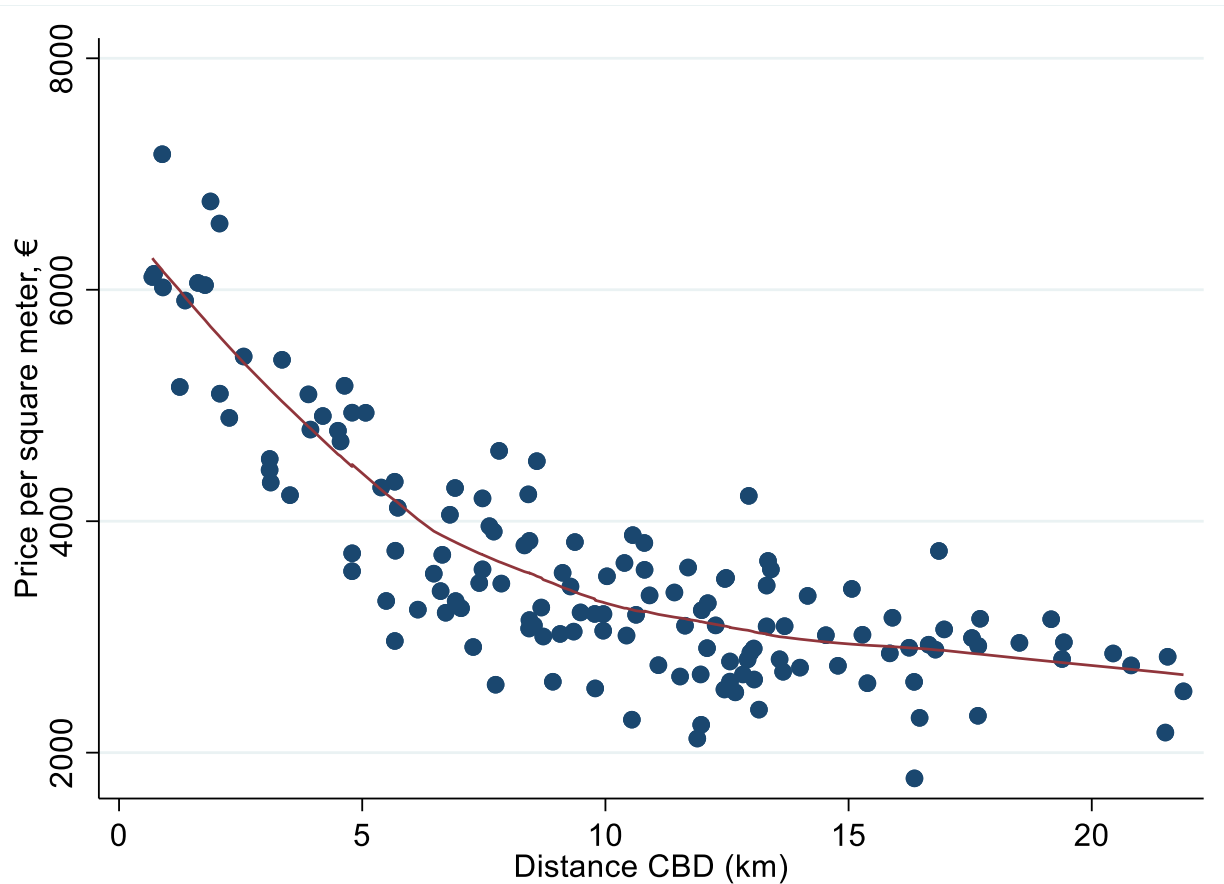
- **So far, the model's two main predictions are that as distance to the CBD increases**
  1. Price per square meter of housing falls;  $p \downarrow$  as  $x \uparrow$
  2. Size of the dwellings increases;  $q \uparrow$  as  $x \uparrow$
- **Two additional results concern the shape of the curve relating housing price  $p$  and distance  $x$  and the total price/rent  $pq$  and distance  $x$**

# Housing price curve



- The **price curve is convex** if housing increases with  $x$
- Consumers substitute cheaper housing for bread, so prices don't have to decline as quickly to compensate consumers

# Housing price curve in HMA postcodes



# Spatial behavior of total rent $pq$

- How does the **total rent ( $pq$ )** for a small central-city dwelling compare to the total rent of a larger suburban house?
- **The answer is ambiguous**
  - Since  $p$  falls with  $x$  while  $q$  increases, the product  $pq$  could either increase or decrease
  - Which is the case, depends on the consumer's preferences or the shape of the indifference curve

