ECON-C4200 - Econometrics II

Lecture 10: Time series III

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Learning outcomes

- At the end of lecture 10, you know

1. what dynamic causal effects are
2. what the assumptions behind such a model are
3. how to estimate a dynamic causal model
4. how to interpet the results of a dynamic causal model
5. what **H**eteroskedasticity and **A**utoregression **C**onsistent (HAC) standard errors are
6. what a **V**ector **A**utoregressive **R**egression (VAR) model is
7. how to estimate a VAR
8. how to estimate a VAR how to and interpret the results of a VAR model

Dynamic causal effect

- Think of macroeconomic data and questions.
- Example: The effect of a change in the ECB interest rate on inflation next month, in 6 months, in 12, in 24,... months.
- Often, we only have one ”subject”, say the country affected by ECB decisions.
- Difficult to think of an RCT.

Dynamic causal effect

- Substitute thought experiment: different treatments to the same (subject / observation unit) at different points in time.
- Treatment = e.g. level of interest rate t periods ago.
- If the different times are drawn from the same distribution (i.e., Y_y, X_t is stationary), then we can estimate the dynamic causal effects from an OLS regression of Y_t on X_t and its lags.
- Such an estimator is called a **distributed lag** estimator.

The distributed lag model

$$Y_t = \beta_0 + \beta_1 X_t + \beta_1 X_{t-1} ... + \beta_1 X_{t-p} + u_t$$

- Interpretation of β_1: The **impact effect of change in** X_t
- Notice that we are keeping the **past** constant!
- β_2: The **1-period dynamic multiplier**. The effect of a change in X_{t-1} on Y_t, keeping X_t, X_{t-2},... X_{t-p} constant.
- β_3: The **2-period dynamic multiplier**. The effect of a change in X_{t-2} on Y_t, keeping X_t, X_{t-1}, X_{t-3}... X_{t-p} constant.
- **j-period cumulative dynamic multiplier** $= \sum_{k=1}^{j-1} \beta_k$
- Example: 3-period cumulative dynamic multiplier $= \beta_1 + \beta_2 + \beta_3 + \beta_4$

  ...that is, the effect of all X_j for $j = t, t-1, t-2, t-3$.

The distributed lag model: assumptions

$$Y_t = \beta_0 + \beta_1 X_t + \beta_1 X_{t-1} ... + \beta_1 X_{t-p} + u_t$$

- **Exogeneity**: $\mathbb{E}[u_t | X_t, X_{t-1}, ...] = 0$
- **Strict exogeneity**: $\mathbb{E}[u_t | ..., X_{t+1}, X_t, X_{t-1}, ...] = 0$
- Notice that if X is strictly exogenous, then it is also exogenous.
- We will proceed with assuming exogeneity.

The distributed lag model: assumptions

A1 $\mathbb{E}[u_t|X_t, X_{t-1}, ...] = 0$

A2 i) Y, X are stationary and ii) (Y_t, X_t) and (Y_{t-j}, X_{t-j}) become independent as j grows large.

A3 (Y_t, X_t) have 8 nonzero finite moments; large outliers are unlikely.

A4 There is no perfect multicollinearity.

The distributed lag model: assumptions

- Assumptions 1 and 4 are familiar.
- Assumption 2 is different. The difference is due to us studying time series.
- Assumption 3 is familiar otherwise, but now need 8 finite moments. This is to ensure "good" (=HAC) standard errors.

The distributed lag model: assumptions

- Assumption 2.i yields
  1. internal validity
  2. external validity
  3. This is the time series equivalent of the "identically distributed" of the cross-section i.i.d assumption
- Assumption 2.ii
  1. enables the use of a version of CLT
  2. This is the time series equivalent of the "independently distribute" of the cross-section i.i.d assumption
- The intuition is that when the time periods are sufficiently separated, we have independent experiments.

Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors

- Under the above assumptions a distributed lag model, estimated via OLS, yields consistent estimates of the coefficients β_k.
- What about the standard errors? The variance of the sampling distribution is different as X_t and u_t may be autocorrelated (think of what the equivalent would mean in a cross-section context).
- $\rightarrow$ need standard errors that are robust not only to heteroskedasticity but also to autocorrelation.

Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors

- In (well-behaving) cross-section data, the variance of β_1 in a univariate regression is

$$var(\hat{\beta}_1) = \frac{\sigma_u^2}{T(\sigma_X^2)^2}$$

- Let's consider a time-series with $T = 2$, with $cov(u_t, u_{t-1}) \neq 0$:

$$\begin{aligned} var\left(\frac{1}{T}\sum_{t=1}^{T} u_t\right) =& var[0.5(u_1 + u_2)] \\ =& 0.25[var(u_1) + var(u_2) + 2cov(u_1, u_2)] \\ =& 0.5(\sigma_u^2 + \rho) \end{aligned}$$

- Autocorrelation means that the usual variance formula for the error term does not apply.

Example: A (sort-of) Phillips curve

$$infl_t = \beta_0 + \beta_1 \delta UE_t + \beta_2 \delta UE_{t-1} + u_t$$

- Let's estimate the model with regular and HAC s.e.

Sort-of Phillips curve

```
. regr dlnpind_euro d.lnue_euro d2.lnue_euro, robust

Linear regression                               Number of obs     =        187
                                                F(2, 184)         =       0.23
                                                Prob > F          =     0.7956
                                                R-squared         =     0.0030
                                                Root MSE          =      .0041

------------------------------------------------------------------------------
             |               Robust
dlnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   lnue_euro |
         D1. |  -.0234946   .0368771    -0.64   0.525    -.0962509    .0492617
         D2. |   .0059077   .0289012     0.20   0.838    -.0511127     .062928
             |
       _cons |    .001453   .0002997     4.85   0.000     .0008616    .0020444
------------------------------------------------------------------------------

. newey dlnpind_euro d.lnue_euro d2.lnue_euro, lag(1)

Regression with Newey-West standard errors      Number of obs     =        187
maximum lag: 1                                  F(  2,        184) =       0.22
                                                Prob > F          =     0.8056

------------------------------------------------------------------------------
             |             Newey-West
dlnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   lnue_euro |
         D1. |  -.0234946    .037917    -0.62   0.536    -.0983027    .0513134
         D2. |   .0059077   .0276402     0.21   0.831    -.0486248    .0604402
             |
       _cons |    .001453   .0003067     4.74   0.000     .0008479    .0020581
------------------------------------------------------------------------------
```

Estimation of the distributed lag model

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} ... + \beta_{p-1} X_{t-p} + u_t$$

- You can estimate the model with OLS.
- You directly get the impact effect of X and the k-period dynamic multipliers & their standard errors
- You do not get the cumulative multipliers directly, nor their standard errors.

Estimating the cumulative dynamic multipliers

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} ... + \beta_{p-1} X_{t-p} + u_t$$

- Option 1: you estimate the above specification and then uses post-estimation commands to calculate the cumulative dynamic multipliers and their standard errors.

Cumulative dynamic multiplier

```
. newey dlnpind_euro d.lnue_euro d2.lnue_euro, lag(3)

Regression with Newey-West standard errors      Number of obs     =        187
maximum lag: 3                                  F(  2,       184) =       0.23
                                                Prob > F          =     0.7916

------------------------------------------------------------------------------
             |             Newey-West
dlnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   lnue_euro |
         D1. |  -.0234946   .0356168    -0.66   0.510    -.0937645    .0467753
         D2. |   .0059077   .0245427     0.24   0.810    -.0425137     .054329
             |
       _cons |    .001453   .0002694     5.39   0.000     .0009215    .0019845
------------------------------------------------------------------------------

. scalar cdm = _b[d.lnue_euro] + _b[d2.lnue_euro]

. scalar list cdm
       cdm = -.01758694

. testnl _b[d.lnue_euro] + _b[d2.lnue_euro]  = 0

  (1)  _b[d.lnue_euro] + _b[d2.lnue_euro] = 0

               chi2(1) =        0.37
           Prob > chi2 =        0.5450
```

Estimating the cumulative dynamic multipliers

- Option 2: Transform the model to yield direct estimates of the cumulative dynamic multipliers.
- Example: A 1-lag distributed lag model:

$$\begin{aligned}
Y_t =& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
=& \beta_0 + \beta_1 X_t - \beta_1 X_{t-1} + \beta_2 X_{t-1} + \beta_1 X_{t-1} + u_t \\
=& \beta_0 + \beta_1 (X_t - X_{t-1}) + (\beta_2 + \beta_1) X_{t-1} + u_t \\
=& \beta_0 + \beta_1 \Delta X_t + (\beta_2 + \beta_1) X_{t-1} + u_t
\end{aligned}$$

- Same trick works for higher order distributed lag models.

