Aalto University
School of Electrical
Engineering

ELEC-E8126: Robotic Manipulation
Constrained and parallel kinematics

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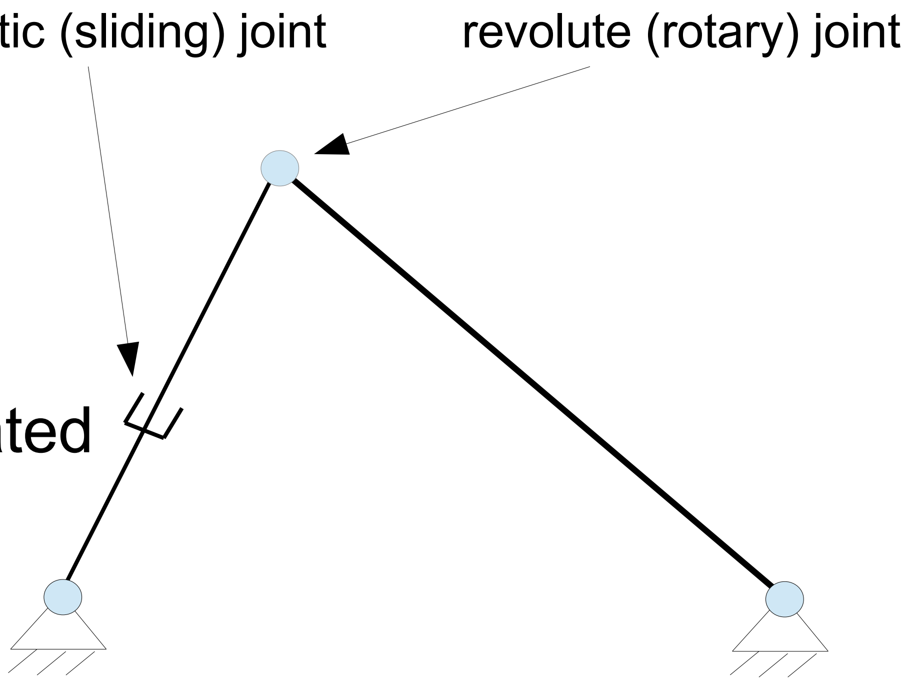
Learning goals

- Understand modeling and characteristics of closed kinematic chains.

- Understand constraints posed by closed chains in contexts of parallel robots, cooperative manipulation and dextrous manipulation.

Terminology

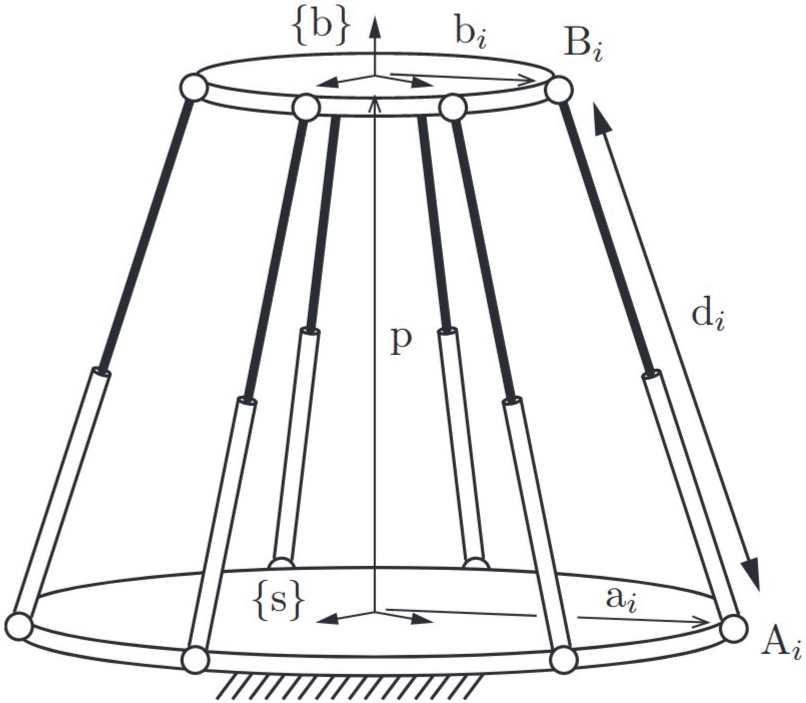
- Closed kinematic chain
- Actuated (active) vs unactuated (passive) joints
  - Why have unactuated joints?



What's the loop here?

