Aalto University
School of Electrical
Engineering

# Lecture 2: Scalar-Controlled Induction Motor Drive

## ELEC-E8402 Control of Electric Drives and Power Converters

Marko Hinkkanen

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# Learning Outcomes

After this lecture and exercises you will be able to:

- Express the dynamic inverse-$\Gamma$ model in synchronous coordinates
- Calculate steady-state operating points and draw the corresponding vector diagrams
- Explain the operating principle of the scalar control

## Motor Model

## Scalar Control

# Model in Stator Coordinates

- Voltage equations

$$\underline{u}_\mathrm{s}^\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s}^\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{s}^\mathrm{s}}{\mathrm{d}t}$$

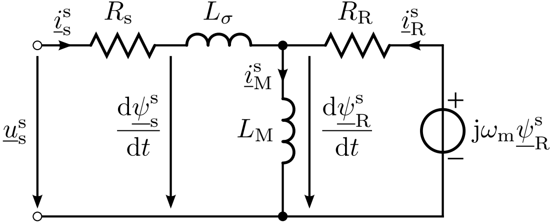
$$\underline{u}_\mathrm{R}^\mathrm{s} = R_\mathrm{R}\underline{i}_\mathrm{R}^\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{R}^\mathrm{s}}{\mathrm{d}t} - \mathrm{j}\omega_\mathrm{m}\underline{\psi}_\mathrm{R}^\mathrm{s} = 0$$

- Flux linkages

$$\underline{\psi}_\mathrm{s}^\mathrm{s} = L_\sigma\underline{i}_\mathrm{s}^\mathrm{s} + \underline{\psi}_\mathrm{R}^\mathrm{s}$$

$$\underline{\psi}_\mathrm{R}^\mathrm{s} = L_\mathrm{M}(\underline{i}_\mathrm{s}^\mathrm{s} + \underline{i}_\mathrm{R}^\mathrm{s})$$

- Steady state: $\mathrm{d}/\mathrm{d}t = \mathrm{j}\omega_\mathrm{s}$



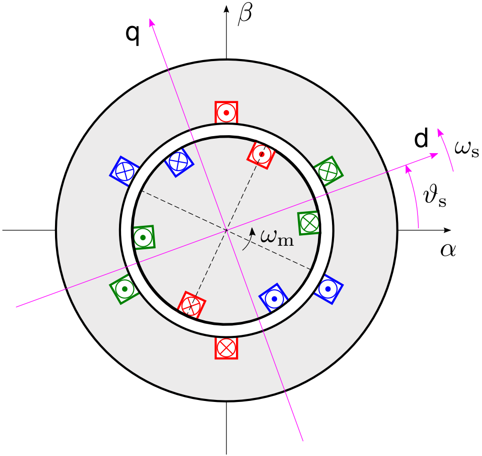
# Model in Synchronous Coordinates

- Synchronous (dq) coordinates rotate at the angular speed $\omega_\mathrm{s}$
- Coordinate transformation $\underline{i}_\mathrm{s}^\mathrm{s} = \underline{i}_\mathrm{s} \mathrm{e}^{\mathrm{j}\vartheta_\mathrm{s}}$, where no superscript is used in synchronous coordinates
- Voltage equations become

$$\underline{u}_\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s}\underline{\psi}_\mathrm{s}$$

$$\underline{u}_\mathrm{R} = R_\mathrm{R}\underline{i}_\mathrm{R} + \frac{\mathrm{d}\underline{\psi}_\mathrm{R}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{r}\underline{\psi}_\mathrm{R} = 0$$

- Angular speed of the coordinate system
  - $\omega_\mathrm{s}$ with respect to the stator
  - $\omega_\mathrm{r} = \omega_\mathrm{s} - \omega_\mathrm{m}$ with respect to the rotor

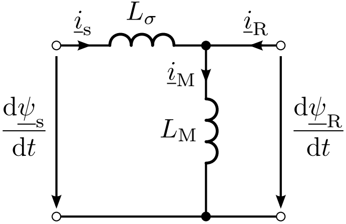


# Model in Synchronous Coordinates

- Voltage equations

$$\underline{u}_\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s}\underline{\psi}_\mathrm{s}$$

$$\underline{u}_\mathrm{R} = R_\mathrm{R}\underline{i}_\mathrm{R} + \frac{\mathrm{d}\underline{\psi}_\mathrm{R}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{r}\underline{\psi}_\mathrm{R} = 0$$




