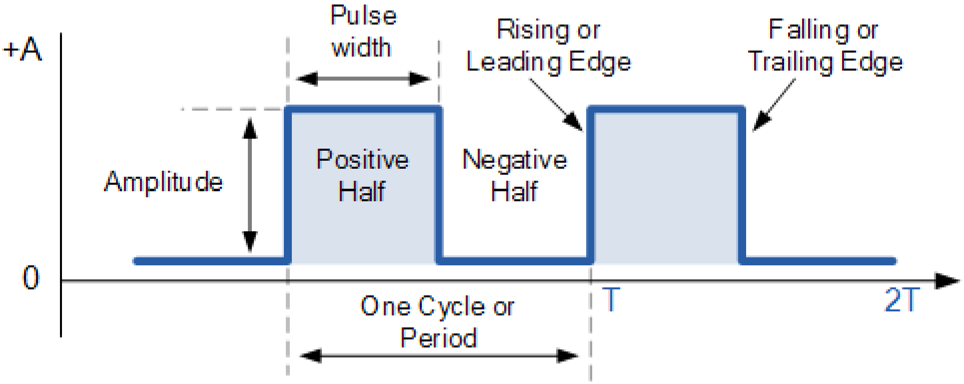
Aalto University

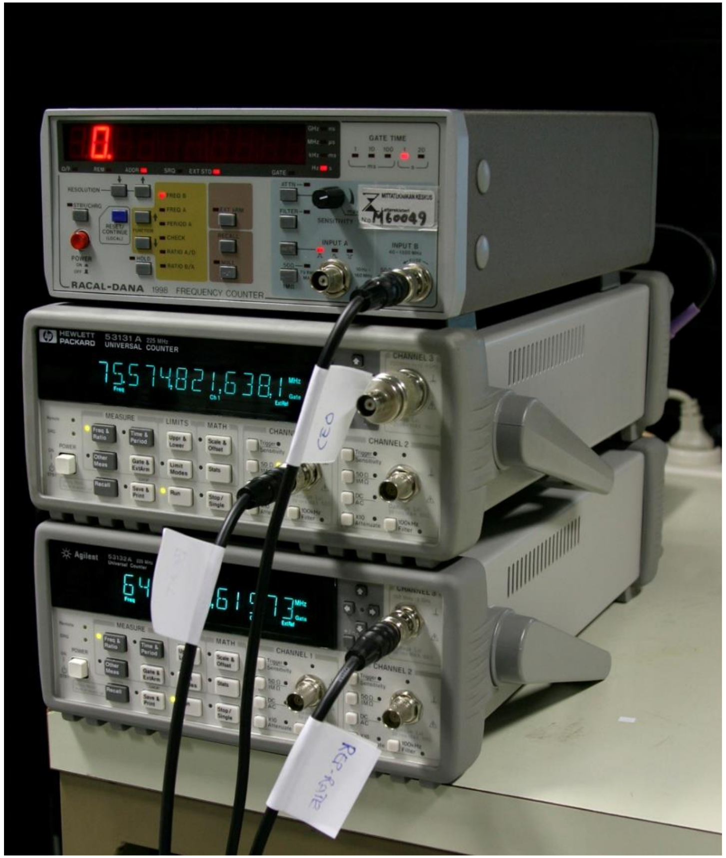
Frequency measurements

ELEC-E5710 - Sensors and Measurement Methods

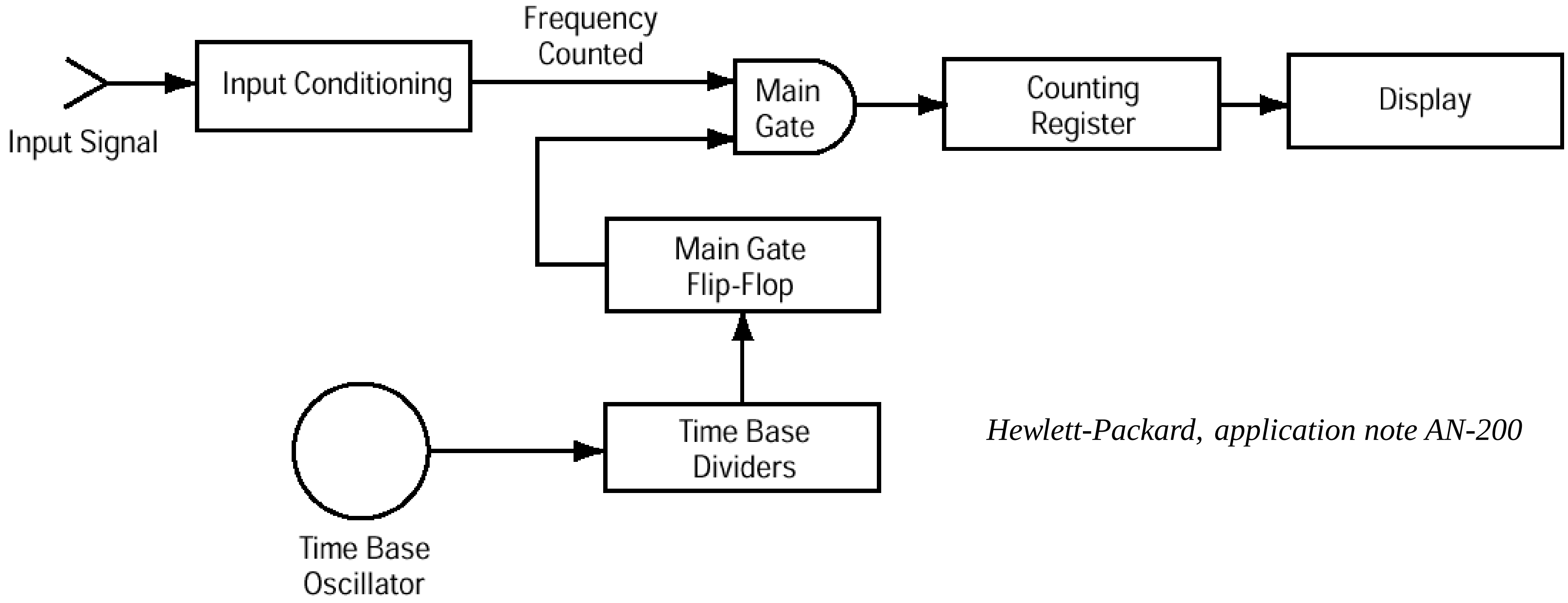
Frequency counter

- Typical frequency counter can measure signal frequency and period, number of pulses within specific time or average interval between pulses
- Some models can measure rise and fall time, jitter, minimum and maximum frequency etc…





Basic principle

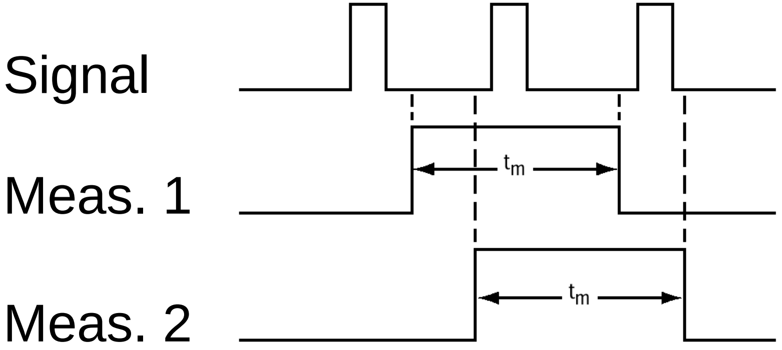


Hewlett-Packard, application note AN-200

- **Direct measurement: count the number pulses within a specific period of time (gate time) and calculate the frequency**
- **Always limited by the accuracy of the time base oscillator**

Limitatioins of direct measurement

- **Maximum counter frequency typically around 3 GHz**
  - Higher frequencies require special techniques
- **Uncertainty of ±1 pulses in the counting**
  - Due to random phase between the internal clock and the signal



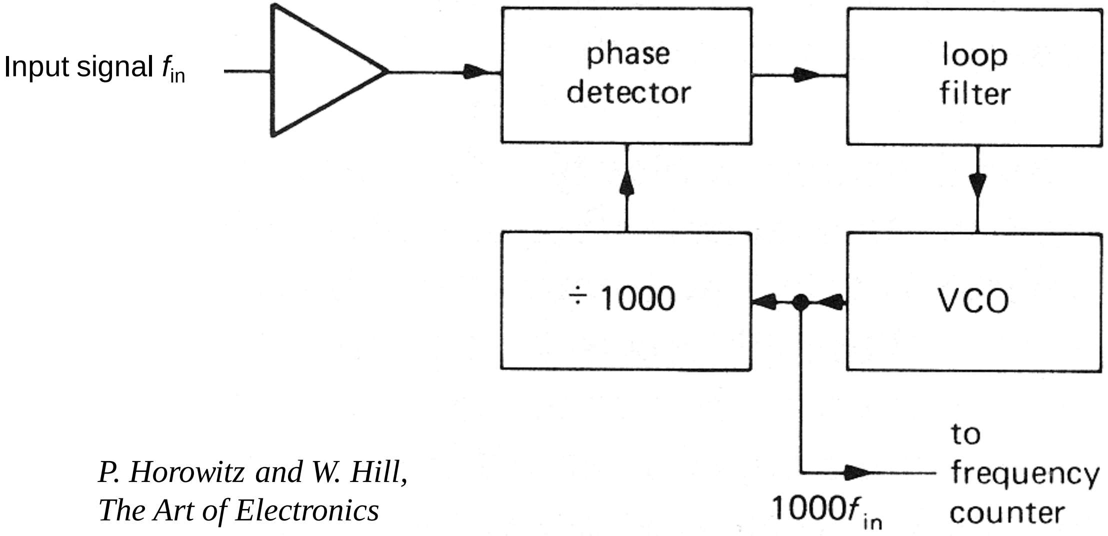
Hewlett-Packard, application note AN-200

- **Relative uncertainty in the measurement is frequency dependent:**

$$\frac{\Delta f}{f_m} = \frac{\Delta N}{N} \approx \frac{1}{Tf_m}$$

Reducing "±1" uncertainty

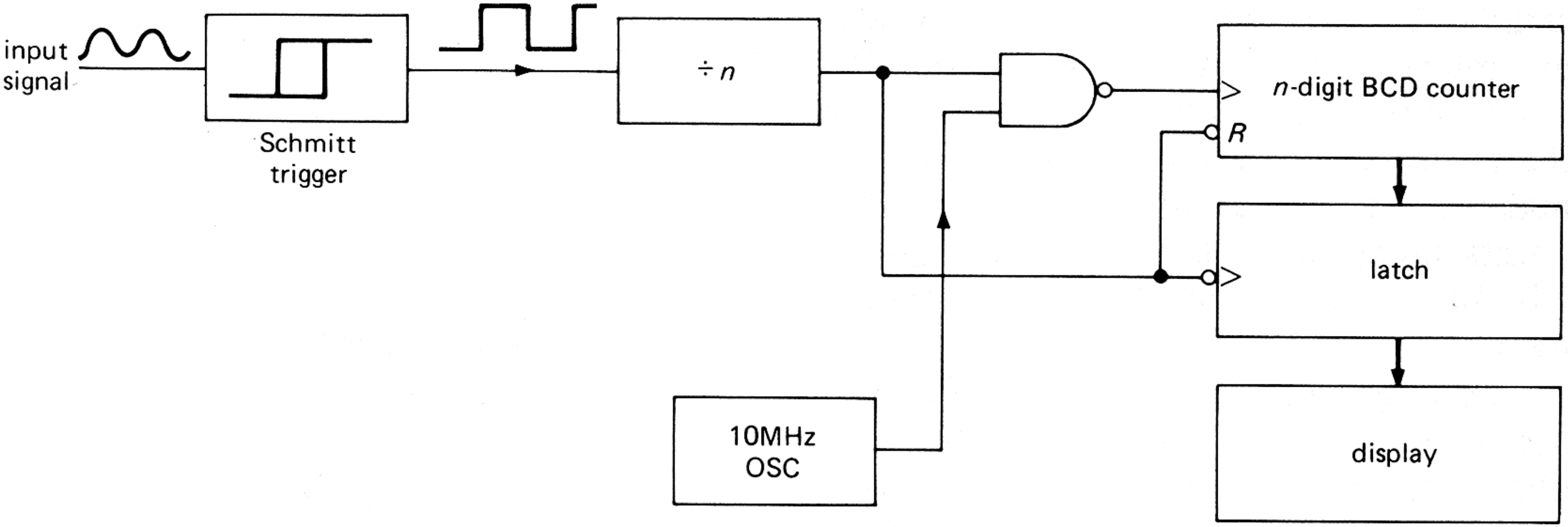
- **Signal period measurement**
- **Averaging many measurements (with added noise)**
- **Phase-locked-loop:**



P. Horowitz and W. Hill, The Art of Electronics

Period measurement

- Count the internal reference clock cycles within a time defined by the period of the input signal



Period measurement

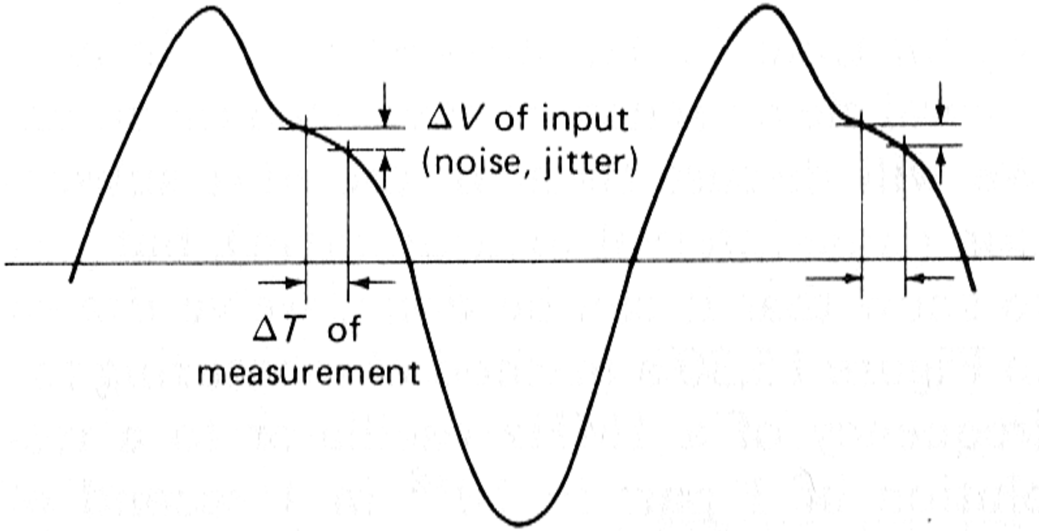
- Uncertainty of ±1 pulses in the counting is still present, now in the reference signal

