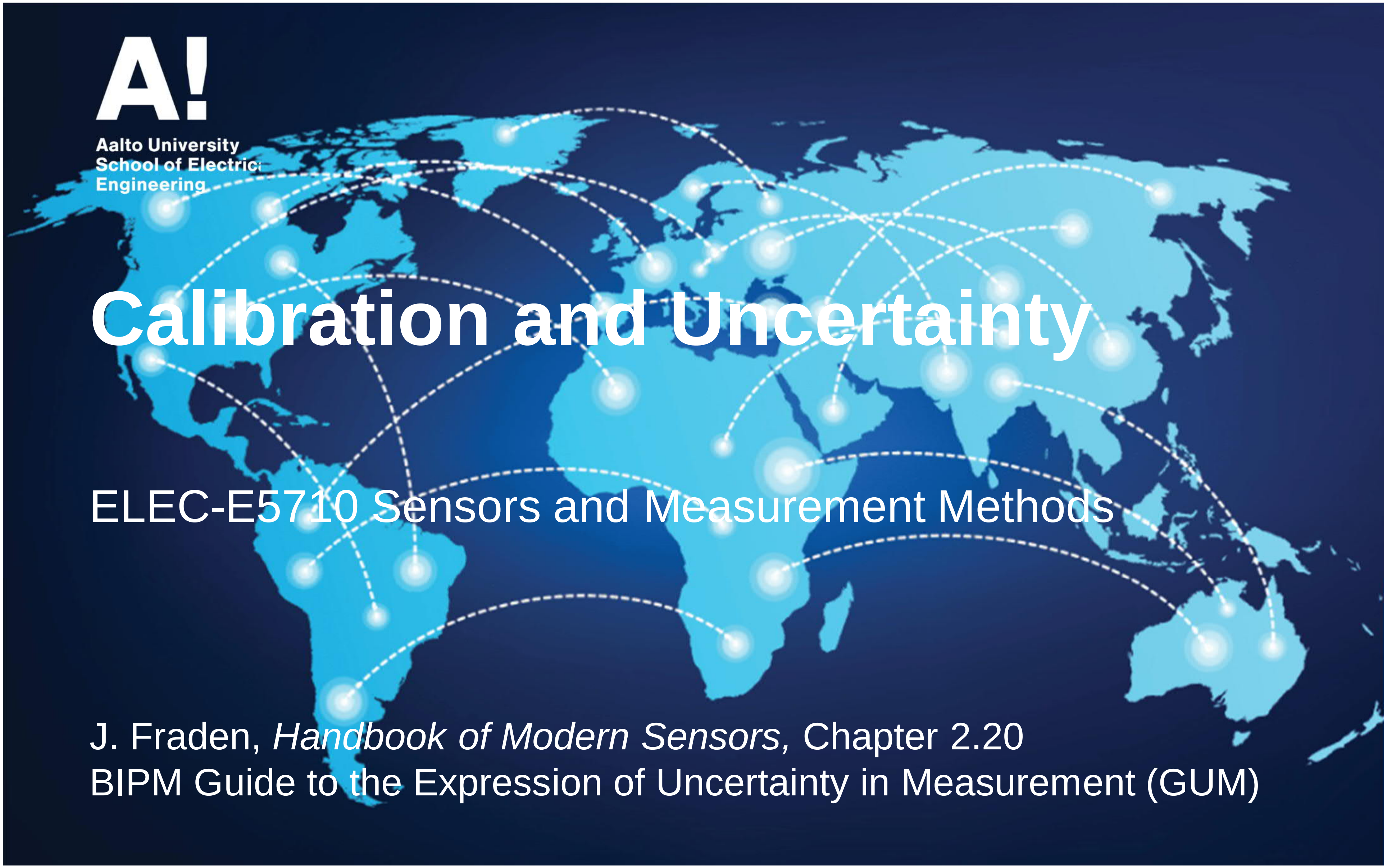


Aalto University
School of Electric
Engineering

Calibration and Uncertainty

ELEC-E5710 Sensors and Measurement Methods

J. Fraden, *Handbook of Modern Sensors,* Chapter 2.20
BIPM Guide to the Expression of Uncertainty in Measurement (GUM)

Calibration of instruments

- In calibration, reading of a device is compared to a more accurate device at one or several measurand levels
- Calibration can be used to reveal and correct for measurement errors due to e.g. Non-linearity, Offset, Gain error, or Drift
- In calibration, device is often adjusted to minimize the noted errors but not necessarily. If not, user of the device needs to correct errors manually
- Traceability to SI system of units guarantees comparable measurements with other measurers
- Calibration result must be accompanied with an uncertainty estimate