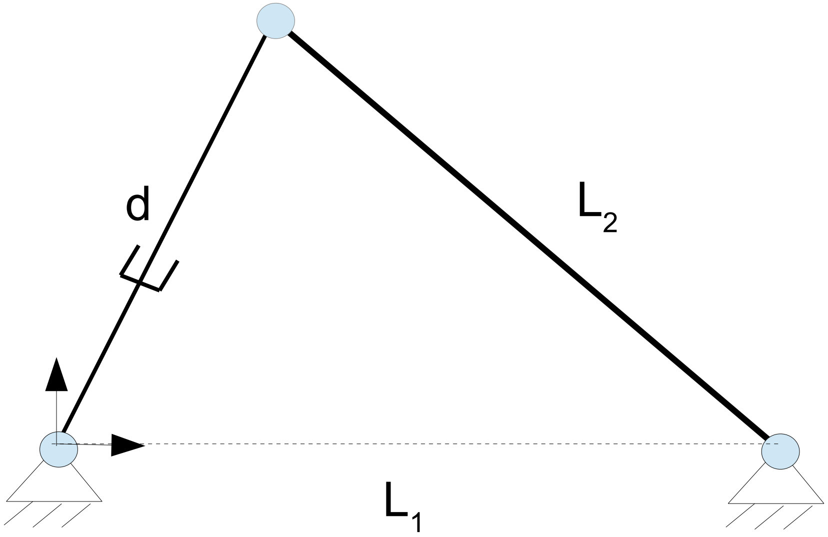
Typical parallel robot characteristics

- Small workspace
- Accurate
- Rigid structure
- More difficult to model than serial.



Kinematic constraints

- What is the position of top point with respect to the length of prismatic joint *d*?
  - What is the constraint equation?



Kinematic constraints

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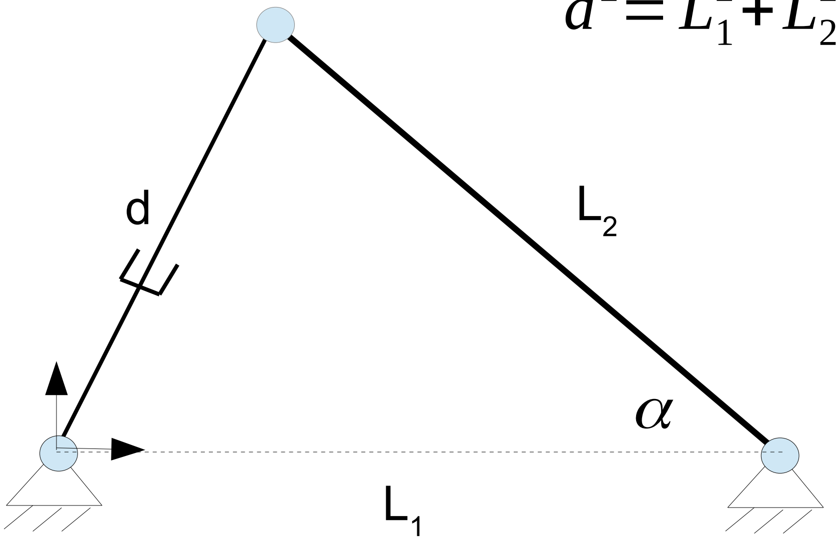
$$d^2 = L_1^2 + L_2^2 - 2 L_1 L_2 \cos\alpha$$



How to now solve the position of the top point?

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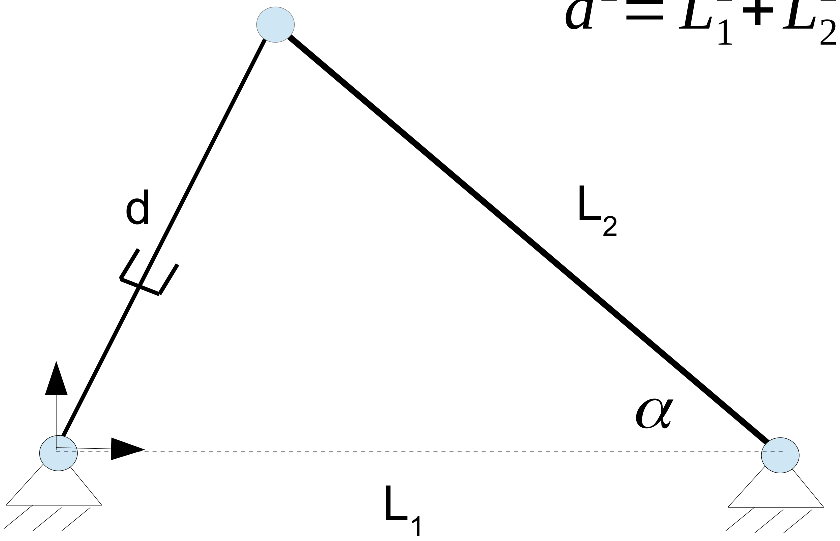
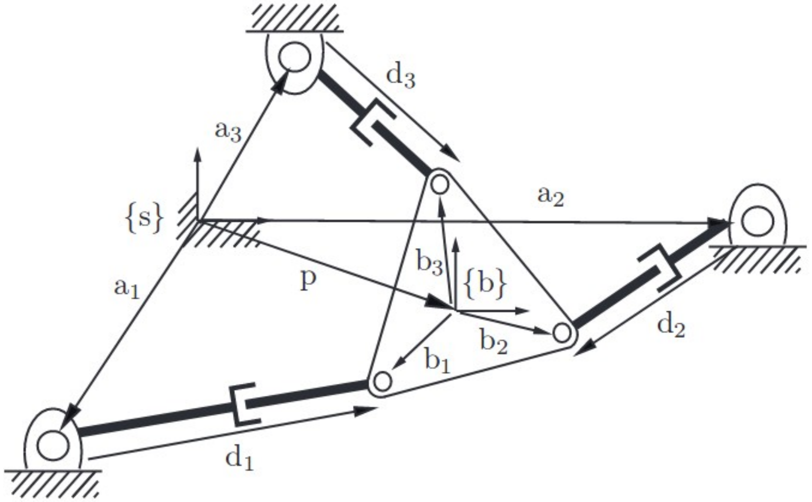


