Aalto-yliopisto
Sähkötekniikan
korkeakoulu

Measuring force

ELEC-E5710 Sensors and Measurement Methods

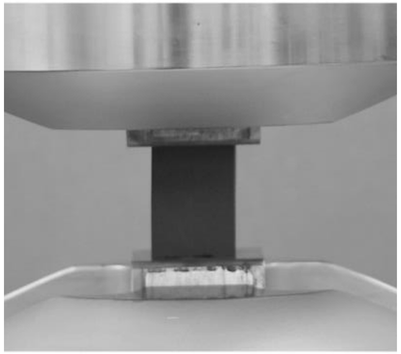
Elastic deformation

- **Stress**

$$\sigma = \frac{F}{A} \quad [\mathrm{Pa}]$$

- **Strain**

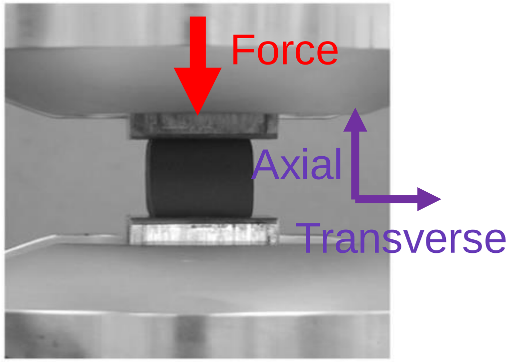
$$\varepsilon = \frac{\Delta L}{L}$$



- **Young's modulus**

  (elastic modulus)
  - Rubber: 10 - 100 MPa
  - Aluminium: 70 GPa
  - Diamond: 1.22 TPa

$$E = \frac{\sigma}{\varepsilon} \quad [\mathrm{Pa}]$$



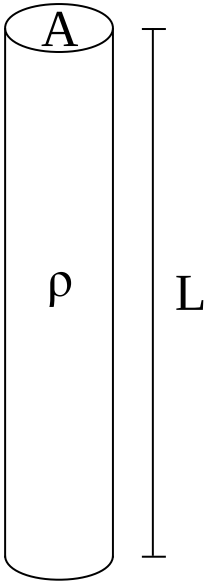
- **Poisson's ratio**
  - Rubber: 0.4999 (no volume change)
  - Aluminium: 0.33
  - Cork: 0.0 (no transverse strain)
  - Auxetic materials: <0 (E.g. paper, and some minerals. Stretching in axial direction, causes expansion in transverse direction.)

$$\nu = -\frac{\varepsilon_{\mathrm{transverse}}}{\varepsilon_{\mathrm{axial}}}$$

Resistance of a wire

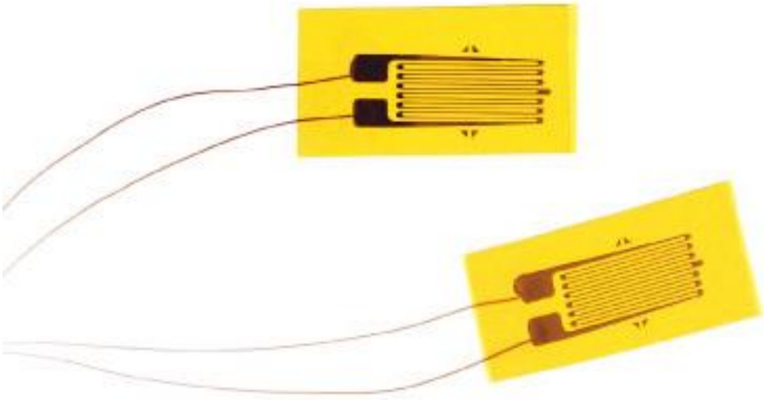
Resistance of a wire depends on its electrical resistivity and **geometry**

$$R = \rho \frac{L}{A}$$



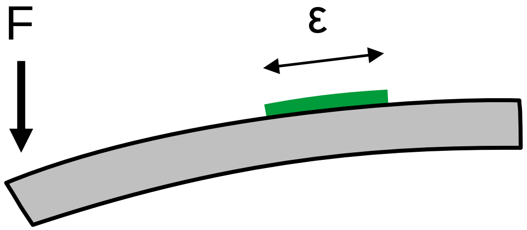
Strain gauge

- Mechanical strain on the strain gauge causes a small change in its electrical resistance.
- **Gauge factor** is the ratio of the relative change in electrical resistance to the mechanical strain.
- Typical gauge factor for metallic foil gauges is around 2.



$$K = \frac{\Delta R / R}{\varepsilon} = \frac{\Delta R / R}{\Delta L / L}$$

Strain gauge as a sensor



- Strain gauge measures strain *ε*.
- By gluing it on the surface of the object with a known Young's modulus *E*, the stress *σ* of the object can be measured.
  - The relationship between strain ε and force *F* depends also on the geometry and direction of applied force.

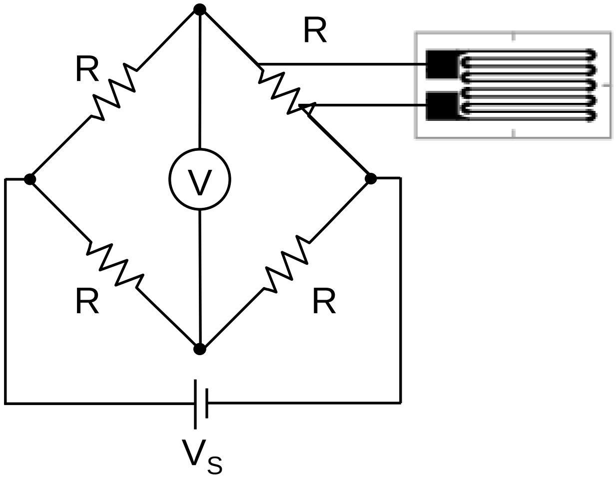
$$\sigma = E\varepsilon$$

$$R = \rho \frac{L}{A} \qquad \varepsilon = \frac{\Delta L}{L} \qquad E = \frac{\sigma}{\varepsilon} \quad [\mathrm{Pa}] \qquad \sigma = \frac{F}{A} \quad [\mathrm{Pa}]$$

Strain gauge as a sensor

- Relative changes in electrical resistance are small

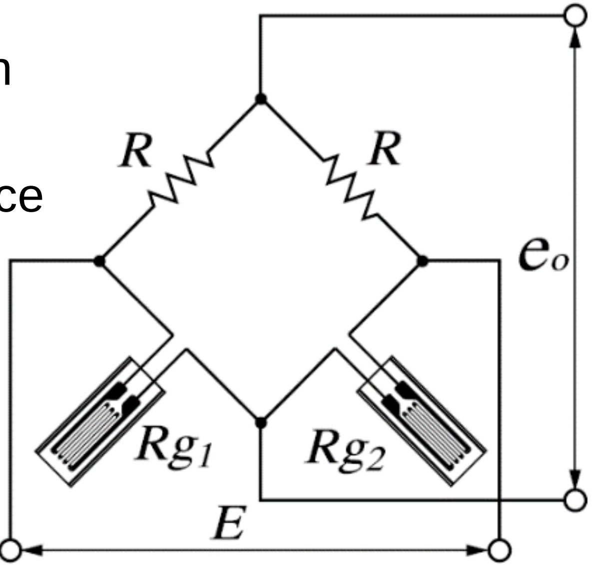
  → **Wheatstone bridge** circuit is often used



Influence of temperature

$$\frac{Rg2}{Rg1+Rg2}=\text{constant}$$

- Thermal expansion and change in resistivity
  - **Quarter bridge** with temperature compensation: A dummy strain gauge in the bridge with no strain (the strain caused by thermal expansion cancels itself out).
  - **Half bridge:** Two strain gauges that experience strain of opposite sign (e.g. glued on the opposite sides of a beam).
  - **Full bridge:** Four strain gauges (e.g. two on each side of a beam).
    - Most sensitive. Linear.

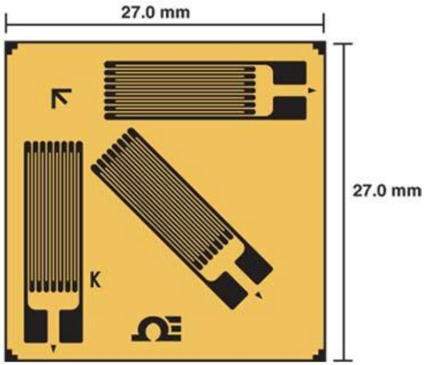


- Self temperature compensation
  - The materials for the strain gauge and the wires can be selected such that the change in resistance due to thermal expansion compensates the change in resistivity due to temperature.
  - STC = self-temperature compensation.

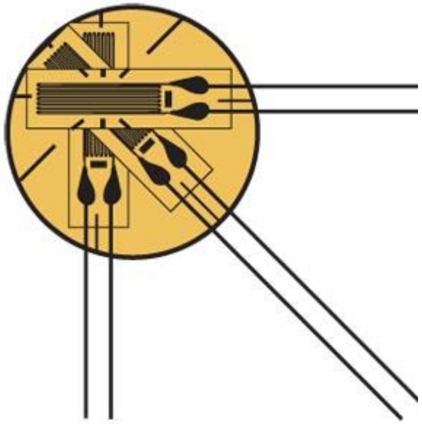
Strain gauge materials

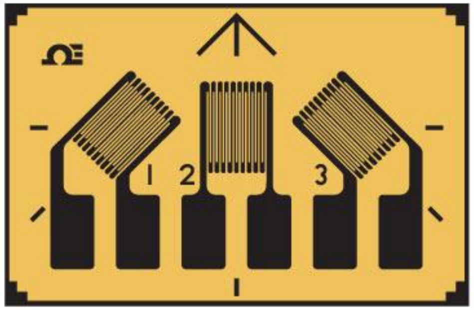
- **Constantan** (copper-nickel alloy), $K = 2.1$
  - The most commonly used, and the cheapest material. Very constant resistivity over a wide temperature range (-30 – 193 ºC).
  - Heat-treated gauges (P alloy) can measure large strains (over 5%). However, P alloy gauges exhibit permanent resistivity change when strained (not recommended in applications with cyclic strain).
- **Karma** (nickel-chrome alloy), $K = 2.0$
  - Very good temporal stability, which makes it suitable for long-term force monitoring. Also suitable for measurements in extremely low temperatures.
- **Isoelastic** (iron-nickel alloy), $K = 3.6$
  - Used in dynamic and cyclic measurements. Good signal-to-noise ratio, but relative high temperature sensitivity.
- **Nichrome V, Armour D, platinum-based**, …
  - For high temperatures (>230 ºC) and other special situations.

Multiaxial strain gauges (strain gauge rosettes)

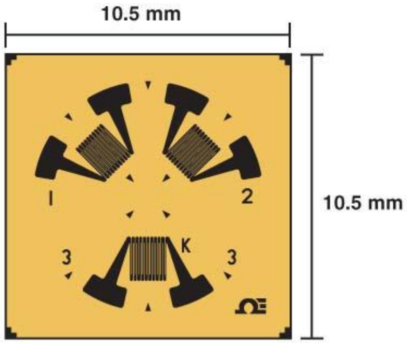


SGD-13/120-RY93

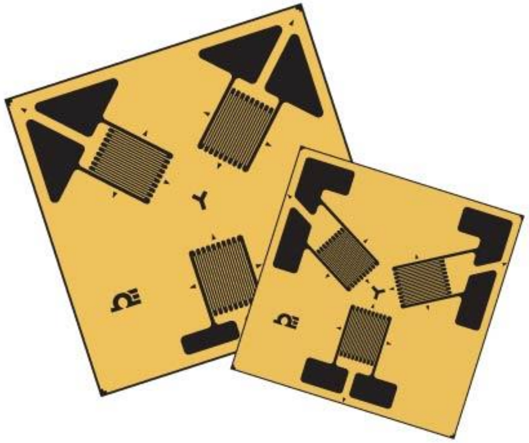




SGD-1/120-RY21

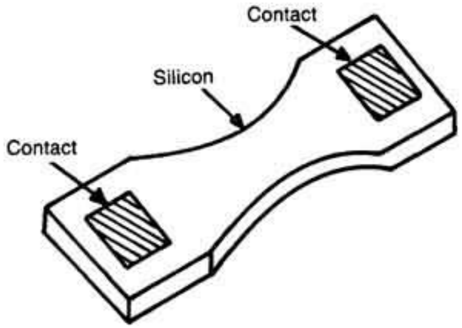


SGD-1.5/120-SR11



Semiconductor strain gauges (piezoresistor)

- Strain causes a change in the electrical resistivity due to piezoresistive effect.
  - Strain affects the bandgap of a semiconductor, the density of current carriers changes.
- Gauge factors as much as 100x higher than with traditional gauges.
- Non-linear, highly temperature dependent, but very sensitive and small.



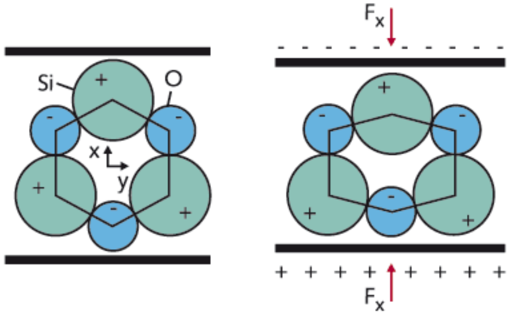
$$\frac{\Delta R}{R} = K\underbrace{\frac{\Delta L}{L}}_{\text{Geometrical component}} + \underbrace{\frac{\Delta \rho}{\rho}}_{\text{Piezoresistive component}}$$

Piezoelectric sensor

- Applied mechanical stress causes electric charge to accumulate at the opposite sides of the material.
- Operational mode is determined by the way the crystal is cut: longitudinal, transverse, and shear.
- High sensitivity compared to strain gauges.
- Static force results in a fixed amount of charge. Only practical in dynamic measurements.
- Lower long-term stability than with strain gauges.

