Experimental realization of a Szilard engine with a single electron

Jonne V. Koski[a,1], Ville F. Maisi[a,b,c], Jukka P. Pekola[a], and Dmitri V. Averin[d]

[a]Low Temperature Laboratory, Aalto University, FI-00076, Aalto, Finland; [b]Centre for Metrology and Accreditation, 02151, Espoo, Finland; [c]Department of Physics, Eidgenössische Technische Hochschule Zürich, CH-8093 Zürich, Switzerland; and [d]Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800

Edited by Peter J. Rossky, Rice University, Houston, TX and approved August 12, 2014 (received for review April 17, 2014)

The most succinct manifestation of the second law of thermodynamics is the limitation imposed by the Landauer principle on the amount of heat a Maxwell demon (MD) can convert into free energy per single bit of information obtained in a measurement. We propose and realize an electronic MD based on a single-electron box operated as a Szilard engine, where $k_BT \ln 2$ of heat is extracted from the reservoir at temperature T per one bit of created information. The information is encoded in the position of an extra electron in the box.

The work of Maxwell introduced what is now known as the "Maxwell demon" (MD) (1). This idea was quantified later on by Szilard (2), initiating interest in the relationship between information and thermodynamics; see, e.g., refs. 3–6. MD extracts heat from a thermal reservoir at temperature T by observing a thermodynamic system to make a spontaneous, thermally induced, transition into a state with larger-than-average free energy (either because of a larger internal energy or a smaller entropy) and using the feedback to collect this extra free energy as work. Szilard demonstrated that by obtaining a single bit of information as a measurement result of the state of the system, one could collect up to $k_BT \ln 2$ useful work, where k_B is the Boltzmann constant. Such a direct conversion of heat into work would by itself violate the second law of thermodynamics, because both the measurement and the feedback part of MD operation can in principle be done without generating extra entropy. In particular, a classical measurement can be viewed as a process of copying the state of the system into the memory of the detector. The only fundamentally unavoidable thermodynamic costs of conversion of heat into work by an MD is the creation of information about the state of the measured system. According to the Landauer principle (7–9), erasure of this information generates at least the extracted amount of heat, $k_BT \ln 2$ per bit, restoring the agreement with the second law.

Whereas these general principles of MD operation are well understood in theory [see, e.g., recent discussions (10–12)], only few experimental realizations of an MD exist (13), and thus far none demonstrates a quantitative connection between the MD output and the obtained information. The goal of this work is to realize a system that demonstrates explicitly the extraction of $k_BT \ln 2$ of heat from a thermal reservoir by an MD per one bit of created information. The operating cycle we use is close to the thought experiment suggested by Szilard. It realizes the MD operation using feedback-controlled molecule in a box as the working system. Fig. 1*A*, from left to right, shows the steps of the operation of such a Szilard engine (SE). The molecule is in equilibrium at temperature T, and the box is divided initially into two equal sections. After the measurement establishes which section the molecule is in, the container is allowed to expand into the full volume, lifting a weight tied to the dividing wall. In doing so, it extracts work from the thermal molecule. Then a dividing wall is introduced again and the cycle repeats. At the beginning of each cycle, the molecule has equal probabilities to be on the right or on the left, so that the measurement produces precisely one bit of information per cycle. As a result, the maximum of the average extracted work per cycle reaches the fundamental value of $k_BT \ln 2$.

Our experimental realization of the SE cycle is shown in Fig. 1*C*. Its main element is the single-electron box (SEB) (14–16), which consists of two small metallic islands connected by a tunnel junction. The SEB is maintained at the dilution-refrigerator temperatures in the 0.1-K range. Physically, there are two main differences between the SEB and the original single-molecule SE. The electrodes of the box contain electron gas of a large number of electrons, and not just one particle. Consequently, what is being manipulated in the engine operation is not this single particle but the charge configuration of the box, which is determined by the position of one extra electron. Also, this manipulation is achieved not by partitioning and reconnecting the electrodes (which for the SEB would correspond to the modulation of the conductance of the tunnel junction connecting the islands) but by changing the potential difference between the electron gases in the two islands. Apart from these differences, the engine follows the steps (illustrated with the potential profiles in Fig. 1*B*) similar to the operation of the original SE. The difference of the chemical potentials between the islands is controlled by the gate voltage V_g applied to one of them. Initially, V_g is such that the extra electron is found equally likely on either of the islands (Fig. 1*C*). This "degeneracy point" is realized when the gate-offset charge $n_g = C_gV_g/e$, where C_g is the capacitance between the gate and the SEB, is half-integer. A single-electron transistor (SET) electrometer, which can be seen in Fig. 1 *C* and *D*, *Bottom Right*, detects which island the electron is on. Then, n_g is changed rapidly to capture electron on the corresponding island by increasing the energy required for tunneling out. Finally, n_g is moved slowly back to the initial degeneracy value, extracting energy from the heat bath in the process, and completing the cycle. An example of four such consecutive experimental cycles is