How to now solve the position of the top point?

e.g. $L_2 \cos\alpha = (L_1^2 + L_2^2 - d^2) / (2 L_1)$

3x RPR – mechanism and kinematics

- Planar mechanism with 3 RPR structures.
- Prismatic joints actuated, revolute joints passive.



- Constraint equations

$$d_i^2=(p_x+b_{ix}\cos\phi-b_{iy}\sin\phi-a_{ix})^2+(p_y+b_{ix}\sin\phi+b_{iy}\cos\phi-a_{iy})^2$$

What do these provide us?

3x RPR – mechanism and kinematics

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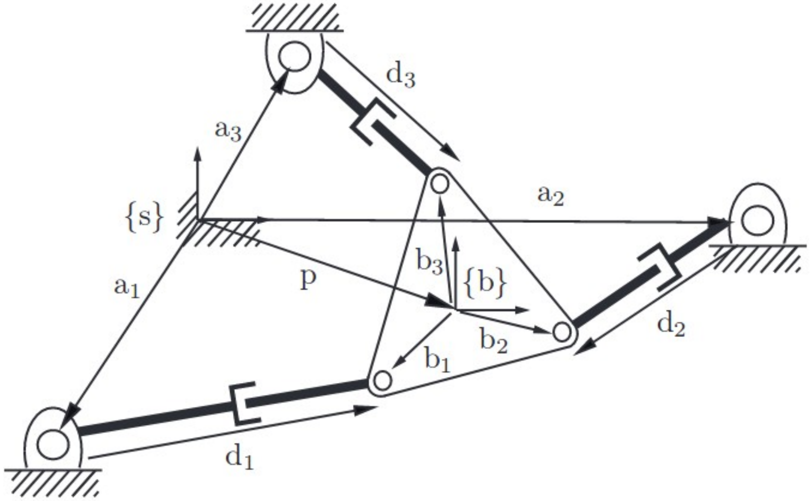

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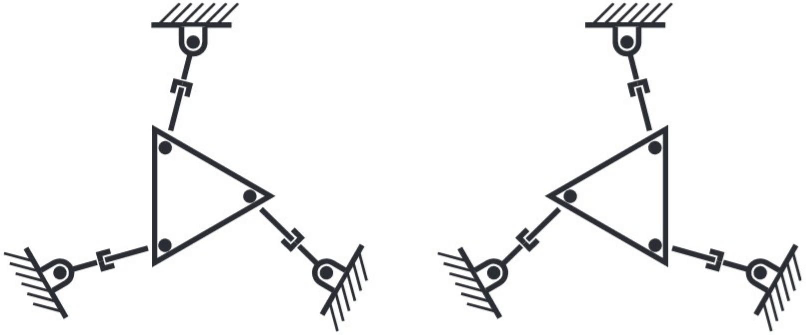
What do these provide us?

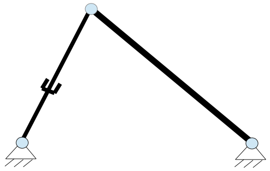
What about forward kinematics?

Forward kinematics

- Fwd kinematics of parallel kinematics often non-unique.



Is fwd kin unique for this?



- Different type of singularity compared to serial mechanisms. But in which way?