Example: Calibration results of a digital voltmeter

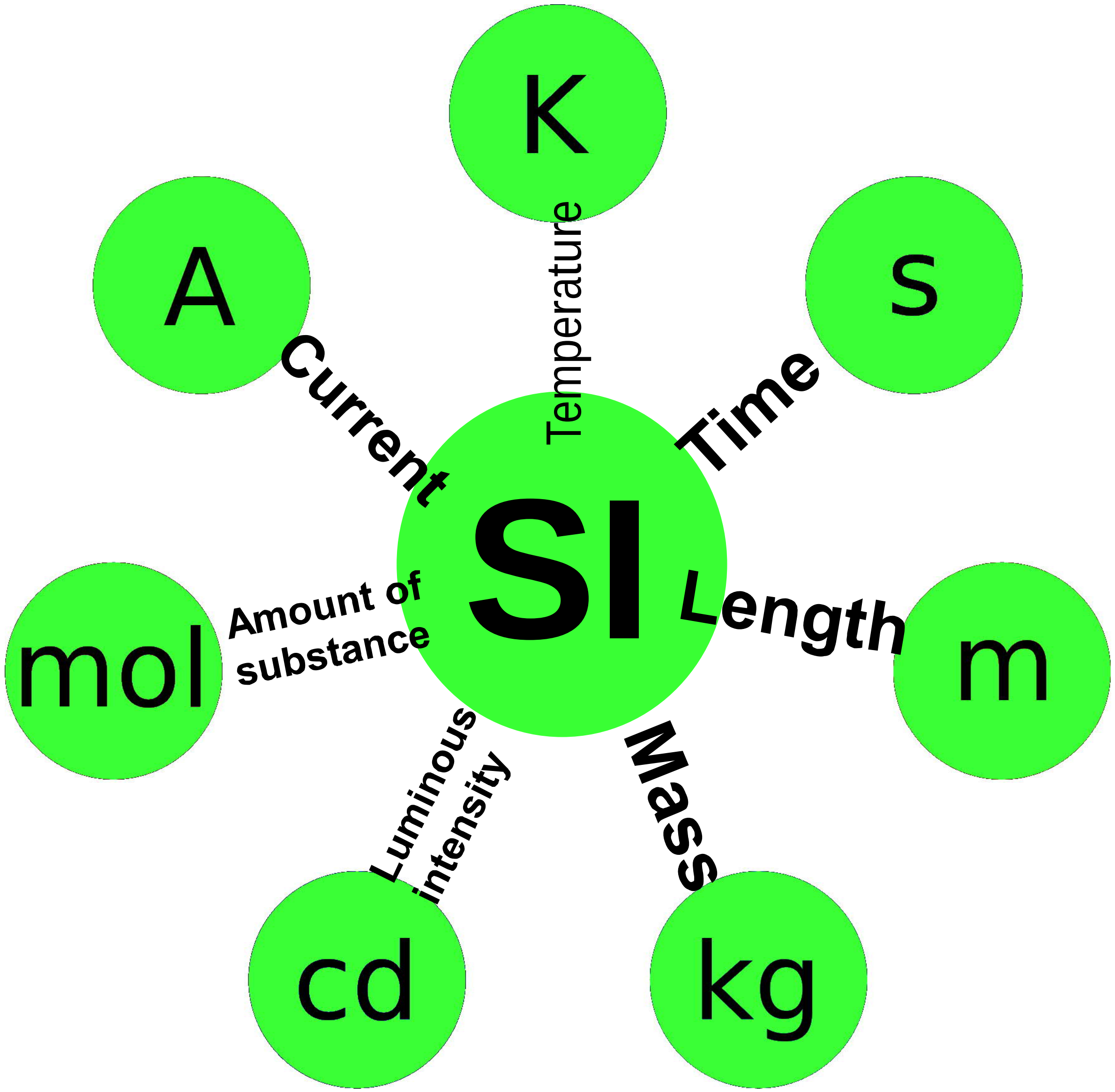
- Digital voltmeter Agilent 34410A is calibrated by comparing with a traceable Keithley Calibrator
- Readings at 0, Full range+, and Full range-
- In addition, midle values to detect linearity

Table 1. Direct voltage. The uncertainty of the multimeter reading is expanded with a coverage factor $k = 2$.

Input voltage	Measurement range	Multimeter reading	Uncertainty
0 mV	100 mV	0.0039 mV	0.0002 mV
20 mV	100 mV	20.00540 mV	0.002 mV
-20 mV	100 mV	-19.99403 mV	0.002 mV
50 mV	100 mV	50.00607 mV	0.002 mV
-50 mV	100 mV	-49.99412 mV	0.002 mV
100 mV	100 mV	100.00494 mV	0.002 mV
-100 mV	100 mV	-99.99480 mV	0.002 mV
0 V	1 V	-0.0000012 V	0.0000003 V
0.2 V	1 V	0.20000 V	0.00001 V
-0.2 V	1 V	-0.19999 V	0.00001 V
0.5 V	1 V	0.50000 V	0.00001 V
-0.5 V	1 V	-0.49999 V	0.00001 V
1 V	1 V	0.99999 V	0.00001 V
-1 V	1 V	-0.99999 V	0.00001 V
0 V	10 V	0.0000003 V	0.0000009 V
2 V	10 V	2.00001 V	0.0001 V
-2 V	10 V	-1.99999 V	0.0001 V
5 V	10 V	4.99998 V	0.0001 V
-5 V	10 V	-4.99996 V	0.0001 V
10 V	10 V	9.99998 V	0.0001 V
-10 V	10 V	-9.99996 V	0.0001 V

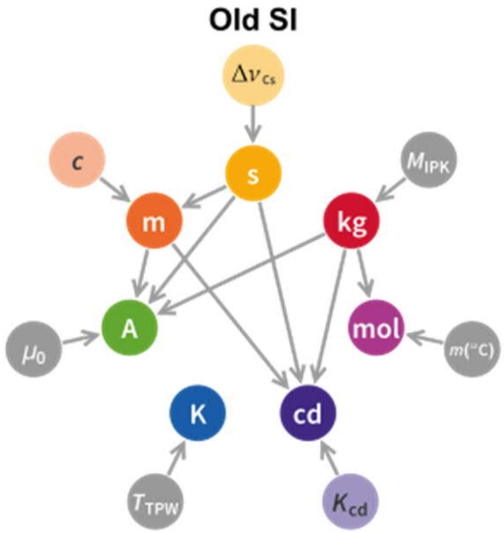
International System of Units (SI)