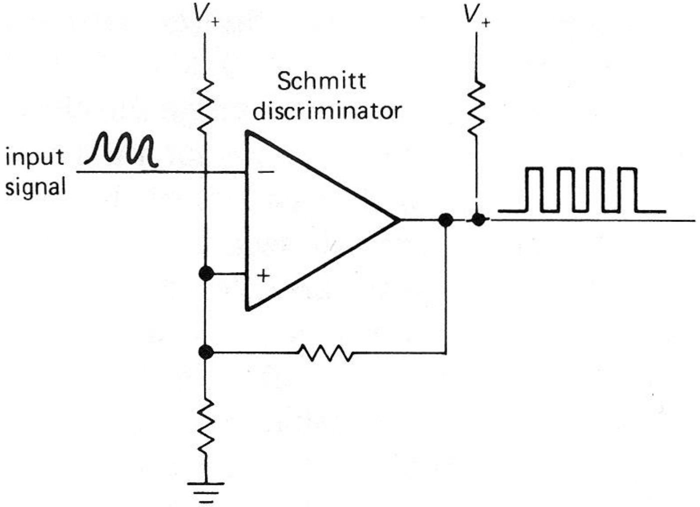
$$\frac{\Delta T}{T_m} = \frac{\Delta N}{N} = \frac{1}{T_m f_{osc}} \xrightarrow{T_m = n / f_m} \frac{f_m}{n f_{osc}}$$

- For specific measurement interval, the uncertainty is constant (regardless of the input signal frequency)
- Typically counters choose the measurement method automatically

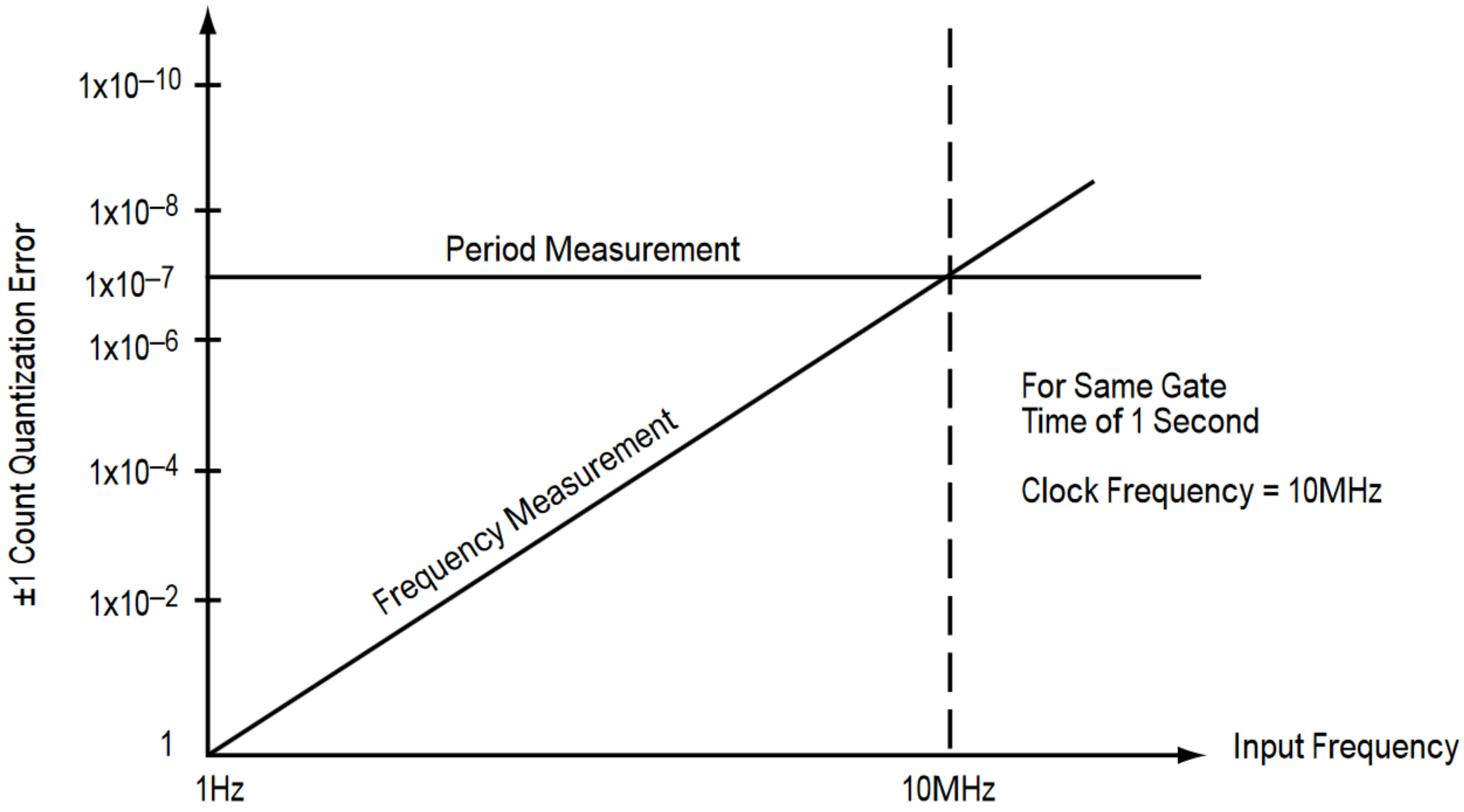
Period measurement

- **Period measurement is more sensitive to noise and jitter**
  - Good SNR is required
  - Good input and triggering circuitry is needed





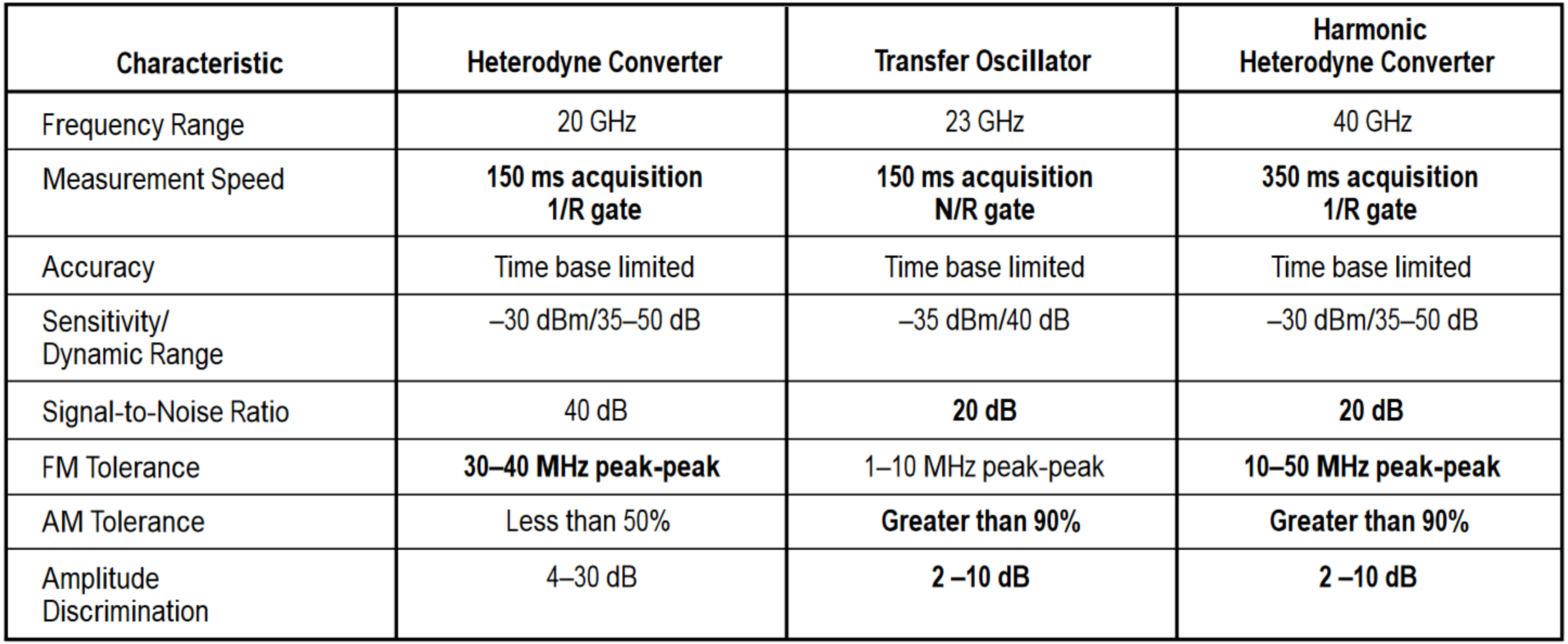
Frequency vs period measurement



Increasing frequency range

- **Prescaling (frequency divider IC)**
  - Up to ~15 GHz
- **Heterodyne-technique (modulation to lower frequency)**
  - Up to ~20 GHz
- **Transfer oscillators (Phased-locked loop)**
  - Up to ~25 GHz
- **Hybrid techniques**
  - Up to ~50 GHz

Comparison of some methods



Characteristic	Heterodyne Converter	Transfer Oscillator	Harmonic Heterodyne Converter
Frequency Range	20 GHz	23 GHz	40 GHz
Measurement Speed	**150 ms acquisition 1/R gate**	**150 ms acquisition N/R gate**	**350 ms acquisition 1/R gate**
Accuracy	Time base limited	Time base limited	Time base limited
Sensitivity/ Dynamic Range	–30 dBm/35–50 dB	–35 dBm/40 dB	–30 dBm/35–50 dB
Signal-to-Noise Ratio	40 dB	**20 dB**	**20 dB**
FM Tolerance	**30–40 MHz peak-peak**	1–10 MHz peak-peak	**10–50 MHz peak-peak**
AM Tolerance	Less than 50%	**Greater than 90%**	**Greater than 90%**
Amplitude Discrimination	4–30 dB	**2 –10 dB**	**2 –10 dB**

Frequency references

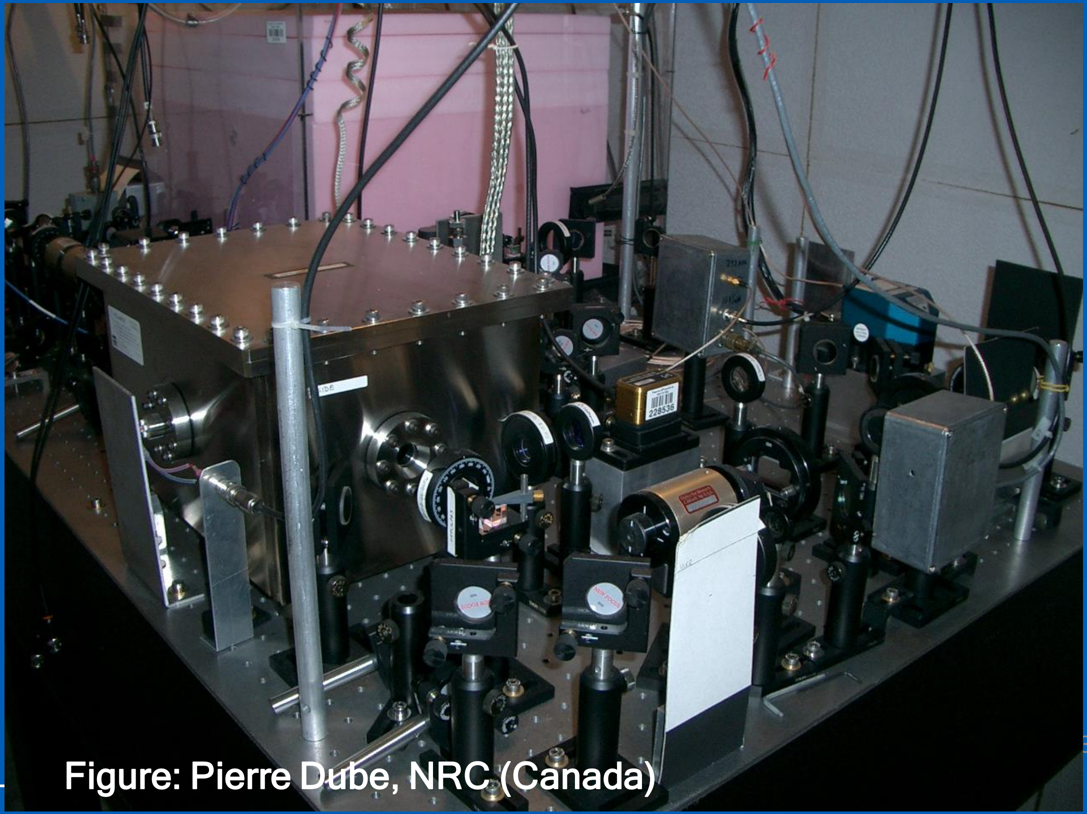
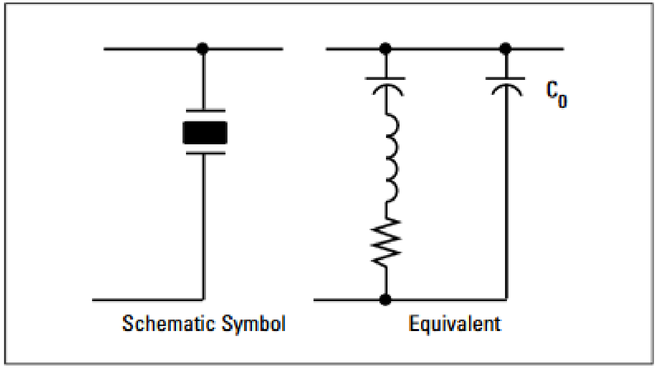
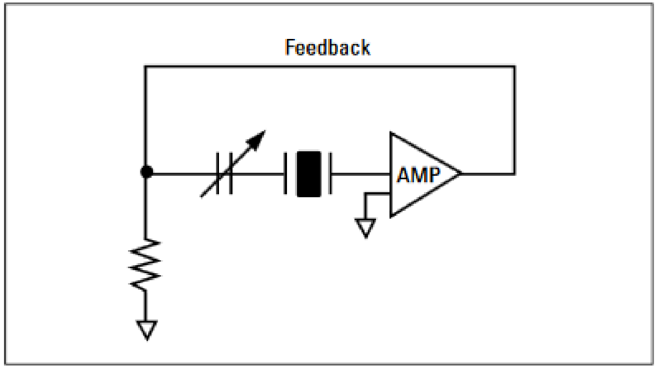