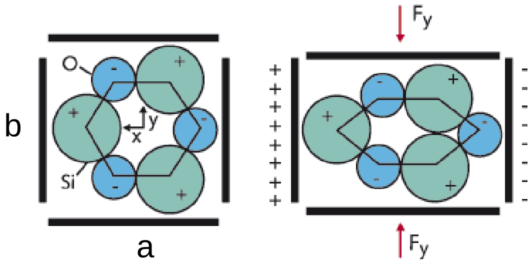
Piezoelectric sensor

Longitudinal



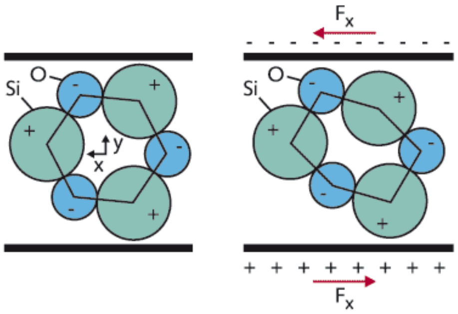
$$q = d_L \cdot F_x$$

Transverse



$$q = d_T \cdot F_y \cdot \frac{b}{a}$$

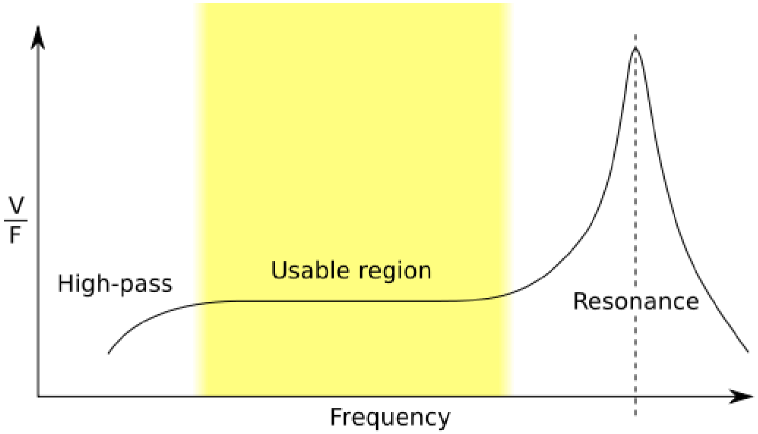
Shear



$$q = 2 \cdot d_L \cdot F_x$$

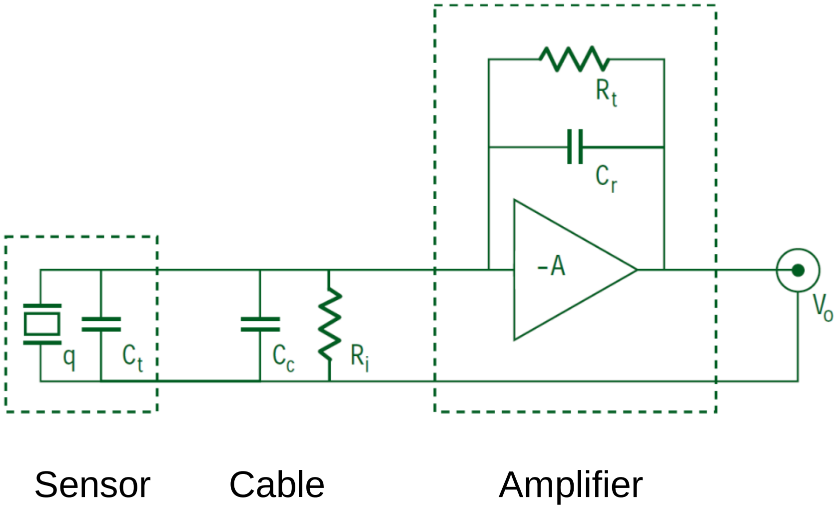
Piezoelectric sensor

- Quartz
  - Naturally piezoelectric material. Highly stable, but quite low charging coefficient: *d* = -2.3 pC/N.
- Ceramic materials (PZT)
  - Artificial materials, high charging coefficient: *d* = -374 pC/N (for PZT-5A). Sensitivity degrades over time.
- Resonance frequency determines usable frequency range.



Piezoelectric sensor

- Charge q (typically some pC) of the piezoelectric sensor can be measured with a charge amplifier
  - High open loop gain and high input impedance



Ignoring the effects of resistances

$$V_o = \frac{-q}{C_r} \frac{1}{1 + \frac{1}{AC_r}(C_t + C_r + C_c)}$$

where A is open loop gain.

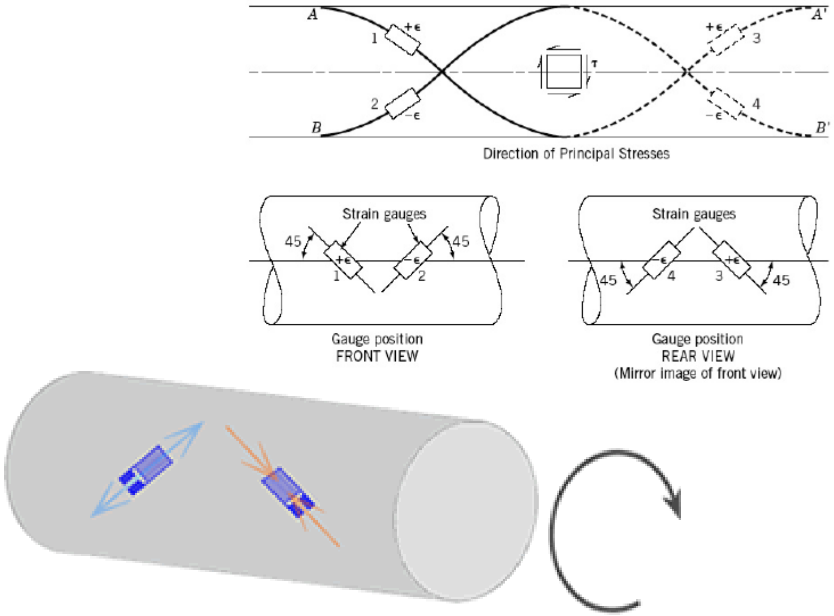
$$V_o \approx \frac{-q}{C_r}, \text{ when } A \to \infty$$

Time constant

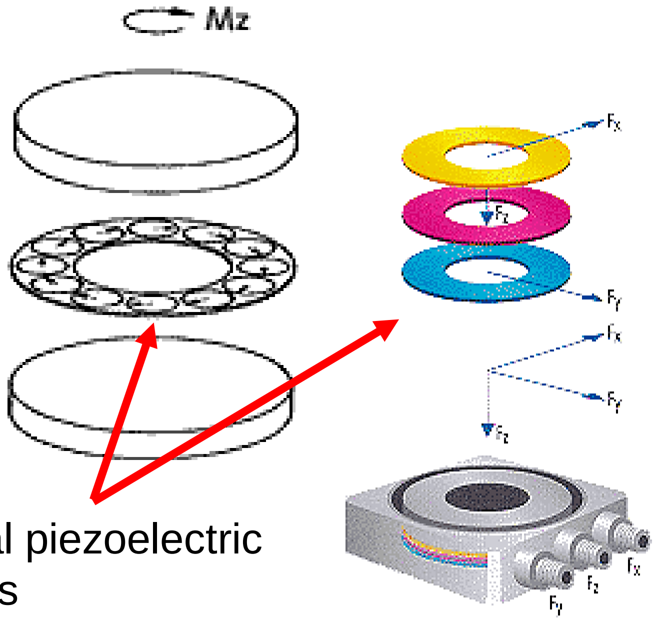
$$\tau = R_t C_r$$

Torque

- Static torque is easy to measure.
- Dynamic torque measurement requires wireless signal transmission from the rotating shaft.



Strain gauge



Several piezoelectric sensors

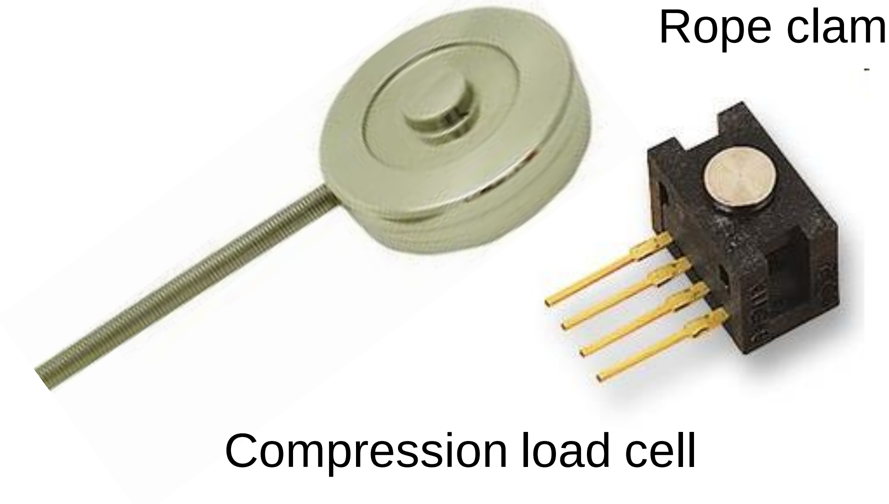
Piezoelectric

Load Cells

Double-ended shear beam

Rope clamp load cell

S-type load cell

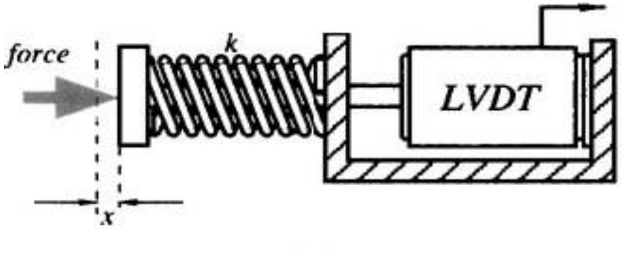


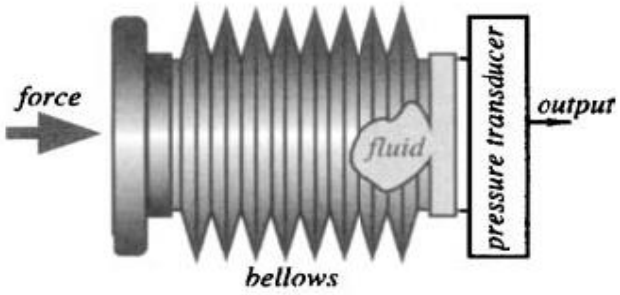
Compression load cell

Tension and compression load cell

Other measurement methods

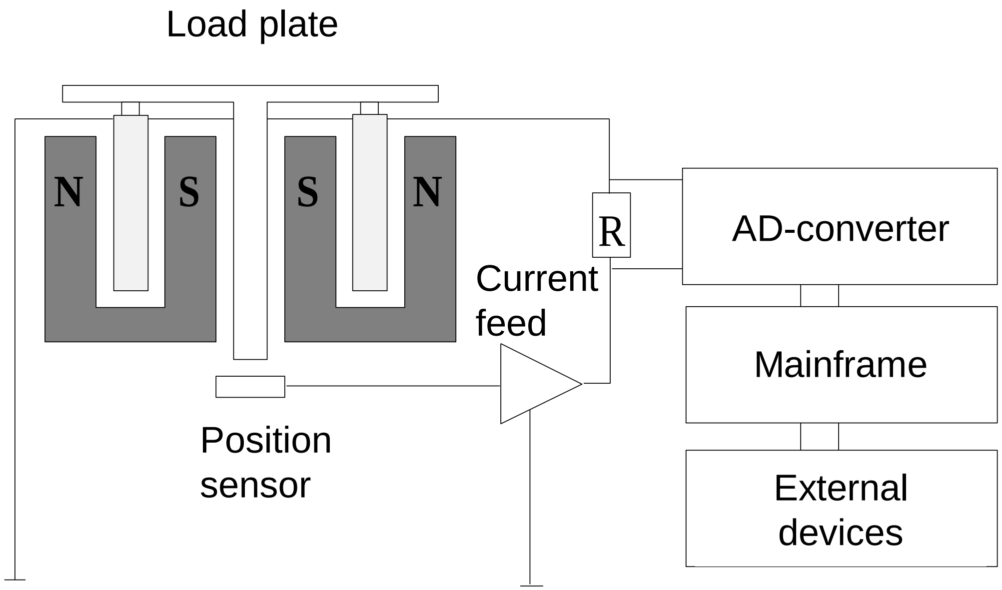
- Linear variable displacement transducer
- Pneumatic pressure
- Hydraulic pressure





Other measurement methods

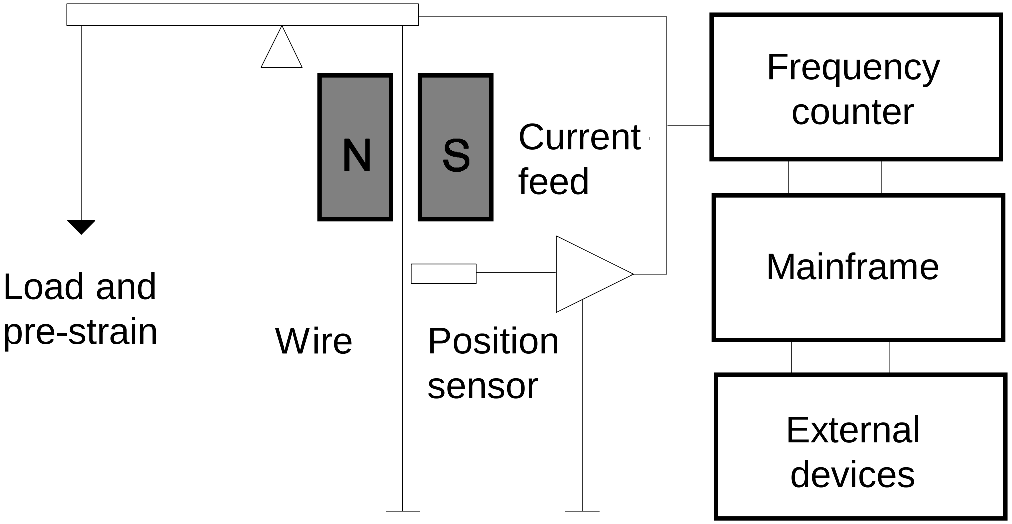
- Equilibrium of forces
  - Feedback is used to balance the force.
  - Force can be calculated from feedback current.



