



Aalto University  
School of Engineering

**MEC-E5003**

**FLUID POWER BASICS**

**Study Year 2020**

# Hydromechanics



Aalto University  
School of Engineering  
Mechanical Engineering / Engineering Design / Mechatronics / Fluid Power

# Lecture themes

Fluid – Does it matter which?

Viscosity – How and why?

Flow – What is it needed for?

Is there a connection between pressure and flow?

# Hydrodynamics

Flowing fluid under internal and external load

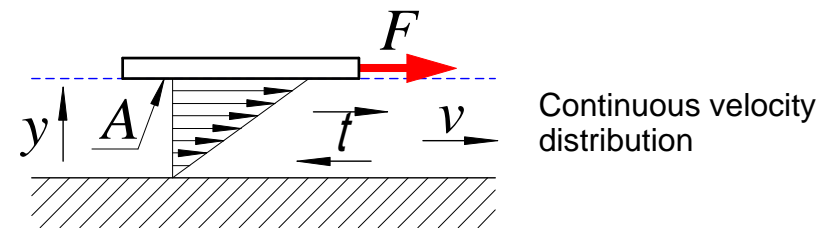
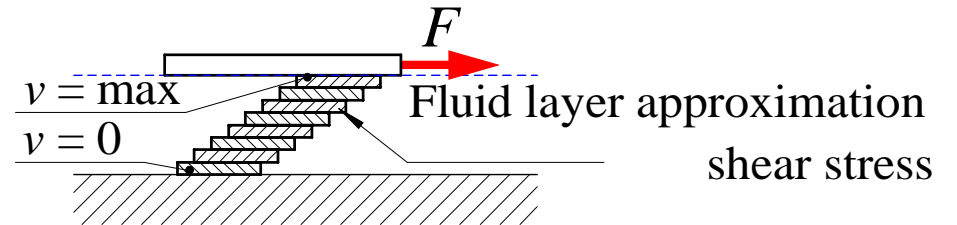
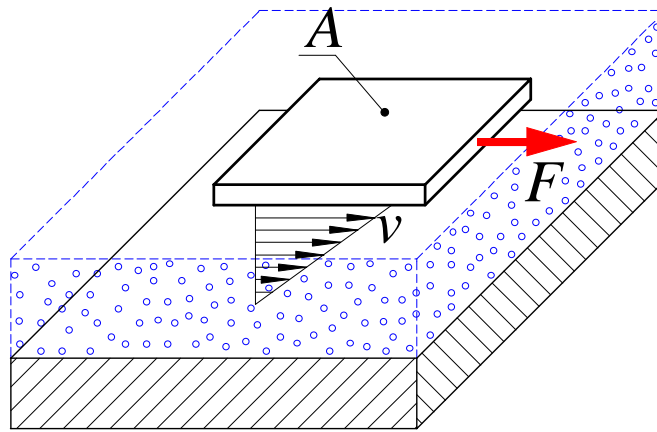
- mass
- internal and external friction
- compressibility

(well, not necessarily all of these in every case...)

# Viscosity

The shear force  $F$  and shear stress  $\tau$  depend on

- (dynamic) viscosity  $\eta$
- velocity gradient



$$F = h \times A \times \frac{v}{y}$$

$$t = h \times \frac{dv}{dy}$$

Newtonian fluid:  
viscosity independent  
of shear rate

Viscosity factor  $h$  represents the properties of fluid, “tenacity”

Dynamic viscosity  $\eta$

Unit [Pa·s]