Jacobian and constraint Jacobian

- Jacobian can be obtained also from inverse kinematics.
  - Why/how? $\boldsymbol{\theta} = f_{ik}(\boldsymbol{x})$ $\dot{\boldsymbol{x}} = J(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}_{\boldsymbol{a}} = \frac{\partial f_{fk}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \dot{\boldsymbol{\theta}}_{\boldsymbol{a}}$ ← active

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- Constraint Jacobian – Jacobian of the set of constraint equations.

$$h(\boldsymbol{\theta}) = \boldsymbol{0}$$

$$H(\boldsymbol{\theta})\,\dot{\boldsymbol{\theta}} = \left[H_a(\boldsymbol{\theta})\,H_p(\boldsymbol{\theta})\right]\begin{bmatrix}\dot{\boldsymbol{\theta}}_{\boldsymbol{a}} \\ \dot{\boldsymbol{\theta}}_{\boldsymbol{p}}\end{bmatrix} = \boldsymbol{0}$$

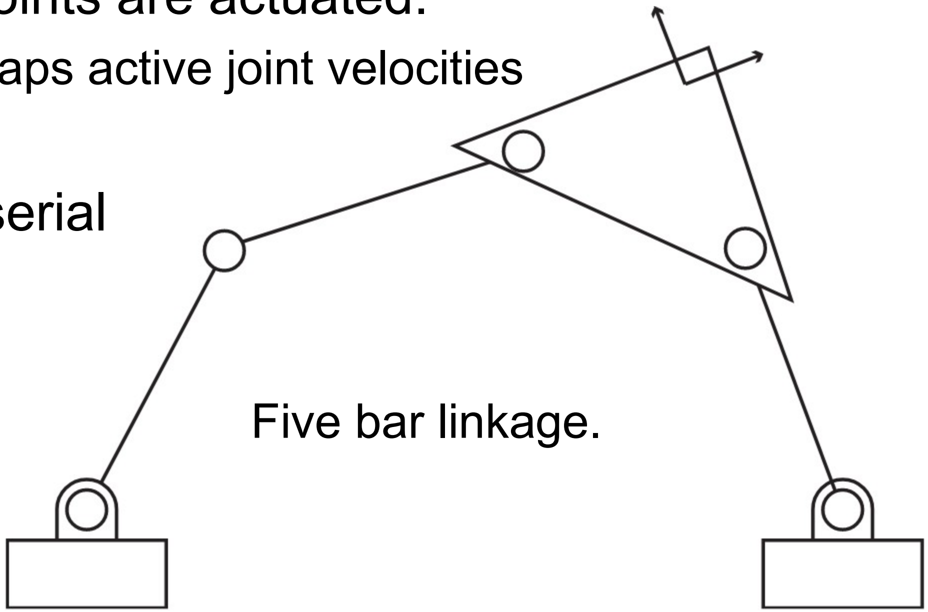
active

passive

What was the relationship of Jacobian to singularities for serial kinematic chains?

Singularities of parallel mechanisms

- End-effector singularities
  - Jacobian loses rank $rank(J) < n$.
  - Do not depend on which joints are actuated.
    - Even though Jacobian maps active joint velocities to Cartesian velocities.
  - Similar to singularities of serial robots.

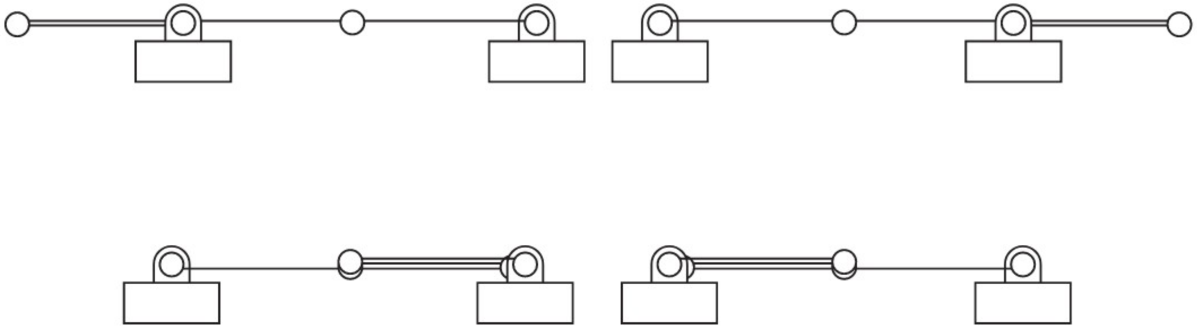


Five bar linkage.

What is the direction of the singularity here?
In which direction the e-e point cannot move?