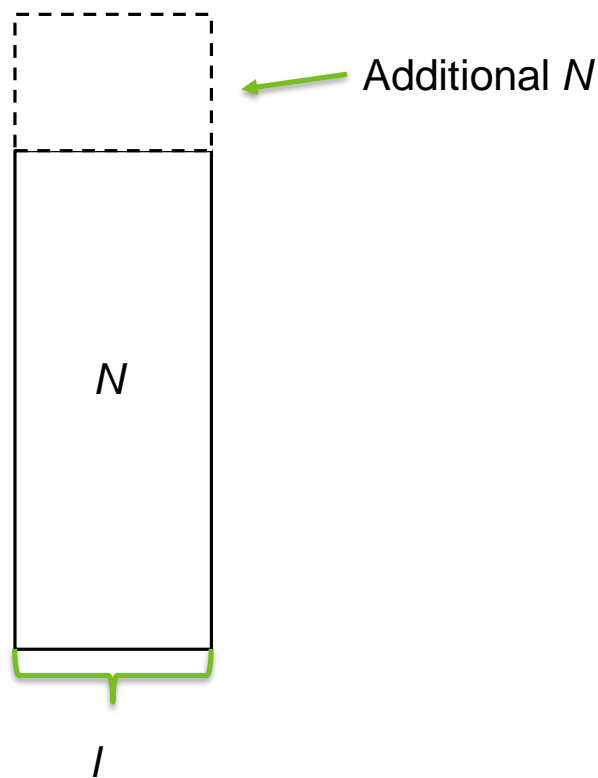
# **Analysis of housing production**

# Analysis of housing production

- Now we turn to analyzing the production side of the market and shift the focus to the activities of housing developers who build structures and rent the space to consumers
- Again, this is a stylized model with several simplifying assumptions
- We assume that housing is produced using only **land ( $l$ )** and **building materials ( $N$ )**
- The production function for housing is  $Q = H(N, l)$ , where  $Q$  is the amount of floor space in the building (remember that  $q$  was dwelling size)

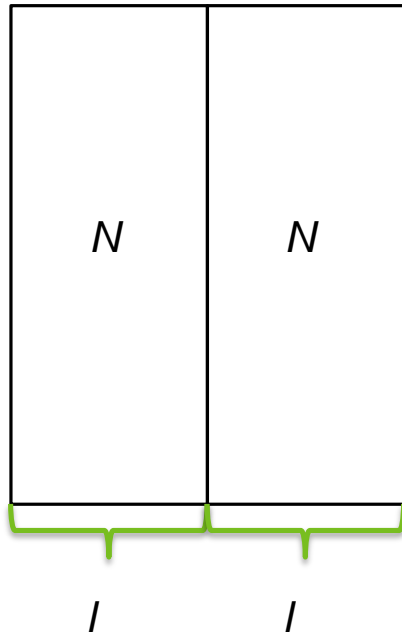


# Diminishing marginal return to capital



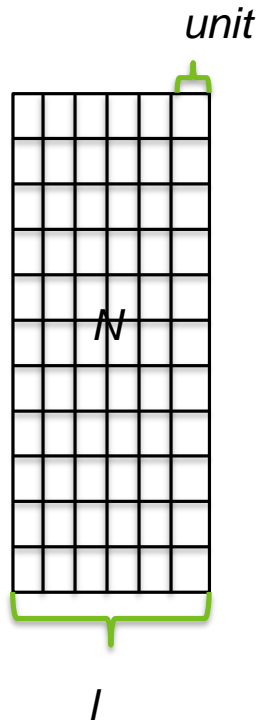
- With land input (size of the lot) held fixed, extra doses of building material lead to a smaller and smaller increases in floor space
- This makes sense as increasing  $N$  with fixed  $l$  makes the building taller
  - *Stronger foundation, thicker beams, elevators...*
- Note that we are assuming that the building completely covers the land area, so there are no yards
- Again, this assumption can be relaxed with the of price of complicating the model

# Constant returns to scale



- Economies of scale are present when doubling both the capital and land inputs leads to more than a doubling of the floor space
- The figure suggests that doubling both inputs leads to exactly doubling of floor space
- Thus, we assume that housing production exhibits “**constant returns to scale**”, at least approximately

# Profit maximization



- The housing developer chooses the capital and land inputs to maximize profits, which leads to a building of particular height
- The developer also implicitly chooses the amount and size of the dwellings in the building
- The latter decision simply responds to consumer choices, i.e. the demand side of the market

# Profit maximization

- The developer's revenue from a building is  $pQ$  or  $pH(N, I)$ , where  $p$  is the price per square meter of housing as before
- Input costs come from capital and land
- We assume that the developer rents the land and capital inputs
  - Land rent per square meter is denoted with  $r$  and rent for capital with  $i$
  - The price of capital is assumed to be fixed, i.e. there are no differences in physical building costs within the city
- Production costs equal  $iN + rI$ , i.e. capital costs + land costs

# Equal profits across space

- In consumer analysis, the utility or welfare of all consumers was the same everywhere in the city
- We have a similar **spatial equilibrium condition** on the producer side: **profits are equal everywhere in the city**
  - If not, developers would not be willing to build housing everywhere
  - Because  $i$  doesn't vary with location, it is **the spatial variation in land rent  $r$  that equates profits** and makes developers willing to build housing throughout the city

# Land rent as a compensating differential

- **Central-city locations offer higher revenue per square meter than suburban locations**
- **This means that land rents must be lower in the suburbs than at central locations**
- **With  $r$  falling as distance  $x$  increases, the disadvantage of lower revenue is offset**