Significance

A Maxwell demon makes use of information to convert thermal energy of a reservoir into work. A quantitative example is a thought experiment known as a Szilard engine, which uses one bit of information about the position of a thermalized molecule in a box to extract $k_BT \ln 2$ of work. The second law of thermodynamics remains valid because, according to Landauer principle, erasure of the information dissipates at least the same amount of heat. Here, we present an experimental realization of a Maxwell demon similar to a Szilard engine, in the form of a single electron box. We provide, to our knowledge, the first demonstration of extracting nearly $k_BT \ln 2$ of work for one bit of information.

Author contributions: J.V.K., V.F.M., J.P.P., and D.V.A. designed research; J.V.K., V.F.M., J.P.P., and D.V.A. performed research; J.V.K., V.F.M., J.P.P., and D.V.A. contributed new reagents/analytic tools; J.V.K. analyzed data; and J.V.K., V.F.M., J.P.P., and D.V.A. wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

[1]To whom correspondence should be addressed. Email: jonne.koski@aalto.fi.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1406966111/-/DCSupplemental.

PHYSICS

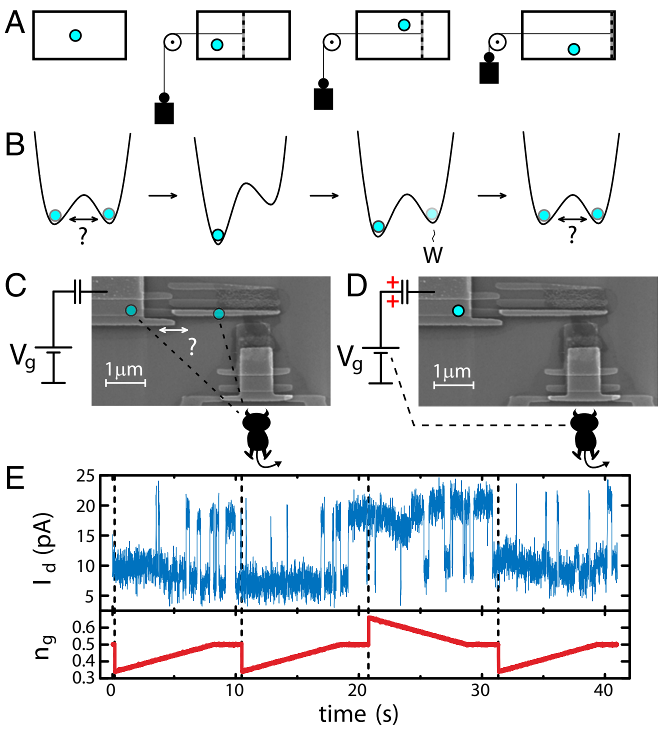

Fig. 1. SE. (*A*) The original proposal of "Szilard engine": a box containing a single molecule is split into two equal sections (*Top Left*). The section holding the molecule is allowed to expand up to the full volume of the box (*Top Right*). Then the partition is introduced again and the process repeats. (*B*) Sketch of the energy diagrams allowing a similar cycle in the SEB. Work is extracted when the particle is thermally excited to the higher energy state. (*C*) Experimental realization of the SE as SEB. An excess electron is located on one of the two metallic islands, corresponding to the first step on *A* and *B*. (*D*) The measurement and feedback parts of our MD operation. An SET electrometer on the bottom detects the electron, while the gate voltage V_g is applied to control the tunneling of the extra electron (to "move the wall") trapping it capacitively. As V_g is slowly driven back to the original setup in *C*, the net extracted work $k_BT \ln 2$ is produced by thermal activation as indicated in the third step of *B*. (*E*) A time trace of the excess electron location, signaled by the SET current I_d. The bottom trace shows the applied gate-voltage signal that provides feedback. Here $n_g = C_gV_g/e$, with C_g being the coupling capacitance between the gate electrode and the gated box island.

shown in Fig. 1*E*. Dotted vertical lines denote the time when the measurement is performed. We observe that the feedback signal indeed locks the extra electron to the measured state (parts of the trace in the Fig. 1*E*, *Upper* with no jumps), but the charge starts to hop again when n_g is moved toward the degeneracy point.