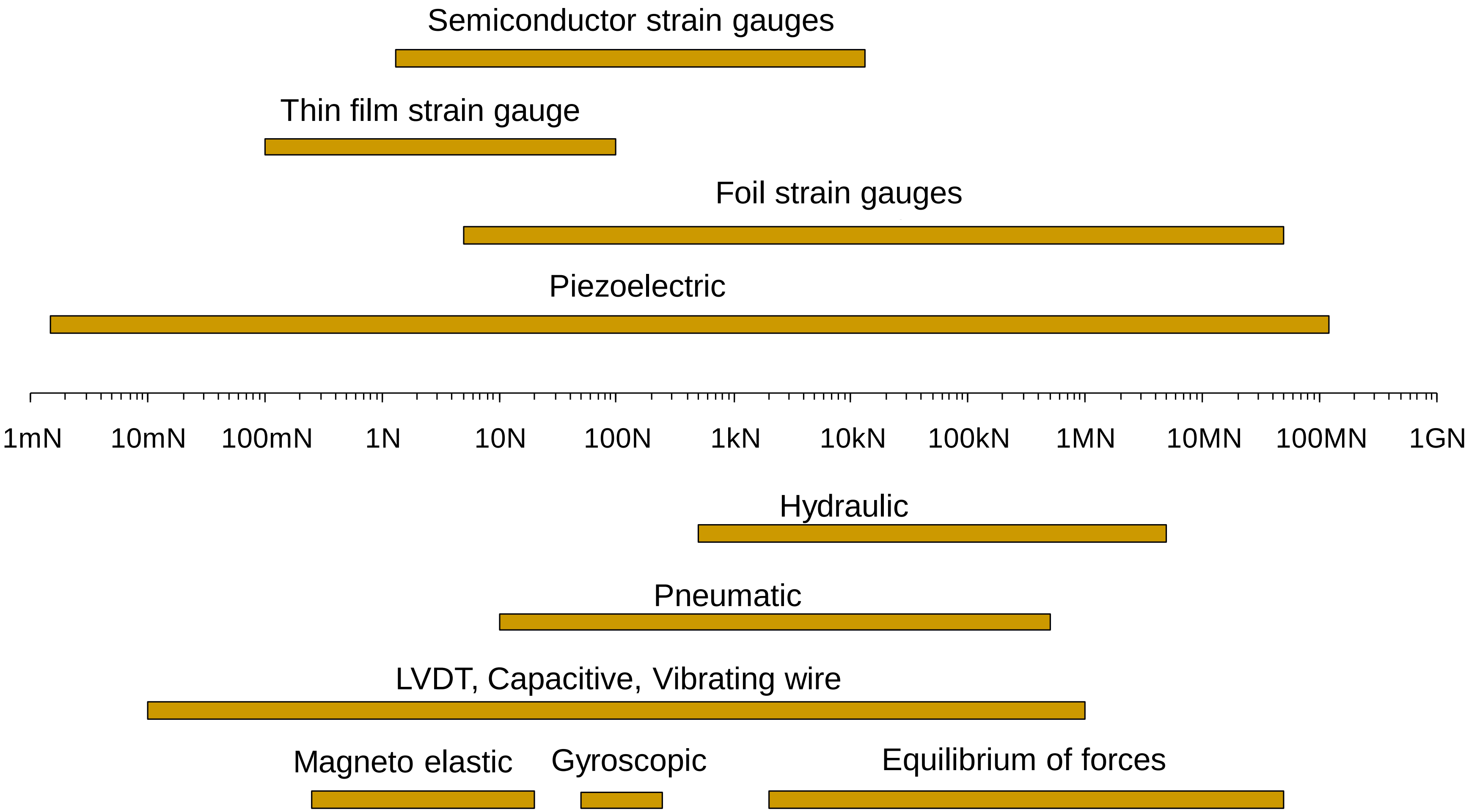
Other measurement methods

- Vibrating wire
  - The tension of the wire changes as a function of applied force.
  - Frequency changes as a function of tension.
  - Excitation required to activate the vibrating wire.

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{m}}$$



Typical force sensors



Aalto-yliopisto
Sähkötekniikan
korkeakoulu

Measuring acceleration

ELEC-E5710 Sensors and Measurement Methods

Speed and acceleration sensors

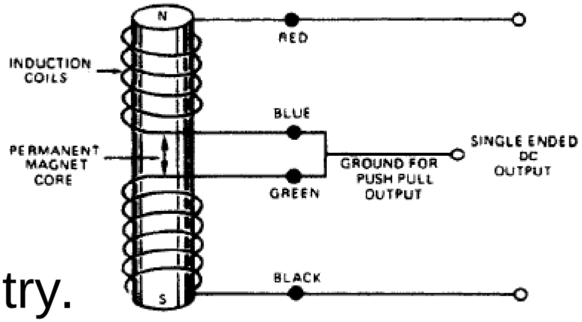
- Speed is the first derivative of position with respect to time. Acceleration is the second derivative.
- However, differentiation and integration increase the measurement error.
- For motions under 10 Hz, the use of position sensors is accurate enough.
  - Strain gauges, motion sensors…
- For 10 Hz – 1 kHz, speed sensor is more accurate.
  - E.g. induction based sensors.
- For over 1 kHz direct acceleration measurement.
  - Force sensor.

Measuring speed

- Indirect speed measurement
  - Differentiation of periodical position measurements
  - Integration of acceleration
- Sensor directly sensitive to speed
  - Magnetic induction

(Note: Linear variable differential transformer (LVDT) is a position sensor!)

- GPS can be used for measuring the speed of larger objects.
  - Based on Doppler effect.
  - Position measurement based on geometry.
    - Position data can be used to determine average speed (less accurate).

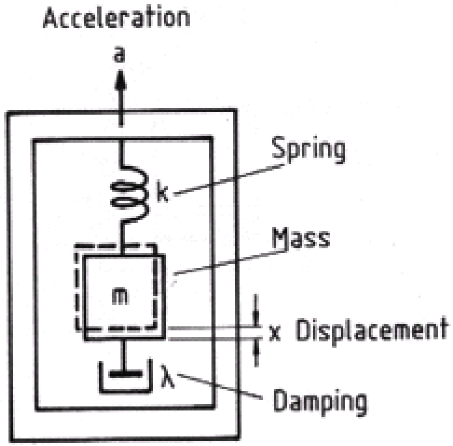


Principle of acceleration measurements

- Spring-mass system
- Acceleration *a* (e.g. gravitational acceleration) related to displacement *x*.
- Equation of motion

$$ma = m\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + kx,$$

Where *k* is the spring constant, and *λ* is the damping constant caused by the viscosity of the medium.



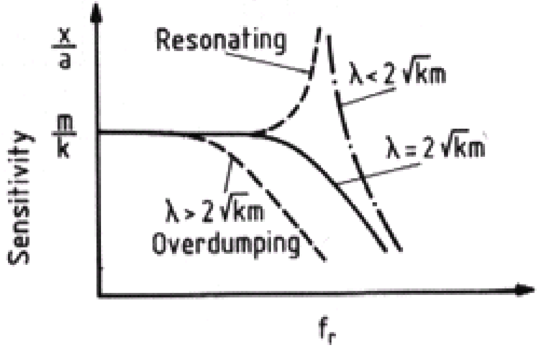
(a)

- Constant acceleration *a*, equilibrium position *x'*(*t*) = *x''*(*t*) = 0:

$$\frac{x}{a} = \frac{m}{k}$$

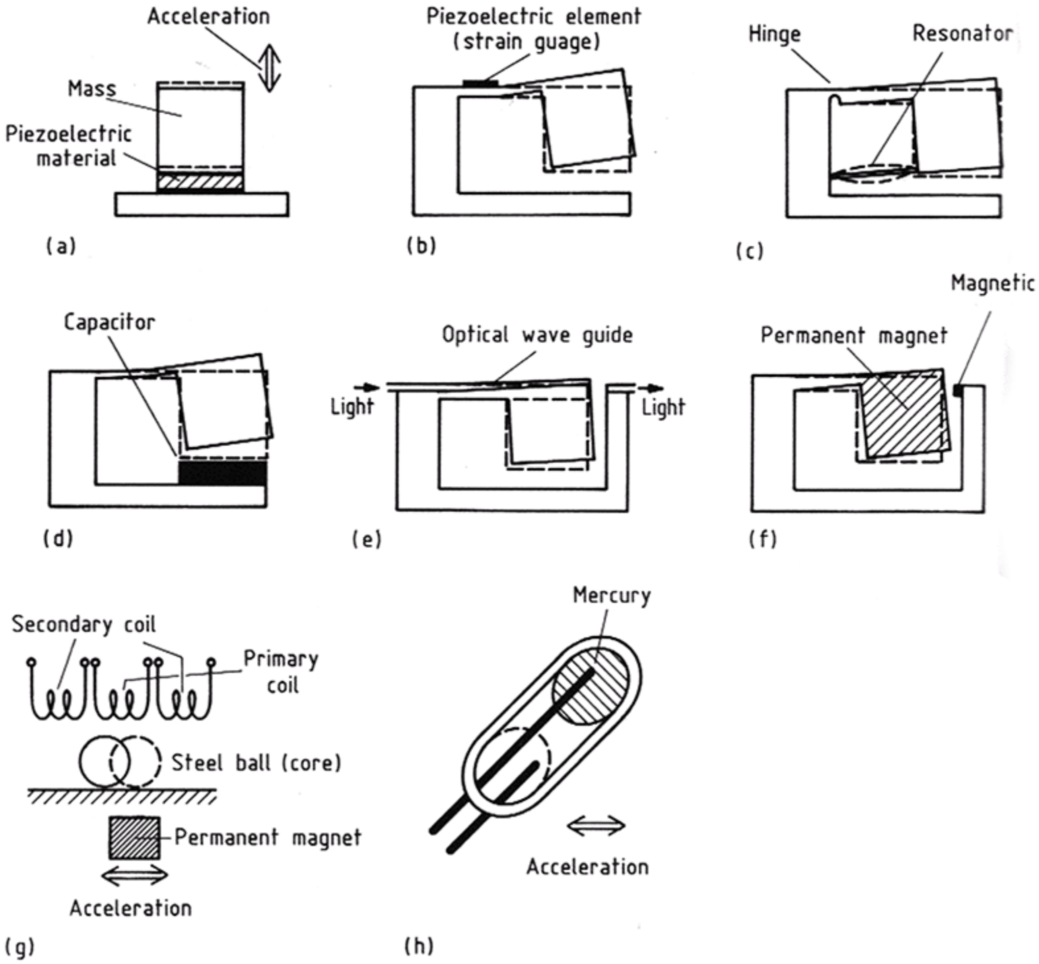
- Resonance frequency in the absence of damping

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



Methods of acceleration measurement

a) piezoelectric
b) piezoresistive
c) resonant
d) capacitive
e) optical
f) magnetic
g) inductive
h) Switch based

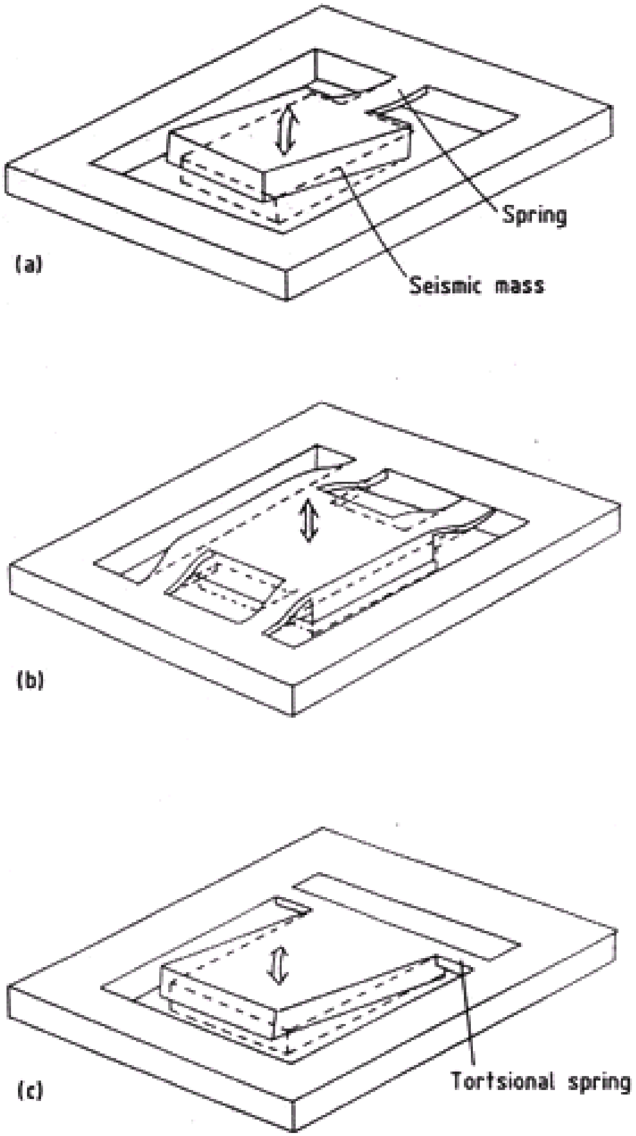


- MEMS = Microelectromechanical systems.

Spring structures

1. Spring (beam)
   - The amount of springs varies
2. Torsional spring

- The spring constant is determined by the structure and dimensions of the spring(s).