# Land rent as a compensating differential

- **We can also think about this as a demand-based phenomenon**
  - Developers compete for prime locations where housing prices are high
  - This higher demand for land and competition among developers bids up land rents near the CBD
  - There is less demand for remote lots and land rents will be lower farther away from the CBD
  - Competition for prime locations drives land rents so high that uniform profits across space imply a zero-profit level (**normal economic profit**)

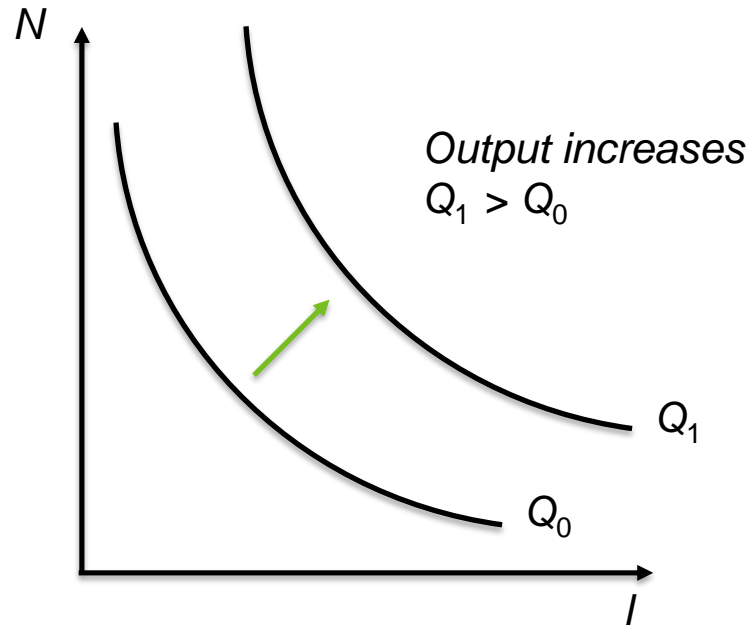
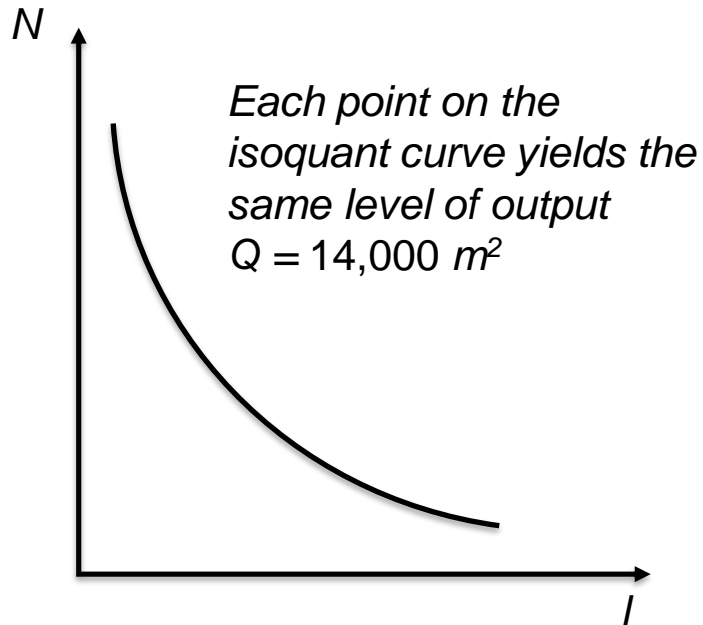
# Building height and distance

- **With the price of capital fixed and land rent rising moving toward the CBD, the land input becomes more expensive relative to capital as distance to CBD declines**
  - This incentivizes developers to economize on land input and use more capital which leads to taller buildings
  - Conversely, as land gets cheaper moving away from CBD, developers use more of it and build shorter buildings
- **Overall, building height decreases as distance to the CBD increases**



# Isoquant curve diagram

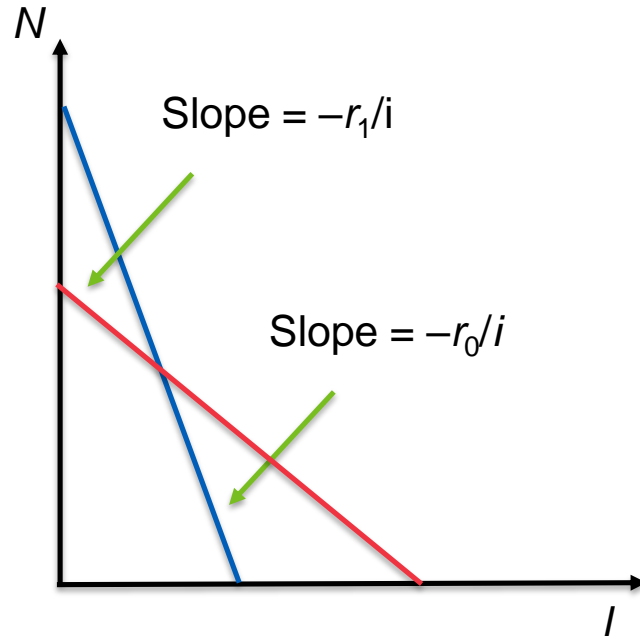
- A graphical way of deriving this result is to use a diagram illustrating **cost minimization** on the part of the developers
- **Isoquant curves** that show the combinations of inputs that yield the same output