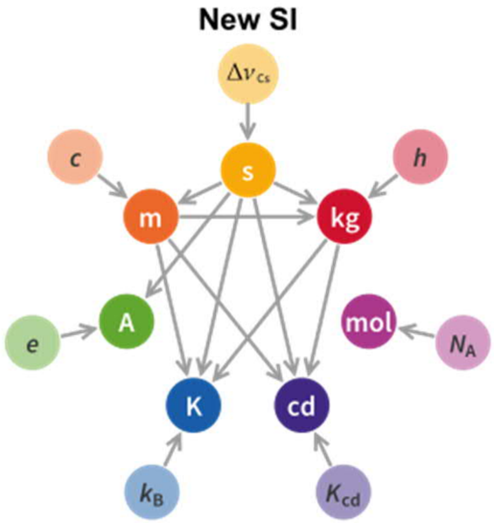
- The most widely used system of measurement
  - *"The foundation of modern science, industry and commerce"*
- Based on the metric system, originally established in France in 1795
- Adopted by international agreement in 1960, redefined in May 2019!
- Maintained by International Bureau of Weights and Measures (BIPM)
- Comprises a coherent system of units of measurement built on seven base units, set of twenty decimal prefixes, and 22+ derived units
- Each unit has a formal definition and sets of instructions that allows unit to be realized in practice
- Since 2019, all base units are derived from invariant constants of nature



Prefix		Base 10
Name	Symbol	
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

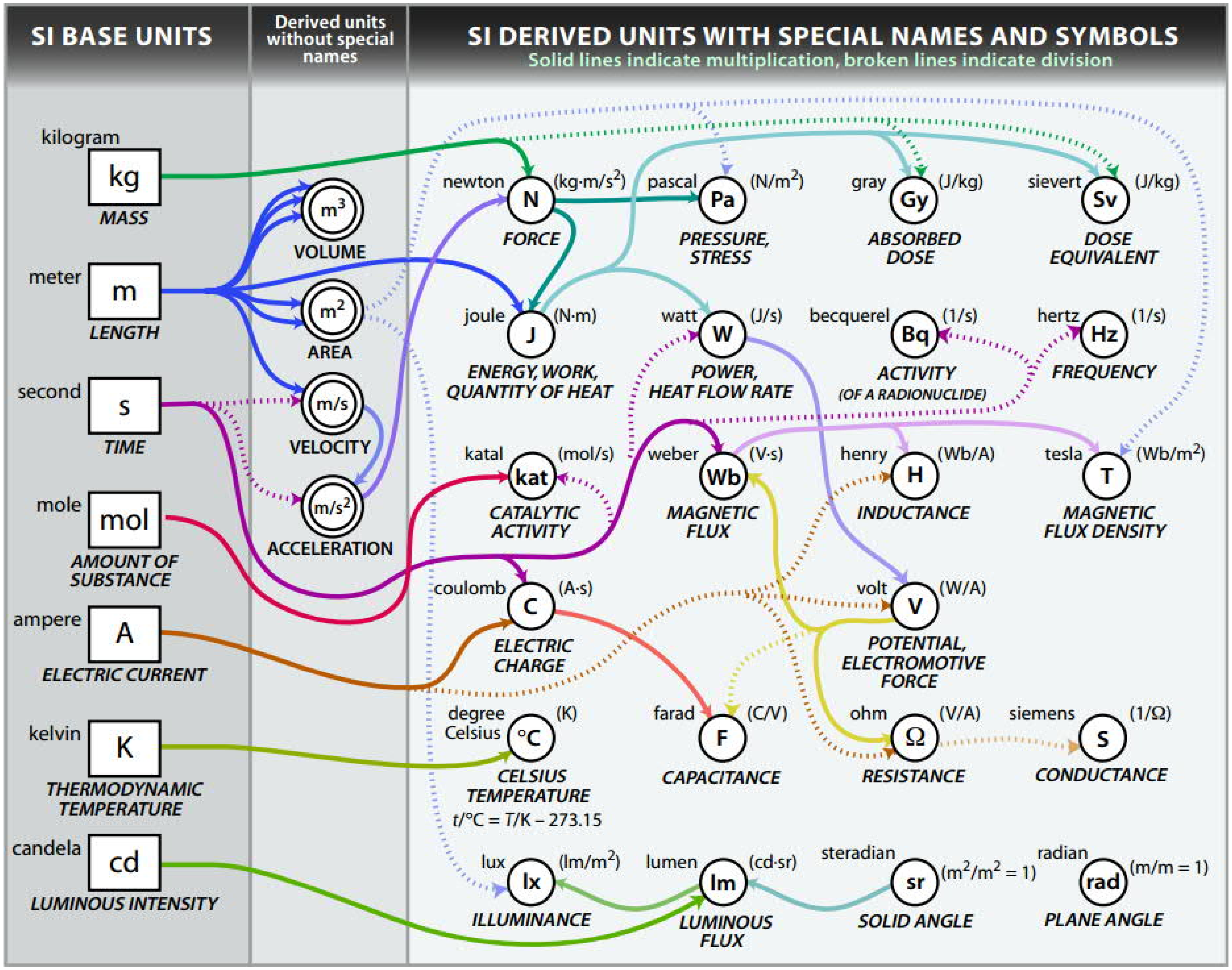
2019 redefinition of SI base units


Old SI


New SI

- Prototypes of [kg] and [m] no longer used!
- Based on fixed values of constants: Planck constant h, elementary charge e, Boltzmann constant k, Avogadro constant N_A, speed of light c, transition frequency of Cs-133 atom $\Delta\nu_{\mathrm{Cs}}$, and luminous efficacy K_{cd}