$$1 \text{ cP} = 10^{-3} \text{ Pa}\cdot\text{s}$$

Kinematic viscosity  $\nu$

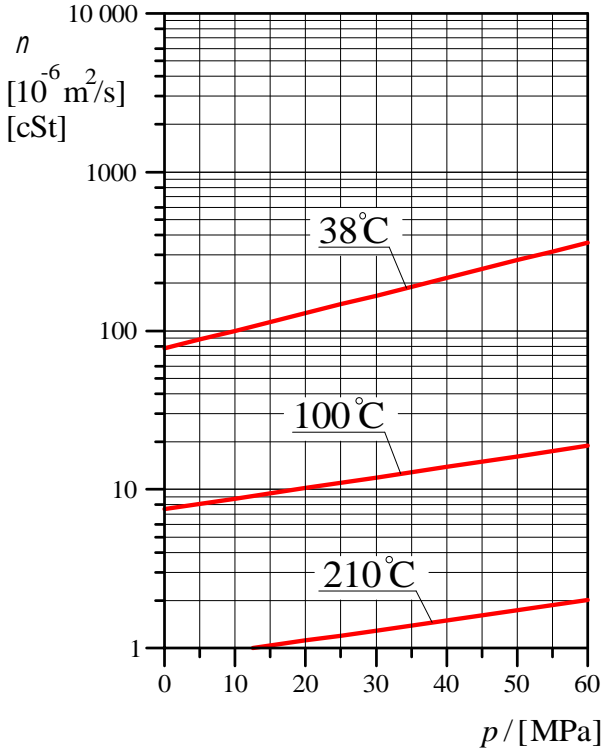
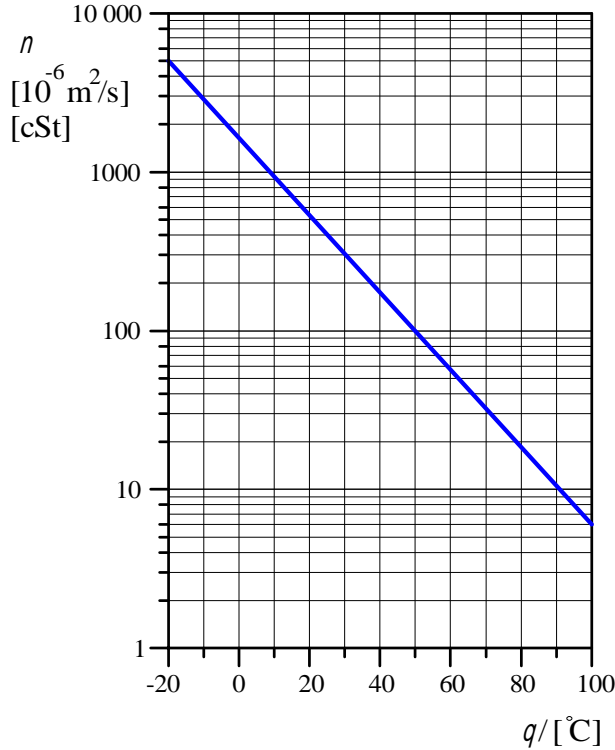
$$\nu = \frac{\eta}{\rho}$$

Unit [m<sup>2</sup>/s]

$$1 \text{ cSt} = 10^{-6} \text{ m}^2/\text{s}$$

# Temperature and pressure dependence of viscosity

Kinematic viscosity



# Impact of viscosity

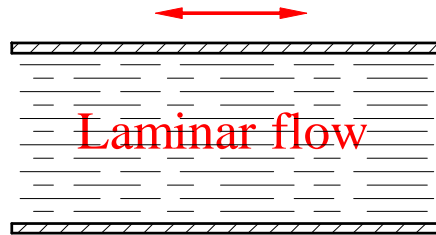
Viscosity affects

- flow induced resistance inside the system
- internal and external leaks of the system
  - ® system efficiency
- lubrication of components
  - ® reliability and life span of the system

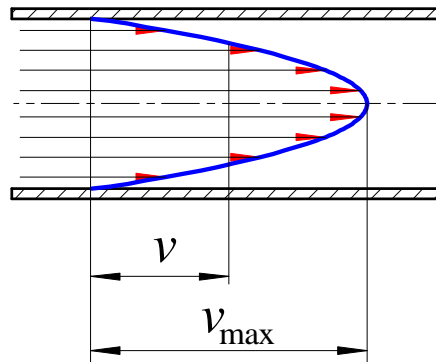


# Flow and flow types

Laminar flow  
fluid moves  
in layers



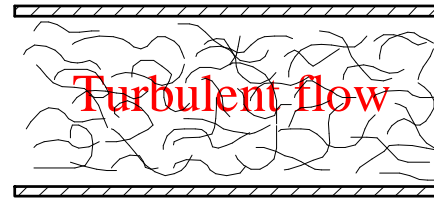
Laminar when  
 $Re < 2300$



Flow friction  $\tau = \eta \frac{dv}{dy}$

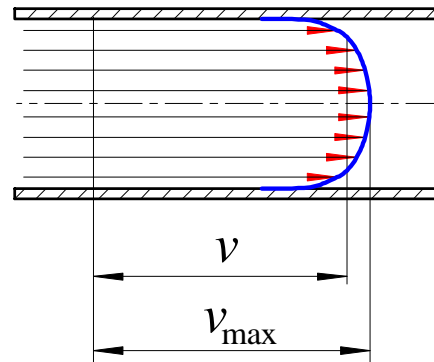
In laminar pipe flow  
 $v_{\text{average}} = 1/2 v_{\text{max}}$   
Parabolic velocity  
profile

Reynolds number



Turbulent flow  
irregular movement of  
particles (chaotic)

Transitional flow  
 $2300 < Re < 4000$   
Turbulent flow  
 $Re > 4000$

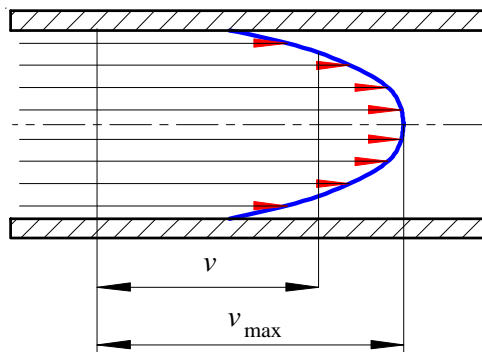


$$Re = \frac{v > D_H}{n}$$

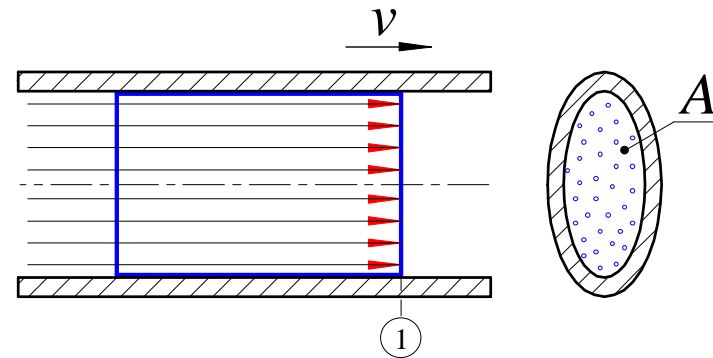
- mean velocity
- (hydraulic) diameter
- kinematic viscosity