# Iso-cost lines

Fix  $iN + r_l$  to some number

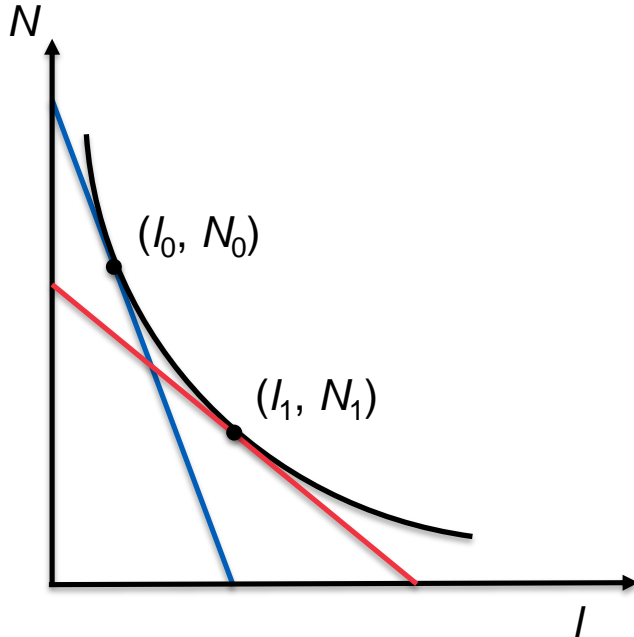
— Central-city  
— Suburb



- **The figure depicts two iso-cost lines**
  - **Iso-cost line** = combination of inputs that cost the same total amount
  - Central-city developer at a distance of  $x_0$  away from CBD with land rent  $r_0$
  - Suburban developer with  $x_1$  and  $r_1$
  - The **slopes of the iso-cost** lines are  $-r_0/i$  and  $-r_1/i$ , respectively
  - The slope for the central-city developer is steeper, because  $r_0$  is high
  - I.e. the central-city developer has to give up more  $N$  in order to acquire additional units of land ( $iN$  buys you less  $l$ )

# Central-city and suburban developer

— Central-city  
— Suburb



- To produce  $Q$  square meters of floor space **as cheaply as possible**, each developer chooses the **input bundle on lowest possible iso-cost line**
- Developers facing different land rents (but same capital rents) use different amounts of inputs
- Since the central-city developer is using more capital and less land, the central-city building is taller than the suburban building

# Predictions from producer analysis

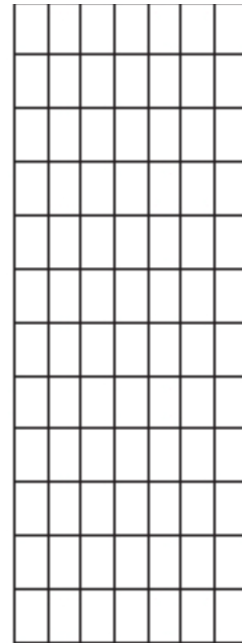
- **Two main predictions are that as distance to the CBD increases**
  1. Price per square acre of land falls;  $r \downarrow$  as  $x \uparrow$
  2. Building height decreases; building height  $\downarrow$  as  $x \uparrow$

# Population density

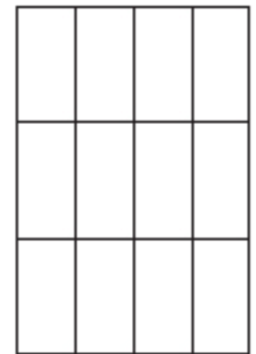
- **Combining the consumer and producer analysis yields a further result regarding population density in different parts of the city**
  - **Population density ( $D$ )** is measured as the number of people per  $\text{km}^2$
  - Central-city location has tall buildings divided into small dwellings, while the suburb has short buildings divided into larger dwellings
  - This implies that **population density is higher in the central-city**
- **Thus,  $D$  falls moving away from CBD;  $D \downarrow$  as  $x \uparrow$**