- Derived units are formed by powers, products and quotients of the base units
- 22 derived units are recognized by the SI with special name



SI BASE UNITS

Derived units without special names

SI DERIVED UNITS WITH SPECIAL NAMES AND SYMBOLS

Solid lines indicate multiplication, broken lines indicate division

kilogram kg MASS

meter m LENGTH

second s TIME

mole mol AMOUNT OF SUBSTANCE

ampere A ELECTRIC CURRENT

kelvin K THERMODYNAMIC TEMPERATURE

candela cd LUMINOUS INTENSITY

m^3 VOLUME

m^2 AREA

m/s VELOCITY

m/s^2 ACCELERATION

newton N ($kg{\cdot}m/s^2$) FORCE

pascal Pa (N/m^2) PRESSURE, STRESS

gray Gy (J/kg) ABSORBED DOSE

sievert Sv (J/kg) DOSE EQUIVALENT

joule J ($N{\cdot}m$) ENERGY, WORK, QUANTITY OF HEAT

watt W (J/s) POWER, HEAT FLOW RATE

becquerel Bq (1/s) ACTIVITY (OF A RADIONUCLIDE)

hertz Hz (1/s) FREQUENCY

katal kat (mol/s) CATALYTIC ACTIVITY

weber Wb ($V{\cdot}s$) MAGNETIC FLUX

henry H (Wb/A) INDUCTANCE

tesla T (Wb/m^2) MAGNETIC FLUX DENSITY

coulomb C ($A{\cdot}s$) ELECTRIC CHARGE

volt V (W/A) POTENTIAL, ELECTROMOTIVE FORCE

degree Celsius °C (K) CELSIUS TEMPERATURE $t/°C = T/K - 273.15$

farad F (C/V) CAPACITANCE

ohm Ω (V/A) RESISTANCE

siemens S (1/Ω) CONDUCTANCE

lux lx (lm/m^2) ILLUMINANCE

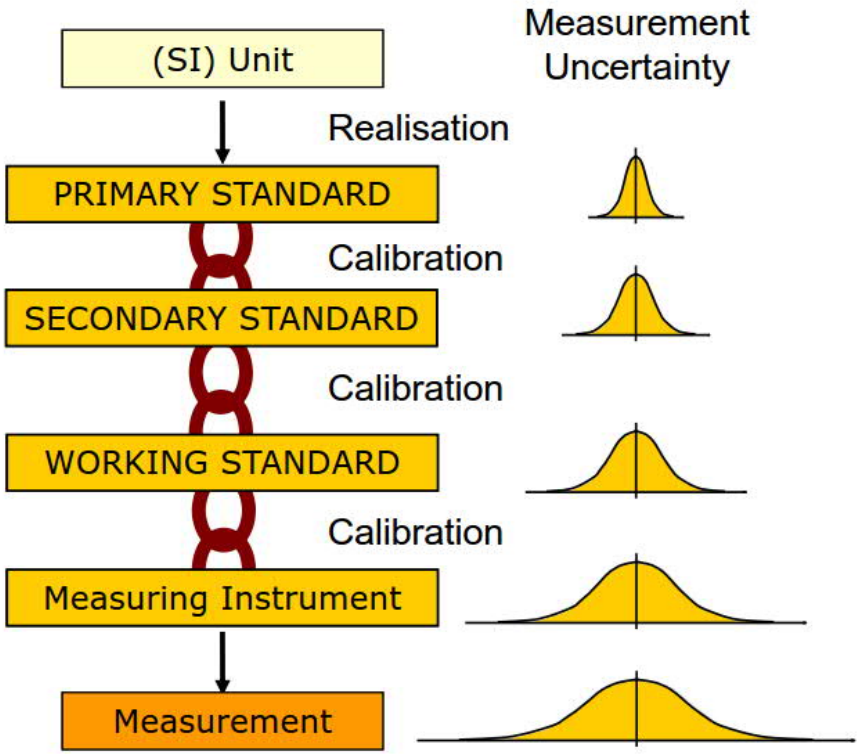
lumen lm ($cd{\cdot}sr$) LUMINOUS FLUX

steradian sr ($m^2/m^2 = 1$) SOLID ANGLE

radian rad (m/m = 1) PLANE ANGLE

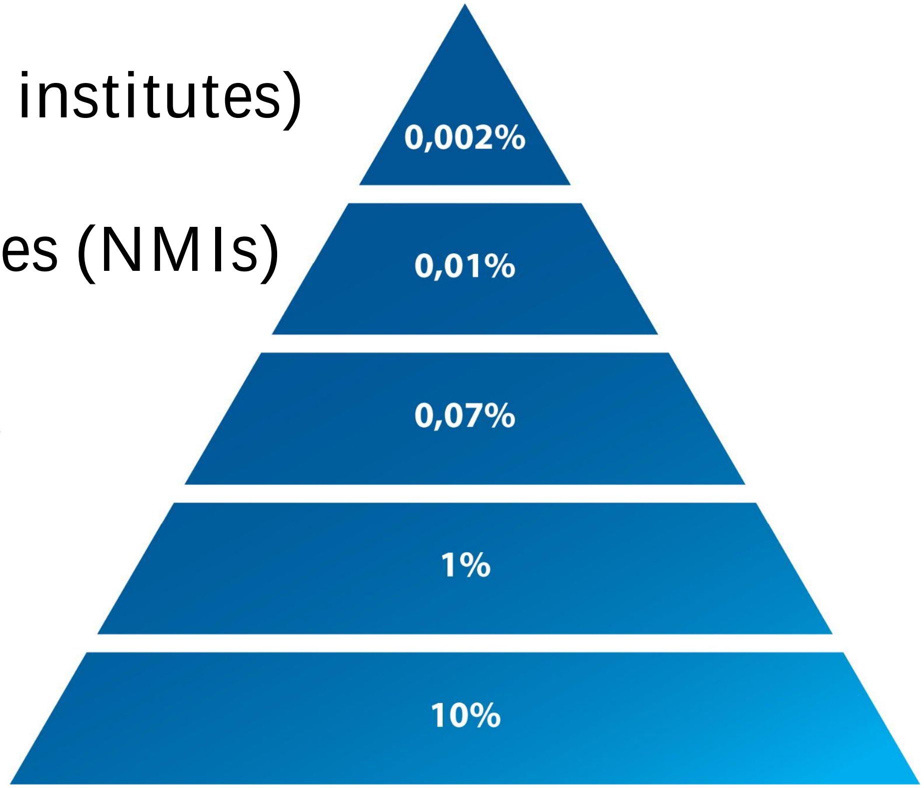
Traceability and standards

- Traceability: property of a measurement result whereby the result can be related to a reference through a *documented unbroken chain of calibrations*, each contributing to the measurement uncertainty
- Primary standard: direct realisation of a unit according to its definition
- Secondary standard: the value is obtained by a comparison to the primary standard.
- Working standard: used for routine calibrations of measurements instruments.



Traceability and standards

- BIPM, (National metrology institutes)
- National metrology institutes (NMIs)
- Calibration laboratories
- Industry, test laboratories
- End users, customers



Measurement uncertainty

- **Uncertainty of measurement** is the doubt about the validity of the result of a measurement
  - *Measure for missing information*
  - Also tells us something about the quality of the measurement
- “Any measurement that you make, without any knowledge of uncertainty, is meaningless”
  – Walter Lewin (1936–), professor emeritus, MIT
- Calibrations, scientific research and other accurate measurements require systematic analysis and estimation of uncertainty
- Useful for practical measurements as well

Terminology

- The objective of a **measurement** is to determine the **value** of the **measurand**, i.e. the value of the **particular quantity** to be measured
- In many cases, a measurand is not measured directly, but is determined from other **input quantities** through known relationship