Spring constants

Spring constant $k = n\frac{Ewt^3}{L^3}$

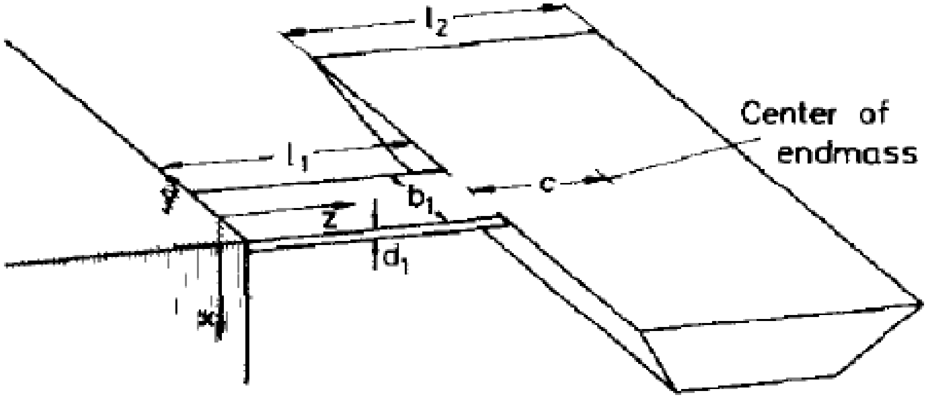
n = Number of springs (beams)

E = Young's modulus

w = width of the beam

t = thickness of the beam

L = length of the beam



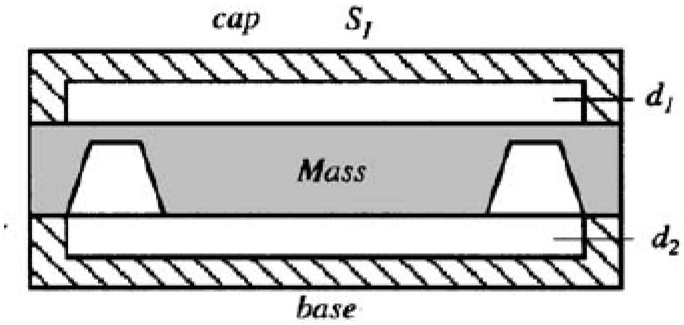
Choosing a sensor for the application

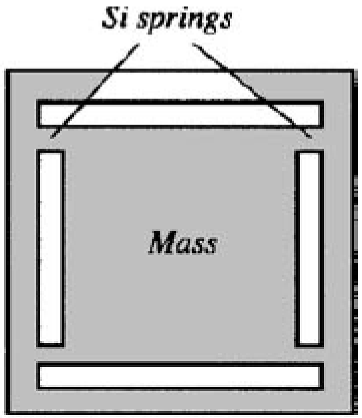
- Vibration or acceleration measurement?
- Operating temperature and temperature fluctuations?
- Operating frequency?
- Requirements for linearity and accuracy?
- Available power supply?
- Corrosive materials?
- Moisture conditions?
- Accelerations beyond the limits?
- Sensitivity for acoustic and electromagnetic fields?
- Grounding?

Capacitive accelerometer

- Sensitivity is better than that of a piezoresistor.
- Small sensor has small capacitance.
- Measurement circuits must be close to the sensor to minimize noise and parasitic capacitances.
- CMOS-circuits often used.
- Frequency range: 0 – 1000 Hz
- Acceleration range: 50 μg – 1000 g

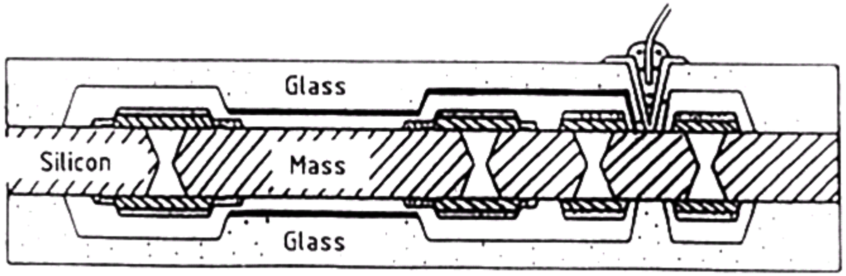
Capacitive accelerometer





Differential silicon based capacitive accelerometer

- A piece of silicon between two sheets of glass.
  - Two capacitances form between the silicon and the sheets of glass.
- The silicon piece is suspended from the edges with multiple beams.
  - Beams are located symmetrically at both sides of the mass.
- Thermal effects and cross-axis sensitivities are minimized with the symmetrical structure.



Differential silicon based capacitive accelerometer

Dependence of acceleration on the displacement of the mass:

$$x = a\frac{m}{k}$$

Acceleration is relative to the difference in capacitances:

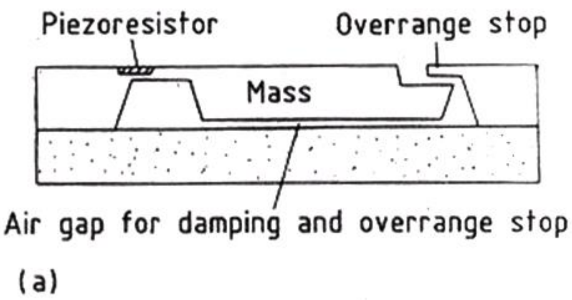
$$a = \frac{kd}{m}\frac{C_1 - C_2}{C_1 + C_2}$$

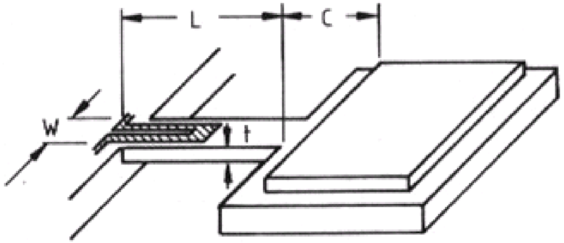
where *d* is the gap between the electrodes in a steady state and *k* is the spring constant.

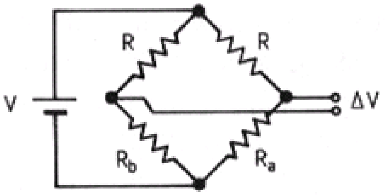
Piezoresistive accelerometer

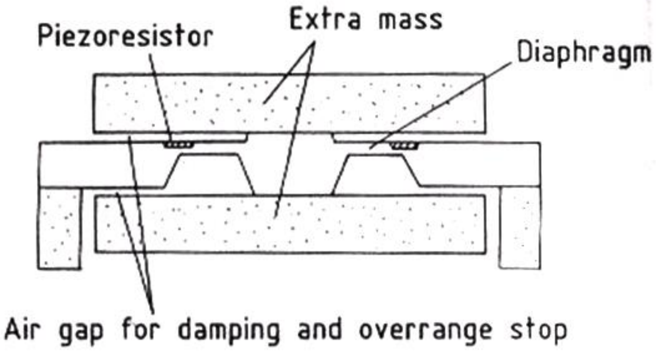
- Piezoresistive elements in a cantilever (spring)
- Can operate in a range of 0 – 13 kHz.
- Dynamic range of -1000 *g* – +1000 *g* when uncertainty of 1% or better is required.
- The sensor can withstand accelerations as high as 10 000 *g*. This over-shock limit is a critical parameter in many applications.
- The relative change in resistance in piezoresistive sensors is smaller than the relative change in capacitance in capacitive sensors, i.e. piezoresistive sensor is less sensitive.
- Sensitive to changes in temperature.

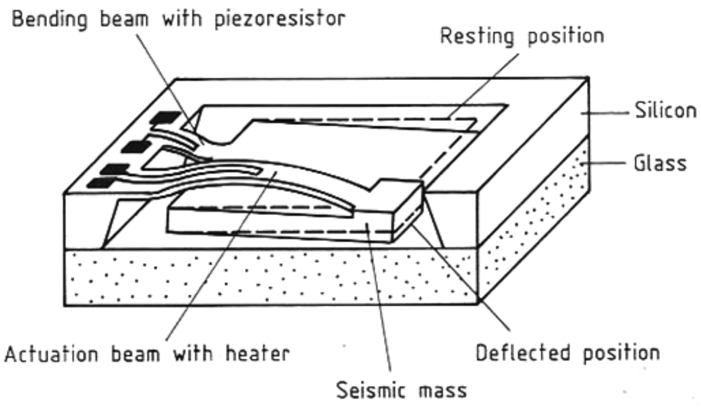
Piezoresistive accelerometer





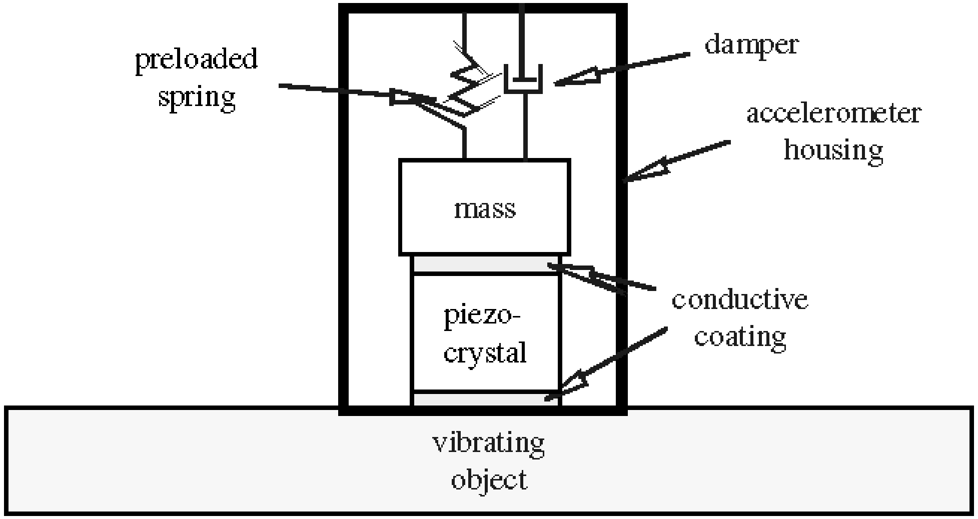






Piezoelectric accelerometer

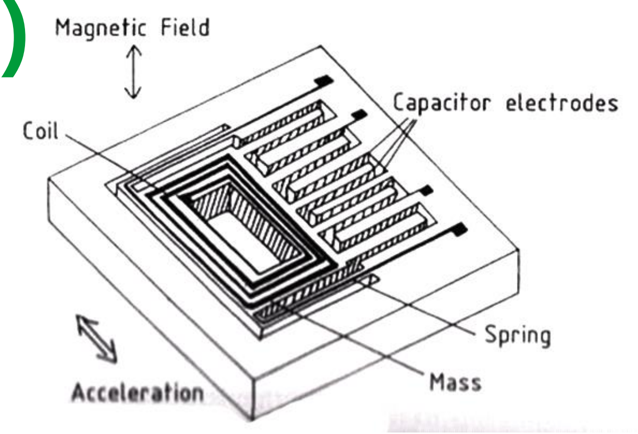
- Piezoelectric crystal connected to a mass
- Changing acceleration/force creates electrical charges in the crystal.
  - Cannot be used to measure static acceleration.
- High natural frequency
- Wide operating temperature range



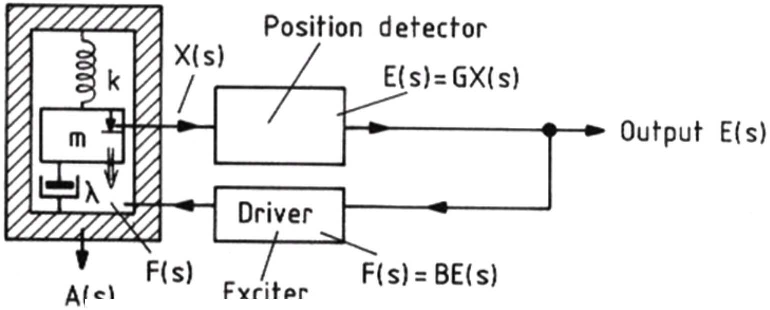
Accelerometer comparison

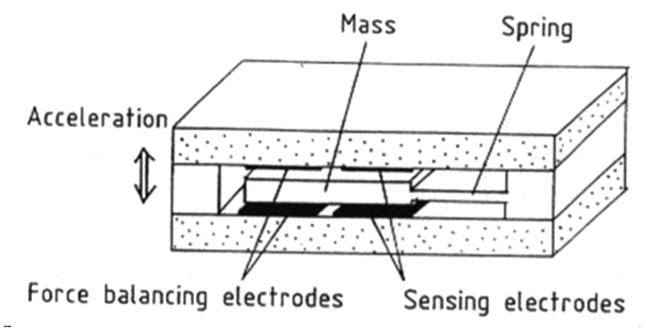
- Capacitive
  - \+ High sensitivity
  - \+ DC
  - – Limited frequency range (1 kHz)
  - – Noise
- Piezoelectric
  - \+ High natural frequency
  - \+ Wide operating temperature range
  - – No DC operation
- Piezoresistive
  - \+ Wide frequency range
  - \+ Over-shock durability
  - – Sensitive to changes in temperature

Feedback-based accelerometer (Closed loop accelerometer)



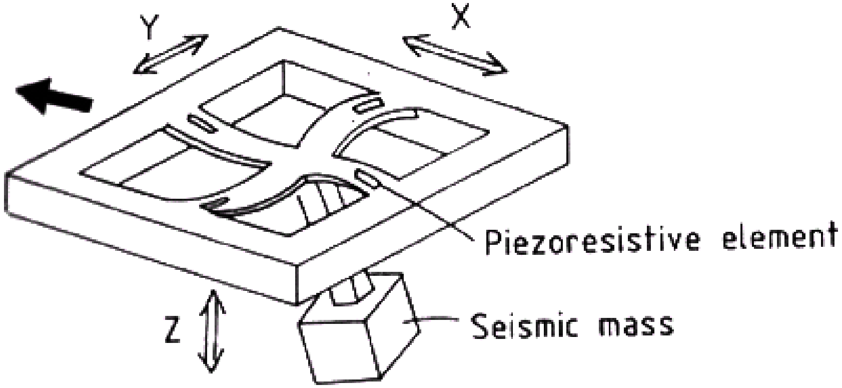
- Electrostatic or magnetic feedback
    - Compensates for the deflection of the seismic mass.
- Benefits
    - Increases the sensitivity.
    - Broadens the frequency range past the resonance frequency.
    - No nonlinearities caused by large deflections.
    - No stiction (seismic mass can get stuck to the structure due to static friction).





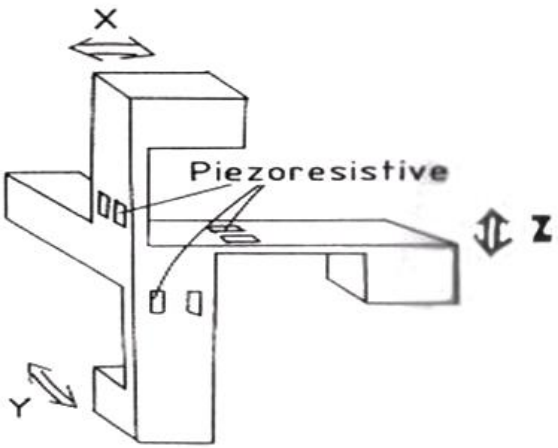
3D accelerometer

- 3D accelerometers measure both magnitude and direction of acceleration.
- The seismic mass in the pictured accelerometer is hanging by the spring with a pole and 4 springs. Piezoresistors measure the bending of the springs.
- z component of the acceleration moves the mass up and down causing symmetrical bending of the springs.
- x and y components of acceleration cause asymmetrical bending of the springs.