More quantitatively, the working space of the engine is spanned by the number n of excess electrons on one of the box islands, whereas equilibrium electron gas in the box islands plays the role of the thermal reservoir at temperature T. Because there is only capacitive coupling between the box and the rest of the circuitry, electron tunneling takes place only between the two box islands. Therefore, the total electric charge on the two islands is conserved, and the state with n excess electrons on one island has $-n$ excess electrons on the other island, as in a regular capacitor made of two electrodes. The internal energy of the engine is given then by the charging energy of these states, $E_n = E_c(n - n_g)^2$, averaged over their occupation probabilities p_n. Here $E_c = e^2/2C_{tot}$ is the usual charging energy of the total capacitance C_{tot} between the box islands. In the low-temperature regime relevant for this work, the charge dynamics is reduced to the two states, $n = 0, 1$. Thermodynamics of the engine cycle described above qualitatively is characterized quantitatively (17) by (*i*) the work done by the gate voltage source, $W = \int (dE_n(n_g)/dn_g)dn_g$, and (*ii*) the heat Q transferred to the electron gas of the box islands, i.e., to the thermal reservoir, by electron tunneling events. Note that electron tunneling events which change the charge state n make the integral in the expression for work W dependent on the specific realization of the history of the tunneling transitions. Each tunneling event produces the heat

$$Q = \pm\left(E_0(n_g) - E_1(n_g)\right) = \pm E_c(2n_g - 1), \qquad [1]$$

where the plus sign describes the n: $0 \to 1$ transitions, and the minus sign describes the n: $1 \to 0$ transitions. These relations enable us to measure Q directly, as was done previously in refs. 18, 19, by detecting the electron tunneling events in real time and evaluating the corresponding energy difference $E_0 - E_1$ at the moments of these events.

In the closed cycle of our experiment, energy conservation makes the total heat $-Q$ extracted from the reservoir equal to the work $-W$ extracted from the engine; see *SI Text* for a proof. The cycle starts with the SEB at degeneracy, and at this point the charge state is measured by the external SET detector. One bit of information represented by the (equally probable) position of the extra electron on one or the other island of the box is copied into the detector and stored for the subsequent feedback process, where it is used to determine the polarity of the rapid gate-voltage drive. If the box is found in the state $n = 0$, the gate voltage is changed rapidly so that the offset charge n_g changes from the degeneracy value $n_g = 1/2$ to $n_g = 0$; if the measured state is $n = 1$, n_g changes from $n_g = 1/2$ to $n_g = 1$. Such a rapid feedback drive traps the electron to the measured state. Ideally, this drive is so fast that no electron transitions have a chance to occur during it and, as a result, no heat is transferred to the reservoir.

The final part of the engine cycle is the quasistatic ramp which returns the box to the degeneracy. In the fully quasistatic limit, the heat Q dissipated in the reservoir can be found by considering the change of the total entropy S of the box. This change, $\Delta S = \Delta S_r + \Delta S_{ch}$, consists of the standard entropy change of the thermal reservoir in equilibrium at temperature T due to heat flow into it, $\Delta S_r = \langle Q\rangle/T$, where $\langle Q\rangle$ is the average dissipated heat, and the change of the Boltzmann entropy of the charge states

$$S_{ch} = -k_B \sum_n p_n \ln p_n. \qquad [2]$$

Using the standard rate equation for the evolution of the occupation probabilities p_n (14), one can find the rate of change of entropy S due to electron tunneling in a general evolution process as

$$\frac{\partial S}{\partial t} = \frac{1}{2}\sum_{n,m} \ln\left[\frac{p_n\Gamma_{mn}}{p_m\Gamma_{nm}}\right](p_n\Gamma_{mn} - p_m\Gamma_{nm}), \qquad [3]$$

