Problem set 4, 05.02.2021:

(20.7) The energy E of a system of three independent harmonic oscillators is given by

$$E = \left(n_x + \frac{1}{2}\right)\hbar\omega + \left(n_y + \frac{1}{2}\right)\hbar\omega + \left(n_z + \frac{1}{2}\right)\hbar\omega$$

Show that the partition function Z is given by

$$Z = Z_{SHO}^3$$

where Z_{SHO} is the partition function of a simple harmonic oscillator. Hence show that the Helmholtz function is given by

$$F = \frac{3}{2}\hbar\omega + 3k_B T \ln(1 - e^{-\beta\hbar\omega})$$

and that the heat capacity tends to $3k_B$ at high temperature.

(22.1) Maximize the entropy $S = -k_B \sum_i P_i \ln P_i$ where P_i is the probability of the ith level being occupied, subject to the constraints that $\sum_i P_i = 1$, $\sum_i P_i E_i = U$, and $\sum_i P_i N_i = N$ to rederive the grand canonical ensemble.

(Problem D) Consider a system of N_0 non-interacting quantum mechanical oscillators in equilibrium at temperature T. The energy levels of a single oscillator are

$$E_m = (m + \frac{1}{2})\frac{\gamma}{V}$$

with $m = 0,1,2, \dots, etc.$ (γ is a constant, the oscillators and volume V are one dimensional).

a) Find U and C_v, as functions of T .
b) Determine the equation of state for the system.

The number of the problem refers to the textbook.

Deadline for Problem set 4: 12th February at 10:00 a.m.
Send the solutions to bayan.karimi@aalto.fi