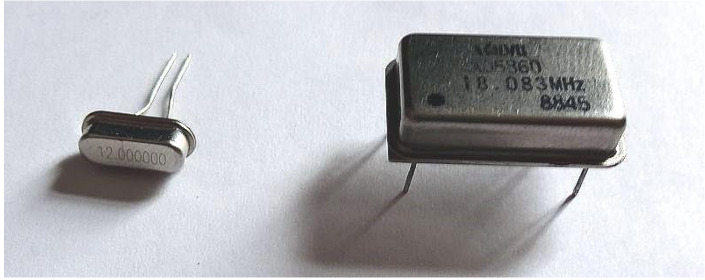
Figure: Pierre Dube, NRC (Canada)

Crystal oscillator

- Mechanical vibration in quartz element
- Most common frequency standard
- Wide range frequencies available from 32.768 kHz to Ghz range
- Most accurate devices around 10 Mhz







Series and parallel resonance

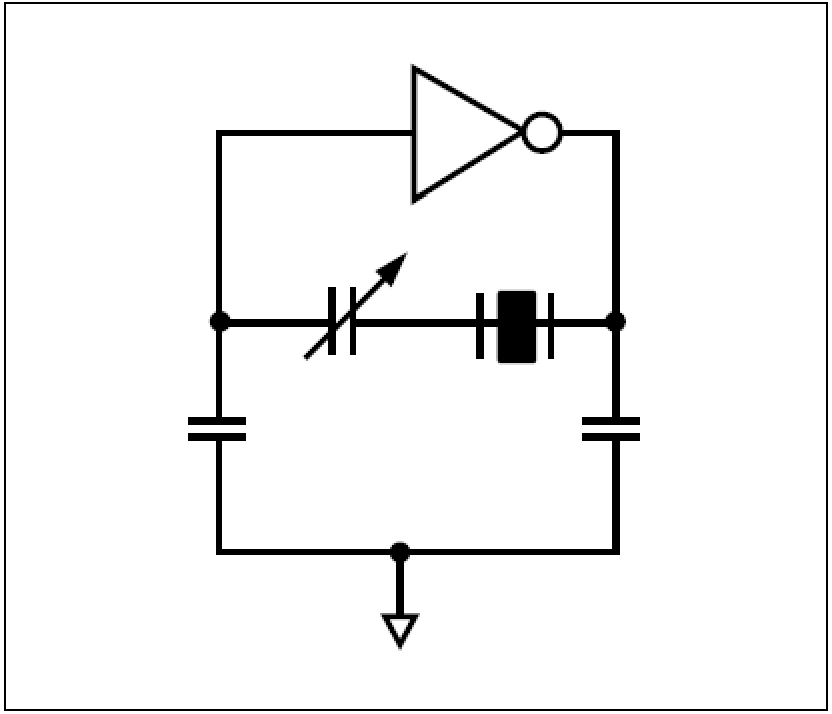


Figure 11. The Series Resonant Oscillator

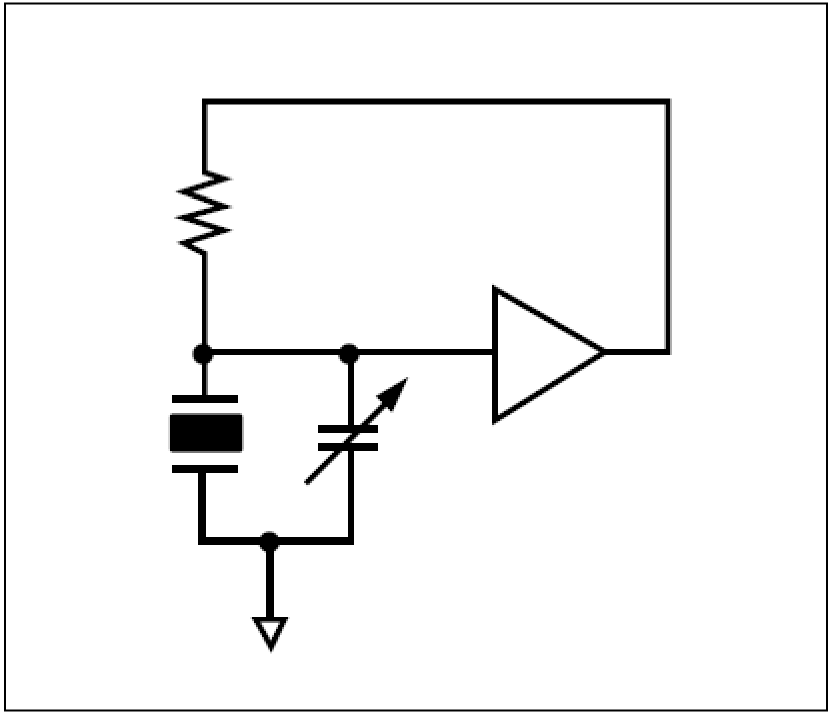
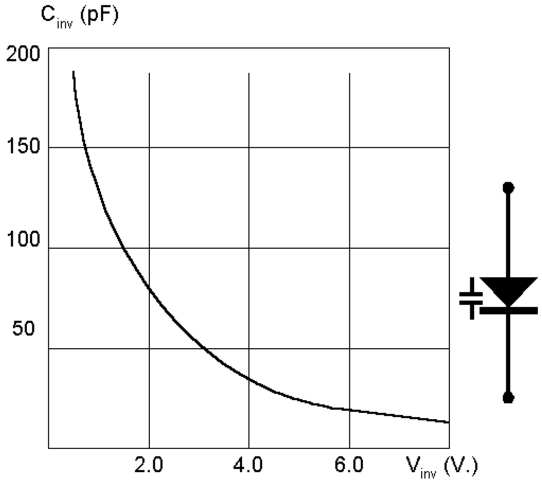
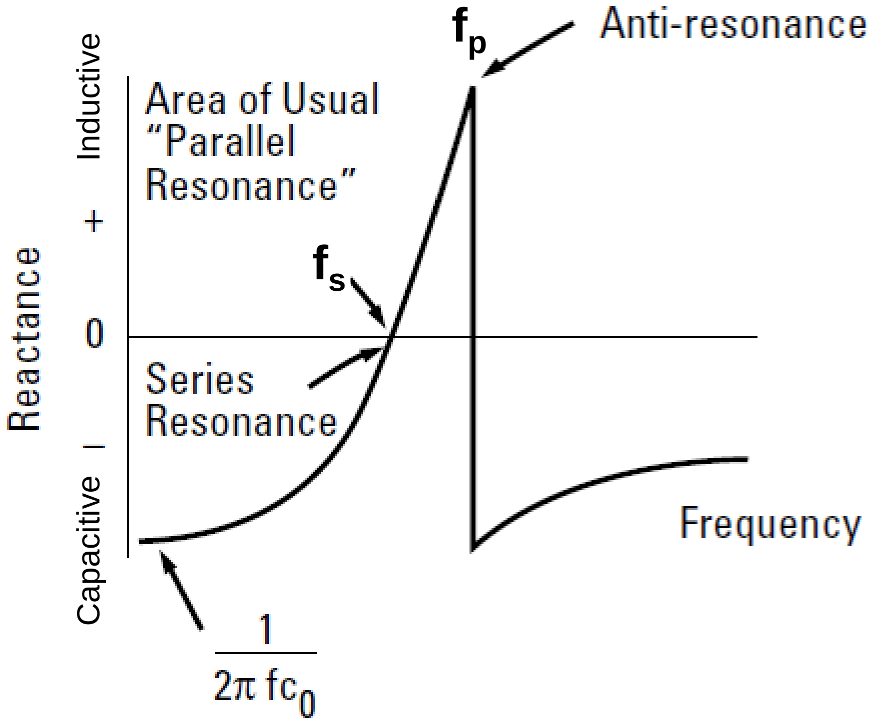


Figure 12. The Parallel Resonant Circuit

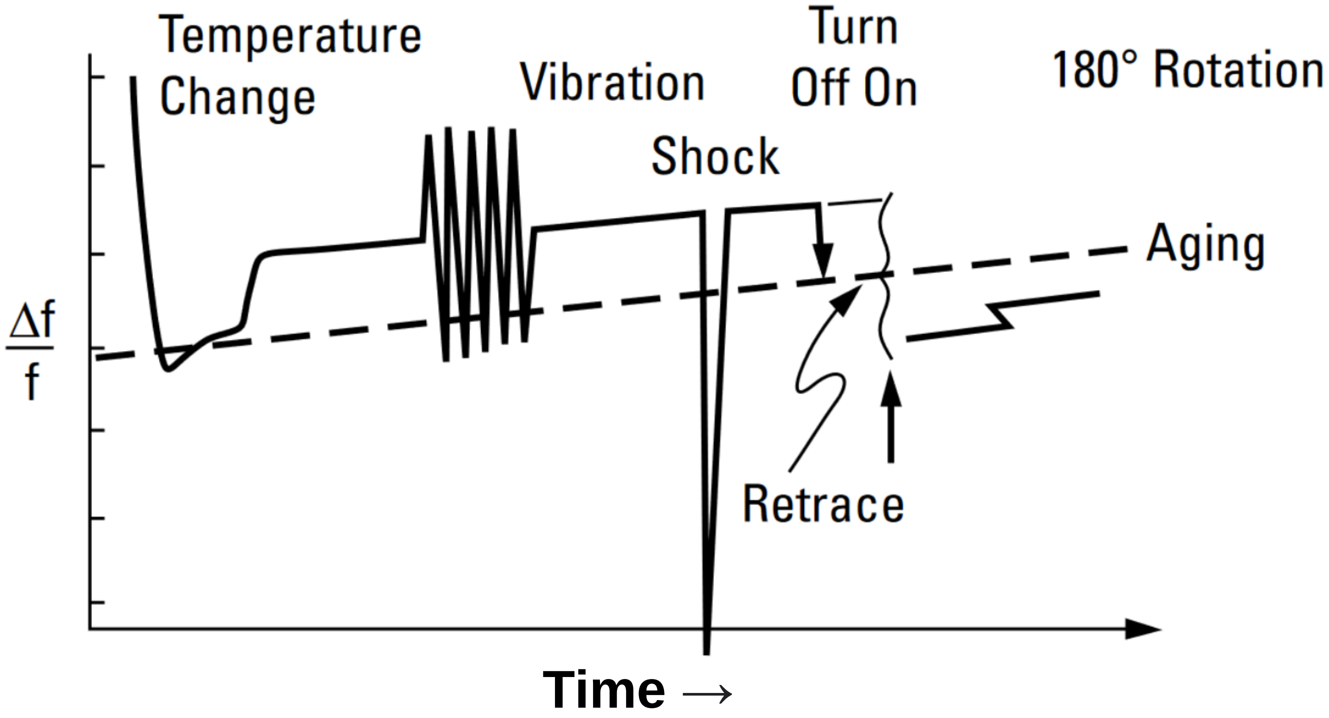
Series and parallel resonance

- **Can be tuned with external capacitances**
- **Voltage controlled tuning with capacitance diodes**



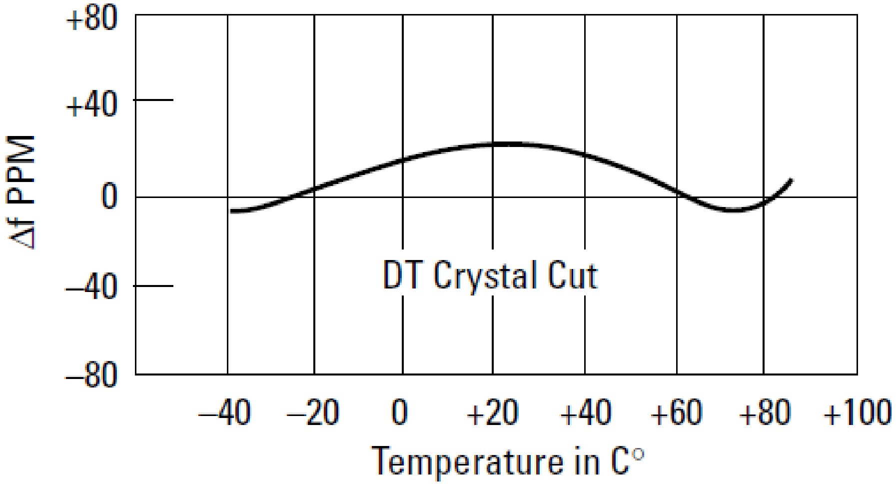


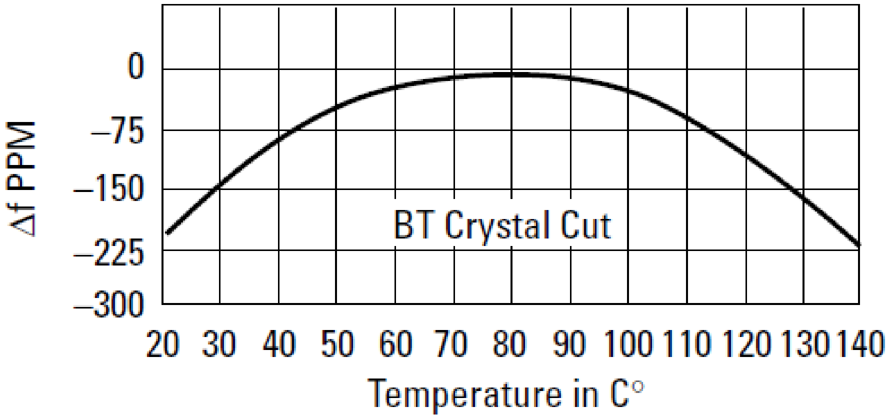
Frequency stability of crystal oscillator



Main uncertainty components in crystal oscillators

- **Drive power: 1×10^{-9}/μW**
- **Gravity (and acceration): ~1×10^{-9}/G**
- **Retrace (on/off switching): ~1×10^{-8}**
- **Long-term and short-term stability**
- **Temperature:**