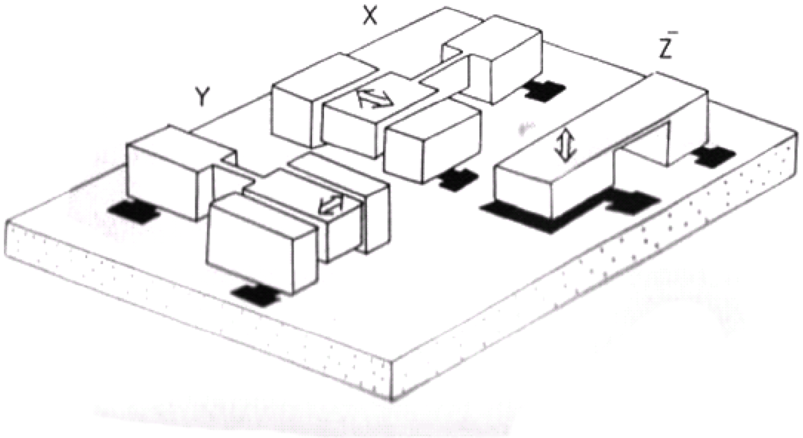


3D accelerometer

- Piezoresistive

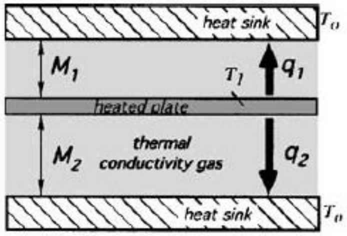


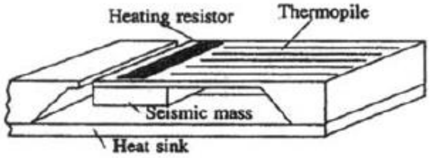
- Capacitive

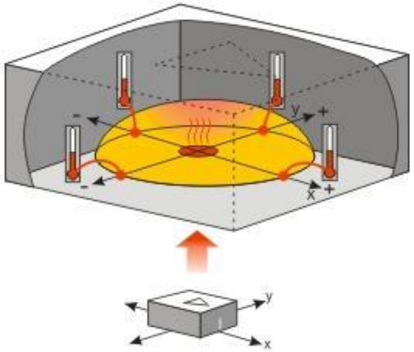


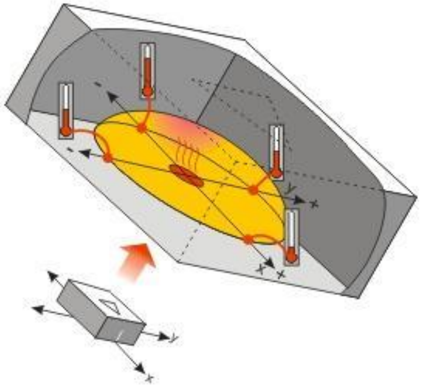
Thermal accelerometers

- Heated plate acts as a mass
  - Measurement of temperature difference
  - Conduction analysis
  - Insensitive to ambient temperature and electromagnetic fields
- Heated gas acts as a mass
  - Multiple temperature sensors
  - Convection analysis
  - No moving parts: no resonant frequency, no stiction (seismic mass can get stuck to the structure due to static friction).







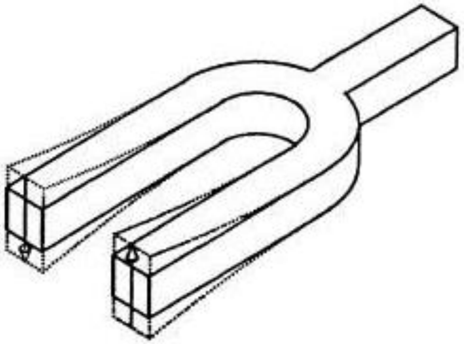


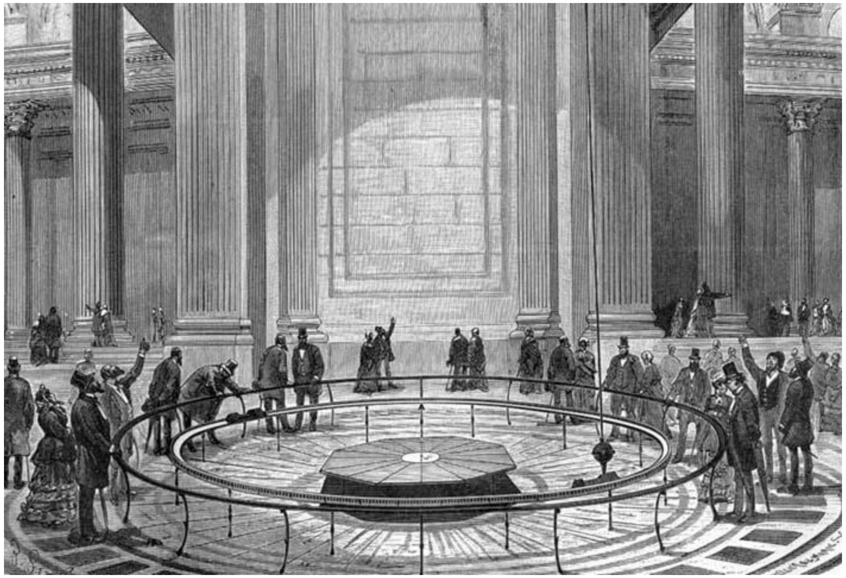
Gyroscope

- A rotating mass which is suspended in such a way that the rotation axis can change freely. (gimbal)
- According to the law of conservation of momentum the mass tends to stay in the same position. (gyroscopic resistance force)
- Mechanical gyroscope is a classic navigation instrument.

Vibrating gyroscope

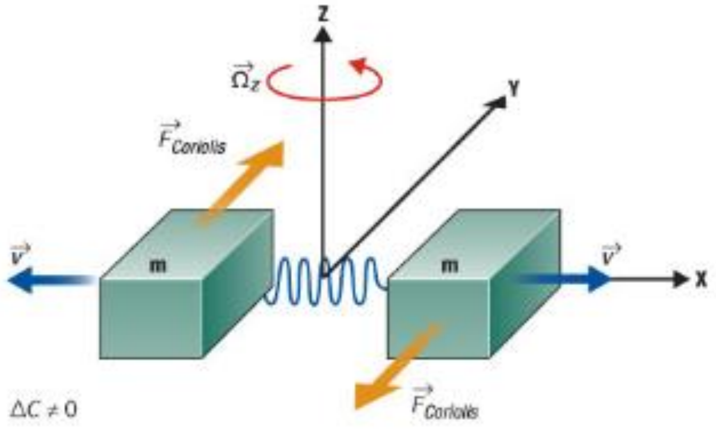
- The rotating mass is replaced with a vibrating element.
- Vibrating element tends to continue vibration in the same plane.
  - Similar to Foucault’s pendulum.





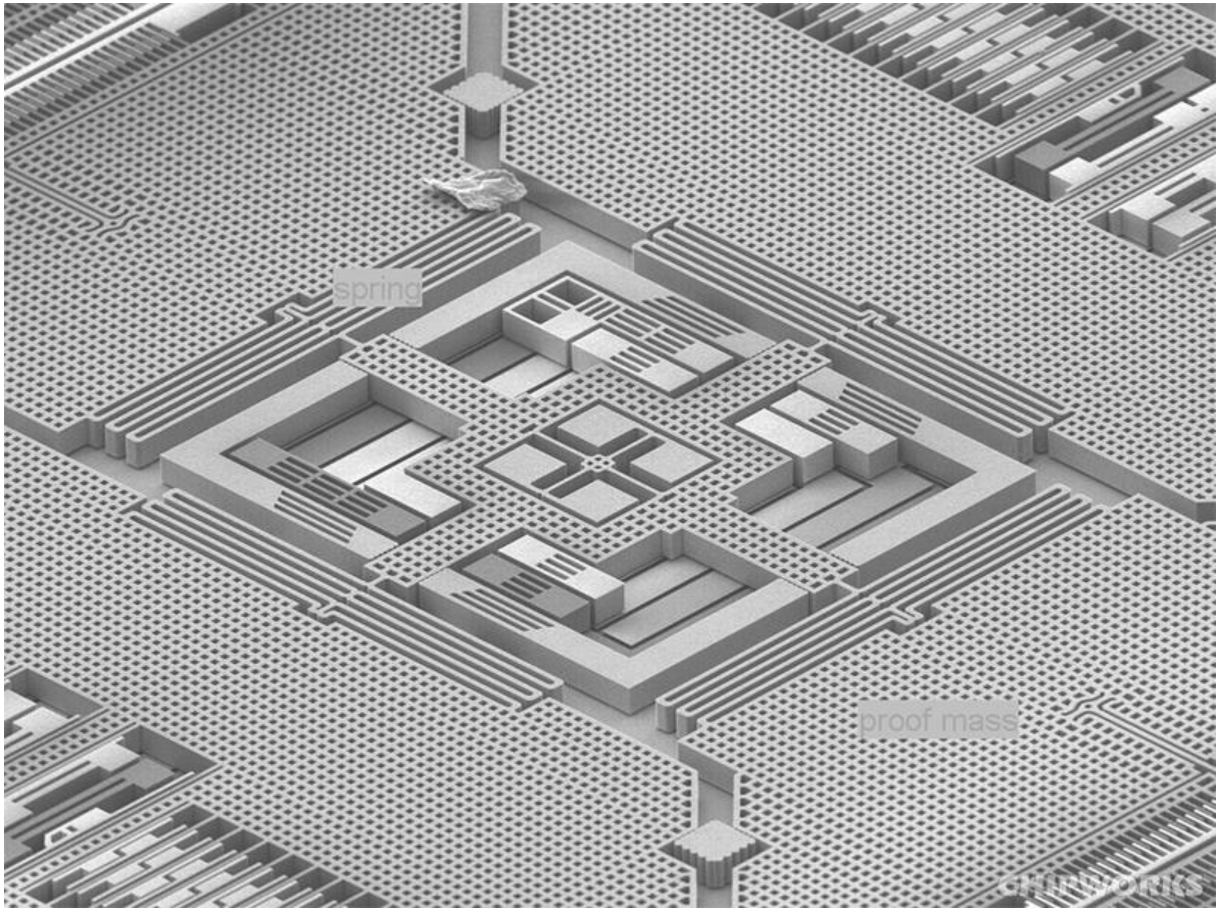
Vibrating gyroscope

- Two masses oscillating in opposite directions.
- When the sensor is rotated (angular velocity is applied), the Coriolis force displaces the masses in opposite directions.
  - The displacement is perpendicular to the oscillation direction and the rotation axis.
- Displacement can be detected e.g. as differential capacitance.
  - Proportional to angular velocity.
  - Linear acceleration moves the masses in the same direction. No differential capacitance.



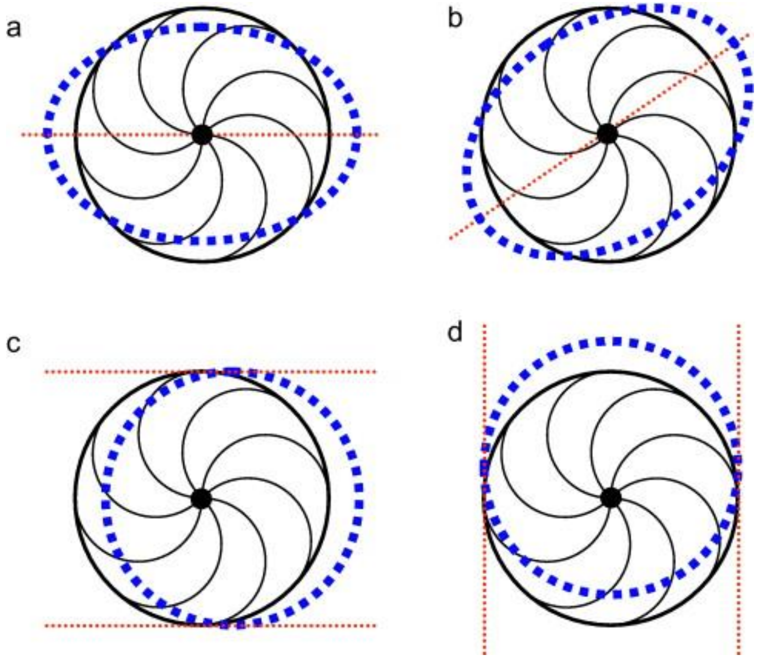
http://electroiq.com/blog/2010/11/introduction-to-mems-gyroscopes/

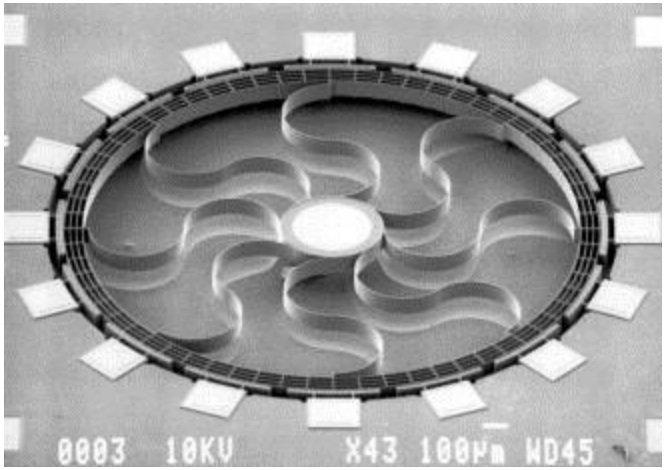
MEMS gyroscopes



MEMS gyroscopes

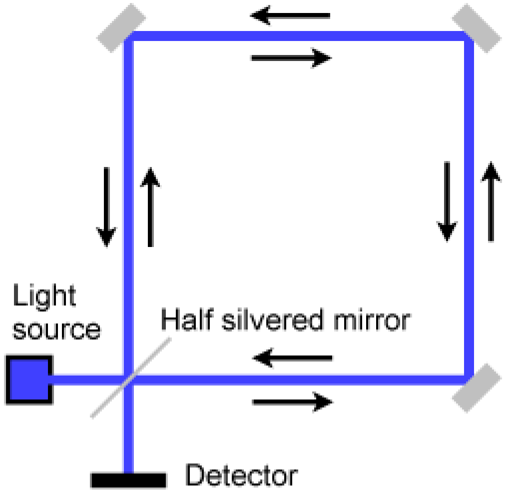
- The vibrational mode of the ring changes due to the Coriolis force.