- Flux linkages

$$\underline{\psi}_\mathrm{s} = L_\sigma\underline{i}_\mathrm{s} + \underline{\psi}_\mathrm{R}$$

$$\underline{\psi}_\mathrm{R} = L_\mathrm{M}(\underline{i}_\mathrm{s} + \underline{i}_\mathrm{R})$$

- Steady state: $\mathrm{d}/\mathrm{d}t = 0$

# Power Balance

$$\frac{3}{2}\,\mathrm{Re}\left\{\underline{u}_\mathrm{s}\underline{i}_\mathrm{s}^* + \underline{u}_\mathrm{R}\underline{i}_\mathrm{R}^*\right\} = \frac{3}{2}R_\mathrm{s}|\underline{i}_\mathrm{s}|^2 + \frac{3}{2}R_\mathrm{R}|\underline{i}_\mathrm{R}|^2 + \frac{\mathrm{d}W_\mathrm{f}}{\mathrm{d}t} + T_\mathrm{M}\frac{\omega_\mathrm{m}}{p}$$

- Electromagnetic torque

$$\boxed{T_\mathrm{M} = \frac{3p}{2}\,\mathrm{Im}\left\{\underline{i}_\mathrm{s}\underline{\psi}_\mathrm{s}^*\right\}}$$

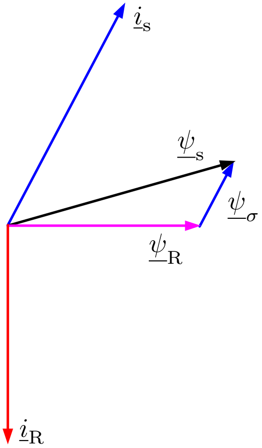
- Rate of change of the magnetic field energy

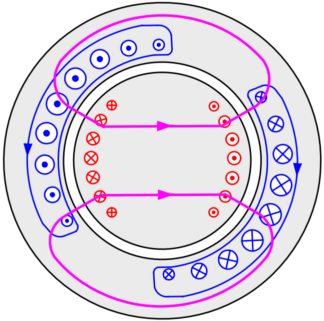
$$\frac{\mathrm{d}W_\mathrm{f}}{\mathrm{d}t} = \frac{3}{2}\,\mathrm{Re}\left\{\underline{i}_\mathrm{s}^*\frac{\mathrm{d}\underline{\psi}_\mathrm{s}}{\mathrm{d}t} + \underline{i}_\mathrm{R}^*\frac{\mathrm{d}\underline{\psi}_\mathrm{R}}{\mathrm{d}t}\right\} = \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}L_\sigma|\underline{i}_\mathrm{s}|^2 + \frac{1}{2}L_\mathrm{M}|\underline{i}_\mathrm{M}|^2\right)$$

is zero in the steady state

# Vector Diagram: Currents and Flux Linkages

- Airgap and leakage flux paths are sketched
- All vectors are constant in synchronous coordinates in the steady state (but the rotor slips at $-\omega_r$)



# Steady-State Torque

- Torque in the steady state

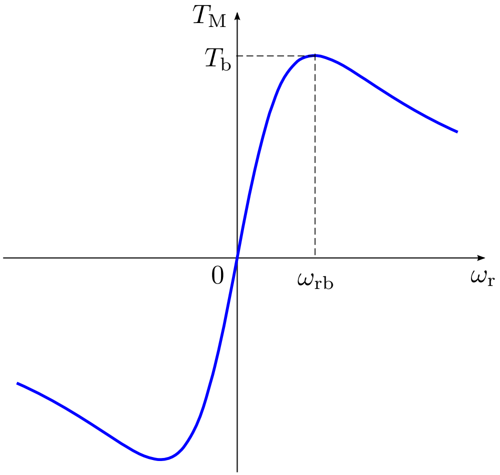
$$T_{\rm M} = \frac{2T_{\rm b}}{\omega_{\rm r}/\omega_{\rm rb} + \omega_{\rm rb}/\omega_{\rm r}}$$

- Breakdown torque

$$T_{\rm b} = \frac{3p}{2}\frac{L_{\rm M}}{L_{\rm M}+L_\sigma}\frac{\psi_{\rm s}^2}{2L_\sigma}$$

- Breakdown slip

$$\omega_{\rm rb} = \frac{R_{\rm R}}{\sigma L_{\rm M}} \quad \text{where} \quad \sigma = \frac{L_\sigma}{L_{\rm M}+L_\sigma}$$




See the derivation in Exercise 1.