where Γ_{mn} is the rate of electron tunneling from state n to m. The tunneling rates satisfy the detailed-balance condition, $\Gamma_{mn} = \Gamma_{nm}\exp\{(E_n - E_m)/k_BT\}$ (see *SI Text* for details). Eq. **3** shows that S never decreases, and remains constant in the fully adiabatic evolution, when the probabilities p_n maintain local equilibrium, $p_n \propto \exp\{-E_n(t)/k_BT\}$ and the detailed-balance condition ensures that the probability fluxes vanish: $p_n\Gamma_{mn} = p_m\Gamma_{nm}$. In this case, the total entropy is conserved, $\Delta S = 0$, and the two components of S change in the opposite directions $\Delta S_r = \langle Q\rangle/T = -\Delta S_{ch}$. The average heat $\langle Q\rangle$ is determined by the change of the entropy of the charge states,

$$\langle Q\rangle = -T\Delta S_{ch}. \qquad [4]$$

In particular, for a perfectly quasistatic ramp in our SE cycle bringing the box from the definite charge state to the degeneracy point, there are no energy fluctuations in the initial or final state, and Eq. **4** gives $Q = -k_B T \ln 2$ for every ramp. Qualitatively, this means that we are extracting $k_B T \ln 2$ of heat from the reservoir by creating a bit of information determined by the electron position on one or the other island of the SEB. In terms of work W, it is first extracted from the box by rapid lowering of the potential, as sketched in Fig. 1*B*. Work is then applied to drive the box back to the degeneracy; however, the required work is lowered by the amount of heat $k_B T \ln 2$ absorbed from the thermal bath. The ability to reach this fundamental maximum distinguishes the SEB setup in this work from other proposed electronic MDs (20–22) and is important for establishing the link between the extracted heat and information.

Fig. 2 shows the results of the measurements that illustrate such an extraction of heat from the reservoir. We drive our SEB starting from $n_g = 0$ toward $n_g = 1$ at various rates $\dot{n}_g$ while monitoring n continuously to measure the total dissipated heat Q with Eq. **1**. We see that as the rate of the drive decreases, the average dissipated heat approaches the prediction of Eq. **4**: $\langle Q \rangle$ tends to $-k_B T \ln 2$ for $n_g = 0.5$. This process can also be viewed as the reversal of the Landauer erasure of one bit of information, in which the system is driven from the degeneracy with two equally occupied states to one certain configuration. Such an erasure produces at least $k_B T \ln 2$ of heat as demonstrated explicitly by recent experiments on a colloidal bead controlled with optical tweezers (9). Because the drive in Fig. 2 starts with $n_g = 0$, such that the SEB is in a definite state $n = 0$ and thus initially $S_{ch} = 0$, the lowest curve in this plot approaching Eq. **4** can be viewed as direct measurement of the equilibrium entropy S_{ch} of the system of the two charge states $n = 0, 1$. When the quasistatic ramp to $n_g = 0.5$, as illustrated in Fig. 2, is complemented with an ideal measurement and immediate feedback that follows our SE protocol, the SEB operates as a reversible MD, abstract models of which have been discussed theoretically recently (10–12).

Fig. 3 demonstrates the experimental performance of our SE; see *SI Text* for details about the measurement protocol. The distribution of W is obtained from an ensemble of measured trajectories of n. The applied work W coincides with Q determined by Eq. **1** for each individual realization. Because the slow part of the cycle is not fully quasistatic, there are cycle-