Optical gyroscope

- Two beams of light travelling in opposite directions create a difference in the transit time when the angle of the mirrors change.
  - The difference can be seen in the interference pattern.
- Ring laser gyroscope and fiber optic gyroscope.



Aalto-yliopisto
Sähkötekniikan korkeakoulu

Measuring mass

ELEC-E5710 Sensors and Measurement Methods

Mass and weight

- Mass m determines the inertia of an object and is defined as the volume V multiplied with the density ρ of the object:

$$m = V\rho \, [\mathrm{kg}]$$

- Mass is a base unit in the International System of Units, SI
- The weight W is the force on the object due to gravity (which depends on latitude φ and height h) :

$$W(\varphi, h) = m g_{\mathrm{g}}(\varphi, h)$$

where g is local acceleration of free fall.

Acceleration due to gravity

- Gravitational force between two objects (masses m_1 and m_2 and distance d) can be expressed with the following equation:

$$F = G\frac{m_1 m_2}{d^2}$$

  - Gravitational constant $G = (6.67428 \pm 0.00067) * 10^{-11}\ \mathrm{Nm^2/kg^2}$

- Gravitational acceleration:

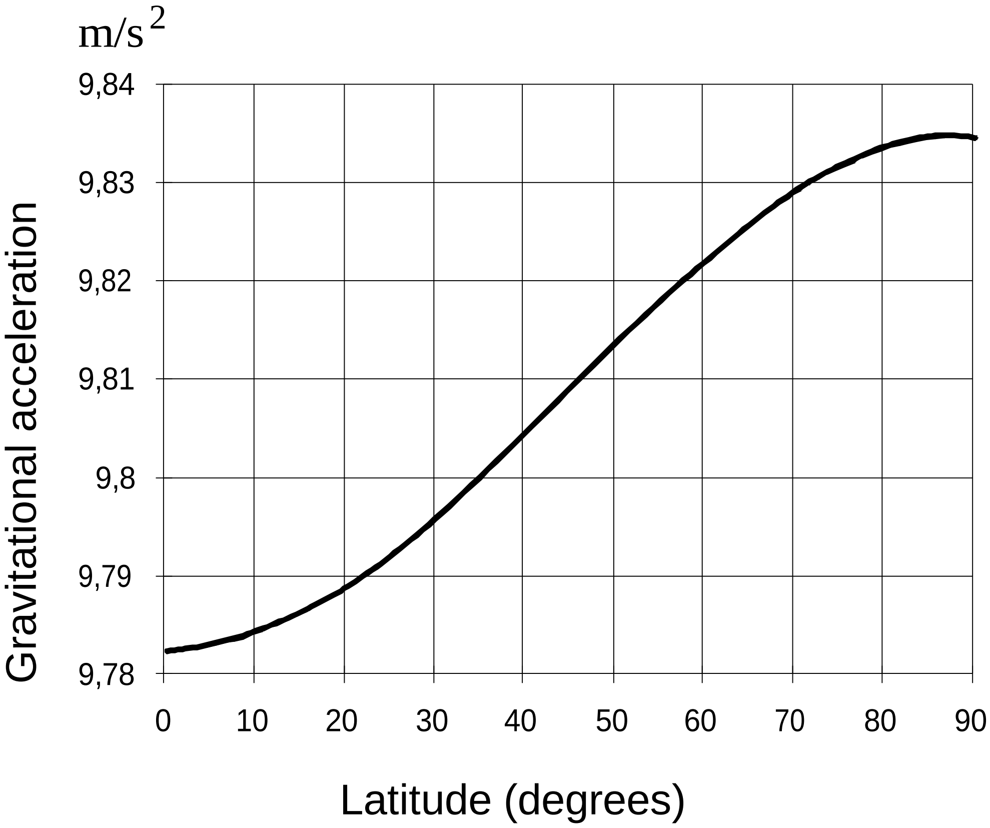
$$g_{\mathrm{g}} = G\frac{m_{\mathrm{earth}}}{\mathrm{r}^2}$$

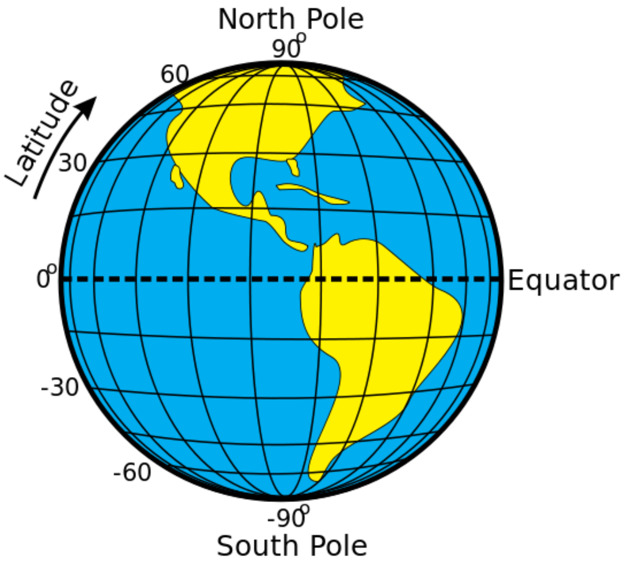
  - m_{earth} and r, the mass and the radius of the Earth

- The Earth is flattened on the poles, therefore acceleration due to gravity is dependent on the latitude.
  - The difference between the radius at the equator (=6376.14km) and the radius at the poles is about 22 km causing a difference of about 0.06 $\mathrm{m/s^2}$ in the acceleration due to gravity.

Acceleration due to gravity

$$g_{\mathrm{g}}(\varphi, h) = 9.78031846\ \left(1 + 0.0053024 \sin^2 \varphi - 0.0000058 \sin^2 2\varphi\right) - 3.086 \cdot 10^{-6} h$$





Centrifugal force

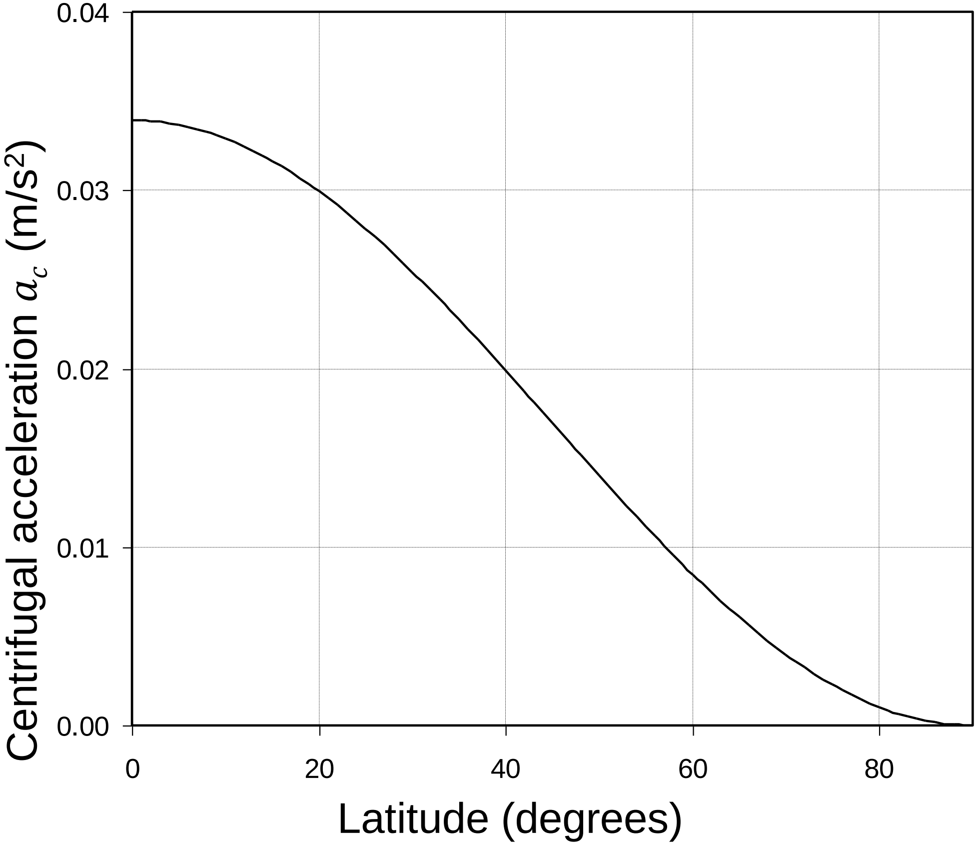
- The rotation of the Earth (angular velocity ω) exerts a centrifugal force on an object.
  - A type of inertial or fictitious force in a rotating frame of reference (as opposed to inertial frame of reference).
- Centrifugal acceleration

$$a_c = \omega^2 r \cos^2 \varphi$$

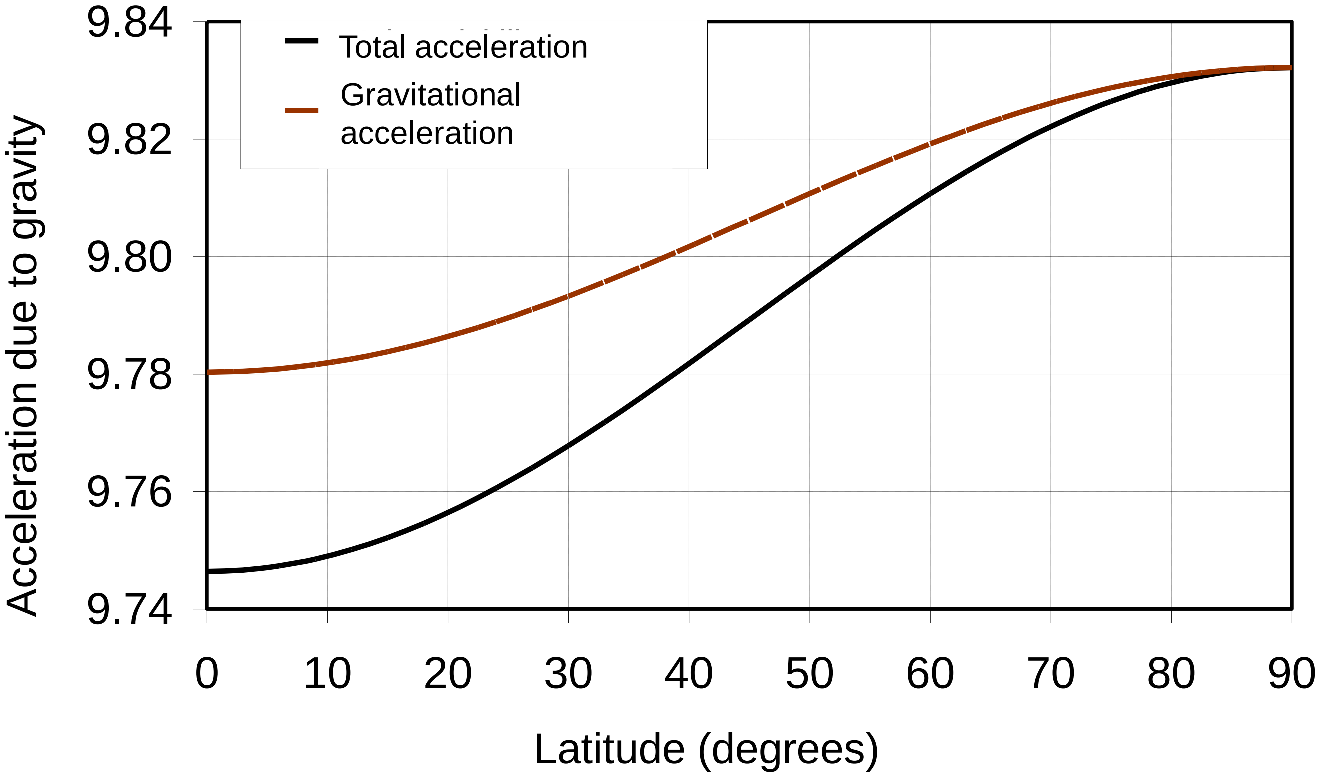
- The effect of centrifugal force is larger at the equator than at the poles.
- Total acceleration measured in gravity measurements

$$g(\varphi, h) = g_{\mathrm{g}}(\varphi, h) - a_{\mathrm{c}}$$

Acceleration due to centrifugal force



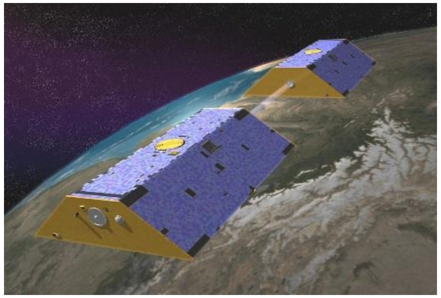
Acceleration due to gravity

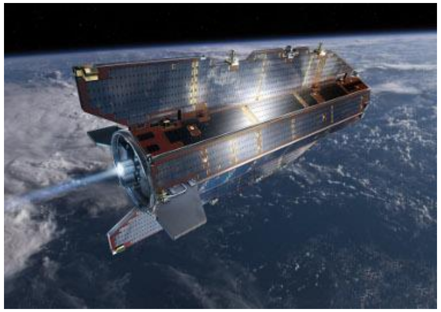


Measuring acceleration due to gravity

- Prevailing acceleration can be determined empirically with a gravimeter either as an absolute or a relative value.
- Relative gravimeter
  - Spring-based gravimeter: A weight on a spring whose stretch is measured.
  - Superconducting gravimeter: Cooled diamagnetic superconducting object (e.g. niobium sphere) is held suspended with a magnetic field. The current generating the magnetic field is proportional to the gravity. (Can detect changes of about $10^{-12}g$)
- Absolute gravimeter
  - Measuring the acceleration of a falling retroreflector with a Michelson interferometer.
  - Measuring small masses free-falling in vacuum