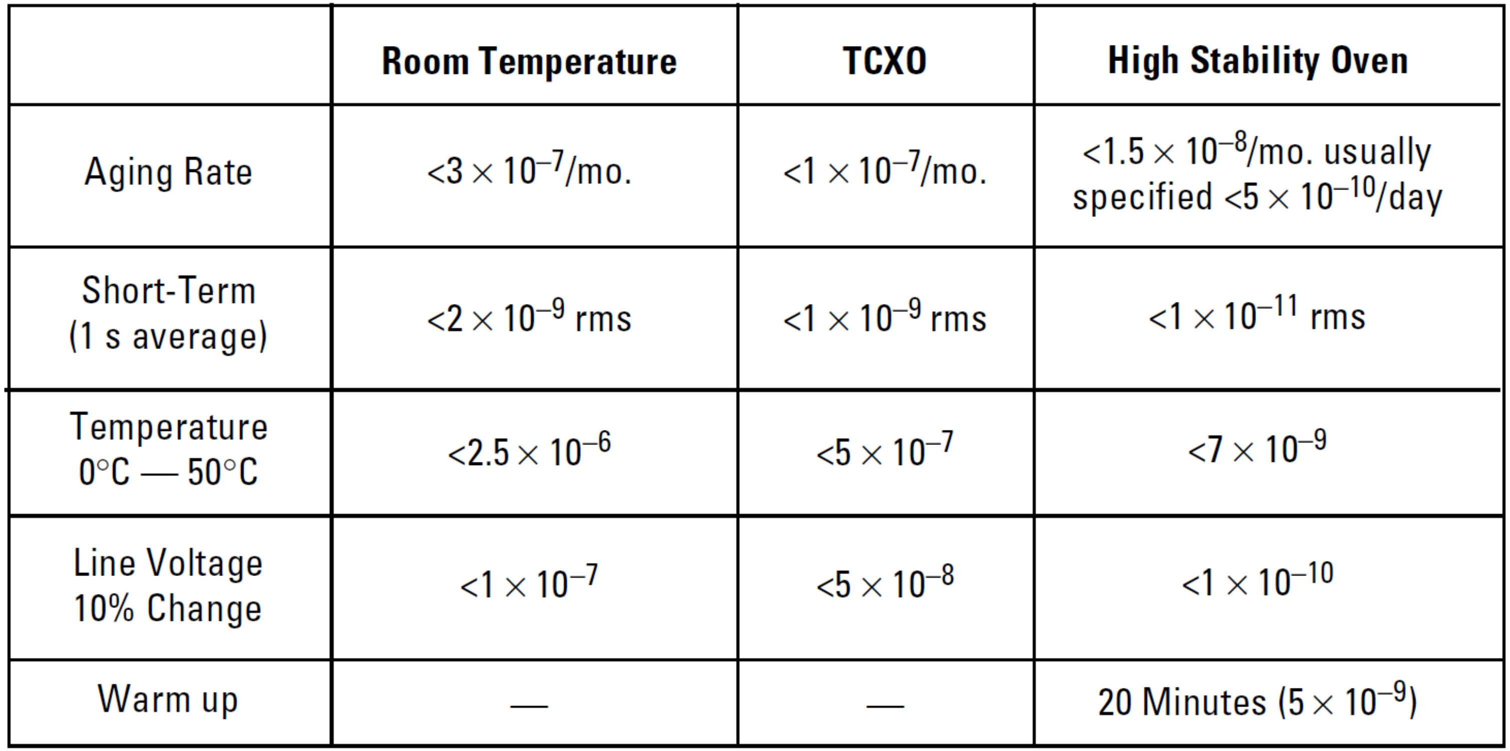




Temperature stabilization

- **RTXO = Room Temperature Crystal Oscillator**
  - No compensation except optimized crystal cut and finish
- **TCXO = Temperature Compensated Crystal Oscillator**
  - Frequency compensated with external components
- **DCXO = Digital Controlled Crystal Oscillator**
  - frequency estimation, frequency correction and tracking algorithms
- **OCXO = Oven Controlled Crystal Oscillator**
  - Crystal and oscillator circuit are operated inside temperature controlled “oven” or even double oven structure

Example performance values



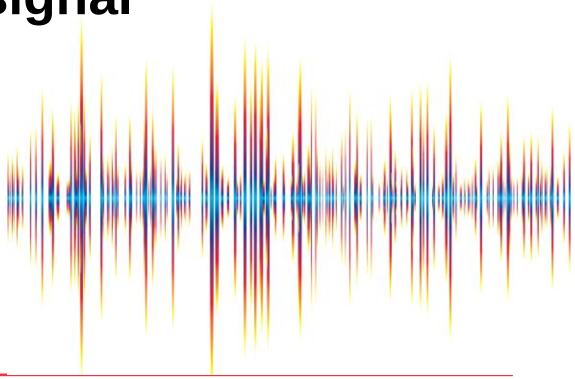
	Room Temperature	TCXO	High Stability Oven
Aging Rate	$<3 \times 10^{-7}$/mo.	$<1 \times 10^{-7}$/mo.	$<1.5 \times 10^{-8}$/mo. usually specified $<5 \times 10^{-10}$/day
Short-Term (1 s average)	$<2 \times 10^{-9}$ rms	$<1 \times 10^{-9}$ rms	$<1 \times 10^{-11}$ rms
Temperature 0°C — 50°C	$<2.5 \times 10^{-6}$	$<5 \times 10^{-7}$	$<7 \times 10^{-9}$
Line Voltage 10% Change	$<1 \times 10^{-7}$	$<5 \times 10^{-8}$	$<1 \times 10^{-10}$
Warm up	—	—	20 Minutes (5×10^{-9})

Noise

Ref : J. Fraden, Handbook of Modern Sensors, chapter 5.9

What is noise?

- In general: all unknown modifications that a signal may suffer during capture, storage, transmission, processing, or conversion
- In this context: spontanious fluctuation of a measurement system that are caused by physical phenomena of devices, components or materials
- Often sets the limit for smallest measurable signal



Noise and statistics

Noise is a random variable that cannot be predicted or removed

- However, it can be minimized and
- It can be examined statistacally

Average

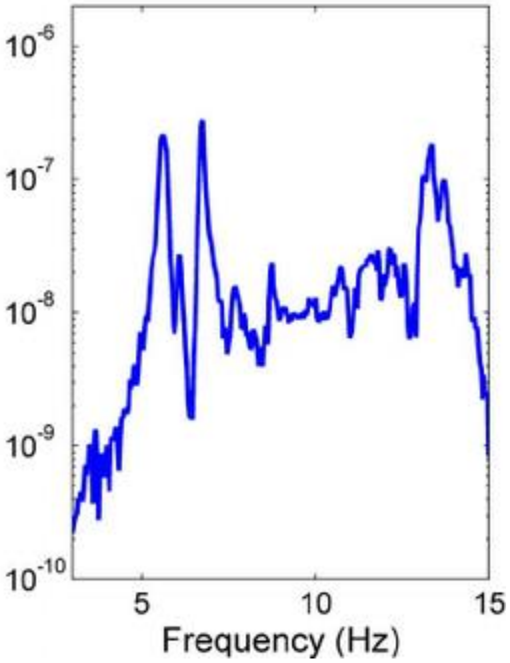
$$\bar{u} = \frac{\int_0^T u(t)\,dt}{T} = 0$$

Deviation

$$\sigma = \sqrt{\frac{\int_0^T \left[u(t)-\bar{u}\right]^2 dt}{T}} = \sqrt{\frac{\sum_{i=1}^{N}\left[u_i-\bar{u}\right]^2}{N}}$$

Measuring noise

- **The RMS value of noise for specific frequency band *B* can be measured for example with**
  - Oscilloscope, RMS voltage meter, power meter
  - Very limited (if any) information about the frequency distribution
- **Frequency information of noise can be measured with spectrum analyzer**
  - Typically gives the noise power spectral density (*i.e.* noise power per unit of frequency)
  - RMS can be obtained as integral of the power spectral density over frequency



From distributions to power or RMS values

- **Power density of noise *S*(*f*) to power P**
  - Units like W/Hz, V²/Hz, A²/Hz

$$P = \int_a^b S(f)\,df$$

- **Voltage density of noise $u_n(f)$ to RMS voltage**
  - Units like V/√Hz, A/√Hz

$$V_{rms} = \sqrt{\int_a^b u_n^2(f)\,df}$$

Similarly for current density $i_n(f)$ etc.

From distributions to power or RMS values

- Note that in theory the limits are *a* = 0 and *b* = ∞
- In practice, however, system is always limited by non-ideal components
- In the case of ideal white noise and ideal filter

$$P = \int_a^b S(f)df$$

$$S(f) = S_0 \rightarrow \; P = S_0\left(b - a\right)$$

$$u_n(f) = u_0 \rightarrow \; V_{rms} = \sqrt{\int_a^b u_0^2 df} = \sqrt{u_0^2(b-a)} = u_0\sqrt{b-a}$$

From distributions to power or RMS values

- **Note that in theory the limits are *a* = 0 and *b* = ∞**
- **In practice, however, system is always limited by non-ideal components**
- **In the case of ideal white noise and ideal filter**

$$P = \int_a^b S(f)\,df$$

$$S(f) = S_0 \rightarrow \quad P = S_0 \left(b - a\right)$$

Noise bandwidth for ideal filter

$$u_n(f) = u_0 \rightarrow \quad V_{rms} = \sqrt{\int_a^b u_0^2\,df} = \sqrt{u_0^2 (b-a)} = u_0 \sqrt{b-a}$$

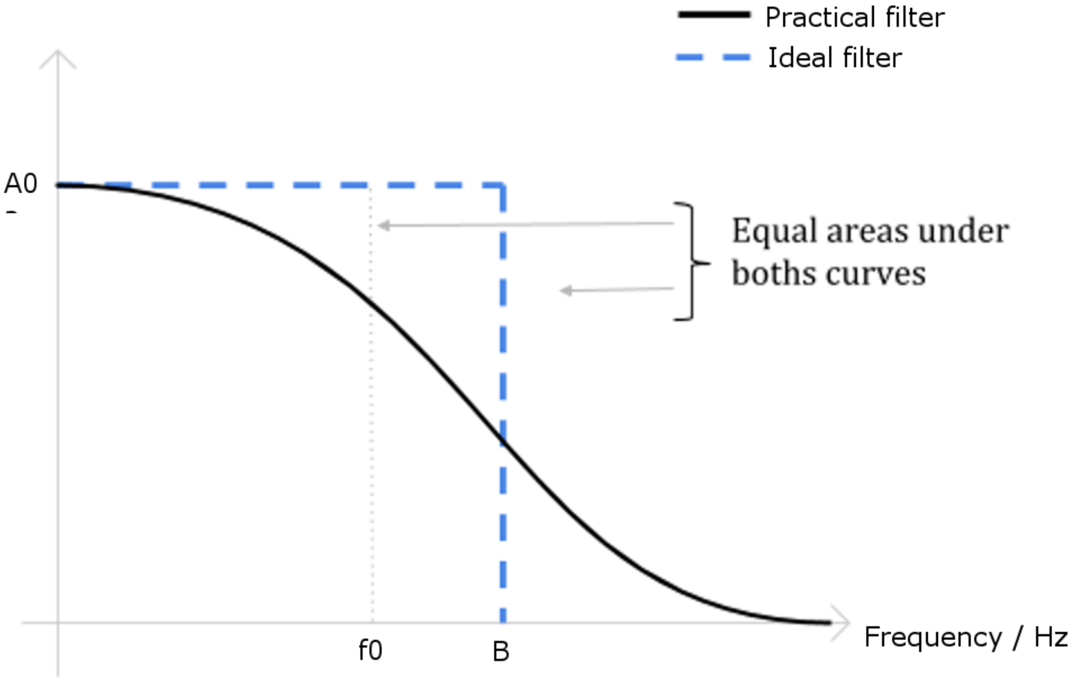
Noise bandwidth for ideal filter

Equivalent noise bandwidth *B*

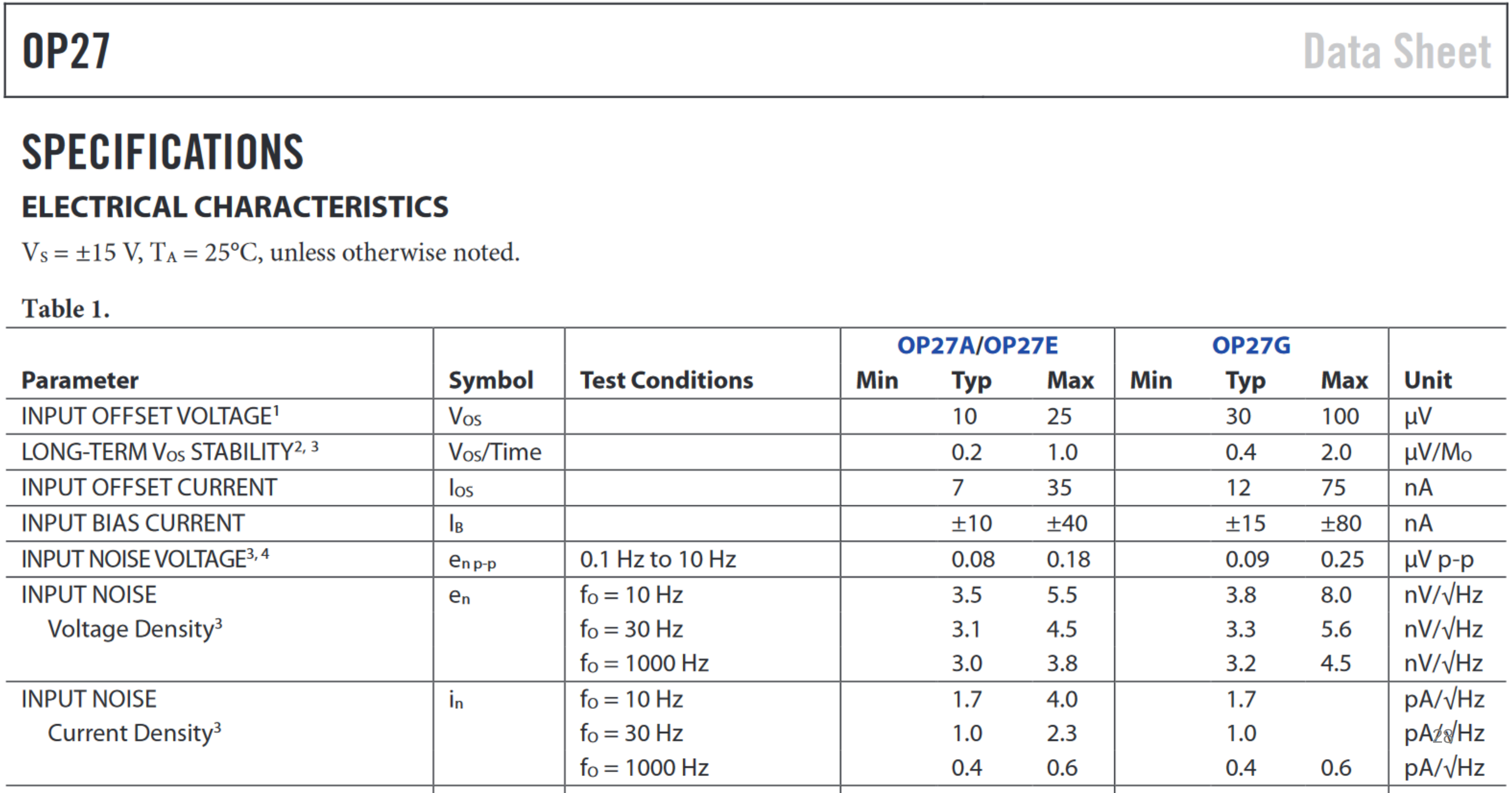
- **Bandwidth of a brickwall (step function) filter which produces same integrated noise power as that of an actual filter**

$$B = \frac{1}{A_0^2}\int_0^\infty |A(f)|^2\, df$$

Filter Order	B / f_0
1	1.57
2	1.22
3	1.15
4	1.13
5	1.11



Practical example: operational amplifier



OP27 Data Sheet

SPECIFICATIONS

ELECTRICAL CHARACTERISTICS

$V_S = \pm 15$ V, $T_A = 25$°C, unless otherwise noted.

