Problem set 5, 12.02.2021:

(Problem E) Based on the discussion on the exercise session on Friday 12.02.21 (second half), calculate the internal energy of the non-interacting three-dimensional electrons in a metal. From this, derive the heat capacity and express it in terms of real material parameters.

(Problem F) Find an analytical expression for the voltage dependence of tunneling rates in a junction between two normal metals (NIN). Show that the detailed balance condition holds, i.e.

$$\Gamma_{\text{reverse}} = e^{-\beta eV}\Gamma_{\text{forward}}$$

For the reverse and forward rates.

(Problem G) Heat in tunneling at finite bias

- **(a)** Normal-insulator-normal (NIN) junction. Show that Joule power (V^2/R_T) is evenly distributed to the two electrodes.
- **(b)** Normal-insulator-superconductor (NIS) junction. Find the voltage dependence of the cooling power of the N electrode at voltage $eV < \Delta$ using the same low T (temperature) approximation that was used on the lecture for calculating the current-voltage dependence of the NIS junction.
- **(c)** S_1IS_2 -junction, i.e. junction between two dissimilar superconductors with gaps Δ_1 and Δ_2. Show that cooling of the smaller gap Δ_1 superconductor diverges at $eV = \Delta_2 - \Delta_1$. Comment on the result.

(Problem H) Write the free energy of a current biased Josephson junction in the limit of $E_C \to 0$. Show that it presents a tilted “washboard” potential with periodic minima at small currents. What is the condition of maximize sustainable supercurrent? (Supercurrent corresponds to a stable ϕ in time.) What happens when one exceeds this current?

Deadline for Problem set 5: 19th February at 10:00 a.m.
Send the solutions to bayan.karimi@aalto.fi