# Flow rate

Actual flow profile



Simplified flow profile



Plug flow

$$DV = A \cdot l = A \cdot v \cdot Dt$$

$$\frac{dV}{dt} = \dot{V} = A \cdot v$$

$$q_v = A \cdot v$$

Flow rate and average flow velocity

# Continuity equation

Simplifying assumption

$$\dot{m} = \frac{dm}{dt} = \text{constant}$$

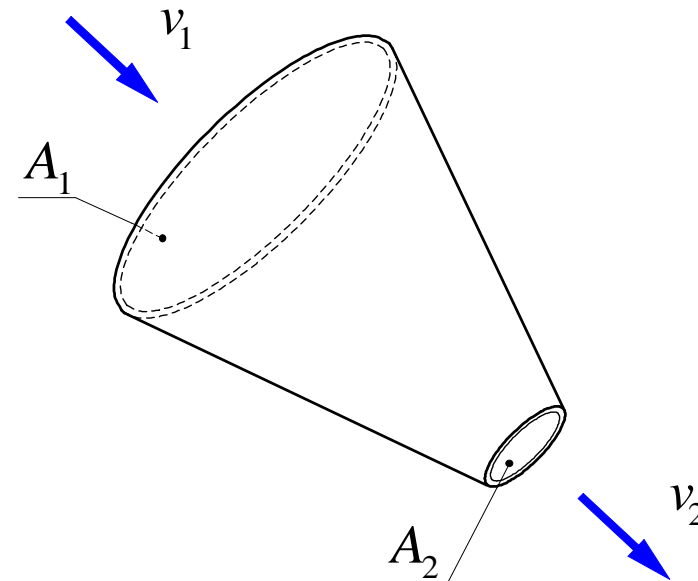
If density does not change

$$q_V = A_1 v_1 = A_2 v_2$$

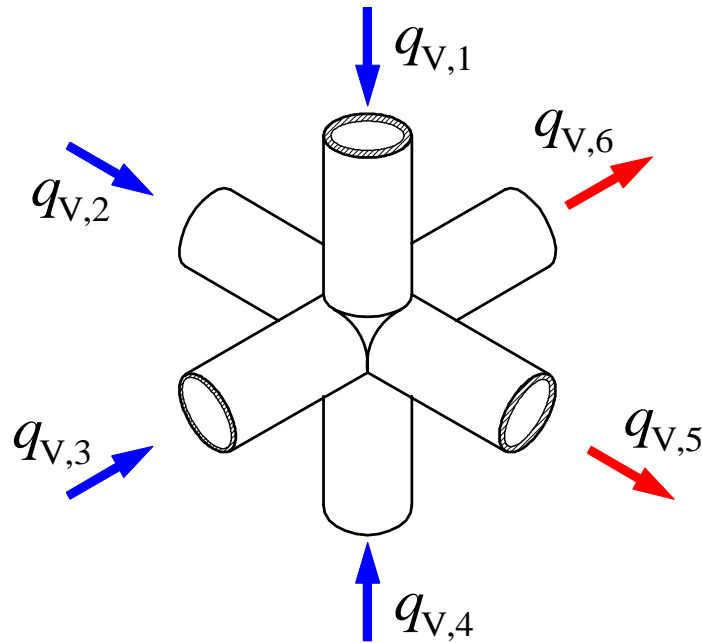
Unit [m<sup>3</sup>/s]

1 l/min = 1/60000 m<sup>3</sup>/s    Engineering unit [l/min]

In reality:  $q_m = r \times \dot{V} + V \times \dot{r}$     mass flow rate



# Division and joining of flow



We assume that the volume of intersection is zero  $\Rightarrow$  no fluid is stored in it.