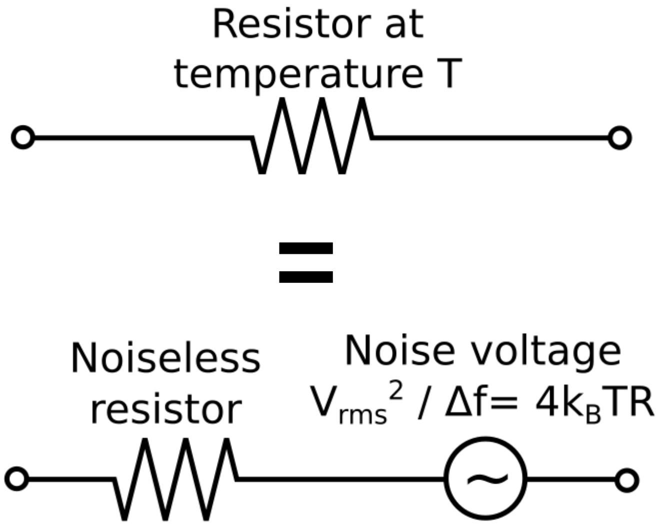
Table 1.

Parameter	Symbol	Test Conditions	OP27A/OP27E Min	OP27A/OP27E Typ	OP27A/OP27E Max	OP27G Min	OP27G Typ	OP27G Max	Unit
INPUT OFFSET VOLTAGE[1]	V_{OS}			10	25		30	100	µV
LONG-TERM V_{OS} STABILITY[2, 3]	V_{OS}/Time			0.2	1.0		0.4	2.0	µV/Mo
INPUT OFFSET CURRENT	I_{OS}			7	35		12	75	nA
INPUT BIAS CURRENT	I_B			±10	±40		±15	±80	nA
INPUT NOISE VOLTAGE[3, 4]	$e_{n\,p\text{-}p}$	0.1 Hz to 10 Hz		0.08	0.18		0.09	0.25	µV p-p
INPUT NOISE Voltage Density[3]	e_n	$f_O = 10$ Hz		3.5	5.5		3.8	8.0	nV/√Hz
		$f_O = 30$ Hz		3.1	4.5		3.3	5.6	nV/√Hz
		$f_O = 1000$ Hz		3.0	3.8		3.2	4.5	nV/√Hz
INPUT NOISE Current Density[3]	i_n	$f_O = 10$ Hz		1.7	4.0		1.7		pA/√Hz
		$f_O = 30$ Hz		1.0	2.3		1.0		pA/√Hz
		$f_O = 1000$ Hz		0.4	0.6		0.4	0.6	pA/√Hz

Noise sources

Thermal noise

- **Intrinsic property of all conductors**
- **Generated by the random motion of charge carriers (electrons)**
- $v_{RMS} = \sqrt{4k_B TRB}$
    - k = Bolzmann constant, 1.381 × 10^{-23} J/K
    - T = Absolute temperature [K]
    - R = Resistance



- **Happens regardless of any applied voltage**
- **Can be higher for some type of resistors**

Reducing thermal noise

$$v_{RMS} = \sqrt{4k_B T R B}$$

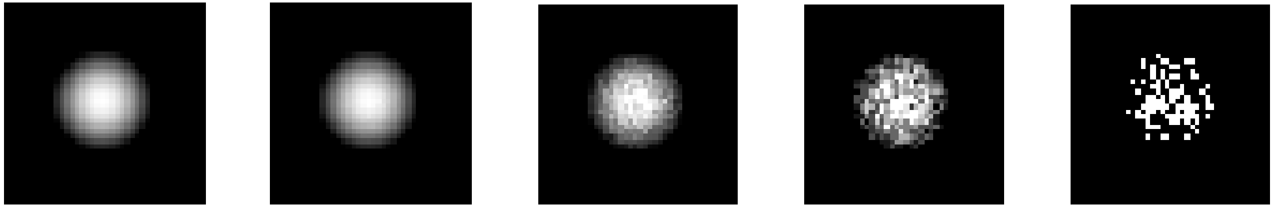
- **Reduce temperature**
  - Cryostats, peltier cooling etc

- **Reduced bandwidth**
  - Averaging, optimized filtering

The heating effect of low power laser measured in liquid helium

Shot noise

- **Originates from the discrete nature of the various phenomena (electrons, photons, particles)**
    - For example transistors, diodes, optical detectors, radiation detectors
    - Insignificant in conventional resistors and wires, as coulomb force even outs the distribution of the flowing electrons
- **As a general rule:** $\boldsymbol{SNR \propto N/\sqrt{N} = \sqrt{N}}$
- **Current *I* over *pn*-junction has a shot noise:** $\boldsymbol{i_{RMS} = \sqrt{2qIB}}$
    - Can be utilized as a noise generator



Microscope image of fluorescent microsphere at various number of collected photons

Reducing electrical shot noise

$$i_{RMS} = \sqrt{2qIB}$$

- Avoid unnecessary *pn*-junctions in the signal path
- Reduce *B* when ever possible
- Reduce *I* when if <u>it does not effect the measured signal</u>
- Often field effect transistors (FETs) have smaller shot noise than traditional bipolar junction transistors (BJTs) due to very small control current

Flicker noise (1/f –noise)

- **General term for noise sources with approximately 1/*f* power spectral density**

$$1/f^{\alpha}, \alpha = 0,5 \ldots 1,5$$

- **Present in practically all electronic devices and measurements systems**
- **Various sources, such as**
  - impurities in a conductive channels, generation and recombination noise in transistors, slow varying temperature effects

Flicker noise

- **First observed in vacuum tube amplifiers (Johnson 1925)**

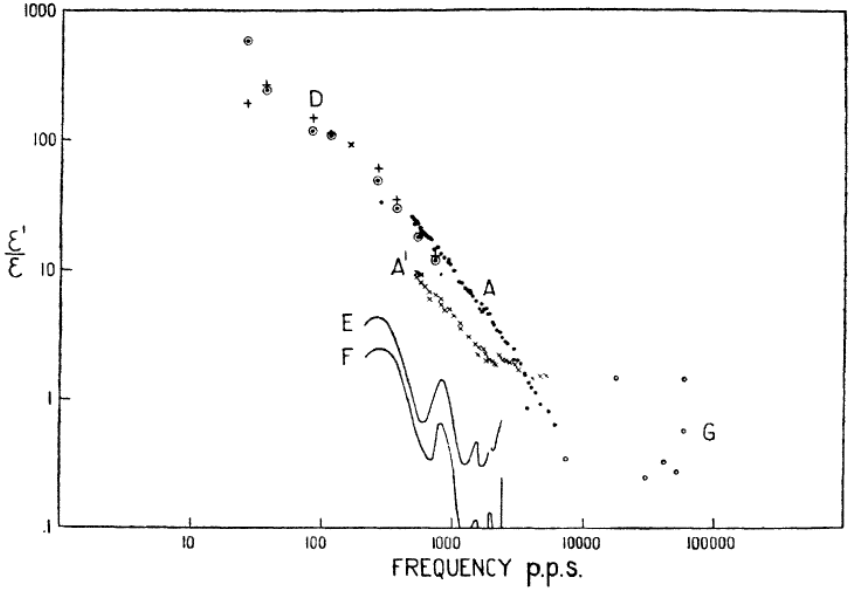


Fig. 6. Frequency variation for tube No. 2, coated filament; same data as in Fig. 4 plotted to a frequency scale; curves E and F give Hartmann's results for 2 m-a. and 20 m-a.; points G were obtained with less steady measuring circuit.

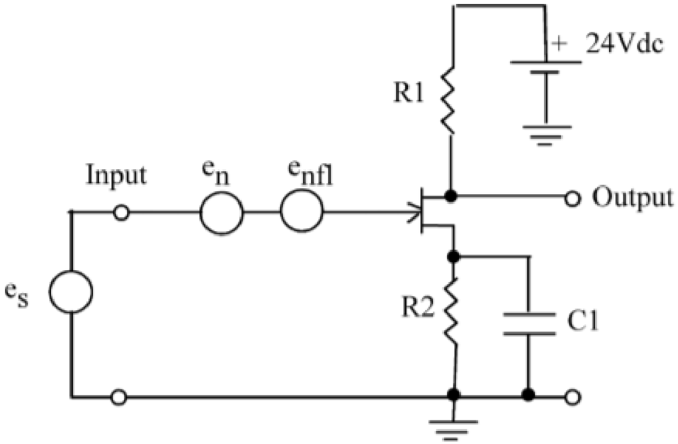
Reducing flicker noise

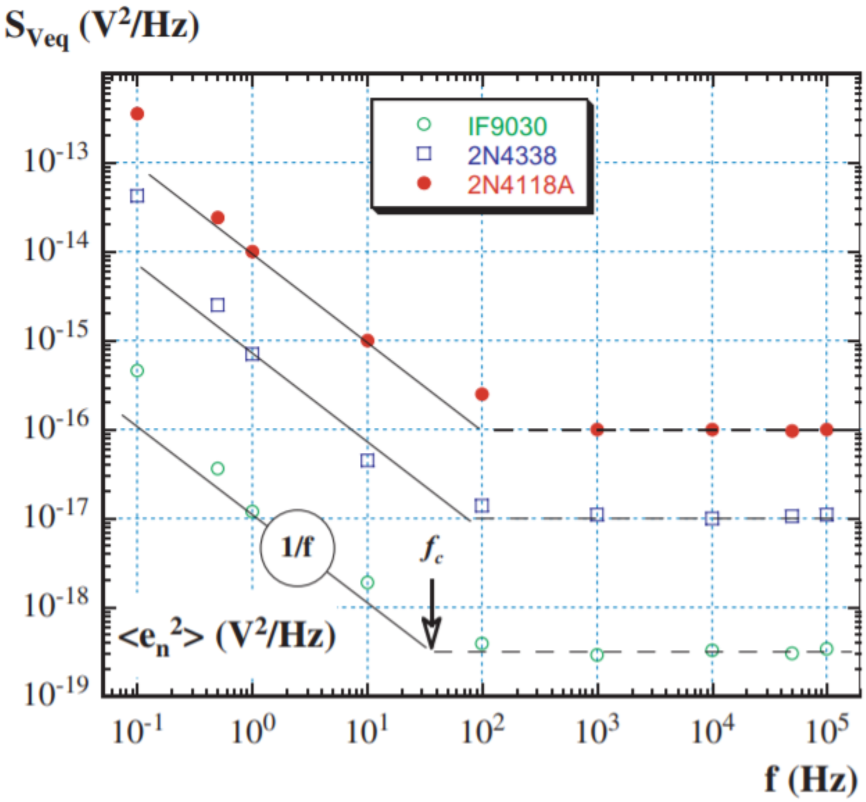
- **Averaging measurements over a long time periods causes or having very long integration times**
  - Low $f \rightarrow 1/f$ –noise becomes significant
- **After certain point, increasing measurement time won't reduce noise but rather increase it (note: also drift components start to effect)**

- **Can be reduced by modulating the measured signal with known frequency, shifting measurement to higher frequencies.**

Reducing flicker noise

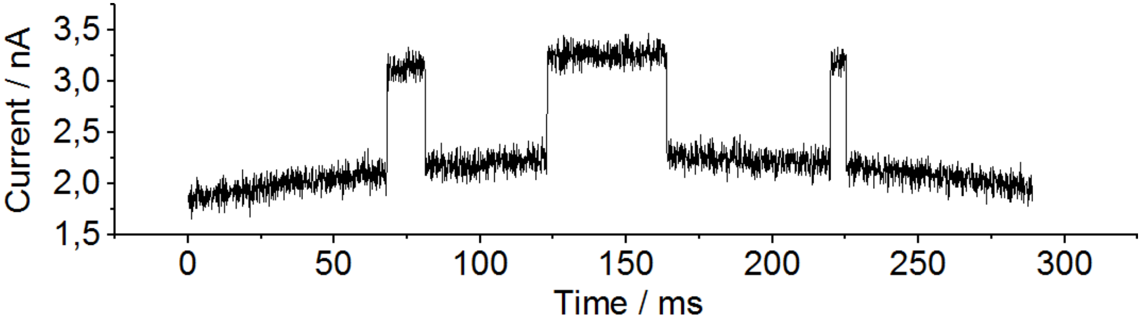
- **1/f –noise can also be effected with component selection**





Burst noise

- **Sudden step-like transitions between two or more discrete voltage or current levels**
- **Present in semiconductors (and other ultra-thin film structures)**
  - Random trapping and release of charge carriers in interfaces
  - For example due to surface contamination in manufacturing process
- **Manufacturers scrap affected devices during tests**
  - Can still occur for example if devices are cooled beyond specified limits



Other local noise sources

- **Capacitive coupling**
  - For example two conductor coupled by stray capacitance
- **Magnetic coupling**
  - Mutual inductance between wire loops
  - Transformers
- **Crosstalk**
  - Electric field between two signal paths running parallel
- **Interfacing and cabling noise**
  - Improper ground loops
  - Digital circuitry near sensitive signal paths

External noise sources

- **Switching current and voltage**
  - High current load, which is switched on/off
  - Power supplies
- **Power line interference**
  - Most problematic when high frequency components are present
- **Sparking**
  - Relays, arching switches, motors with commutator, etc.
- **Environmental and atmospheric noise**

- **How to avoid?**
  - Proper shielding and grounding, filtering, distance to noise source