Motor Model

Scalar Control

# Stator Voltage vs. Stator Frequency

- Steady-state stator voltage

$$\underline{u}_\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}\underline{\psi}_\mathrm{s}$$

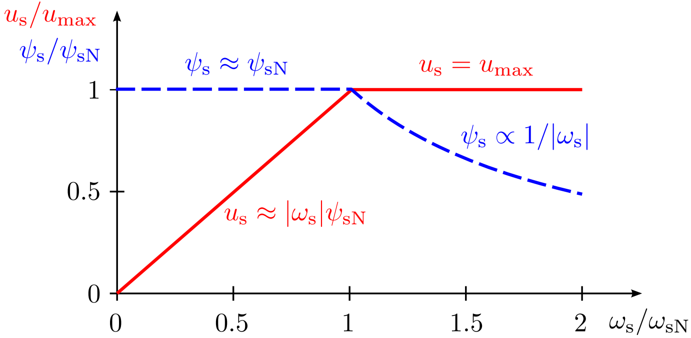
- Approximate magnitude

$$u_\mathrm{s} = |\omega_\mathrm{s}|\psi_\mathrm{s}$$

  where $u_\mathrm{s} = |\underline{u}_\mathrm{s}|$ and $\psi_\mathrm{s} = |\underline{\psi}_\mathrm{s}|$

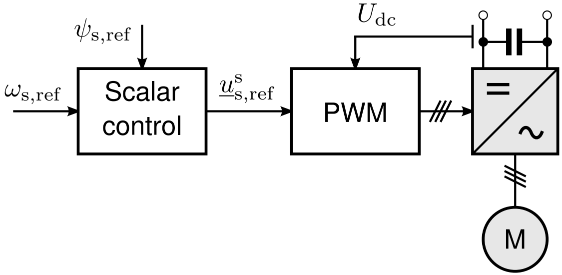
- Maximum voltage is limited

$$u_\mathrm{s} < u_\mathrm{max}$$



# Scalar Control or Constant-Volts-per-Hertz Control

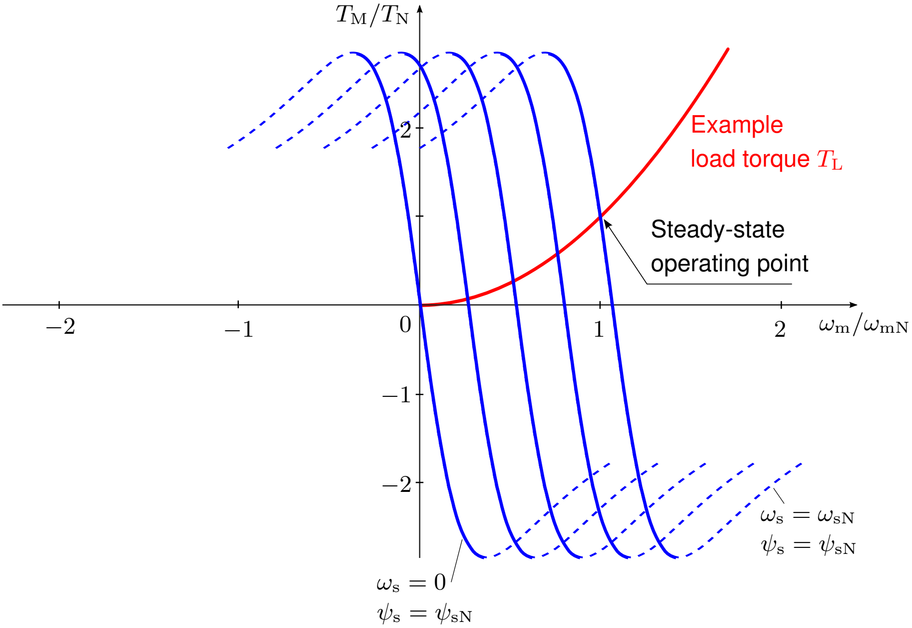
- Based on the steady-state equations
- Supply frequency $\omega_{\mathrm{s,ref}}$ corresponds to the desired rotor speed
- Some speed error due to the slip (can be partly compensated for)
- Slow or oscillating dynamics
- Torque cannot be controlled
- Current cannot be limited
- For simple applications