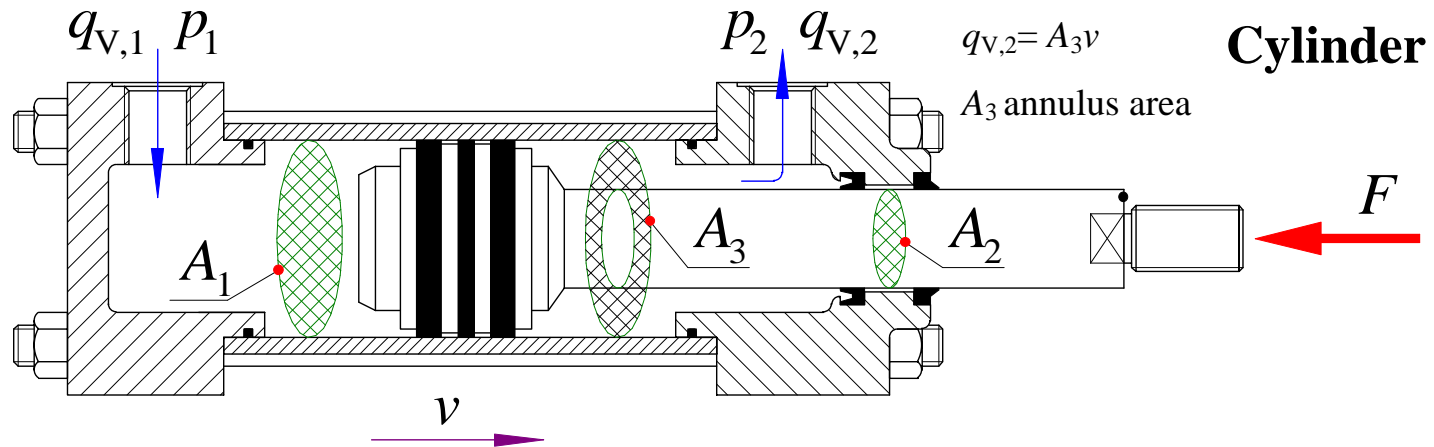
Fluid can be "stored" in "volumes" like pipes, cylinder chambers and accumulators  $\Rightarrow$  the amount of fluid stored affects the pressure.

Kirchhoff's I law

$$q_{V,1} + q_{V,2} + q_{V,3} + q_{V,4} = q_{V,5} + q_{V,6}$$

# Flow: An application example

- Stationary flow case:
- flow rate does not change
  - piston velocity does not change
  - pressures ( $p_1, p_2$ ) do not change



Continuity equation

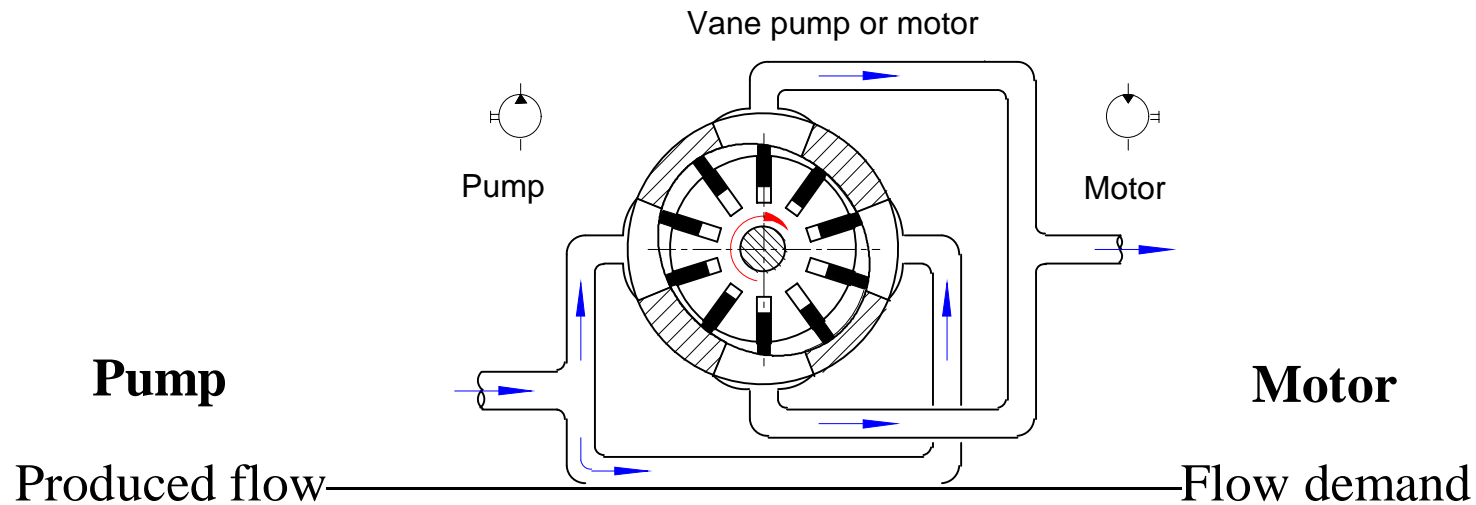
$$q_{v,1} = A_1 v$$

Flow demand

$$q_{v,1} = A_1 v$$

What will happen if  $q_{v,1} \neq A_1 v$  ?

# Flow rate: An application example



$$q_{V,p} = n_p \times V_{g,p}$$

No leakage! Theoretical!