Aalto University

Measuring small signal levels

ELEC-E5710 - Sensors and Measurement Methods

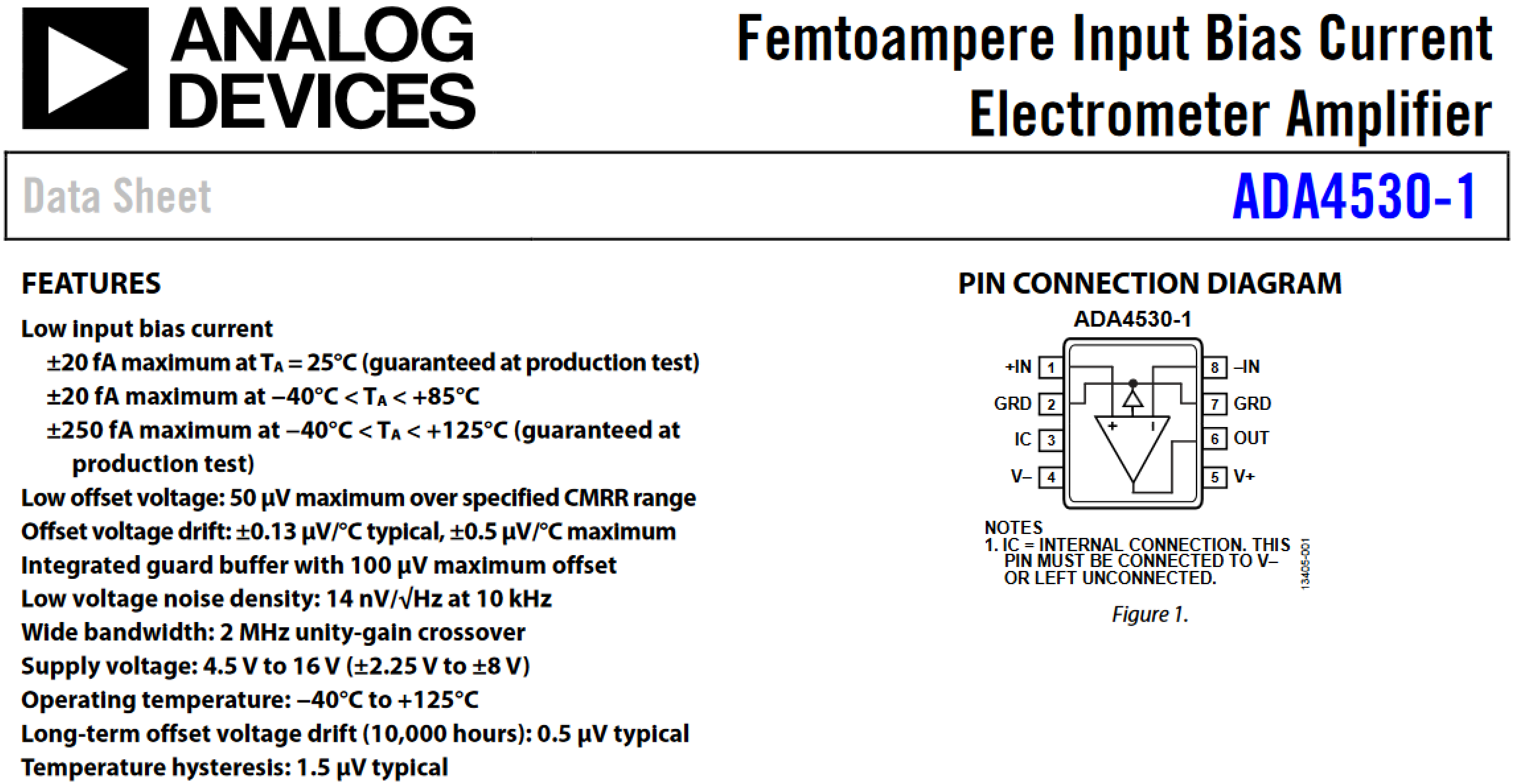
Ref: Keithley, Low Level Measurements Handbook

General aspects

When small signals are measured, one should pay attention to:

- Ripple of the supply voltages
- Noise optimization (bandwidth, temperature if possible/necessary)
- Enclosures
- Cabling issues, grounding, ground loops
- Electromagnetic interference
  - *Resistive, inductive, capacitive, radiowaves*
- Component selection

Classic example: very high quality operational amplifier



ANALOG DEVICES

Femtoampere Input Bias Current Electrometer Amplifier

Data Sheet **ADA4530-1**

FEATURES

- Low input bias current
  - ±20 fA maximum at $T_A = 25°C$ (guaranteed at production test)
  - ±20 fA maximum at $-40°C < T_A < +85°C$
  - ±250 fA maximum at $-40°C < T_A < +125°C$ (guaranteed at production test)
- Low offset voltage: 50 µV maximum over specified CMRR range
- Offset voltage drift: ±0.13 µV/°C typical, ±0.5 µV/°C maximum
- Integrated guard buffer with 100 µV maximum offset
- Low voltage noise density: 14 nV/√Hz at 10 kHz
- Wide bandwidth: 2 MHz unity-gain crossover
- Supply voltage: 4.5 V to 16 V (±2.25 V to ±8 V)
- Operating temperature: –40°C to +125°C
- Long-term offset voltage drift (10,000 hours): 0.5 µV typical
- Temperature hysteresis: 1.5 µV typical

PIN CONNECTION DIAGRAM

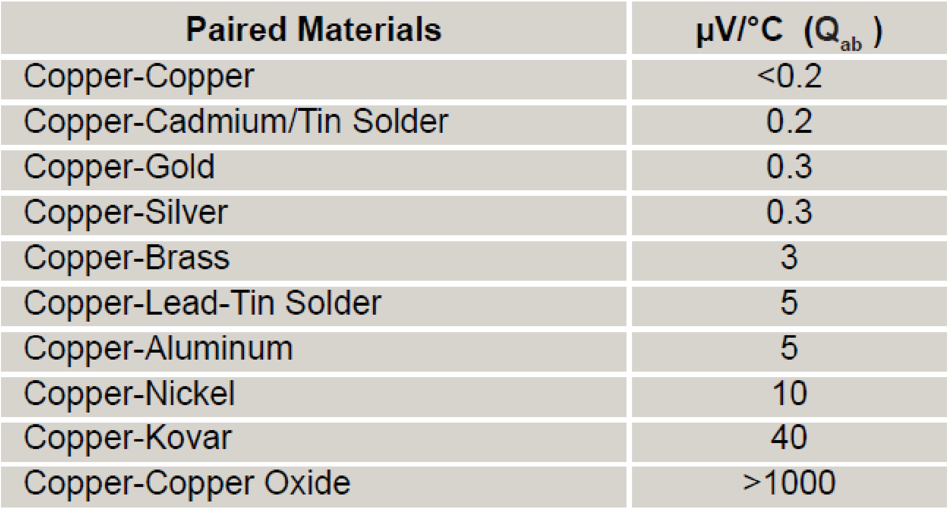
NOTES
1. IC = INTERNAL CONNECTION. THIS PIN MUST BE CONNECTED TO V– OR LEFT UNCONNECTED.

Figure 1.

Measuring small voltages

Seebeck effect

- **Thermoelectric potential difference that occurs when**
  - Different materials are in contact
  - Temperature difference is present
- **Avoid temperature gradients in devices if possible**
- **Use solderless connection (copper to copper crimp)**



Paired Materials	µV/°C (Q_{ab})
Copper-Copper	<0.2
Copper-Cadmium/Tin Solder	0.2
Copper-Gold	0.3
Copper-Silver	0.3
Copper-Brass	3
Copper-Lead-Tin Solder	5
Copper-Aluminum	5
Copper-Nickel	10
Copper-Kovar	40
Copper-Copper Oxide	>1000

Magnetic fields

- **Wire movement in magnetic field generates voltage**
- **Can be nanovolts in earths magnetic field**
- **Can be avoided with:**
  - Shortened wiring
  - Twisted pair cables
  - Differential inputs and balanced lines
  - Magnetic protection (mu-metal)

Chopper amplifier

- **Small DC-signal requires high gain**
  - Hard to achieve low offset and low $1/f$ noise with high gain and DC operation
- **Much easier to build a high gain AC amplifier!**
- **Solution: chop the input signal so that it can be processed as if it were an AC signal**
  - integrated back to a DC signal at the output
- **Often combined with auto-zero circuitry, which measures and compensates input offset periodically**

ANALOG DEVICES

Zero-Drift, Single-Supply, Rail-to-Rail Input/Output Operational Amplifier

Data Sheet

AD8628/AD8629/AD8630

FEATURES

- Lowest auto-zero amplifier noise
- Low offset voltage: 1 µV
- Input offset drift: 0.002 µV/°C
- Rail-to-rail input and output swing
- 5 V single-supply operation
- High gain, CMRR, and PSRR: 130 dB
- Very low input bias current: 100 pA maximum
- Low supply current: 1.0 mA
- Overload recovery time: 50 µs
- No external components required
- Qualified for automotive applications

PIN CONFIGURATIONS

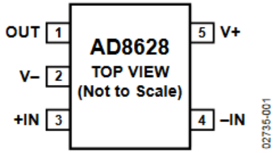
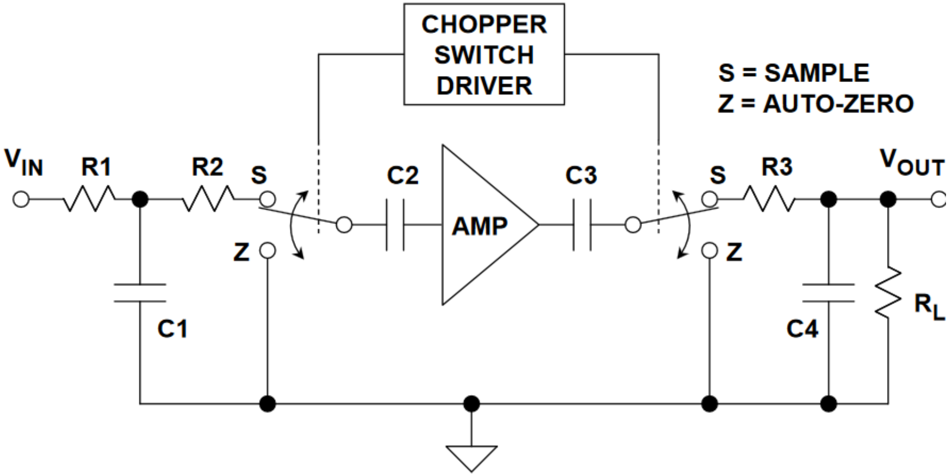


Figure 1. 5-Lead TSOT (UJ-5) and 5-Lead SOT-23 (RJ-5)

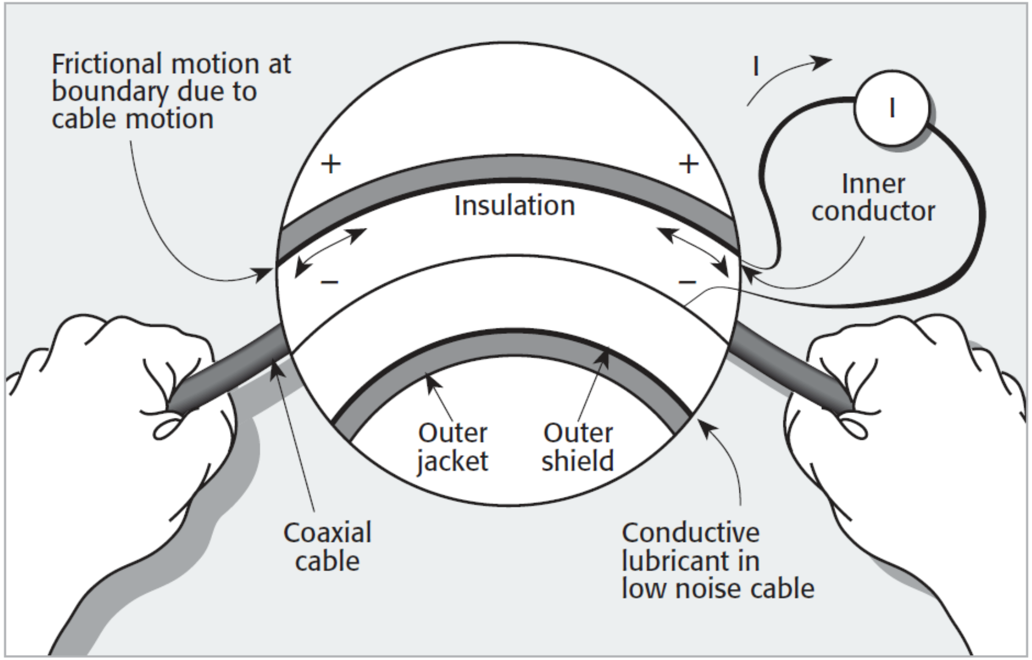


- **Modern device can be used in similar way as conventional op amp**

Measuring small currents

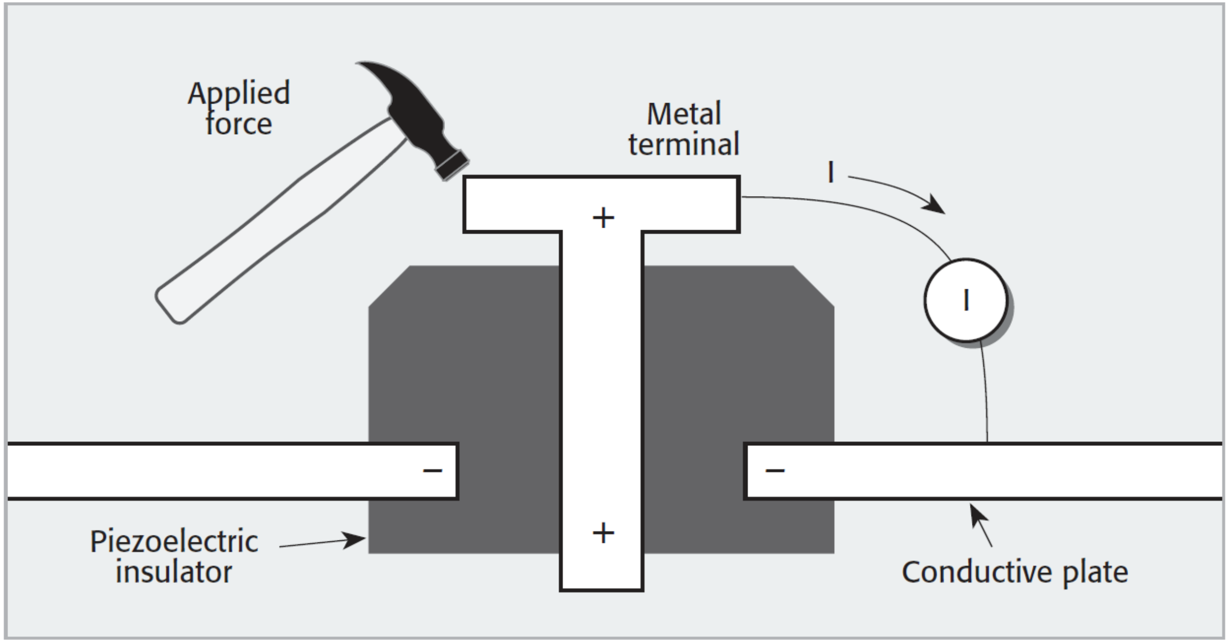
Triboelectric effect

- Electric charge is generated due to frictional moment between different materials
- Movement of cables should be avoided
- Special cables available with low triboelectric effect