# Population density

**Buildings and dwelling sizes in central-city and the suburb**

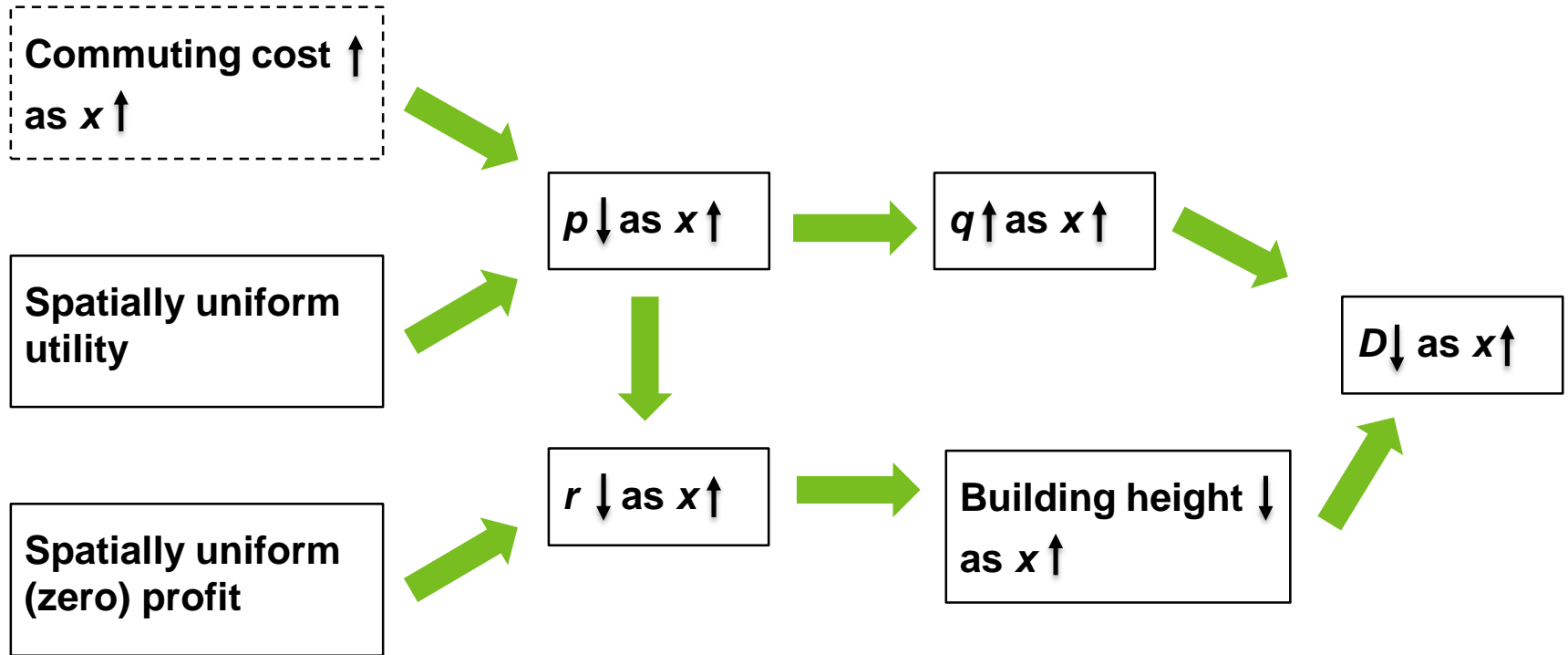


Central city  
(many dwellings per acre)



Suburbs  
(fewer dwellings per acre)

# Summary of the model logic



# Empirical example – New metro line

OSKARI HARJUNEN

**METRO INVESTMENT  
AND THE HOUSING  
MARKET ANTICIPATION  
EFFECT**





# Do people value accessibility and by how much?

- One of the central predictions of the model was that house prices per square meter are higher in locations closer to the CBD with low commuting costs
- But the model is very stylized. In reality, locations differ in several other ways besides accessibility
- How can we know whether and by **how much** people really value accessibility?
- **Solution:** look at how prices change when accessibility changes and compare this to price changes in places where accessibility does not change (DID)