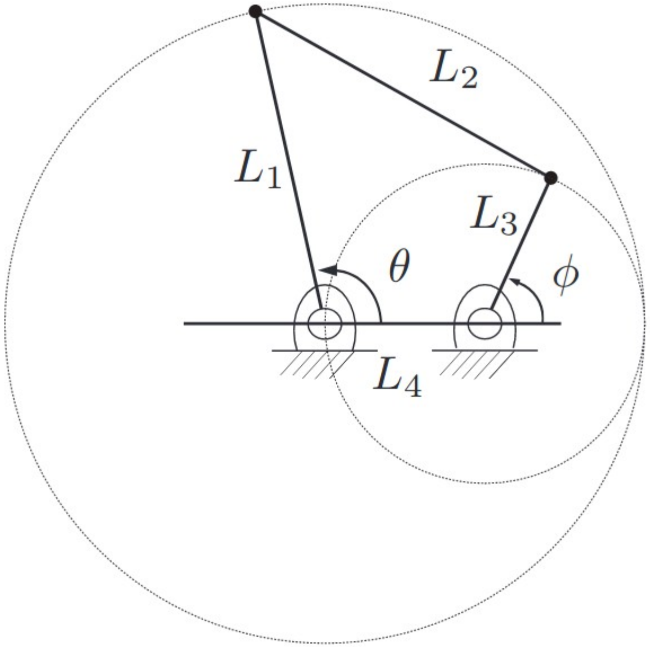
Singularities of parallel mechanisms

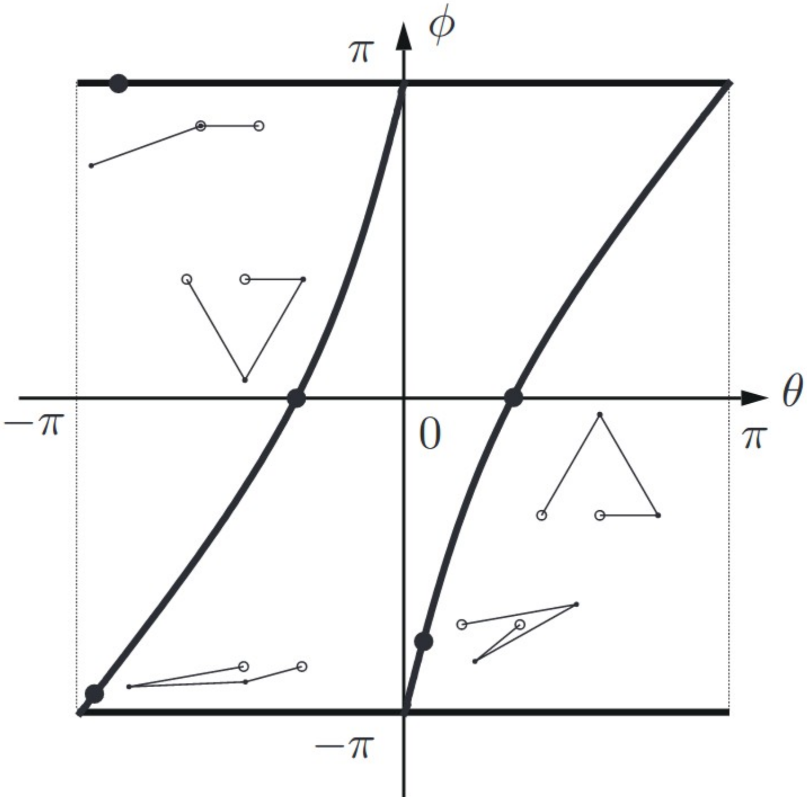
- Configuration space singularities
  - Constraint Jacobian loses rank $rank(H) < p$.
  - Do not depend on which joints are actuated.
  - Branching points/regions in full configuration space.



Configuration space singularity example

- Four bar linkage

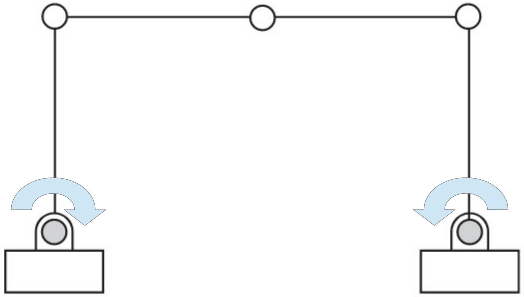




Where are the singularities?

Singularities of parallel mechanisms

- Actuator singularities
  - Constraint Jacobian of passive joints loses rank $rank(H_p) < p$
  - Changing the set of actuated joints will eliminate the singularity.
    - But new one(s) may be created.



What happens?

Can you avoid by changing actuated joints? How?

Closed chains and manipulation

- Are there situations where manipulation creates closed chains?

Closed chains and manipulation

- Are there situations where manipulation creates closed chains?

