General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. The (attempted) solutions should be submitted via email to the assistant at the start of the exercise session on January 20. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.
The fusion reaction

a) The chemical reaction describing the complete combustion of gasoline is

$$2\ C_8H_{18} + 25\ O_2 \rightarrow 16\ CO_2 + 18\ H_2O + 94\ \text{eV},$$

releasing an average energy of 0.9 eV per atom. The D-T fusion reaction

$$\text{D} + \text{T} \rightarrow {}^4\text{He} + \text{n} + 17.6\ \text{MeV}$$

releases, on average, 8.8 MeV per atom, a factor of 10^7 times more than the cemical combustion of gasoline per atom. What are the reasons for this difference, and how do these values compare to the ionization potential for e.g. hydrogen?

b) Show that the neutron carries 14.05 MeV of the 17.6 MeV produced in D-T fusion. What are the implications for fusion control of this? It is advisable to use a center of mass coordinate system for the calculations.

Exercise 2.
Ignition (Lawson) criterion

a) **Ideal ignition:** Find the ignition temperature of D-T fuel in a thermonuclear fusion reaction, only considering losses due to Bremsstrahlung. Is this a stable or unstable equilibrium? Assume a 50-50 D-T fuel mix and the low temperature ($T < 25$ keV) approximation for the mean collisionalities

$$\langle \sigma v \rangle_{DT} \approx 3.68 \times 10^{-18} T^{-2/3} \exp\left(-19.94 T^{-1/3}\right)\ \text{m}^3\ \text{s}^{-1},$$

for $[T] = \text{keV}$.

b) **Ignition:** Derive the ignition (Lawson) criterion for $n\tau_E$, where τ_E is the energy confinement time, considering power losses due to Bremsstrahlung radiation and thermal conduction $P_{therm} \approx W/\tau_E$, were W denotes the plasma internal energy. Spectral line radiation losses are not considered for simplicity. Plot the obtained relation, using the same approximation for the mean collisionality as above. How does the obtained curve relate to the temperature found above?

c) **Ideal operation:** What is the minimum value of $n\tau_E$, and at which temperature does the minima occur? Assuming the hydrostatic plasma pressure to be $p = 2nT$, calculate the pressure required for ITER-like confinement times of 8 s. How does this pressure compare to the pressure at the center of the sun and the atmospheric pressure? What factors do you need to consider when analysing your answers from this exercise?

Exercise 3.
Plasma power balance and impurity concentration

a) **Carbon contamination:** Calculate the fusion power reduction due to fuel dilution and impurity radiation density for a $T = 10$ keV, $n_e = 10^{20}$ m^{-3} plasma with 4% $^6C^{6+}$ carbon concentration ($n_C/n_e = 0.04$), and compare the values to each other and to the power density of a pure D-T plasma. The power radiated by carbon at $T = 10$ keV, assuming coronal equilibrium, is $R_C = 10^{-34}$ Wm^3 [1] and the impurity radiation losses are given by

$$P_{rad} = n_Z n_e R_Z \quad [\mathrm{Wm}^{-3}]$$

Set out from the quasi-neutrality condition for plasmas and assume the electron density to have an upper density limit. Assume the carbon to be fully ionized.

b) **Tungsten contamination:** Perform the same calculations and analysis as above, assuming $n_W/n_e = 10^{-5}$ and that all tungsten atoms are have charge state 50 ($Z = 50$). The power radiated by tungsten at $T = 10$ keV, assuming coronal equilibrium, is $R_W = 10^{-31}$ Wm^3 [1].

c) What can be said about impurity contamination and fuel dilution based on your findings?

Exercise 4.
The Lawson criterion and fuel dilution and impurity radiation.

For realistic approximation of the Lawson criterion fuel dilution and impurity radiation must be considered when evaluating the operating space igniting the plasma. To investigate these effects, download the .m files in the exercise repository of MyCourses. The main script plotting_script_for_ntaue.m plots the viable ignition parameter regime for given helium confinement time parameter ρ^*, impurity concentration f_Z, and impurity charge state Z, with a switch whether to include radiation losses or not. As the rate of fusion reactions and the helium concentration are coupled, the function solve_fhe.m solves the helium concentration f_{He} from equation (2.8) in [2].

a) Investigate the parameter space for ignition for a pure plasma (no impurities, $f_Z = 0$). Try running the script with and without the radiation contribution for different values of ρ^*. What drives the closure of the ignition domain for a pure plasma?

b) Investigate the impact of carbon ($Z = 6$) and tungsten ($Z = 74$) impurities on the size of the ignition domain. Run the plotting script with $\rho^* = 5$ and try to replicate the figures on the page 24 in the lecture slides Fusion Principles. The result should look similar, but the models applied are necessarily not identical. Assume that the carbon impurities are fully ionized, and that the dominant charge state for the tungsten impurities is about 50. How does this reflect to the calculations in the previous exercise?

c) Based on your findings, outline the physical processes causing the closing and shrinkage of the parameter space for ignition. Sections 2 and 3 of the Reiter *et al.* paper [2] might be of assistance.

Constants:
1 eV $= 1.602 \times 10^{-19}$ J
$m_p = 1.673 \times 10^{-27}$ kg
$m_n = 1.675 \times 10^{-27}$ kg
$N_A = 6.022 \times 10^{23}$ mol^{-1}
$k_B = 1.381 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
$\sigma_{SB} = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
$c_B = 1.71 \times 10^{-38}$ W m^3 eV$^{-1/2}$

Power equations assuming pure hydrogenic (Z=1) plasma:

Fusion power density: $P_f = \alpha n_i n_j \langle \sigma v \rangle E_f$,
where E_f represents the produced energy per a fusion reaction, n_i and n_j are the fuel isotope densities, n_e the electron density, T_S the surface temperature of the black body, and A the surface area of the black body. The α parameter in the fusion power density equation is 1 for D-T fusion, and 1/2 for D-D fusion. This parameter arises due to the fact that when calculating the fusion cross-section integral ($\langle \sigma v \rangle$) for like particle collisions (D-D), every collision is counted twice. This should not be confused with the 1/4-factor that arises in the D-T fusion cross-section with 50-50 % fuel mixture due to $n_D = n_T = n_e/2 \rightarrow n_D \times n_T = (1/4) \times n_e$. More information can be found in e.g. [3].

Bremsstrahlung radiation: $P_{Br} = c_B n_e^2 T_e^{1/2}$

Black body radiation: $P_{bb} = \sigma_{SB} T_S^4 A$

References:
[1] J. Wesson, Tokamaks 3rd or 4th edition, Chapter 1.9
[2] D. Reiter, G.H. Wolf, H. Kever, "Burn condition, helium particle confinement and exhaust efficiency", Nucl. Fusion, 30, (1990), 2141
[3] J. P. Freidberg, Plasma Physics and Fusion Energy, Cambridge University Press, 2007, p.44