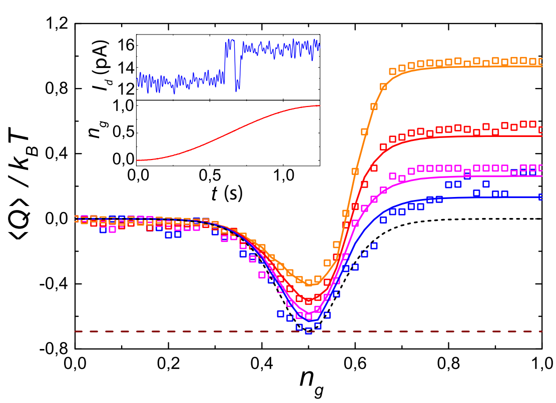

Fig. 2. Quasistatic drive. The average total heat transferred to the reservoir in a ramp starting from $n_g = 0$ up to n_g indicated on the x axis. Symbols show the measured and solid lines the theoretical results. See *SI Text* for details about the theoretical model (for all figures). Dashed curve gives the fully quasistatic limit of Eq. **4**; dashed straight line gives the fundamental $-k_B T \ln 2$ limit. The maximum drive rates are $\dot{n}_g = 0.22\Gamma_0$ for orange, $0.11\Gamma_0$ for red, $0.055\Gamma_0$ for magenta, and $0.027\Gamma_0$ for blue, where $\Gamma_0 = 22$ Hz is the tunneling rate at degeneracy. The averages are taken over $n = 2{,}105$, 1,764, 333, and 160 repetitions, respectively. (*Inset*) Example of realization of the measurement.

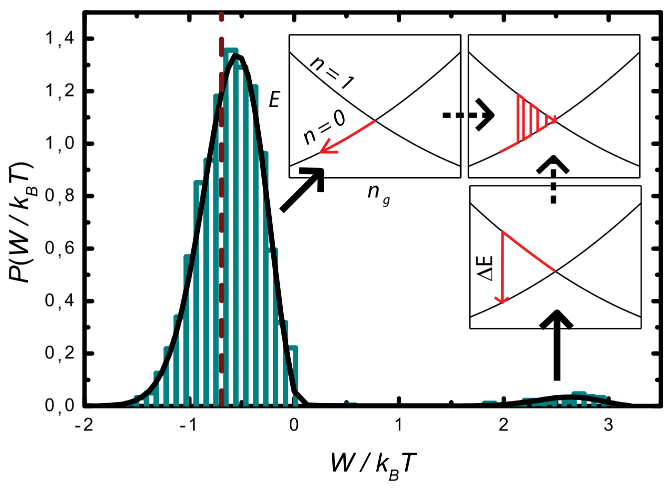

Fig. 3. Distribution of work under feedback protocol. The bars show the measured distribution, whereas the black line shows what is expected numerically. (*Insets*) Sketch of the two processes corresponding to the two peaks in the distribution in the case n was measured to be 0. (*Left Inset*) Cycle with correctly performed feedback which contributes to the large peak at $W < 0$ around the ideal value $-k_B T \ln 2$ indicated by the dashed line. Cycles with an error in the feedback (*Lower Inset*) send the box into the large-energy state producing extra dissipation and contributing to the peak at $W > 0$. The overall work distribution shown here is obtained from 2,944 cycles. The average extracted work for successful feedback response (the peak on the left-hand side) is $\langle -W \rangle \sim 0.9 \times k_B T \ln(2)$, and the average of the full distribution is $\langle -W \rangle \sim 0.75 \times k_B T \ln(2)$.

to-cycle fluctuations in W, such that W forms a continuous distribution. The cycles with correct gate-voltage feedback (Fig. 3, *Left Inset*) trap the electron on the SEB island, on which it actually sits at degeneracy after the measurement. Then no electron tunneling occurs in the feedback process, and W is close to the ideal limit $-k_B T \ln 2$. Such successful cycles produce the large peak at negative values of W in Fig. 3, around the ideal value that is indicated by the vertical dashed line. An error in the measurement or feedback drives the SEB to the excited charge state with excess energy $\Delta E = 2E_C|\Delta n_g|$, where Δn_g is the total change in n_g during the fast drive. Subsequent tunneling to the low-energy state (Fig. 3, *Lower Inset*) dissipates energy $\Delta E \gg k_B T \ln 2$ extracted in the quasistatic part. Such cycles produce the small peak at positive values of W in Fig. 3. For this measurement, we have chosen the optimized $|\Delta n_g| = 0.125$ to keep the contribution of the positive W as small as possible, without significantly reducing the heat extracted from the thermal bath during the quasistatic drive. With this choice, we obtain an average extracted work per cycle of $\langle -W \rangle \sim 0.75 \times k_B T \ln 2$. For comparison, if no measurement were performed, only 50% of the cycles would be successful, and one would do positive work $\langle W \rangle \sim 1.55 \times k_B T \ln 2$ on the average.