Cumulative dynamic multiplier

```
. gen diff = d.lnue_euro - d2.lnue_euro
(6 missing values generated)

. newey dlnpind_euro diff d2.lnue_euro, lag(3)

Regression with Newey-West standard errors      Number of obs     =        187
maximum lag: 3                                  F(  2,       184) =       0.23
                                                Prob > F          =     0.7916

------------------------------------------------------------------------------
             |             Newey-West
dlnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        diff |  -.0234946   .0356168    -0.66   0.510    -.0937645    .0467753
             |
   lnue_euro |
         D2. |  -.0175869   .0290564    -0.61   0.546    -.0749134    .0397396
             |
       _cons |    .001453   .0002694     5.39   0.000     .0009215    .0019845
------------------------------------------------------------------------------
```

Distributed lag model with autocorrelation

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} ... + \beta_{p-1} X_{t-p} + u_t$$

- Now assume that

$$u_t = \tilde{u}_t + \phi u_{t-1}$$

  in other words, u_t is autocorrelated (AR(1)).

- Besides using HAC standard errors, we could consider transforming the regression equation so that the error term in the transformed equation is i.i.d.

- As an example, let's work with the following distributed lag model:

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t$$

Estimating the cumulative dynamic multipliers

- We can proceed as follows:

$$
\begin{aligned}
Y_t =& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
Y_t - \phi Y_{t-1} =& \beta_0 + \beta_1 X_t - \beta_1 X_{t-1} + \beta_2 X_{t-1} + \beta_1 X_{t-1} + u_t - \phi Y_{t-1} \\
=& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
& - \phi \underbrace{\left(\beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + u_{t-1}\right)}_{=Y_{t-1}} \\
=& (\beta_0 - \phi\beta_0) + \beta_1 X_t + (\beta_2 - \phi\beta_1) X_{t-1} - \phi\beta_2 X_{t-2} + \tilde{u}_t \\
& + \phi u_{t-1} - \phi u_{t-1} \\
Y_t =& \delta_0 + \phi Y_{t-1} + \delta_1 X_t + \delta_2 X_{t-1} + \delta_3 X_{t-2} + \tilde{u}_t
\end{aligned}
$$

- The outcome is an ADL(1,2) model with i.i.d error term.

Vector autoregressive (VAR) models

- What if you want to model two or more series simultaneously?
- What if they interact, i.e., the past values of Y affect current value of X and vice versa?
- VAR models are designed for these situations. Below an example:

$$\begin{aligned} Y_t =& \beta_{10} + \beta_{11} Y_{t-1} + \beta_{12} Y_{t-2} + \gamma_{11} X_{t-1} + \gamma_{12} X_{t-2} + u_{1t} \\ X_t =& \beta_{20} + \beta_{21} X_{t-1} + \beta_{22} X_{t-2} + \gamma_{21} Y_{t-1} + \gamma_{22} Y_{t-2} + u_{2t} \end{aligned}$$

Vector autoregressive (VAR) models

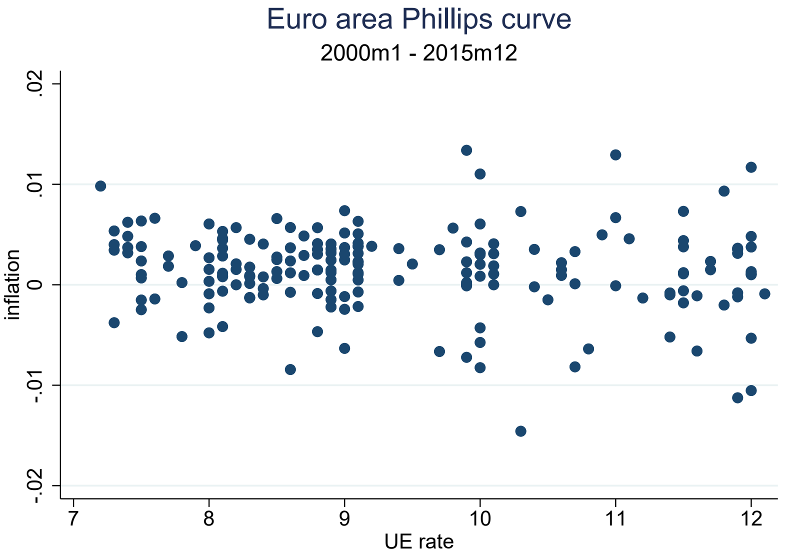
- Without any constraints on the coefficients, one could estimate the VAR equation by equation.
- Often, theory suggests some constraints on the coefficients, e.g., in macroeconomic models.
- Example of VAR: The Euro-area Phillips curve.
- We use data from 2000m1 - 2015m12.