- **Influence quantity** is not a measurand but still affects the result of measurement
  - For example temperature, humidity, pressure, vibration
- **Calibration** establishes a relation between the indication of the measurement device and the quantity value provided by a measurement standard, with corresponding uncertainties.
  - Comparison with a reference value with a smaller uncertainty

Error vs. uncertainty

- **Error** is the difference between the measured value and the ‘true value’ of the measurand. Errors should be corrected, not added to uncertainty (if they are significant):
  - Corrections from calibration certificates
  - Measured or modelled characteristics
  - Use of calibration artefacts before measurements

- **Uncertainty** is a quantification of the doubt about the measurement result
  - Uncertainties of calibrations, characteristics, and corrections
  - Error with unknown values are sources of uncertainty

Uncertainty budget

- Uncertainty components analysed as standard uncertainties (1δ, 68.3% probability)
- Combined standard uncertainty gives range, where measurand is with 68.3% confidence
- Expanded uncertainty increases the confidence to 95% (normal distribution)

Table 2.2. Uncertainty Budget for Thermistor Thermometer

Source of Uncertainty	Standard uncertainty (°C)	Type
Calibration of sensor	0.03	B
Measured errors		
Repeated observations	0.02	A
Sensor noise	0.01	A
Amplifier noise	0.005	A
Sensor aging	0.025	B
Thermal loss through connecting wires	0.015	A
Dynamic error due to sensor's inertia	0.005	B
Temperature instability of object of measurement	0.04	A
Transmitted noise	0.01	A
Misfit of transfer function	0.02	B
Ambient drifts		
Voltage reference	0.01	A
Bridge resistors	0.01	A
Dielectric absorption in A/D capacitor	0.005	B
Digital resolution	0.01	A
Combined standard uncertainty	0.068	
Expanded uncertainty (k = 2)	0.136	

Two types of uncertainty components

- **Type A uncertainty:** uncertainty component is obtained from statistical analysis of series of observations
  - “Five sequential measurements had the standard deviation of...”
  - “With a 10 second integration time per sample, a signal-to-noise ratio of 12 dB was achieved, resulting in...
- **Type B uncertainty:** uncertainty component is obtained using other methods than statistical analysis, for example:
  - Accuracy guaranteed by device manufacturer
  - Value obtained from calibration certificate
- **Increasing the number of measurements will not decrease type B uncertainty**

Procedure for analyzing uncertainty

(adapted from GUM)

1. Formulate how the measurand is determined from input quantities
2. Identify error sources and correct when possible (or necessary)
3. List all (relevant) sources of uncertainty
4. Calculate standard uncertainty of mean for Type A uncertainties
5. Use other methods to estimate Type B uncertainties
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1. Formulate how the measurand is determined from input quantities