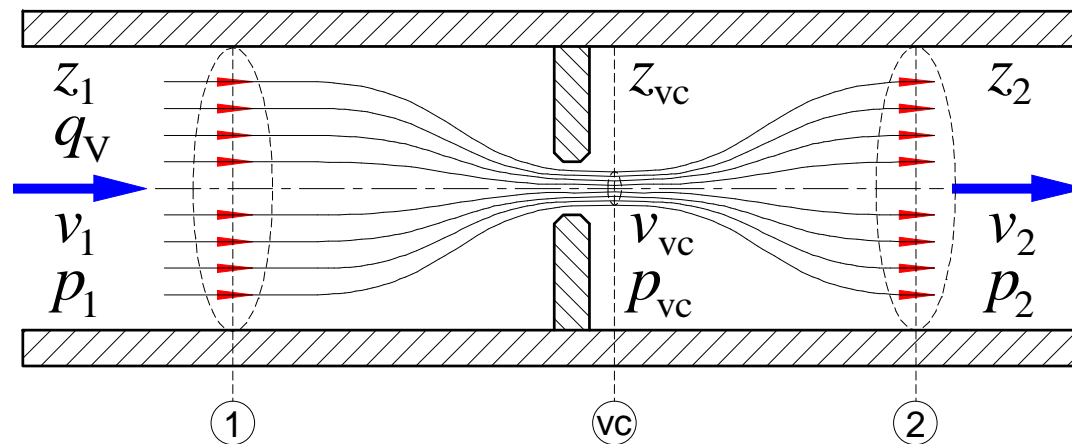
$$q_{V,m} = n_m \times V_{g,m}$$

1. Rotational speed
2. Swept volume (displacement)

# Energy equation

## Flow through orifice

Flow rate remains the same

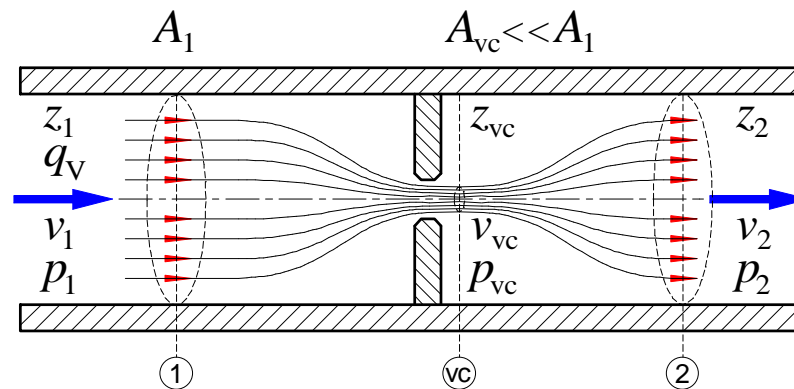


VC - Vena Contracta

$$\text{Bernoulli: } p + \rho \times g \times z + \frac{\rho \times v^2}{2} = \text{constant}$$

- static
- hydrostatic
- dynamic

vc =  
vena contracta  
diameter of flow  
is the least



Using  
observation  
points  
1 and vc:

$$p_1 + \rho \times g \times z_1 + \frac{\rho \times v_1^2}{2} = p_{vc} + \rho \times g \times z_{vc} + \frac{\rho \times v_{vc}^2}{2}$$

Simplification ③

$$p_1 = p_{vc} + \frac{\rho \times v_{vc}^2}{2}$$

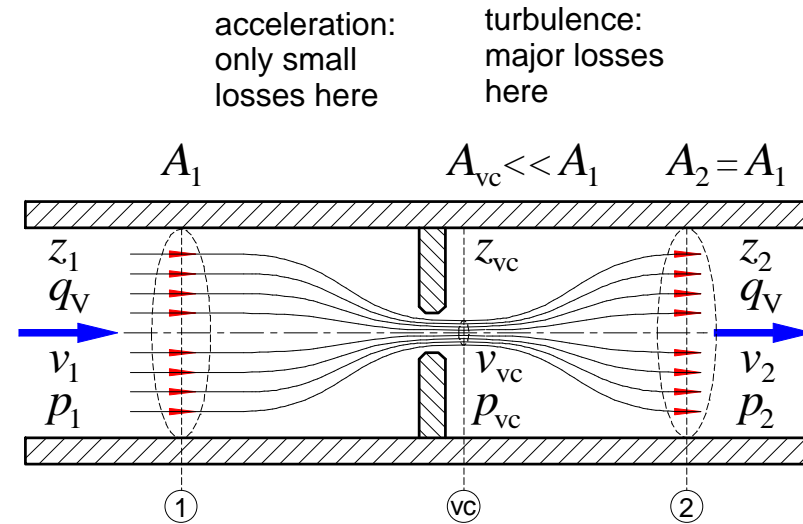
$v_1$  is small compared with  $v_{vc}$   
also squared!  
 $\Rightarrow p_{vc}$  must be smaller