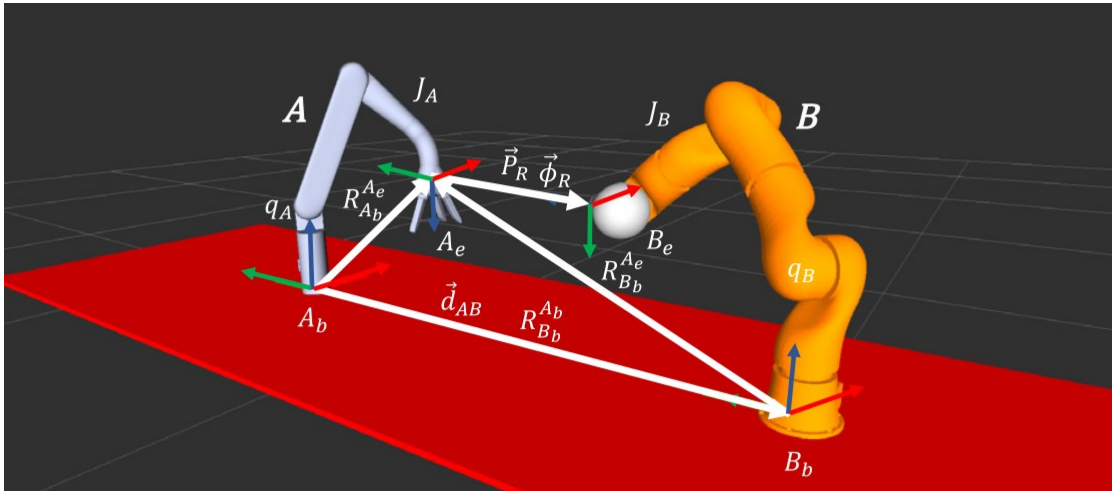
- Cooperative (dual-arm) manipulation
- Dextrous (in-hand) manipulation

- Let’s take a quick look at these.

Cooperative manipulation

- Cooperative manipulation: Multiple arms manipulate a tightly grasped (rigid) object.

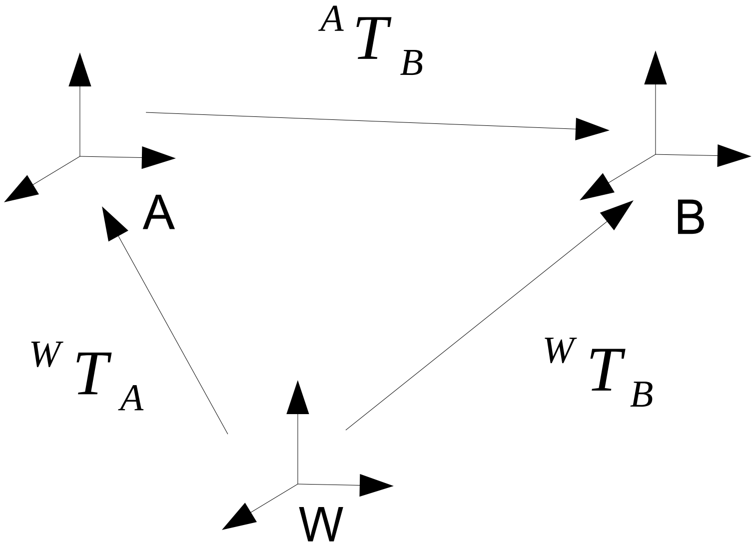
- What kind of constraints exist?



Frame transforms?
Total wrench on object?

Kinematic constraints

- Chain remains closed (and rigid) →
  - Robot velocities need to match in object frame.
  - Alternatively, relative pose remains constant.



Closed kinematic chain?

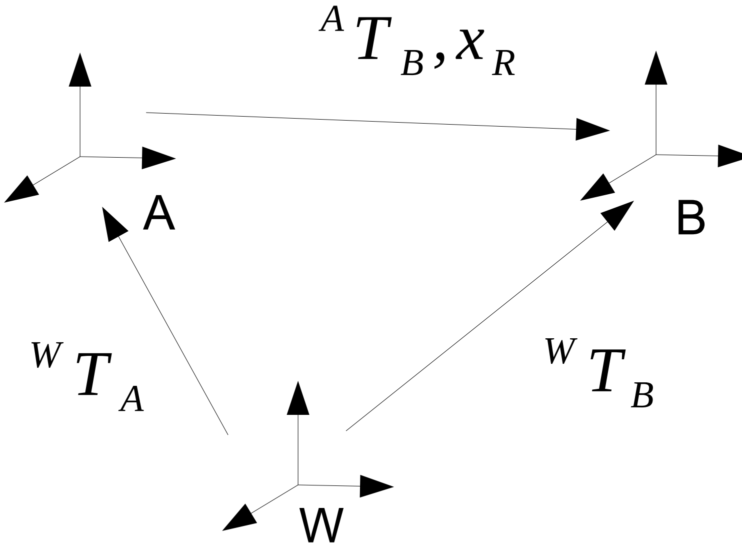
Relative Jacobian

- Jacobian of relative pose is called relative Jacobian

$$\dot{x}_R = J_R \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix}$$

Size?