- Usually measurand Y is not measured but determined of *N* amount of input variables: $Y = f(X_1, X_2, \ldots, X_N)$
  - Output power *P* of a DC power supply is determined by measuring voltage *U* and current *I*

$$P = UI$$

  - Temperature *T* is determined by measuring the resistance *R* of a platinum sensor, and by using the R_0 value provided by the sensor manufacturer and coefficients *A* and *B* based on commonly used resistivity model

$$T = \frac{-A + \sqrt{A^2 - 4B\left(1 - \frac{R_T}{R_0}\right)}}{2B}$$

1. Formulate how the measurand is determined from input quantities

- Typically, the governing equations consist of basic operations
- But it is possible that they involve complex mathematics…

$$\alpha = \frac{\int \Phi_{\mathrm{e,s}}(\lambda) R_{\mathrm{s}}(\lambda) \frac{\alpha_{\mathrm{s}}(\lambda)}{1+\alpha_{\mathrm{s}}(\lambda)}\,\mathrm{d}\lambda + \int \Phi_{\mathrm{e,p}}(\lambda) R_{\mathrm{p}}(\lambda) \frac{\alpha_{\mathrm{p}}(\lambda)}{1+\alpha_{\mathrm{p}}(\lambda)}\,\mathrm{d}\lambda}{\int \Phi_{\mathrm{e,s}}(\lambda) R_{\mathrm{s}}(\lambda) \frac{1}{1+\alpha_{\mathrm{s}}(\lambda)}\,\mathrm{d}\lambda + \int \Phi_{\mathrm{e,p}}(\lambda) R_{\mathrm{s}}(\lambda) \frac{1}{1+\alpha_{\mathrm{p}}(\lambda)}\,\mathrm{d}\lambda}$$

$$\eta_m(\lambda) = \eta_{a,m}(\lambda, \theta_1) + \sum_{i=2}^{N} \left[\eta_{a,m}(\lambda, \theta_i) \prod_{j=1}^{i-1} \rho_{r,m}(\lambda, \theta_j) \right]$$

- …or numerical methods, computer simulations, extrapolation etc.

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2. Identify error sources and correct when possible (or necessary)

- Based on instrument calibrations:
  - “According to the calibration of a digital voltage meter, the voltage reading must be corrected with a factor of 1.00021”

- Some error sources can be measured
  - “When the water flow was cut off, the flow meter showed the value of 0.2 l / min. This offset was substracted from the recorded values
  - “The signal loss of the photodetector due to reflection from the front surface was measured to be 10.2 %”

2. Identify error sources and correct when possible (or necessary)

- Calculations
  - “It was calculated that the input impedance of the oscilloscope attenuates the measured signal source by 0.3 %

- Computer simulations
  - “Computer simulations indicate that the grooves in mechanical parts induce an underestimation around 20 ppm in the diameter measurement”

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3. List all (relevant) sources of uncertainty

- Uncertainty components are the basis of so called uncertainty budget, which lists all uncertainty components and their contribution to the total uncertainty
- The more demanding the measurement, the more detailed the list of uncertainty components needs to be
- Typically, in an uncertainty budget there are a couple of dominating components. If these are identified, the smaller components may be skipped
- In simple measurements, uncertainties due to calibration of the measurement instrument and Type A uncertainty (repeatability or reproducibility) may be enough

3. List all (relevant) sources of uncertainty (list from GUM)

a) incomplete definition of the measurand;

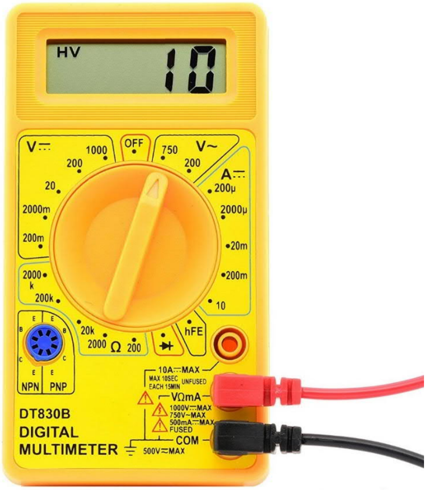
b) imperfect realization of the definition of the measurand;

c) nonrepresentative sampling — the sample measured may not represent the defined measurand;

d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;

e) personal bias in reading analogue instruments;

3. List all (relevant) sources of uncertainty



f) finite instrument resolution or discrimination threshold;

g) inexact values of measurement standards and reference materials;

h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;

i) approximations and assumptions incorporated in the measurement method and procedure;

j) variations in repeated observations of the measurand under apparently identical conditions. (Repeatability)

3. List all (relevant) sources of uncertainty

- Repeatability: The agreement between consecutive measurements when the same measurand is measured in unchanged conditions. Same method of measuring, device, measurer, place and repeats in a short period of time.

- Reproducibility: The agreement between consecutive measurements when the same measurand is measured in changed conditions.
  - Specification of changed conditions should be included

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4. Calculate standard uncertainty of mean for Type A uncertainties

- Input variable X. The best estimate $\bar{x}$ of its expected value is the arithmetic mean of *n* amount of independent measurement results:

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

- Single measurements differ from each other. Standard deviation of a measurement result distribution describes the variation of single values around the mean:

$$s(x_k) = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2}$$

- Standard deviation $s(x_k)$ describes the repeatability of measurements.