$$p_1 = p_{vc} + \frac{\rho v_{vc}^2}{2}$$

Flow velocity  
in vena contracta

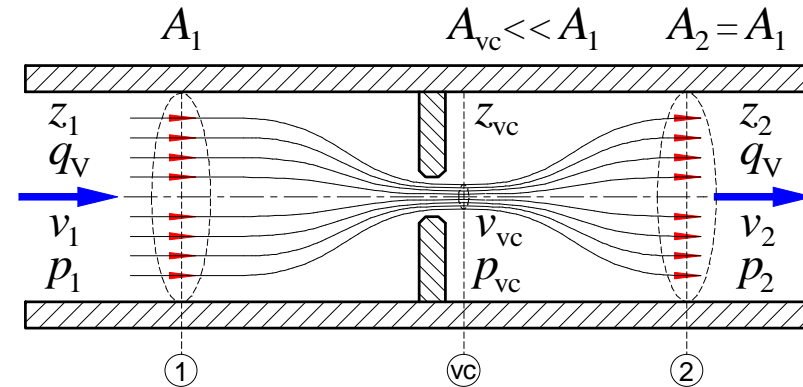
$$\textcircled{R} \quad v_{vc} = \sqrt{\frac{2(p_1 - p_{vc})}{\rho}}$$



Flow rate  $q_V = A v$

$$\textcircled{R} \quad q_{Vvc} = A_{vc} \times \sqrt{\frac{2(p_1 - p_{vc})}{\rho}}$$

$$q_{Vvc} = A_{vc} \times \sqrt{\frac{2 \times (p_1 - p_{vc})}{\rho}}$$



Point of vena contracta is difficult to measure

- transfer latter observation point (vc) to point 2, much more meaningful
- change requires a correction factor  $C_q$  to the equation

$$\textcircled{R} \quad q_V = C_q \times A \times \sqrt{\frac{2 \times (p_1 - p_2)}{\rho}}$$

Denote  $\Delta p = p_1 - p_2$

$$\textcircled{R} \quad q_v = C_q \times A \times \sqrt{\frac{2 \times \Delta p}{\rho}}$$

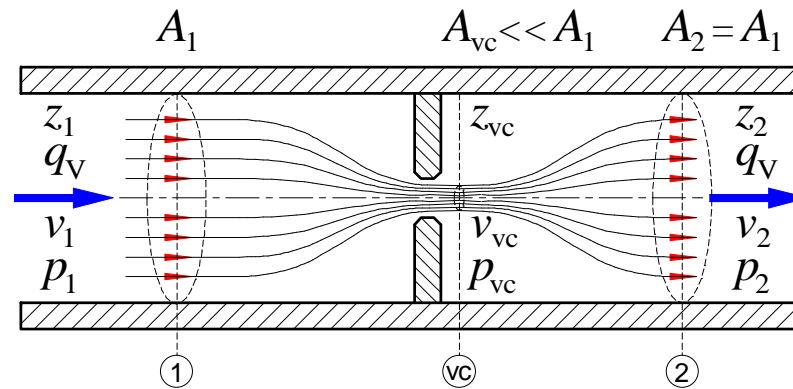
Flow coefficient  $C_q$

According to Bernoulli  $p + \rho \times g \times z + \frac{\rho \times v^2}{2} = \text{constant}$

pressures  $p_1 = p_2$ , since flow channel properties in observation points 1 and 2 are equivalent

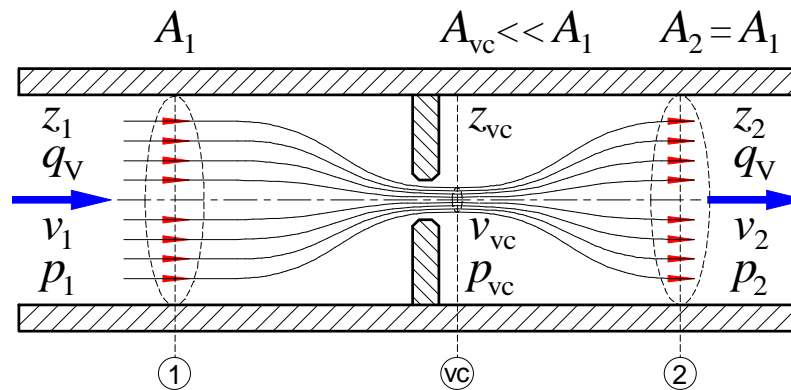
, so why  $\Delta p$ ?

Static pressure  $p$  does not normalize (reach its original value) after energy conversion although flow channel properties normalize



An energy loss takes place and is manifested as pressure loss  $Dp$

$$\textcircled{R} \quad p_1 + r \times g \times z_1 + \frac{r \times v_1^2}{2} = p_2 + r \times g \times z_2 + \frac{r \times v_2^2}{2} + Dp \quad \text{Energy equation}$$



$$p_1 + r \times g \times z_1 + \frac{r \times v_1^2}{2} = p_2 + r \times g \times z_2 + \frac{r \times v_2^2}{2} + Dp$$

In hydrostatic systems the heads of elevation and flow velocities are typically low