# West Metro extension

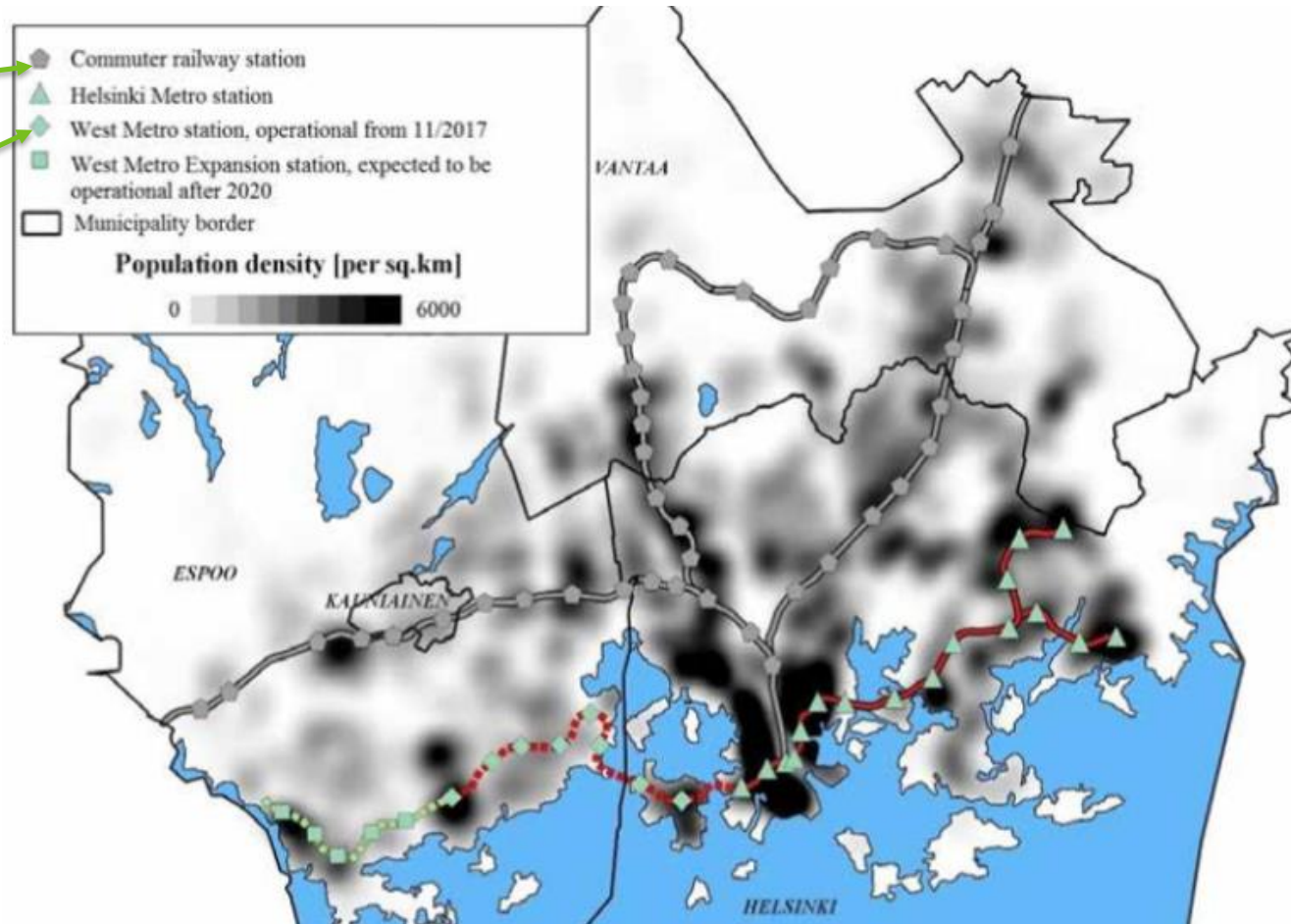
- **Harjunen (2018, chapter of his PhD thesis) analyzes the price effects of the West Metro extension in the HMA**
  - [https://www.hel.fi/hel2/tietokeskus/julkaisut/pdf/18\\_01\\_25\\_tyopapereita\\_02\\_Harjunen.pdf](https://www.hel.fi/hel2/tietokeskus/julkaisut/pdf/18_01_25_tyopapereita_02_Harjunen.pdf)
- **The West Metro introduced eight new metro stations – two in Helsinki and six in Espoo**
- **The study focuses on the time period when the construction of the new line started, but before it became operational**
- **The idea is to see whether the is anticipated in the housing market already before the new line was operational**



# Research design

Control

Treatment



# Timing

- **The construction of the West Metro was finally approved in the city councils of Espoo and Helsinki in September 2008**
- **The underground master plan of the West Metro was approved in January 2009 and the official ceremony initiating the construction works took place 11th of November**
  - But in large scale the constructions began in 2010 after delays caused by the appeal process
- **In the study the “treatment period” begins at the start of 2010**
- **At this time, it was clear where the new metro stations will be located**

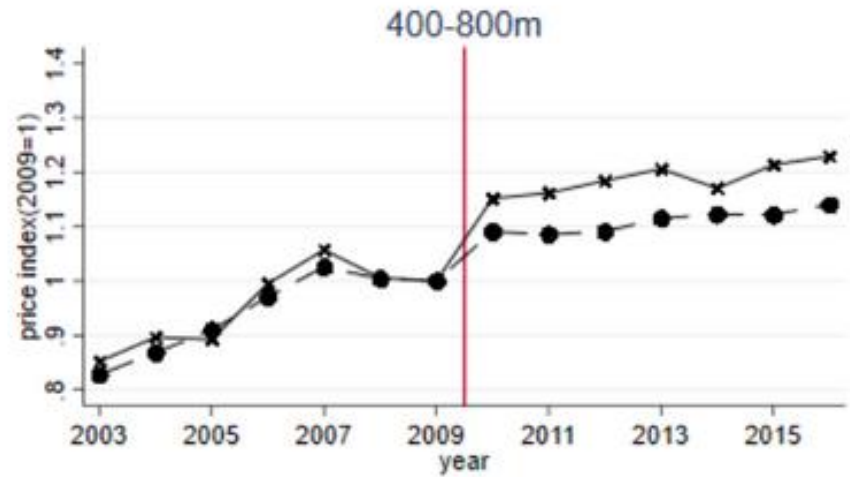
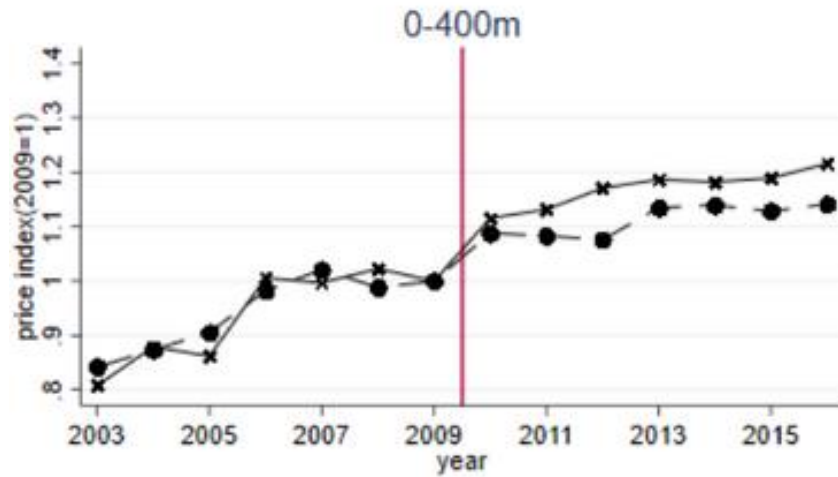
# Data