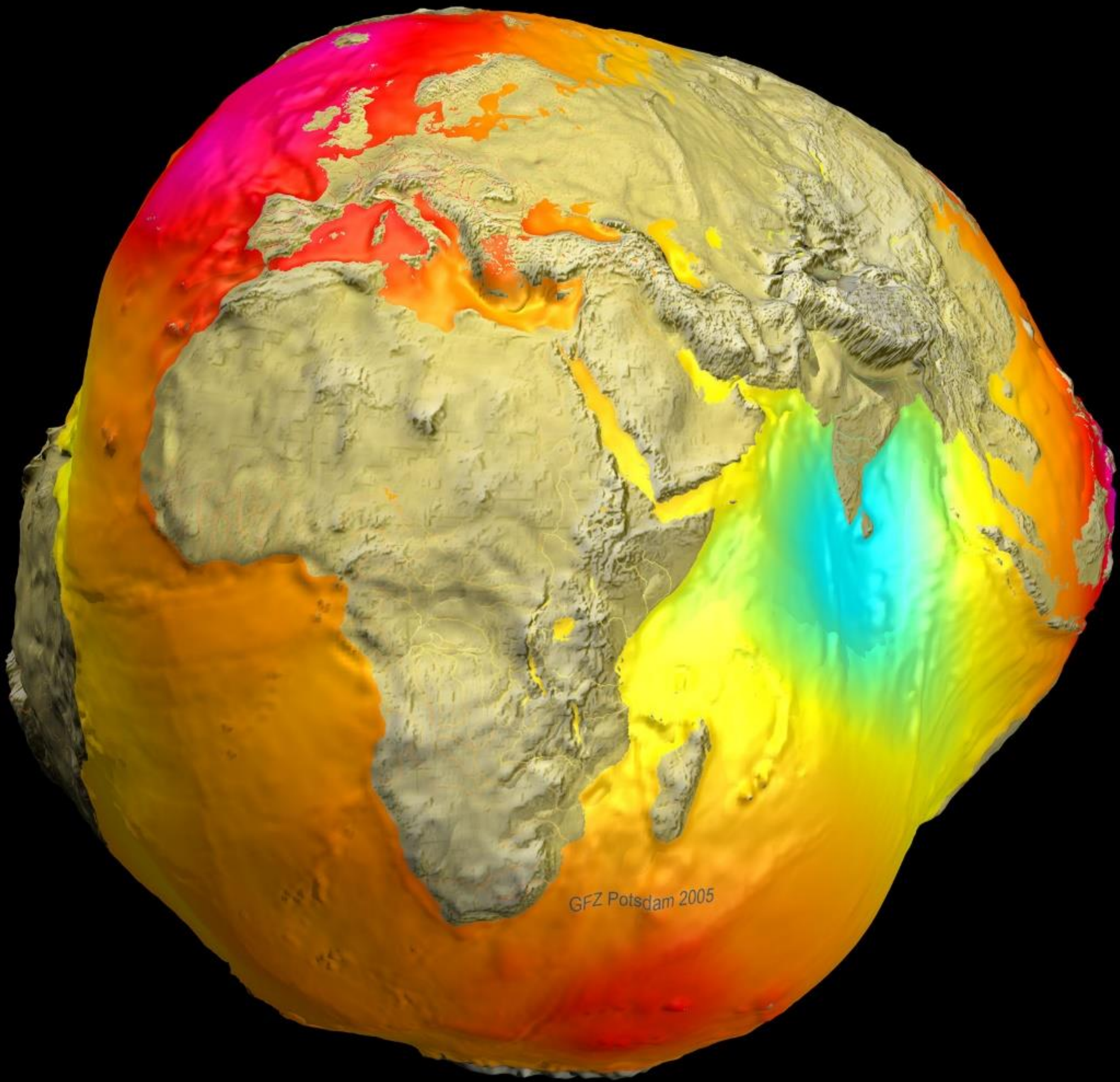
Measurements from orbit

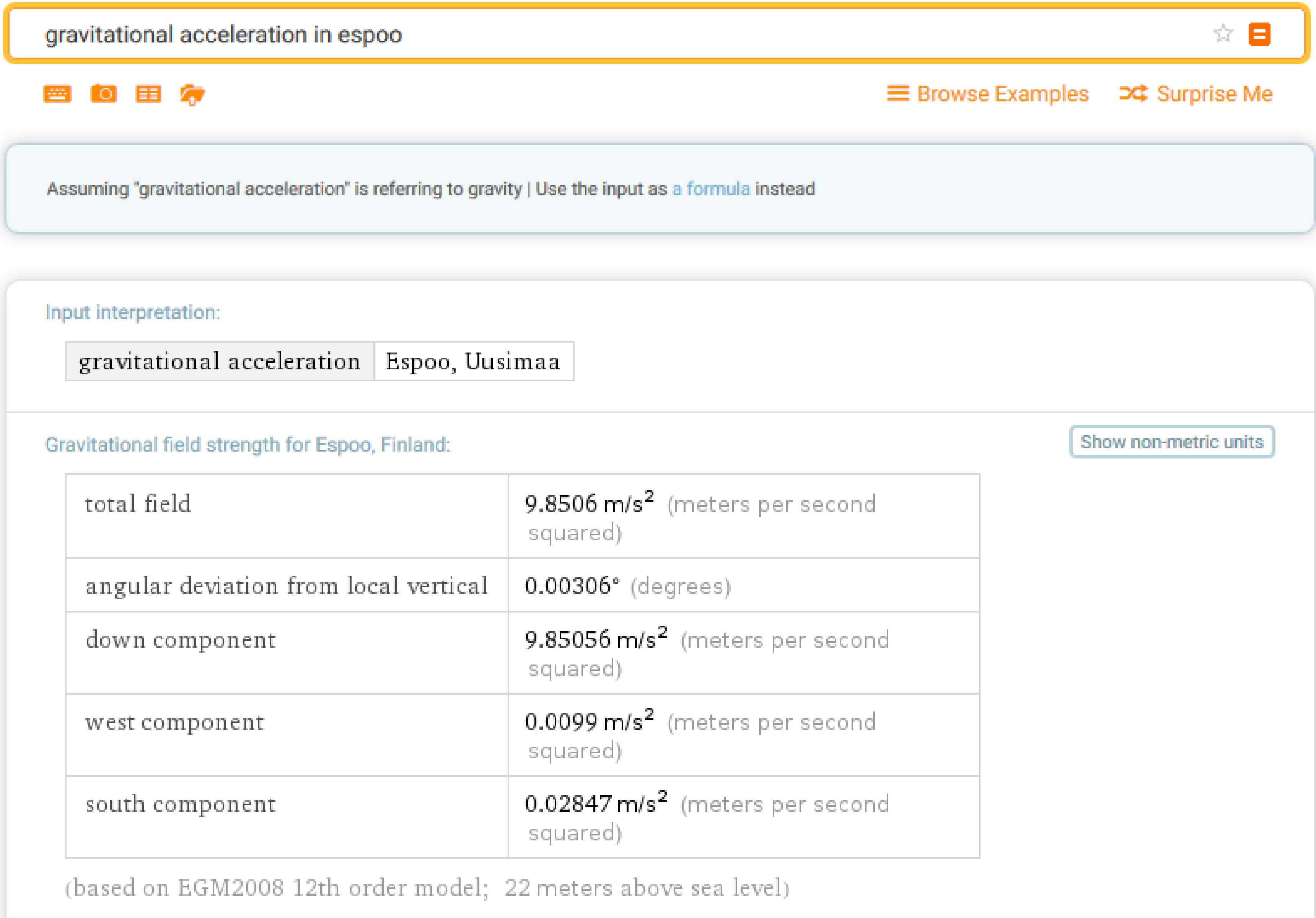




- Twin satellites (GRACE / NASA & DLR)
  - Map of Earth’s gravity field anomalies, distribution of mass.
    - Oceans, ice sheets, the gradual rise of land masses since ice age…
  - Two satellites in a polar orbit about 220 km apart.
  - Changes in gravity affect the distance between the satellites. The distance between the satellites monitored continuously.
- Measuring with gradiometer (GOCE / ESA)
  - Highly detailed map of Earth’s gravity field.
  - Measurement of the gravitational gradient along three orthogonal axes using three pairs of accelerometers.

”Potsdam gravity potato”





gravitational acceleration in espoo

Assuming "gravitational acceleration" is referring to gravity | Use the input as a formula instead

Input interpretation:

gravitational acceleration	Espoo, Uusimaa

Gravitational field strength for Espoo, Finland:

total field	9.8506 m/s^2 (meters per second squared)
angular deviation from local vertical	0.00306° (degrees)
down component	9.85056 m/s^2 (meters per second squared)
west component	0.0099 m/s^2 (meters per second squared)
south component	0.02847 m/s^2 (meters per second squared)

(based on EGM2008 12th order model; 22 meters above sea level)

Measuring mass

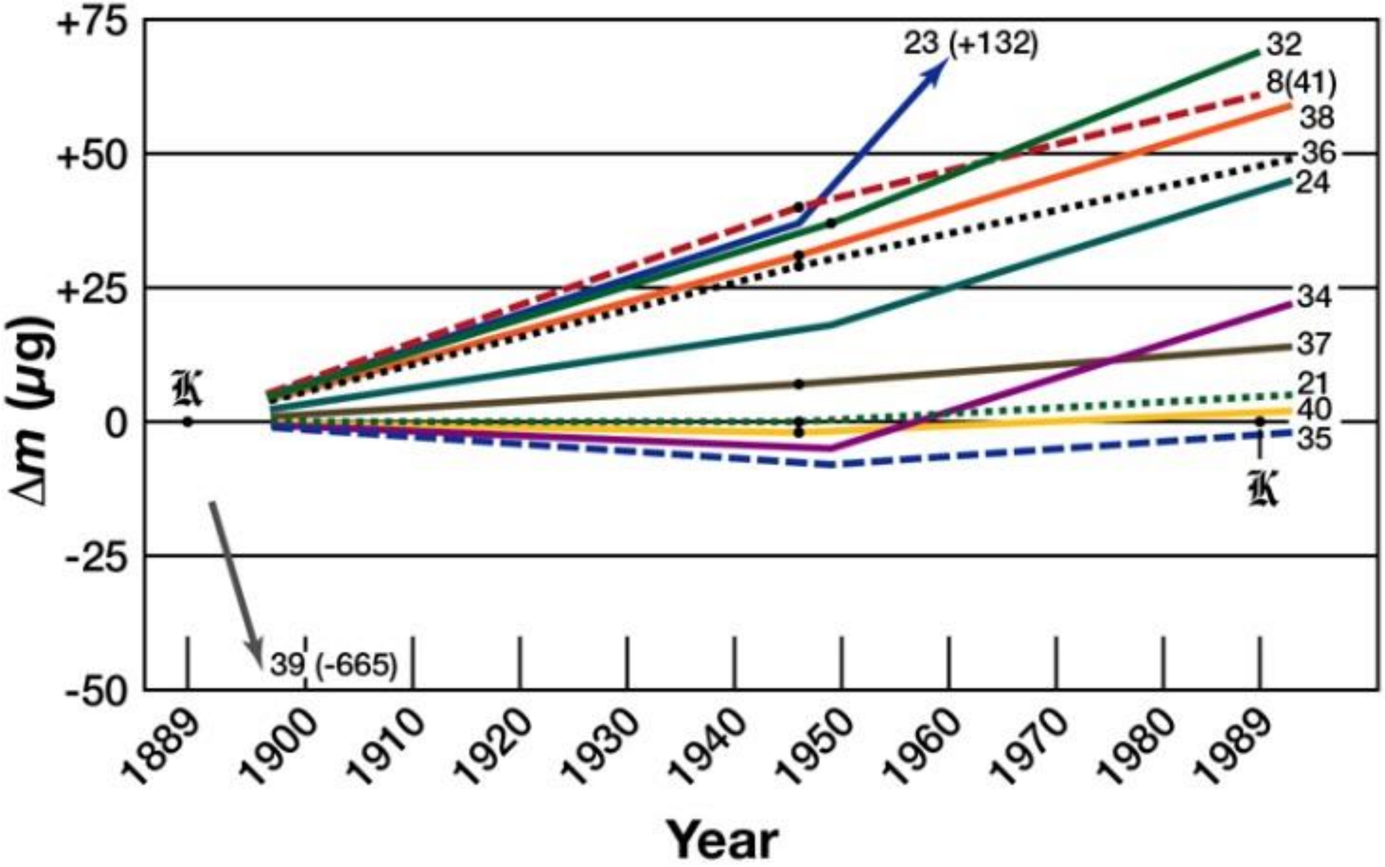
- Comparing masses on a weighing scale.
- Determined using force sensors.
- Most of the electric weighing systems are based on the use of a strain gauge.

International Prototype of Kilogram

- Prototype made of platinum-iridium alloy
  - “Le Grand K”
  - The only SI unit not yet defined by a fundamental physical property.
  - IPK and its 6 sister copies stored in a vault at the International Bureau of Weights and Measures (BIPM) in France.
  - Several national prototypes and additional copies
    - Finnish kilogram (#23) could be the heaviest in the world.
  - Three verifications 1889, 1948, 1989

International Prototype of Kilogram

International Prototype of Kilogram



Need a new definition of kilogram based on a fundamental constant

Planck's constant (h)= 6.626 069 … x 10^{-34} Js

A physical constant that relates the photon energy to its frequency.

For comparison: the metre…

Definition of metre

National prototype metre

Speed of light

Since 1983,

1 metre = distance of light travels in 1/299,792,458 s

Linking kg to Planck's constant

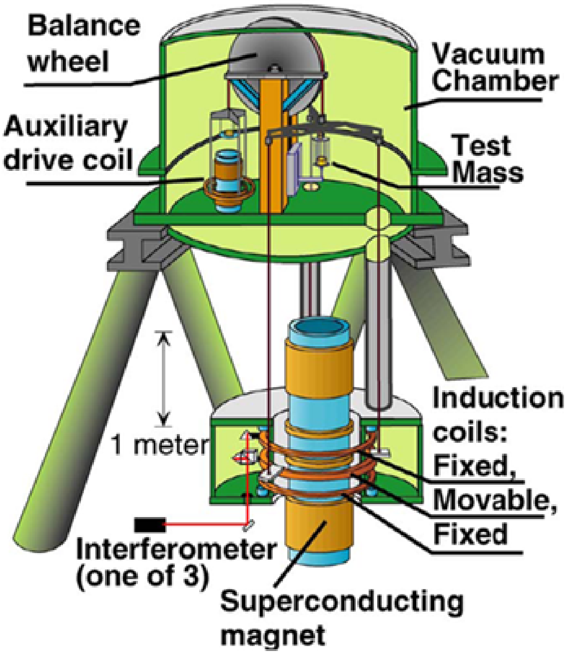
1. Avogadro's project

- Counting atoms in an isotopically pure silicon-28 sphere that weighs the same as the reference kilogram and obtain Avogadro's constant and convert to Planck's constant.
- Sphere costs about 1 million Euro each, plus measurement capabilities.