- Can be calculated using individual robot Jacobians.
- What is the loop closure constraint given the relative Jacobian?



$$J_R \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix} = J_R \dot{\theta} = 0$$

Can be used to derive a closed-loop controller
Is there anything strange with that?

$$\dot{\theta} = J_R^+ K (x_R - x_R^*)$$

Relative Jacobian and coordinated motion control

- Let’s define a (hybrid) velocity controller using relative Jacobian.

$$\dot{\theta}=\underbrace{J_R^+ K_R(x_R-x_R^*)}_{\textit{relative pose fb}}+\underbrace{\overbrace{(I-J_R^+ J_R)}^{P_R}[J_A \quad 0]^+(\dot{x}_A^*+K_P(x_A-x_A^*))}_{\textit{pose fb}}$$

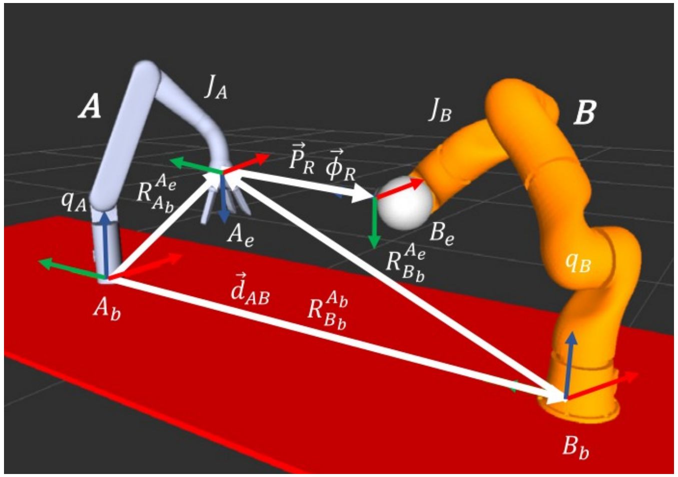
Check position+force control lecture!

Dynamics of cooperative manipulation

- What is the total wrench applied on the object?

$$ {}^{O}F = {}^{O}F_A + {}^{O}F_B = G_A F_A + G_B F_B = G \overbrace{\begin{bmatrix} F_A \\ F_B \end{bmatrix}}^{F} $$

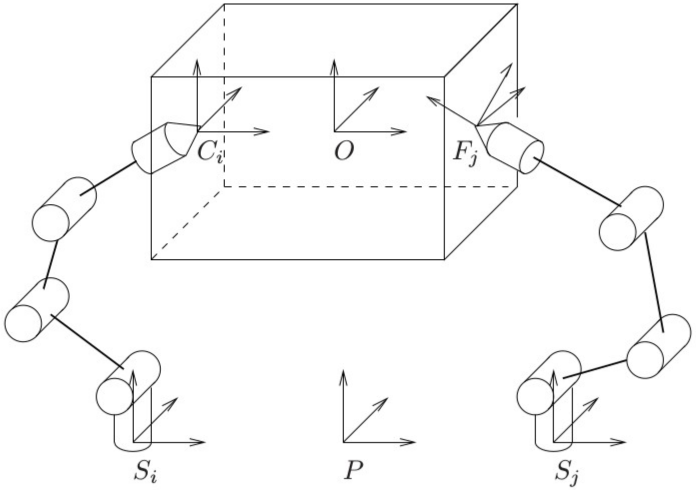
$$ F = G^{+}\,{}^{O}F + V_S F_I $$



- External wrench ${}^{O}F$
- Internal forces in $V = null(G)$
  - How do we want this to behave?
  - Would internal forces be useful in some case?

Dextrous manipulation

- To manipulate grasped objects in hand,
  - finger motions need to be coordinated for grasp to remain stable.
  - object needs to be manipulable.
- What does this mean beyond force closure?



Coordinated motion in grasping

- Finger motions have to correspond to object motion at contacts

$$J \underbrace{\dot{\theta}}_{finger\ joint\ vels} = G^T \underbrace{V_O}_{object\ twists}$$

Why is this more than the relative Jacobian?