- **Data comprises of transactions in Helsinki and Espoo from 2003 to 2016**
- **The data are collected by a consortium of Finnish real estate brokers and the dataset is refined and maintained by the Central Federation of Finnish Real Estate Agencies (KVKL)**
  - As not all real estate agencies participate, the dataset represents a sample (albeit rather large) of the total volume of transactions
- **The data include the transaction price and sale date for each dwelling as well as a rich set of dwelling characteristics including its exact location**
  - The sample is restricted to multi-story and row house sales

# Data

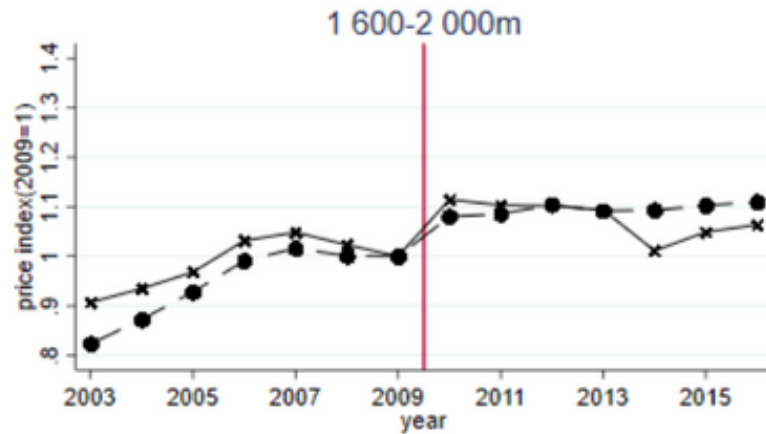
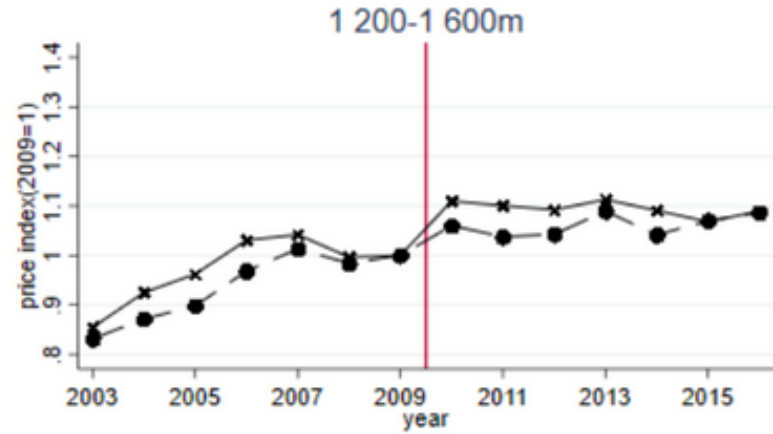
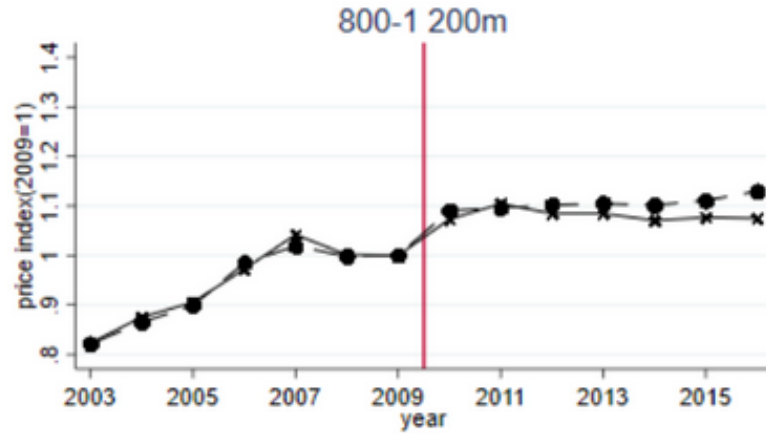
Sample	Whole data (Helsinki and Espoo)				
	0 to 800m		800 to 1 600m		
Status		Treated	Control	Treated	Control
<b>N</b>	<b>43 025</b>	<b>6 868</b>	<b>15 640</b>	<b>4 429</b>	<b>11 267</b>
<b>Sale price</b>	223 668 [110 007]	252 024 [119 458]	196 154 [78 980]	311 661 [156 343]	199 122 [82 107]
<b>Square price</b>	3 506 [918]	4 181 [951]	3 325 [805]	3 877 [919]	3 242 [805]
<b>Area</b>	66 [29]	62 [27]	61 [25]	82 [38]	64 [27]
<b>Age</b>	37 [17]	43 [17]	32 [17]	32 [13]	39 [18]
<b>Maint. Charge (€/m2)</b>	3,5 [1.2]	3,8 [1.1]	3,5 [1.2]	3,5 [1.2]	3,5 [1.3]
<b>Floor number</b>	2,4 [1.6]	2,7 [1.7]	2,5 [1.5]	2,3 [1.5]	2,3 [1.4]
<b>Floors in building</b>	3,8 [3.0]	4,4 [2.2]	3,8 [2.1]	3,6 [2.3]	3,4 [1.9]
<b>Dist. to nearest station (m)</b>	869 [489]	482 [190]	484 [185]	1 168 [239]	1 134 [239]
<b>Dist to CBD (km)</b>	12 [4.6]	9 [3.6]	13 [4.8]	11,2 [3.2]	12,5 [4.6]