To summarize, our experiment is a realization of an MD, similar to an SE, with an SEB. We demonstrate quantitatively the extraction of $k_B T \ln 2$ of heat by creating a bit of information encoded in the position of the extra electron on one of the two islands of the box. Under a practical feedback cycle, our engine achieves a fidelity of about 75%. The heat transfer measurements performed as a part of MD demonstration provide also a direct measurement of the equilibrium entropy of a two-state system.

ACKNOWLEDGMENTS. This work has been supported in part by the European Union Seventh Framework Programme Information, Fluctuations and Energy Control in Small Systems (INFERNOS) (FP7/2007-2013) under Grant Agreement 308850, Academy of Finland (Projects 139172, 250280, and 272218), Väisälä

Foundation, the National Doctoral Programme in Nanoscience (NGS-NANO) (to V.F.M.), and the National Science Foundation Grant PHY-1314758 (D.V.A.). We acknowledge The Otaniemi Research Infrastructure for Micro- and Nanotechnologies for providing the processing facilities and technical support.

1. Leff HS, Rex AF, eds (2003) *Maxwell's Demon* (IOP, Bristol, UK).
2. Szilard L (1929) Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Z Phys* 53(6):840–856.
3. Brillouin L (1960) *Science and Information Theory* (Academic, New York), Chap 16.
4. Bennett CH (2003) Notes on Landauer's principle, reversible computation, and Maxwell's Demon. *Stud Hist Philos Mod Phys* 34(3):501–510.
5. Sagawa T, Ueda M (2008) Second law of thermodynamics with discrete quantum feedback control. *Phys Rev Lett* 100(8):080403.
6. Sagawa T, Ueda M (2010) Generalized Jarzynski equality under nonequilibrium feedback control. *Phys Rev Lett* 104(9):090602.
7. Landauer R (1961) Irreversibility and heat generation in the computing process. *IBM J Res Develop* 5(3):183–191.
8. Landauer R (1988) Dissipation and noise immunity in computation and communication. *Nature* 335(6193):779–784.
9. Bérut A, et al. (2012) Experimental verification of Landauer's principle linking information and thermodynamics. *Nature* 483(7388):187–189.
10. Horowitz JM, Parrondo JMR (2011) Thermodynamic reversibility in feedback processes. *Europhys Lett* 95(1):10005.
11. Mandal D, Jarzynski C (2012) Work and information processing in a solvable model of Maxwell's demon. *Proc Natl Acad Sci USA* 109(29):11641–11645.
12. Barato AC, Seifert U (2013) An autonomous and reversible Maxwell's demon. *Europhys Lett* 101(6):60001.
13. Toyabe S, Sagawa T, Ueda M, Muneyuki E, Sano M (2010) Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality. *Nat Phys* 6(12):988–992.
14. Averin DV, Likharev KK (1986) Coulomb blockade of tunneling, and coherent oscillations in small tunnel junctions. *J Low Temp Phys* 62(4):345–372.
15. Büttiker M (1987) Zero-current persistent potential drop across small-capacitance Josephson junctions. *Phys Rev B Condens Matter* 36(7):3548–3555.
16. Lafarge P, et al. (1991) Direct observation of macroscopic charge quantization. *Z Phys B* 85(3):327–332.
17. Averin DV, Pekola JP (2011) Statistics of the dissipated energy in driven single-electron transitions. *Europhys Lett* 96(6):67004.
18. Saira O-P, et al. (2012) Test of the Jarzynski and Crooks fluctuation relations in an electronic system. *Phys Rev Lett* 109(18):180601.
19. Koski JV, et al. (2013) Distribution of entropy production in a single-electron box. *Nat Phys* 9(10):644–648.
20. Averin DV, Möttönen M, Pekola JP (2011) Maxwell's demon based on a single-electron pump. *Phys Rev B* 84(24):245448.
21. Strasberg P, Schaller G, Brandes T, Esposito M (2013) Thermodynamics of a physical model implementing a Maxwell demon. *Phys Rev Lett* 110:040601.
22. Bergli J, Galperin YM, Kopnin NB (2013) Information flow and optimal protocol for a Maxwell-demon single-electron pump. *Phys Rev E Stat Nonlin Soft Matter Phys* 88(6):062139.