- Each contact only in friction constrained directions

$$H \hat{J} \dot{\theta} = H \hat{G}^T V_O$$

Selection matrix to choose constrained directions

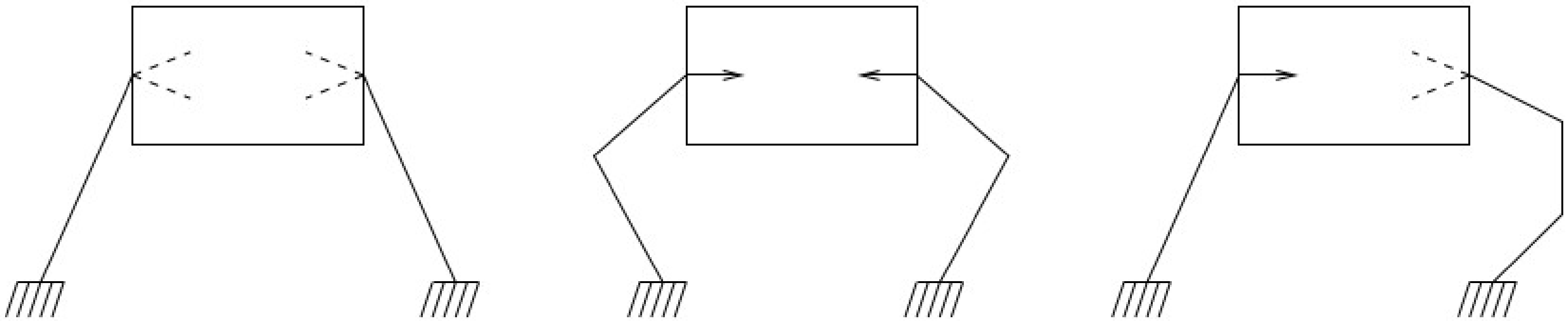
- What is then the constraint for fingers to be able to generate any twist V_O?

$$rank(G) = 6 \qquad rank(GJ) = 6$$

force closure (almost)

Finger motions can create any object motion.

Manipulability vs force closure



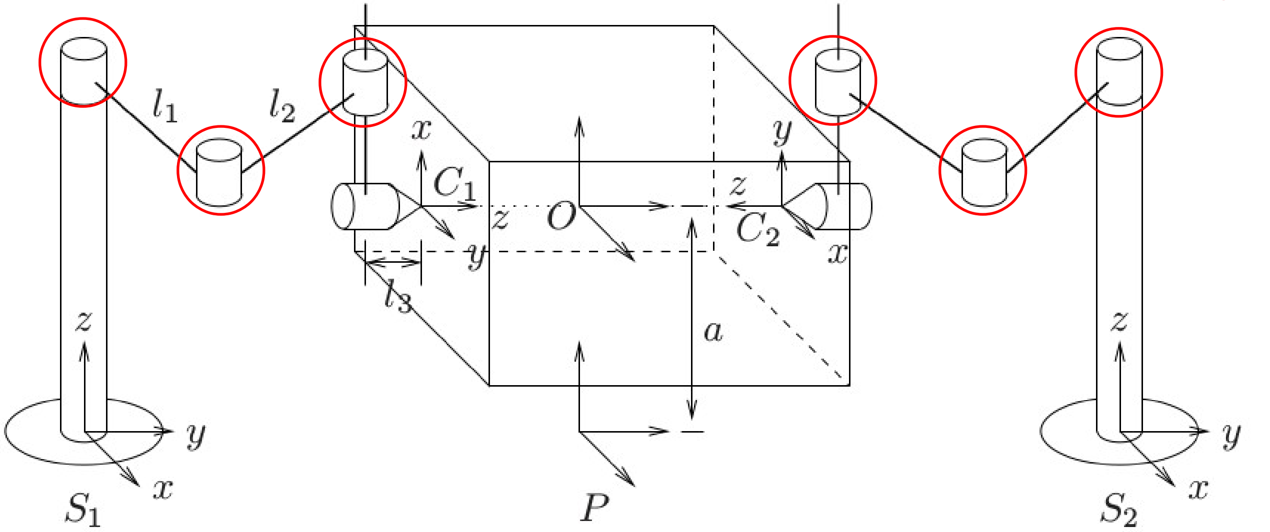
force closure
not manipulable

not force-closure
manipulable

not force-closure
not manipulable

Manipulability example: 2x SCARA

- Assume soft fingers.
- Is the grasp force closure?
- Is the grasp manipulable?


controllable joints

Summary

- Parallel robots have typically both actuated and unactuated joints in closed chains.
  - Inverse kinematics for typical parallel robots are often unique.
  - Forward kinematics often yields multiple solutions.

- Closed chains also appear in cooperative and dextrous manipulation.

Next time: Redundancy

- Readings:
  - Chiaverini et al., “Redundant robots”, in Springer Handbook of Robotics, 2nd ed., ch. 10-10.2.2.
    - Freely available through library webpage lib.aalto.fi. Log-in first and then search for “Springer Handbook of Robotics”.