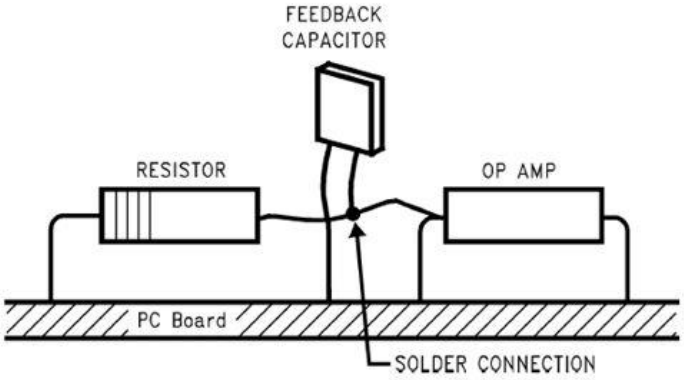
Piezoelectric effect

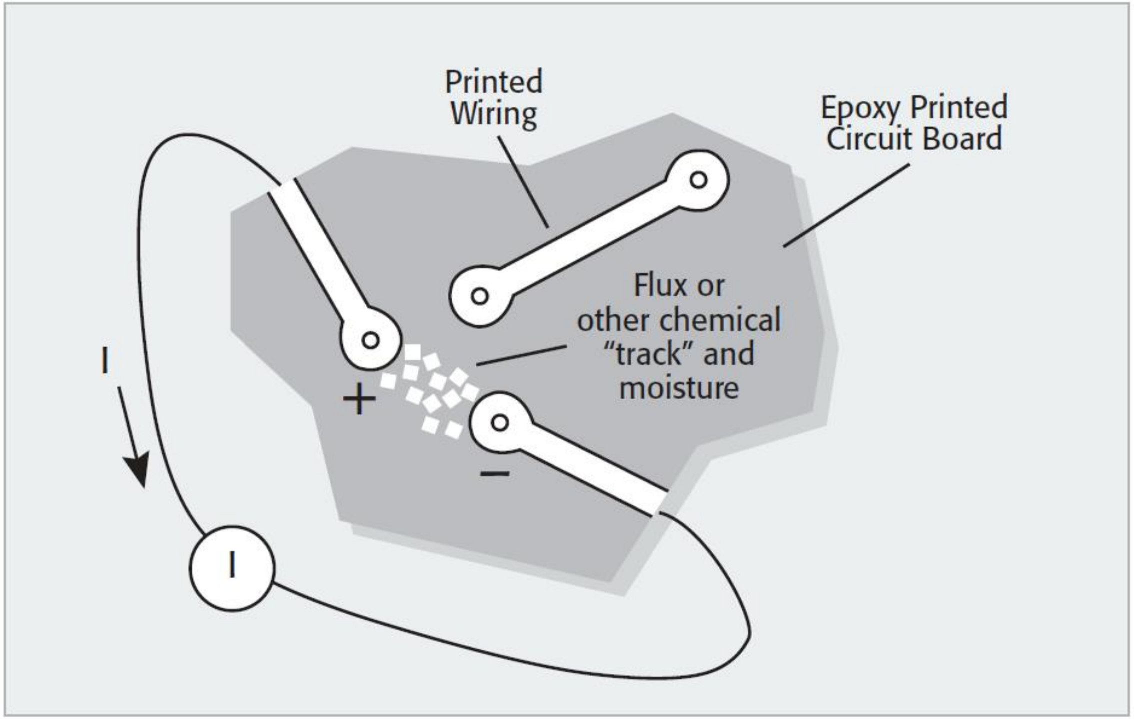
- **Charge caused by mechanical stress in some insulator materials**
- **Vibrations and mechanical stress should minimized**



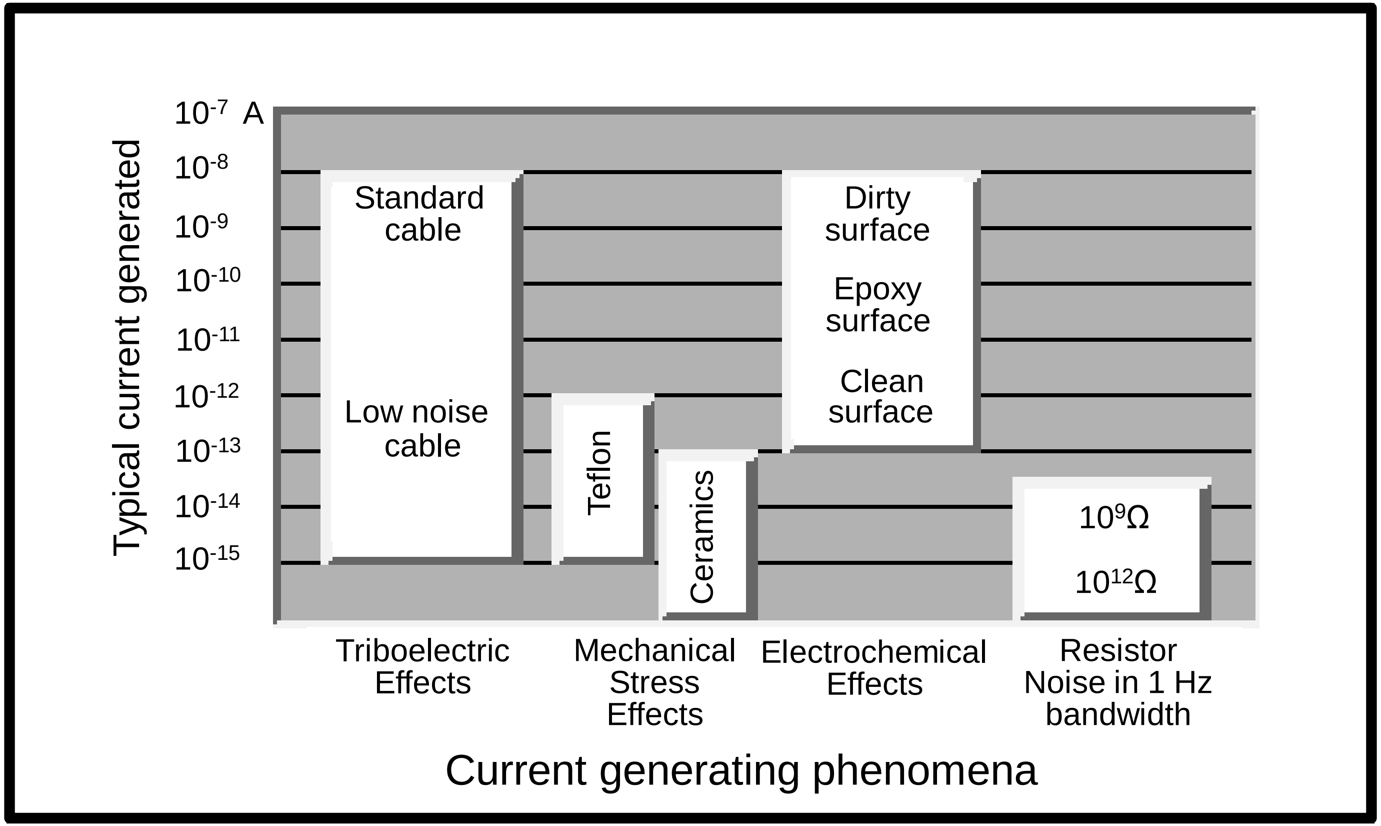
Electrochemical effects

- **Careful cleaning of etching solution, flux or other contamination**
- **Epoxy can cause nanoamperes of current in circuits**
- **Sometimes airwires need to be used**



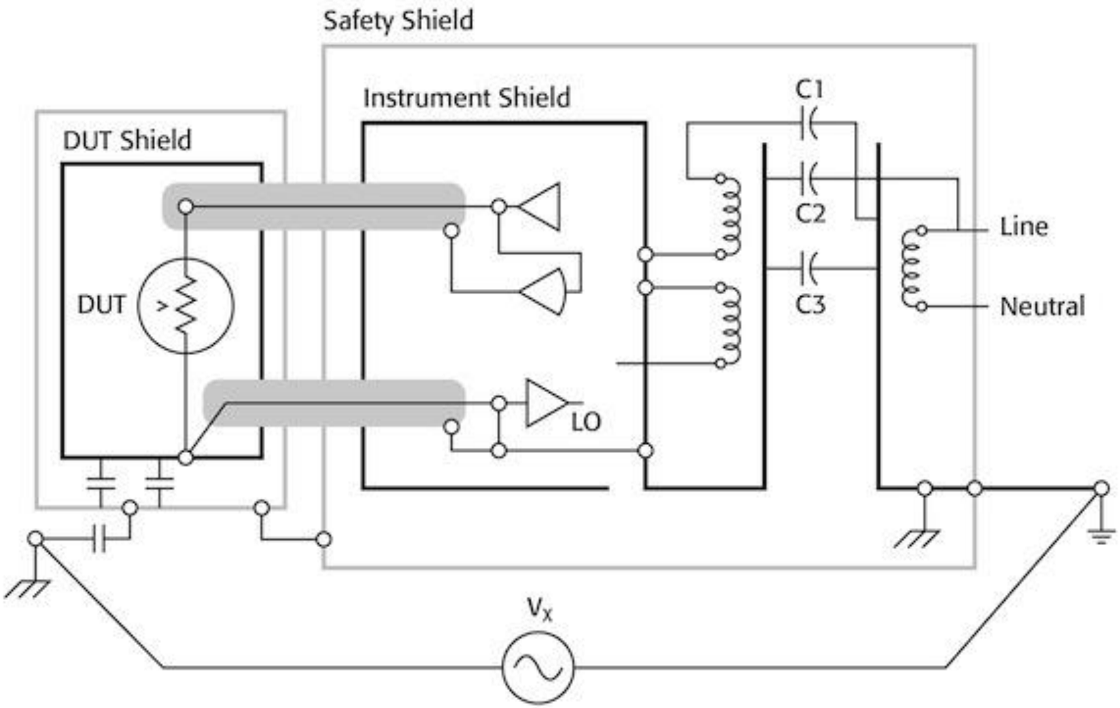


Typical magnitudes of currents generated by low current phenomena



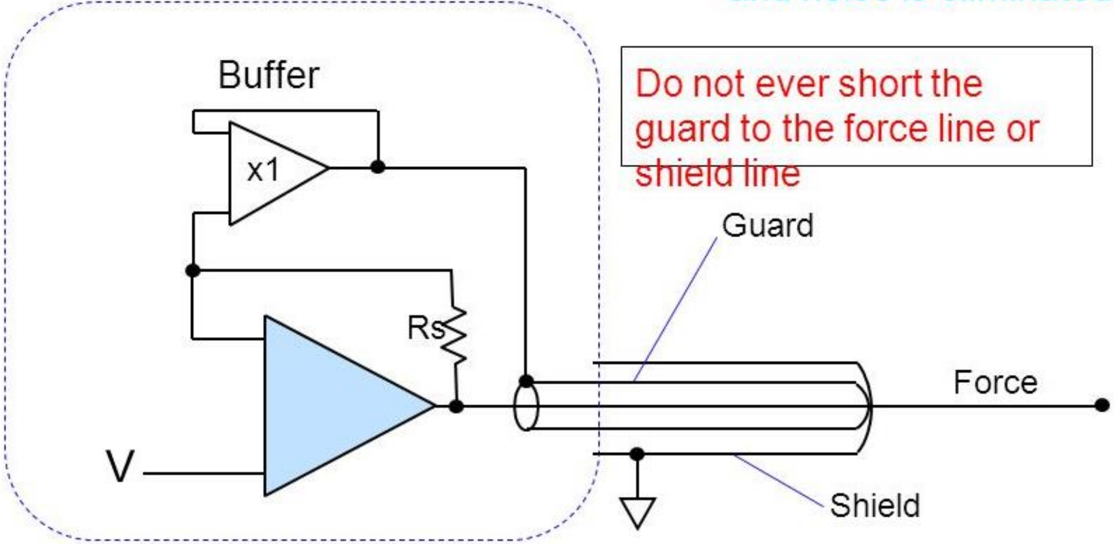
Guarding techniques

System level shielding



Guarding techniques

Triaxial cables



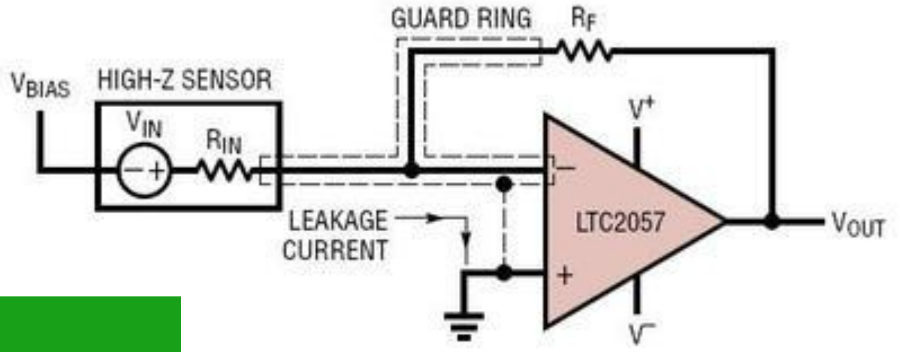
The guard voltage tracks the force voltage exactly.

Cable charging current and noise is eliminated..