Plotting the Phillips curve

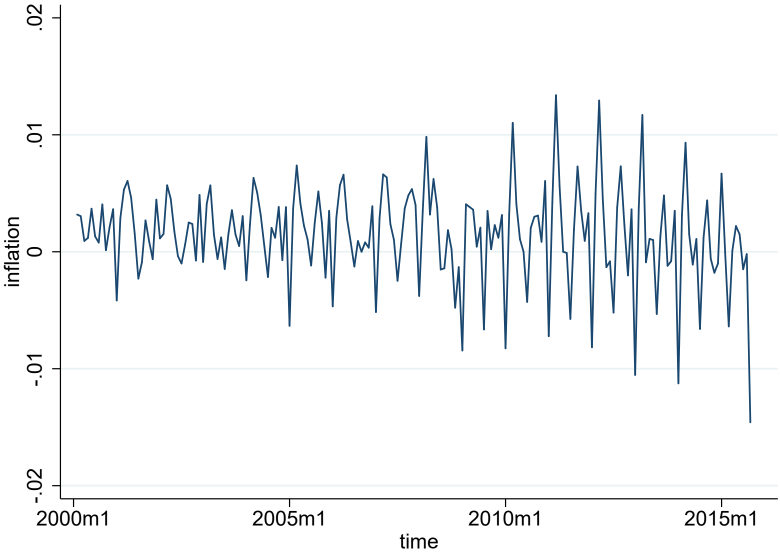
Stata code

```
use "mytimeseries.dta", clear
tsset time_ind
label var dlnpind_euro "inflation"
label var ue_euro "UE rate"
scatter dlnpind_euro ue_euro , ///
    title("Euro area Phillips curve") ///
    subtitle("2000m1 - 2015m12") ///
    graphregion(color(white)) bgcolor(white)

  graph export "L_II_10_4_phillips.pdf", replace
```

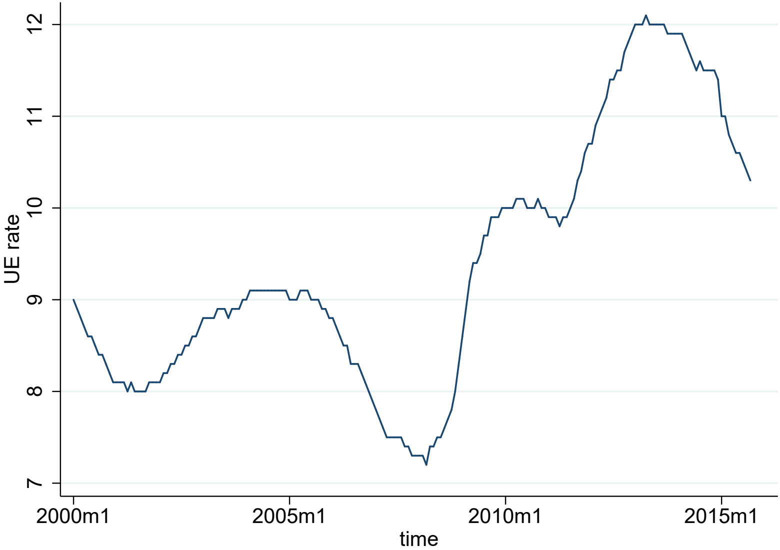
Plotting the Phillips curve



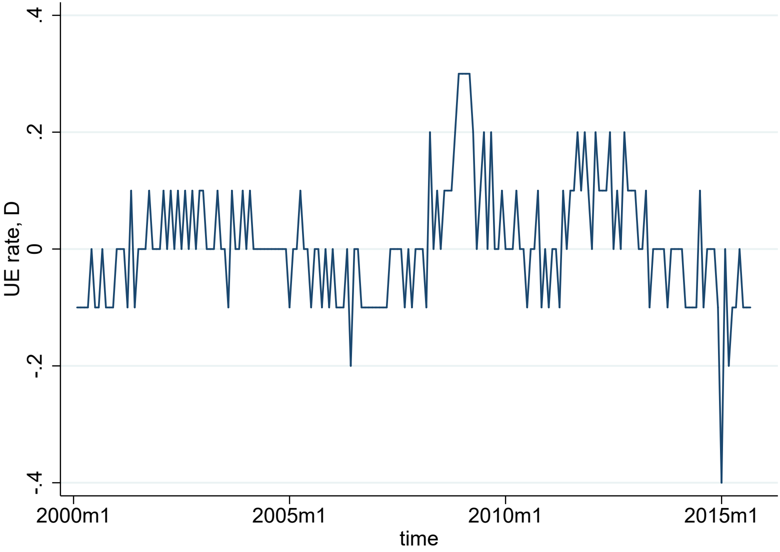
Inflation



Unemployment



Change in unemployment



VAR on inflation and UE

Stata code

```
use "mytimeseries.dta", clear
forvalues t = 1/10{
  dfuller ue_euro, lags(`t')
  }
forvalues t = 1/10{
  dfuller d.ue_euro, lags(`t')
  }
forvalues t = 1/10{
  dfuller dlnpind_euro, lags(`t')
  }
```