# Results – graphical



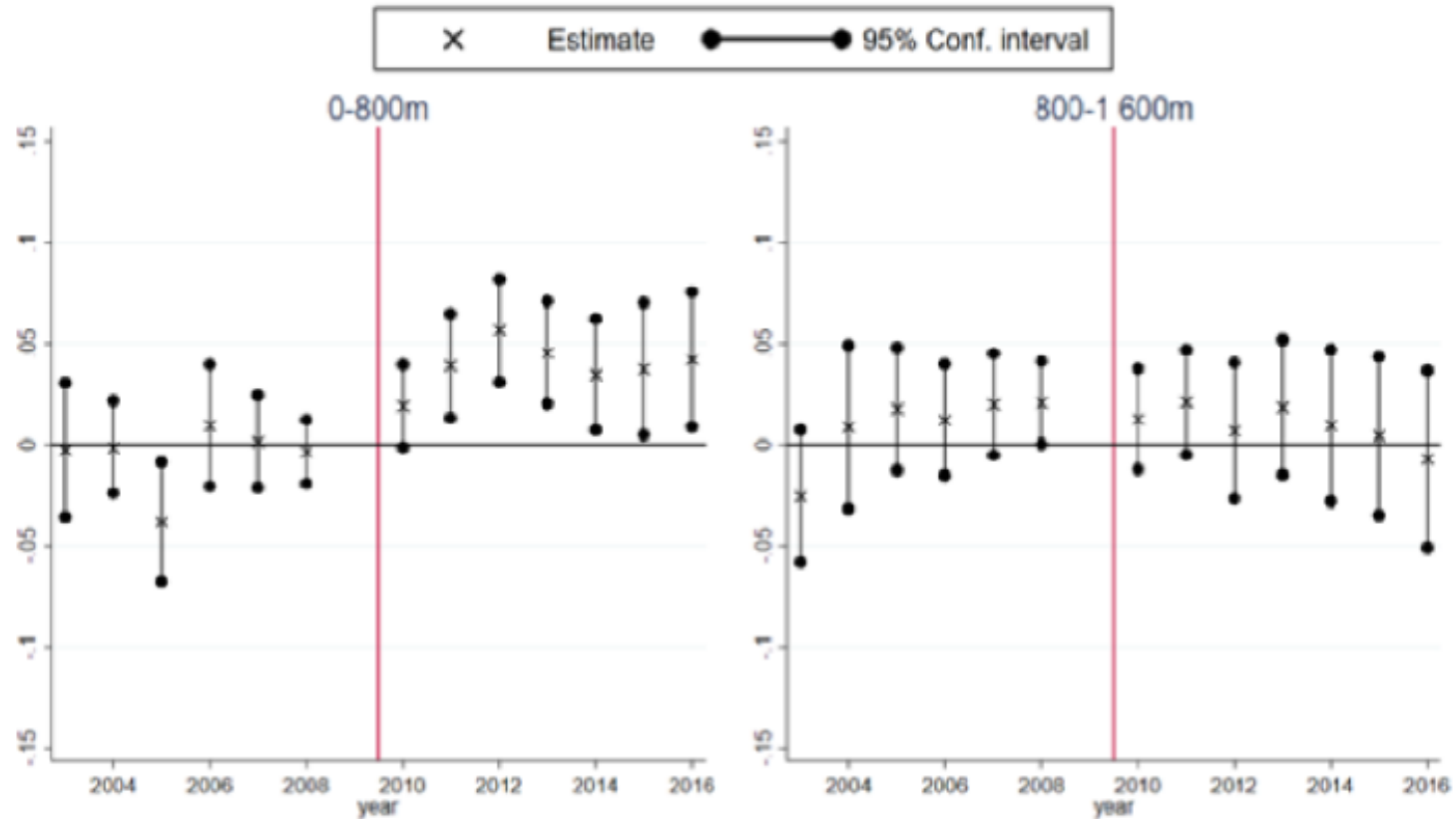


# Results – graphical



# Results – regression model

Fig. 3. Coefficients of yearly estimates, year 2009 omitted



# Conclusion from the study

- **Housing markets start adjusting to the information about the infrastructure investment swiftly after the construction begins, years before the line becomes operational**
- **Apartments within 800 meters from the new metro stations, where the accessibility will be increased the most, experience a positive price increase that converges to around 4%**
- **Question: What are the likely further effects of this price increase in the old housing stock?**