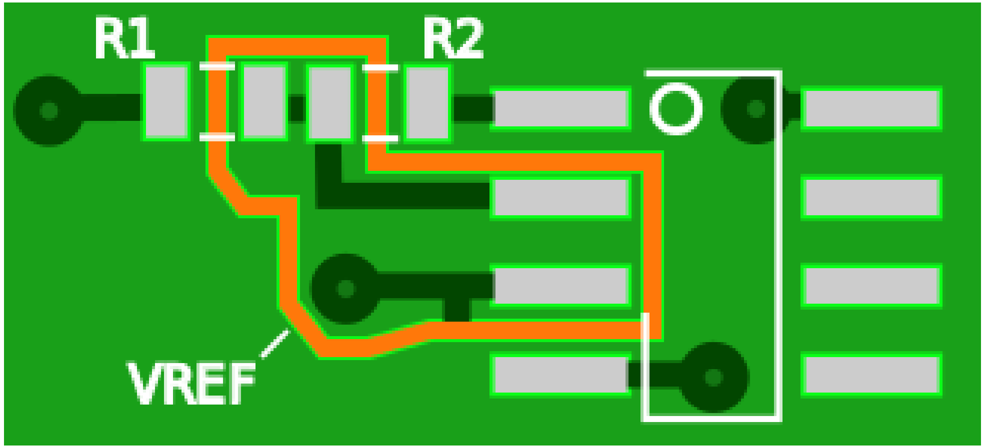
Do not ever short the guard to the force line or shield line

Guarding techniques

Component level

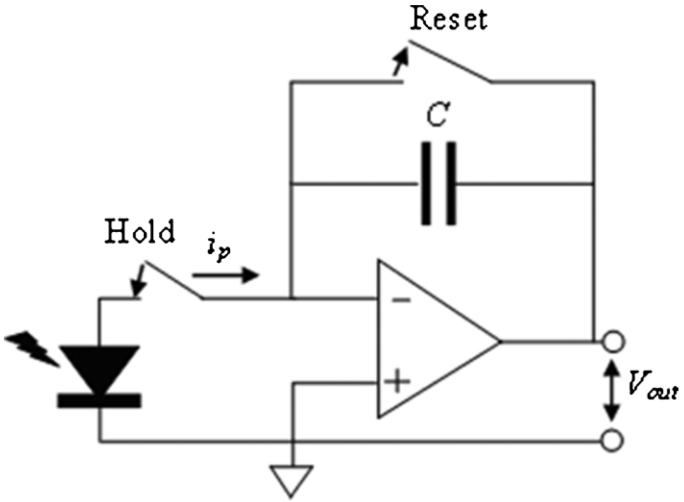


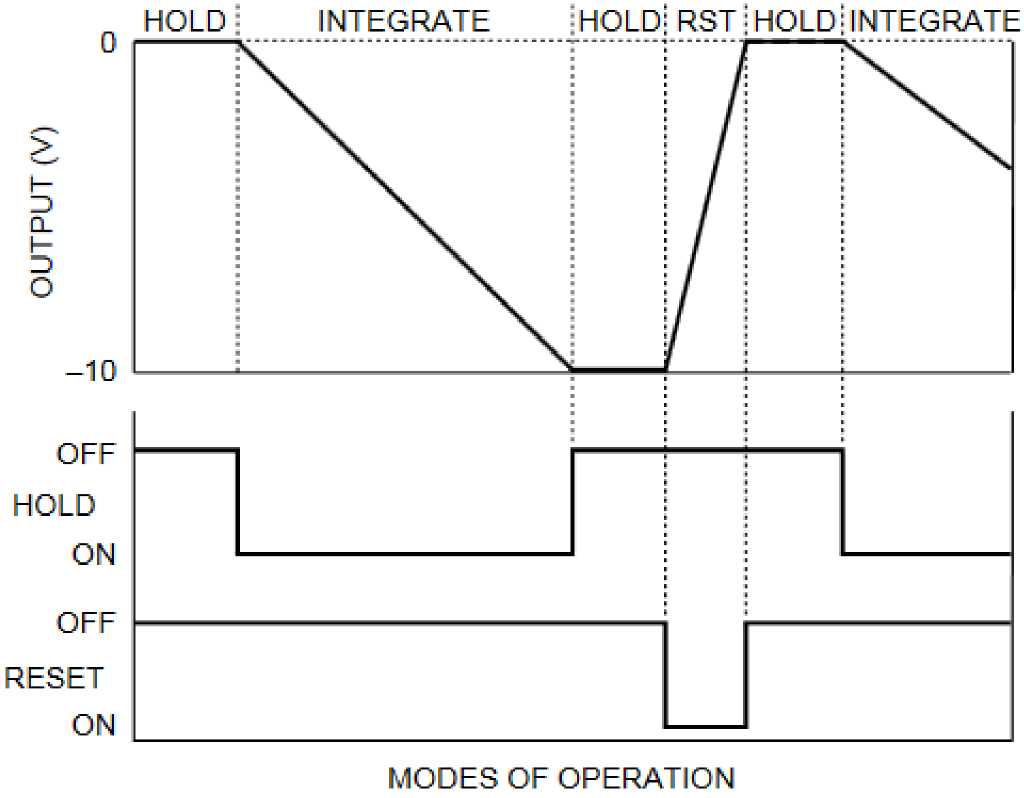
LEAKAGE CURRENT IS ABSORBED BY GROUND INSTEAD OF CAUSING A MEASUREMENT ERROR.



Switched integrator amplifier

- **Capacitive feedback only**
  - Virtually no thermal noise
  - Cheap and fast as compared to other methods in fA to pA current scale



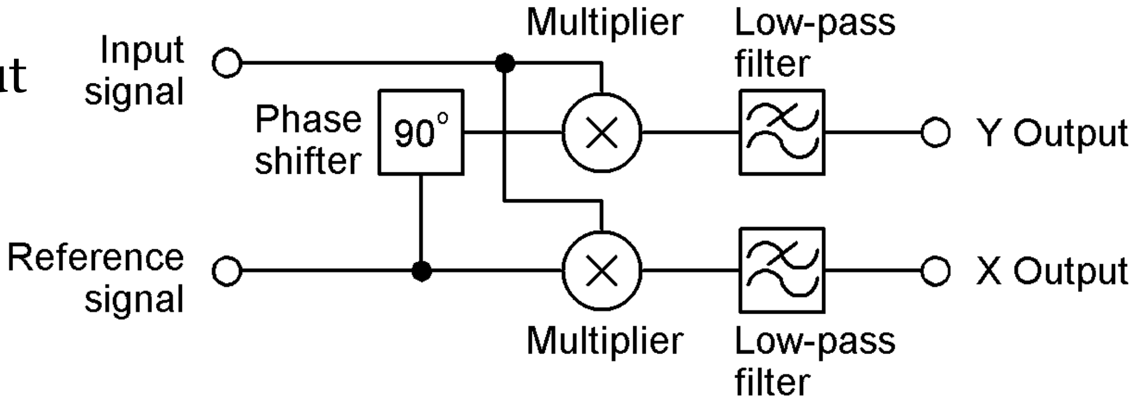


Lock-in amplifier

(And other carrier wave -based techniques)

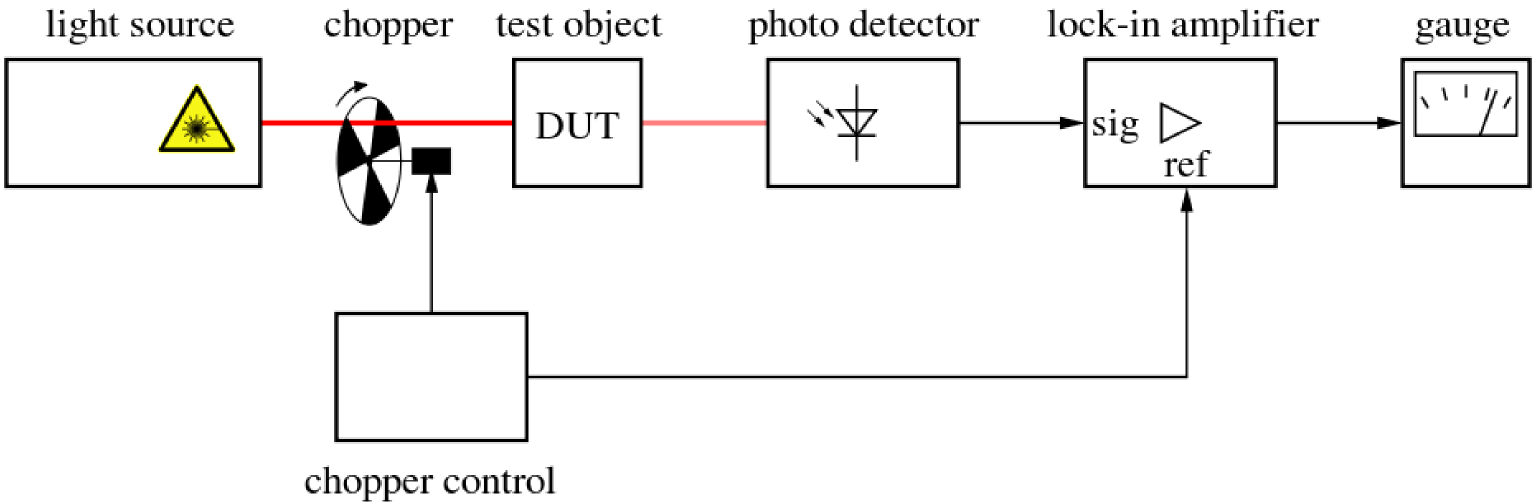
Lock-in amplifier

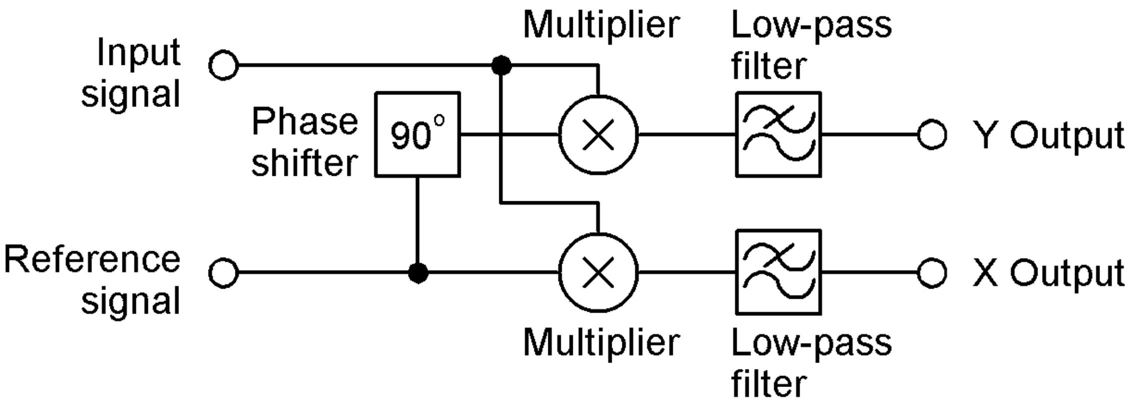
- **Measurements of signals in extremely noisy environment**
- **Basic idea:**
  - Modulate measurement signal with known frequency
  - Multiply the measured signal with two signals at the modulation frequency that have 90 degree phase difference
  - Low pass filter the multiplied signals X and Y
  - Magnitude $\boldsymbol{R} = \sqrt{\boldsymbol{X}^2 + \boldsymbol{Y}^2}$ is directly proportional to the amplitude of the input signal



Signal modulation

- Modulation is typically done electrically
- In optical measurements mechanical chopper is also used



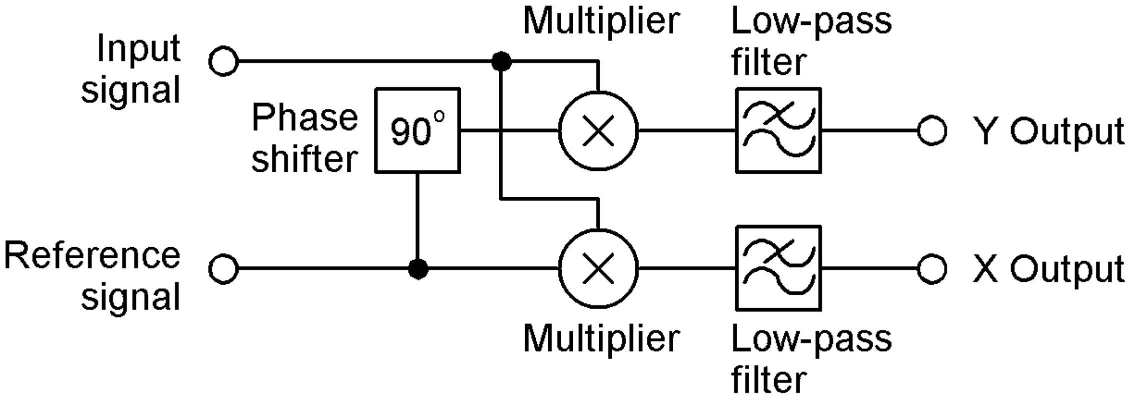


The reference signal u_r is a sine wave with an amplitude A_r and frequency f_r. The phase shifter is used to generate another reference signal $\tilde{u}_r$ with 90 degrees or π/2 radians phase difference:

$$u_r = A_r \sin \omega_r$$

$$\tilde{u}_r = A_r \sin\left(\omega_r - \frac{\pi}{2}\right) = -A_r \cos \omega_r$$

$$\omega = 2\pi f t$$



The input signal u_{in} is a sum of possible very large noise component u_n and the desired measured signal u_m that has an unknown amplitude A_m and is being modulated with reference frequency ω_r.

The measurement setup may cause a delay between reference and measured signal, which is seen as phase shift φ_m

$$u_{in} = u_m + u_n = A_m \sin(\omega_r - \varphi_m) + u_n$$

Using trigonometric identities one can obtain the result from input and reference signal multiplication

$$u_{in}u_r = [A_m \sin(\omega_r - \varphi_m) + u_n]A_r \sin \omega_r$$

$$= A_m A_r \sin(\omega_r - \varphi_m) \sin \omega_r + u_n \, A_r \sin \omega_r$$

$$= \frac{A_m A_r}{2}[\cos \varphi_m - \cos(2\omega_r - \varphi_m)] + u_n A_r \sin \omega_r$$

$$2 \sin \theta \sin \varphi = \cos(\theta - \varphi) - \cos(\theta + \varphi)$$

And similarly

$$u_{in}\tilde{u}_r = [A_m \sin(\omega_r - \varphi_m) + u_n](-A_r \cos \omega_r)$$

$$= -A_m A_r \sin(\omega_r - \varphi_m) \cos \omega_r - u_n\, A_r \cos \omega_r$$

$$= -\frac{A_m A_r}{2}[\sin(-\varphi_m) + \sin(2\omega_r - \varphi_m)] - u_n\, A_r \cos \omega_r$$

$$= \frac{A_m A_r}{2}[\sin(\varphi_m) - \sin(2\omega_r - \varphi_m)] - u_n\, A_r \cos \omega_r$$

$$2 \sin \theta \cos \varphi = \sin(\theta + \varphi) + \sin(\theta - \varphi)$$

The multiplied signals are then being filtered with low-pass filter.

$$u_{in}u_r = \frac{A_m A_r}{2}[\cos\varphi_m - \cos(\xcancel{\quad\quad\quad}\varphi_m)] + u_n\xcancel{\quad\quad\quad}\omega_r$$

$$u_{in}\tilde{u}_r = \frac{A_m A_r}{2}[\sin(\varphi_m) - \sin(\xcancel{\quad\quad\quad}\varphi_m)] - u_n\xcancel{\quad\quad\quad}\omega_r$$

When the cut-off frequency is much lower than the reference frequency, ideally only DC signal is left.

$X = \frac{A_m A_r}{2}\cos\varphi_m$ **and** $Y = \frac{A_m A_r}{2}\sin\varphi_m$

$$R = \sqrt{X^2 + Y^2} = \sqrt{\left(\frac{A_m A_r}{2}\cos\varphi_m\right)^2 + \left(\frac{A_m A_r}{2}\sin\varphi_m\right)^2}$$

$$= \frac{A_m A_r}{2}\sqrt{(\cos\varphi_m)^2 + (\sin\varphi_m)^2} = \frac{A_m A_r}{2}$$