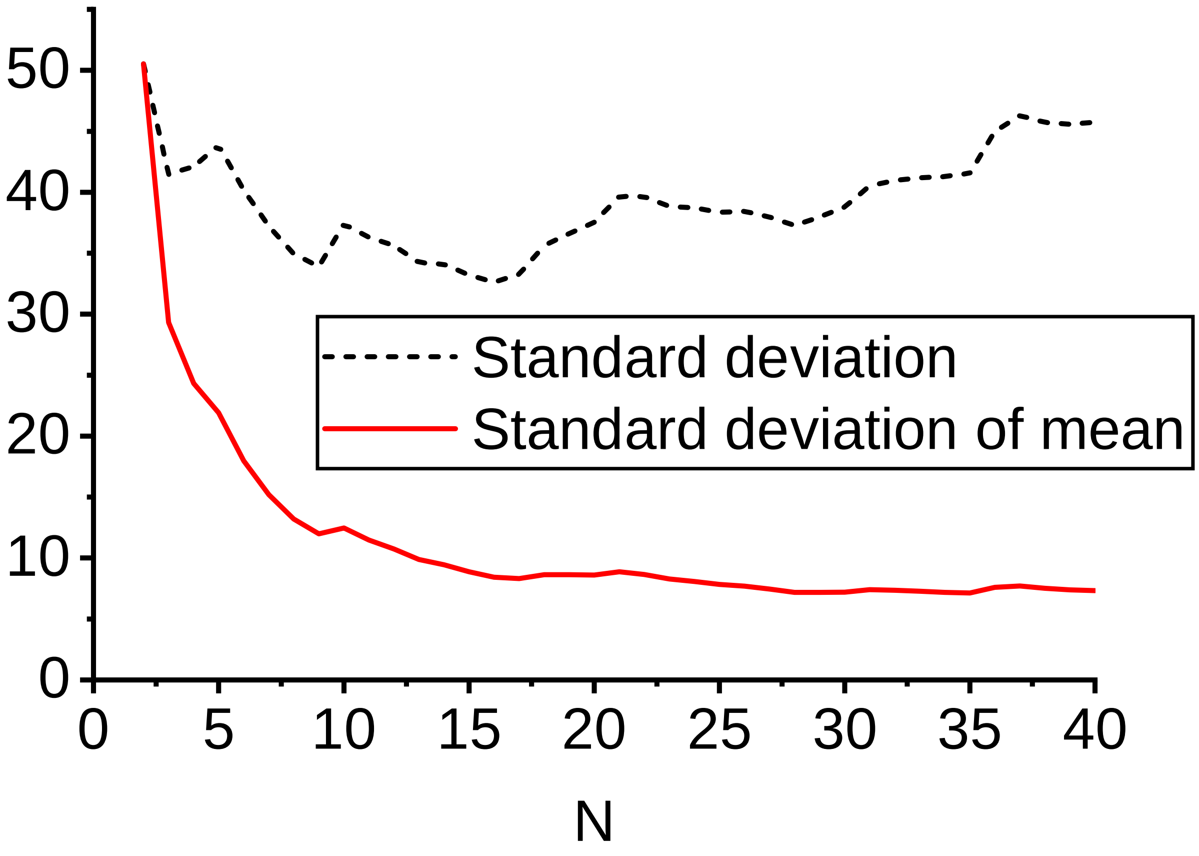
4. Calculate standard uncertainty of mean for Type A uncertainties

- 68% of results lie within $\pm s(x_k)$
- Standard deviation of measurement does not decrease even when the number of measurements *N* increases
- Instead, Type A uncertainty is better evaluated using the standard deviation of a mean, written $s(\bar{x})$,

$$s(\bar{x}) = \frac{s(x_k)}{\sqrt{n}}$$

- Standard deviation of the mean $s(\bar{x})$ describes the reproducibility of measurements

4. Calculate standard uncertainty of mean for Type A uncertainties



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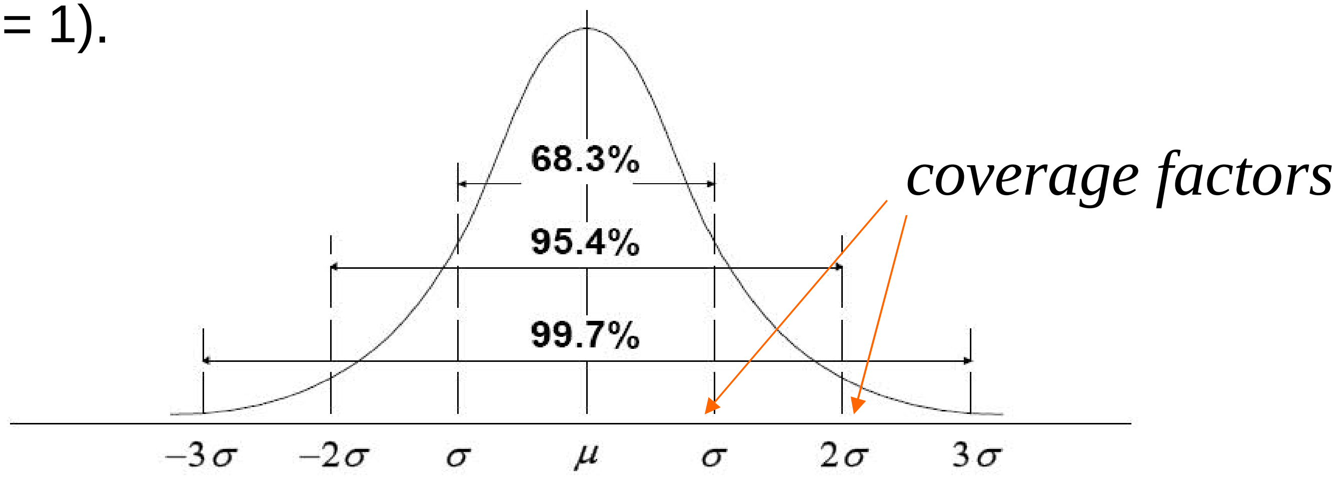
5. Use other methods to estimate Type B uncertainties