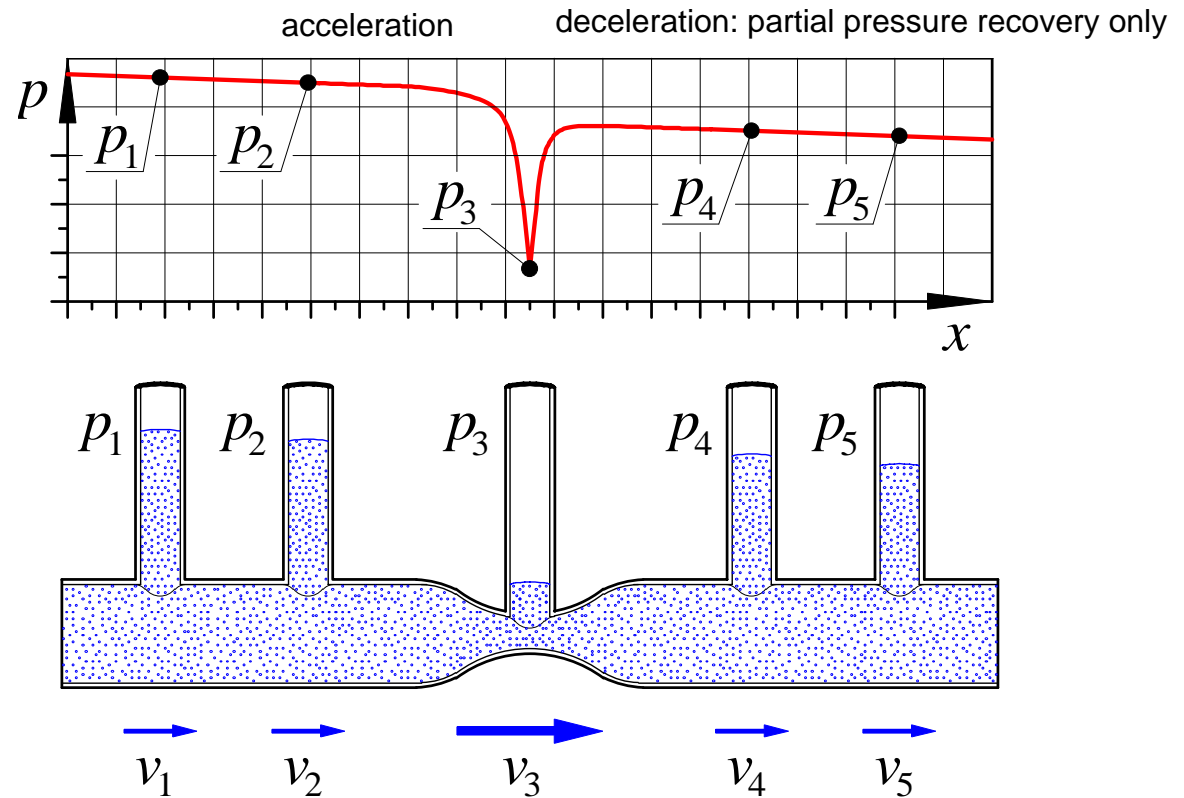
$$\textcircled{R} \quad p_1 = p_2 + Dp \quad \text{or} \quad p_1 = p_2 + p_s$$

## Interconnectedness of pressure and flow

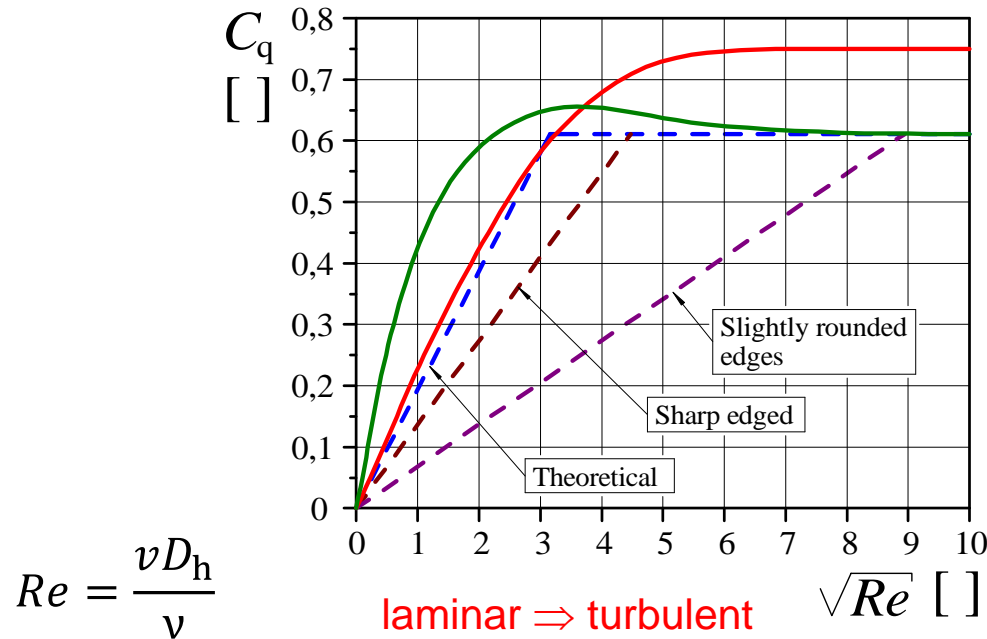
$$p_1 = p_2 + \Delta p$$

$$(\Delta p = p_1 - p_2)$$

Pressure losses in  
 - pipes  
 - orifice



At small velocities (pressure differences) also orifice flow is laminar.