Linking kg to Planck's constant

2. Kibble balance

- Determine Planck's constant by comparing gravitational force on a reference mass with the electromagnetic force on a coil carrying current in a magnetic field.
- UK, USA, Canada, France, Switzerland



Avogadro project

- Mass is dependent on the amount of substance of the particular atom when Avogadro constant is fixed.
- Nearly perfect sphere made of silicon. The number of atoms can be evaluated based on the dimensions of the sphere.
- Very sophisticated processes for ultra-pure silicon due to semiconductor industry.
- Better long term stability compared to IPK needs to be demonstrated (but is difficult).
  - Silicon dioxide formation?

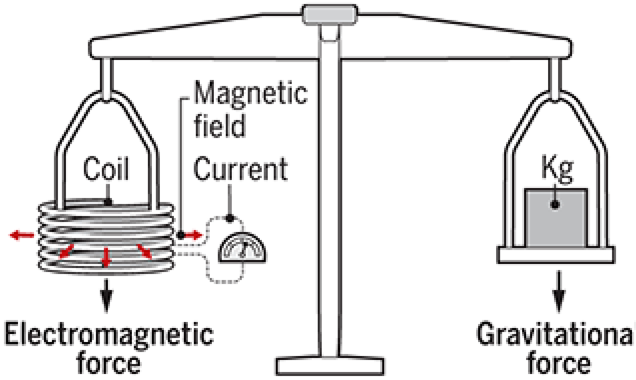
Photo: Master optician Achim Leistner

Kibble balance (previously Watt balance)

- Two steps:
  - Weighing mode: Force caused by a mass is compensated by driving current to a coil located in a static magnetic field.
  - Velocity mode: Coil is moved in the magnetic field and the induced voltage is measured.
    - Eliminates the need to measure the density of magnetic flux or length of coil.

A balance of forces

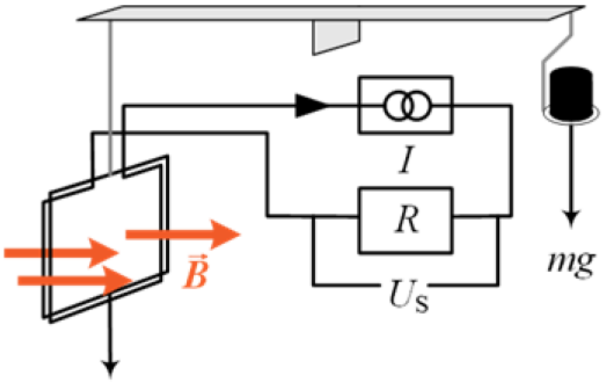
In a Kibble balance, the magnetic force on a current-carrying coil offsets the force of gravity on a weight.



- Balance and velocity is measured with an interferometer.
- Gravitational acceleration measured with a gravimeter.
- Voltage measurement based on Josephson effect
  - Relates voltage to frequency in a superconducting circuit.
  - Definition of kilogram in terms of Planck constant *h*.

Kibble balance – operation modes

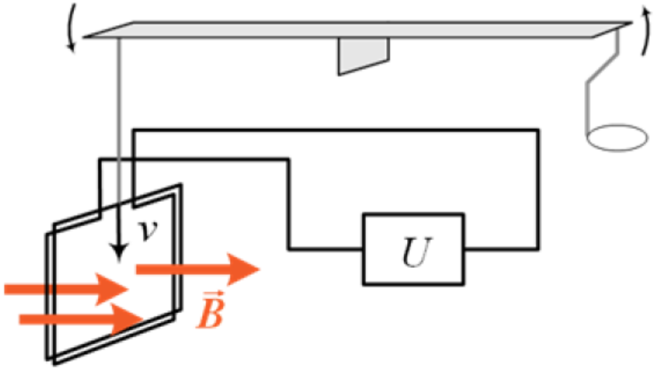
Weighing mode:



current pass through coil in the magnetic field and is adjusted until weight of kg is equal and opposite to electromagnetic force on coil.

$$mg = BLI$$

Moving mode:



no weight, move coil back and forth at constant velocity through magnetic field, which induces voltage in the coil.

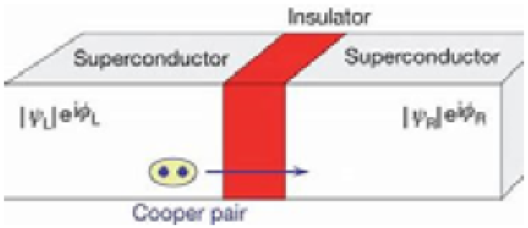
$$V = BLv$$

$$VI = mgv$$

Essential measurements

$$VI = mgv$$

V, voltage: can accurately measure using a macroscopic quantum effect that involves Josephson junctions, apply microwave frequency f across the junctions and create a voltage across the device with stack of N junctions.



$$V = \frac{Nhf}{2e}$$

I, current: measure V on R, the resistance. R can be measured very accurately using quantum Hall effect.

$$I = \frac{V}{R}$$

$$R = \frac{h}{ne^2}$$

Essential measurement

$$VI = mgv$$

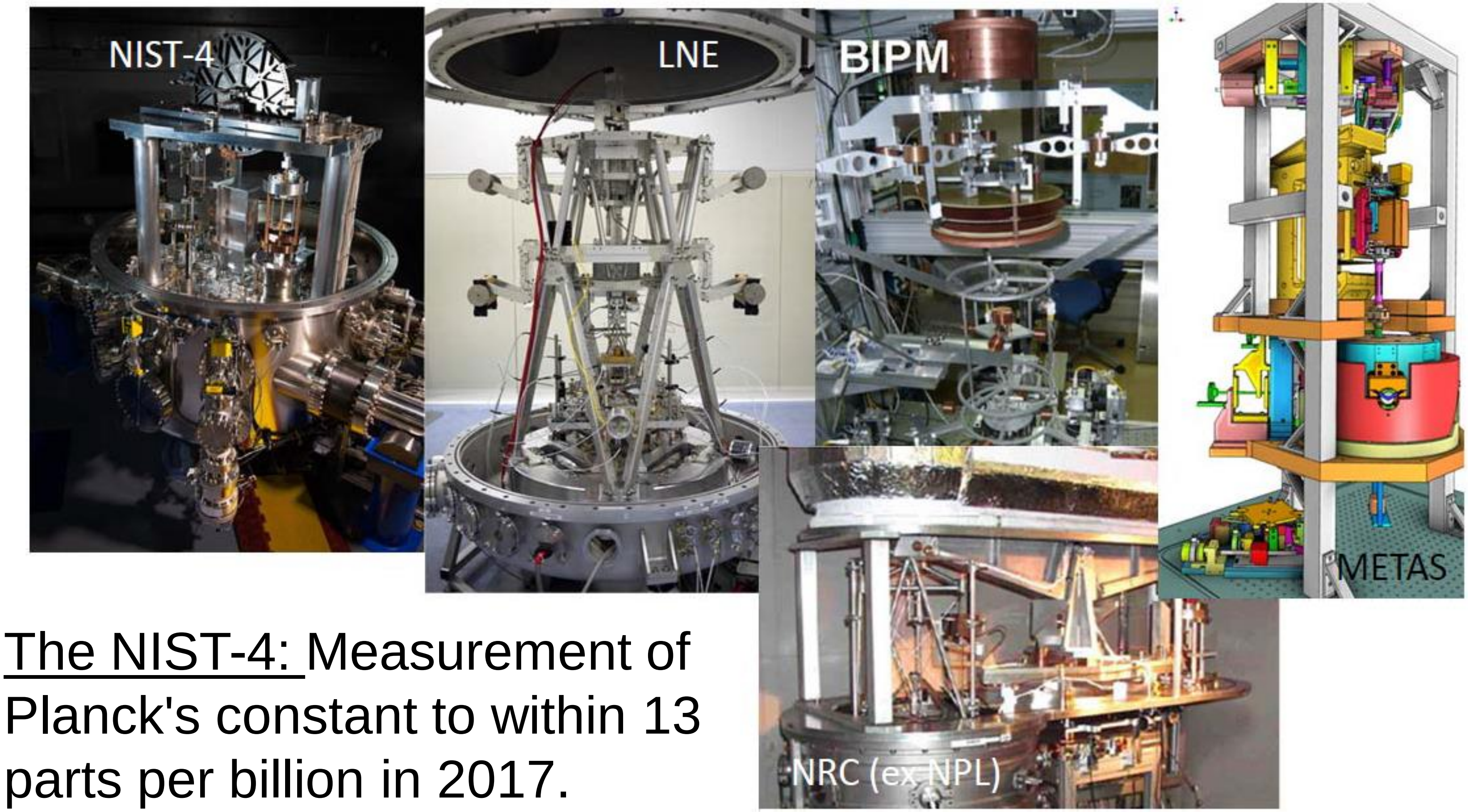
Force difference between gravitational and electromagnetic forces: comparator

Local gravitational acceleration: gravimeter

Coil velocity via
position : laser interferometer
time : atomic clock

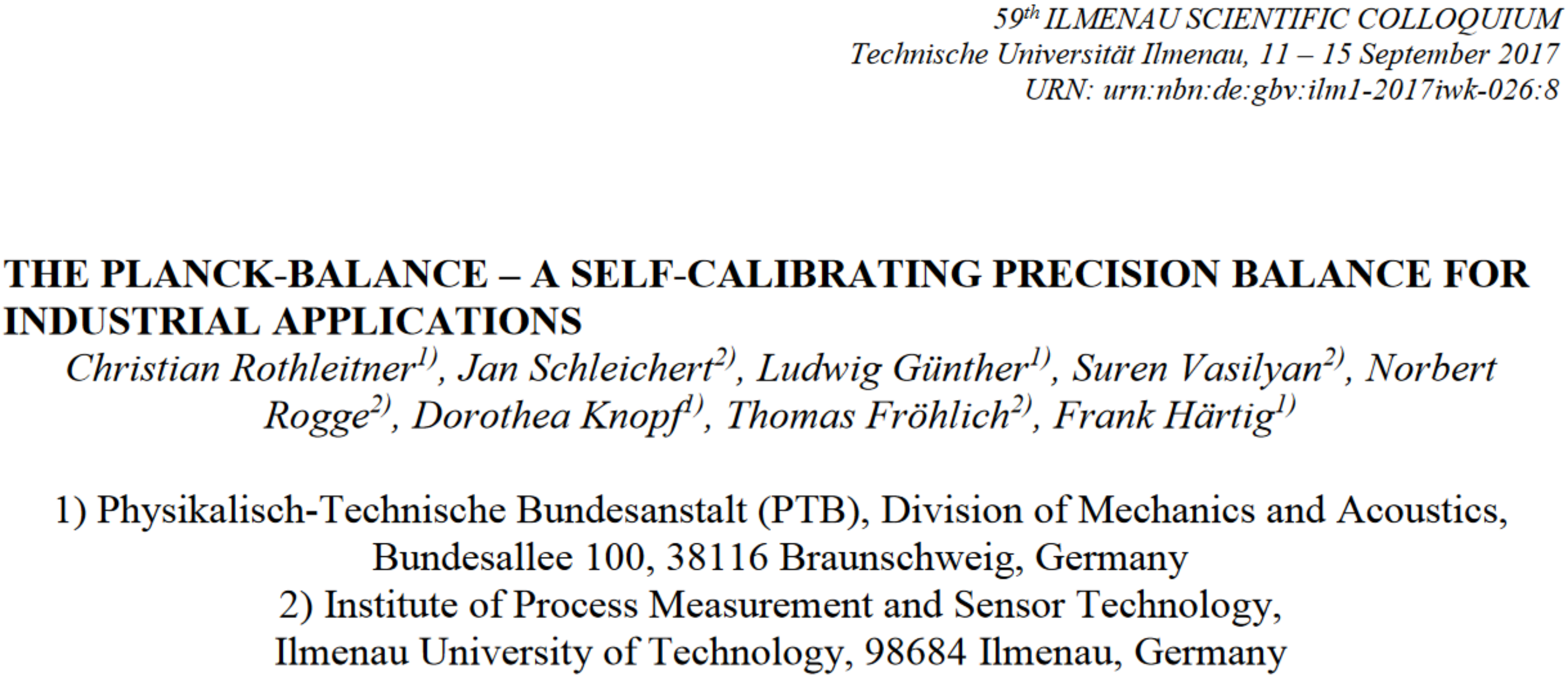
Barry Woods of NRC, Canada and CODATA: "A Kibble balance is easy, all you have to do is measure six quantities each with an uncertainty of 1 part in 10^8 !"

Kibble balances around the world…



The NIST-4: Measurement of Planck's constant to within 13 parts per billion in 2017.

Simplified version proposed for industrial applications



59^{th} ILMENAU SCIENTIFIC COLLOQUIUM
Technische Universität Ilmenau, 11 – 15 September 2017
URN: urn:nbn:de:gbv:ilm1-2017iwk-026:8

THE PLANCK-BALANCE – A SELF-CALIBRATING PRECISION BALANCE FOR INDUSTRIAL APPLICATIONS

Christian Rothleitner(1), Jan Schleichert(2), Ludwig Günther(1), Suren Vasilyan(2), Norbert Rogge(2), Dorothea Knopf(1), Thomas Fröhlich(2), Frank Härtig(1)

1) Physikalisch-Technische Bundesanstalt (PTB), Division of Mechanics and Acoustics,
Bundesallee 100, 38116 Braunschweig, Germany
2) Institute of Process Measurement and Sensor Technology,
Ilmenau University of Technology, 98684 Ilmenau, Germany

Actually… simple enough for a LEGO project

Chao, L. S., et al. "A LEGO Watt balance: An apparatus to determine a mass based on the new SI." *American Journal of Physics* 83.11 (2015): 913-922.