- Most uncertainties are of type B
  - Uncertainty of calibration from calibration certificate
  - Measured characteristics for devices such as non-linearity, frequency response, wavelength dependence
  - Characteristics taken from datasheets, manuals, literature, such as ageing, non-linearity, temperature dependence
- Uncertainties need to be converted to standard deviations, 68.3% confidence level
  - Expanded uncertainties (k=2) with normal distribution divided with 2
  - Unity distributions (100% confidence) divided with $\sqrt{3}$.

5. Use other methods to estimate Type B uncertainties

- Type B uncertainty components are often normally distributed
  - Device calibrations, ambient conditions, modelled corrections etc.
  - Safest assumption to make about type B uncertainty
- Standard uncertainty equals a confidence level of 68,3 % (coverage factor *k* = 1).

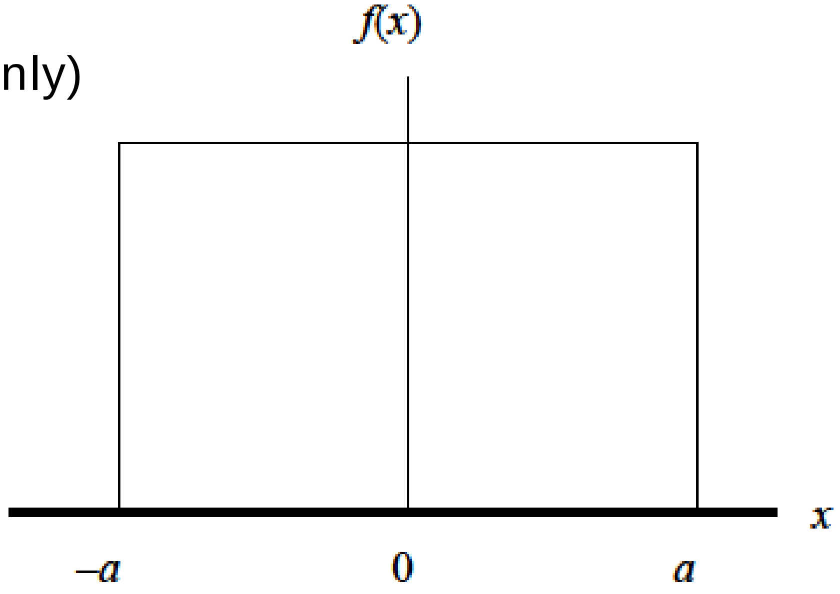


coverage factors

5. Use other methods to estimate Type B uncertainties

- Some uncertainty components have uniform distribution
  - Those who have strict upper and lower limits (-a and a)
  - Display resolution, counter value
  - Specifications from datasheets (mainly)
- Type B standard uncertainty of a uniform distribution can be calculated using statistics:

$$u(x_i) = \frac{a}{\sqrt{3}}$$



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6. Calculate/estimate how much every uncertainty source contributes to uncertainty of the measurand

- Goal is to find out how much the standard deviation of every uncertainty component affects the standard uncertainty of the measurand
- If the uncertainty component is one of the input quantities, the contribution can be easily calculated
  - Vary input parameter by its standard uncertainty and calculate how much the obtained value changes
- Other uncertainty components must be analyzed using
  - Test measurement, literature data, computer simulations, “special know-how of the measurer”

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7. Calculate the combined standard uncertainty

- If the uncertainty components are uncorrelated, the combined standard uncertainty of the measurand can be calculated by taking the sum in quadrature of each contribution
- That is “square root of the sum of the squares”

$$u(Y) = \sqrt{u(x_1)^2 + u(x_2)^2 + u(x_3)^2 + \cdots u(x_4)^2}$$

- All uncertainty contribution should be either in absolute values (seconds, amperes, grams etc.) or relative values (%)
- Typically all uncertainty components are assumed to be uncorrelated
  - *If some are significantly correlated, the correlations must be taken into account*
  - *This requires more sophisticated analysis*

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8. Calculate the expanded uncertainty for desired level of confidence

- Very often, the measurement uncertainties are expressed using expanded uncertainty
- Expanded uncertainty: $U = k \cdot u$
- Coverage factor $k=2$, is equal to confidence level of 95 %.
- There is a probability of 95% that the real value of Y belongs to the interval of

$$y - 2\sigma \leq Y \leq y + 2\sigma$$

- Using a coverage factor k=3 we have a probability of 99,7 % that the real value is in the interval of $\pm\ 3\sigma$.

Recap

- **Calculate standard uncertainties of uncertainty components**
- **Determine the effect of uncertainty components to the uncertainty of the measurand**
- **Obtained uncertainty components are summed quadratically**

  → Combined standard uncertainty of the measurand
- **Combined standard uncertainty is multiplied by a coverage factor (typically k=2)**

  → Expanded uncertainty
- **Uncertainty often reported is either absolute (e.g. millivolt or kelvin) or relative (%)**