Flow coefficient  $C_q$

Theoretical value 0,611

$$q_v = C_q \times A \times \sqrt{\frac{2 \times Dp}{\rho}}$$

laminar flow

$Dp \propto \eta v$  to the 1st power of  $v$

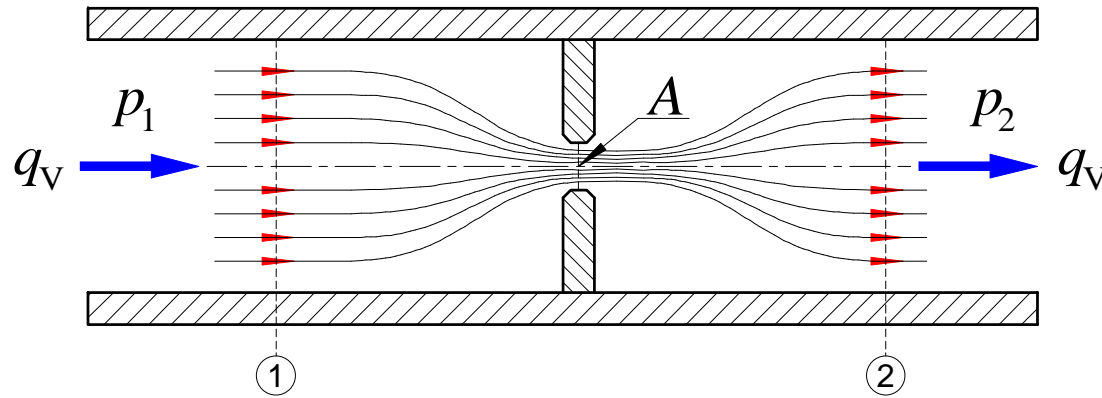
turbulent flow

$Dp \propto 1/2 \rho v^2$  to the 2nd power of  $v$

Flow coefficient  $C_q$  <sup>1</sup> Discharge coefficient  $C_d$

Flow coefficient  $C_q \approx$  Velocity coefficient  $C_v$   $\times$  Contraction coefficient  $C_c$   
 Actual velocity of Jet / Theoretical velocity    Area of Jet / Area of Orifice

So,



Pressure difference induces flow

«

(Flow induces pressure difference)

$$q_v = C_q \times A \times \sqrt{\frac{2 \times \Delta p}{\rho}}$$

$$\Delta p = \frac{\rho}{2} \times \left( \frac{q_v}{C_q \times A} \right)^2$$

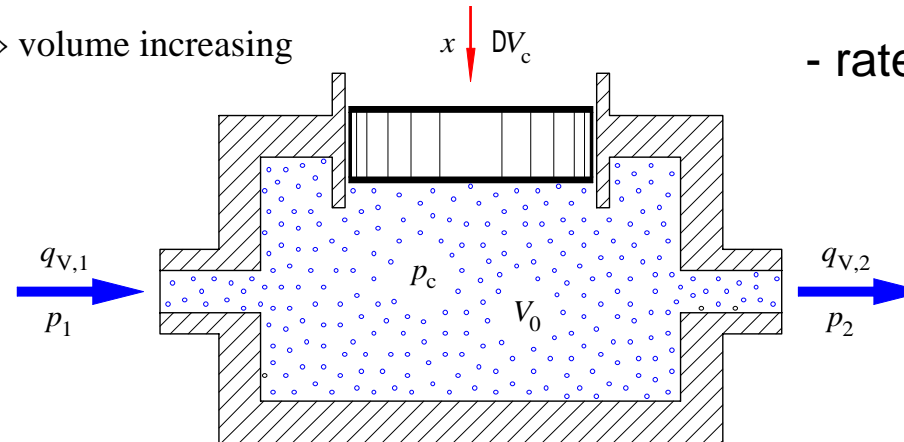
$1/2 \rho v^2$  included  
(dynamic pressure)



# Flow and pressure in volume

Pressure induced by flow into and out of a volume

$\Delta V_c > 0 \Rightarrow$  volume increasing



Throttle restricting the inflow

Throttle restricting the outflow

Two mechanisms to change pressure

- net flow rate  $\neq 0$

- rate of change of volume  $\neq 0$

Time derivative of pressure: "how fast does the pressure change" [Pa/s]

Pressure generated in volume

$$\dot{p} = \frac{K_e}{V_{0,c}} \times [\dot{a} q_v - \dot{V}_c]$$

hydraulic stiffness

Thermal expansion coefficient  $\approx 0.7 \cdot 10^{-3} \text{ 1/K}$  (mineral oil)

Integrate this to get the pressure changes

# Fluid properties

Where do we need the information?

Density	turbulent pressure losses in orifices and losses in pipes
Viscosity	leakage, lubrication, laminar flow pressure losses
Bulk modulus	hydraulic stiffness, mechanical stiffness

# Lecture themes - Recap

Do the fluid properties have impact on the system?

Flow rate, from where?

Is there any use for flow rate?

Interconnection between flow and pressure,  
does it exist?