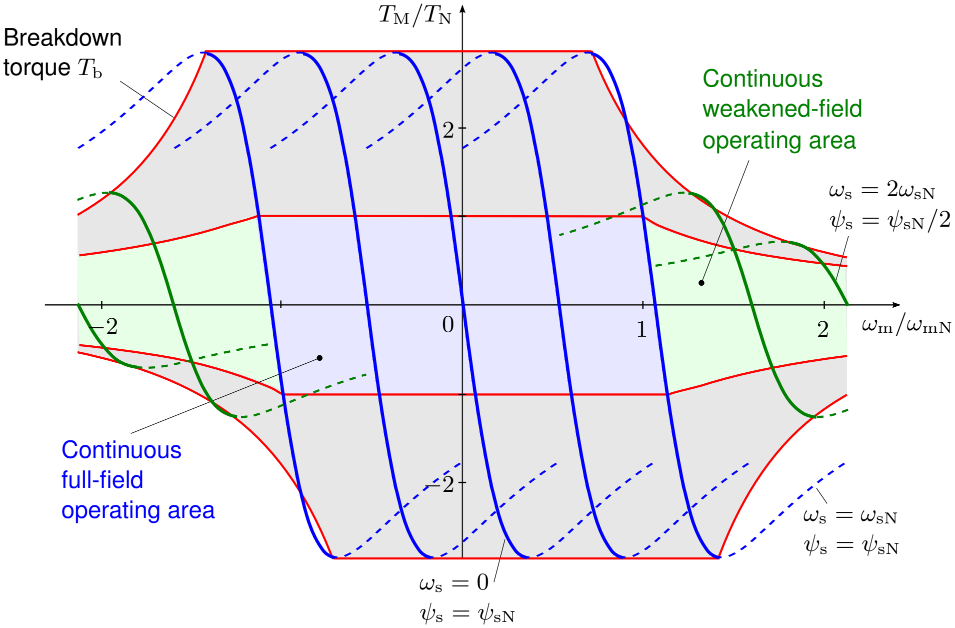

$u_{\mathrm{s,ref}} = \omega_{\mathrm{s,ref}}\psi_{\mathrm{s,ref}}$ ($+R_{\mathrm{s}}i_{\mathrm{s}}$ compensation)

$\vartheta_{\mathrm{s}} = \int \omega_{\mathrm{s,ref}} \mathrm{d}t$

$\underline{u}^{\mathrm{s}}_{\mathrm{s,ref}} = u_{\mathrm{s,ref}} \mathrm{e}^{\mathrm{j}\vartheta_{\mathrm{s}}}$



$T_\mathrm{M}/T_\mathrm{N}$
$\omega_\mathrm{m}/\omega_\mathrm{mN}$
$-2$
$-1$
0
1
2
2
$-1$
$-2$
Example
load torque $T_\mathrm{L}$
Steady-state
operating point
$\omega_\mathrm{s} = \omega_\mathrm{sN}$
$\psi_\mathrm{s} = \psi_\mathrm{sN}$
$\omega_\mathrm{s} = 0$
$\psi_\mathrm{s} = \psi_\mathrm{sN}$


$T_M/T_N$
Breakdown torque $T_b$
Continuous weakened-field operating area
$\omega_s = 2\omega_{sN}$
$\psi_s = \psi_{sN}/2$
2
0
$-2$
1
2
$\omega_m/\omega_{mN}$
Continuous full-field operating area
$\omega_s = \omega_{sN}$
$\psi_s = \psi_{sN}$
$\omega_s = 0$
$\psi_s = \psi_{sN}$