DF tests

```
. forvalues t = 1/10{
  2.         dfuller ue_euro, lags(`t')
  3.         }

Augmented Dickey-Fuller test for unit root         Number of obs   =       187

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -0.724            -3.481            -2.884            -2.574
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.8405
```

```
. forvalues t = 1/10{
  2.         dfuller d.ue_euro, lags(`t')
  3.         }

Augmented Dickey-Fuller test for unit root         Number of obs   =       186

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -4.188            -3.481            -2.884            -2.574
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0007
```

DF tests

```
. forvalues t = 1/10{
  2.         dfuller dlnpind_euro, lags(`t')
  3.         }

Augmented Dickey-Fuller test for unit root         Number of obs   =       186

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -11.302            -3.481            -2.884            -2.574
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

VAR on inflation and UE

Stata code

```
var dlnpind_euro d.ue_euro
var dlnpind_euro d.ue_euro, lags(1 2 3)
var dlnpind_euro d.ue_euro
fcast compute f_, step(12)
fcast graph f_dlnpind_euro, ///
    title("Forecast for Euro area inflation") ///
    graphregion(color(white)) bgcolor(white)
  graph export "L_II_10_8_fcast_inflation.pdf", replace

fcast graph f_D_ue_euro, ///
    title("Forecast for Euro area UE") ///
    graphregion(color(white)) bgcolor(white)
  graph export "L_II_10_9_fcast_ue.pdf", replace
```

VAR estimation results

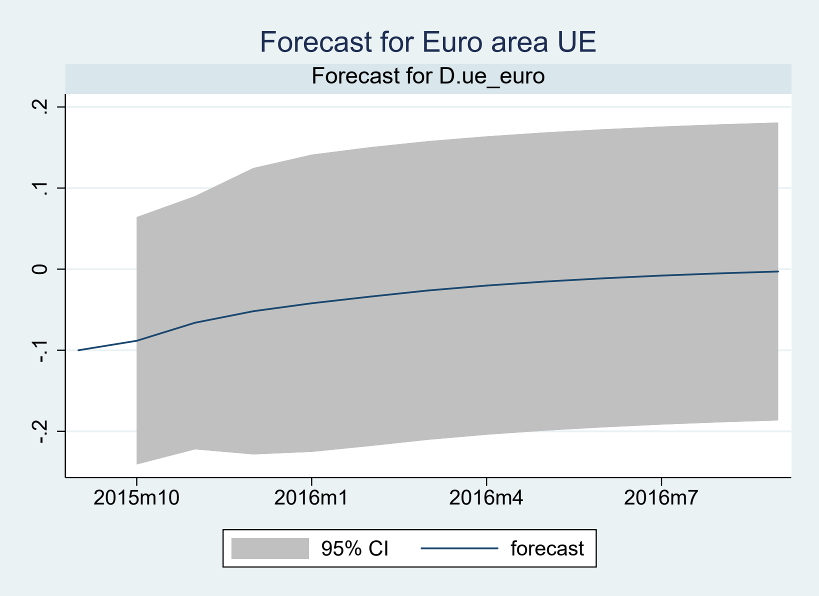
- HQIC = Hannan-Quick information criterion

```
. var dlnpind_euro d.ue_euro

Vector autoregression

Sample:  2000m4 - 2015m9                        Number of obs     =       186
Log likelihood =    978.682                     AIC               = -10.41594
FPE            =   1.03e-07                     HQIC              = -10.34566
Det(Sigma_ml)  =   9.22e-08                     SBIC              = -10.24251

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dlnpind_euro         5     .004021   0.0567   11.18402   0.0246
D_ue_euro            5     .077844   0.3899   118.8916   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dlnpind_euro |
dlnpind_euro |
         L1. |   .0607082   .0745786     0.81   0.416    -.0854631    .2068795
         L2. |  -.2318106   .0746343    -3.11   0.002    -.3780911     -.08553
             |
     ue_euro |
         LD. |  -.0019017   .0033073    -0.57   0.565     -.008384    .0045806
        L2D. |   .0024801   .0033123     0.75   0.454    -.0040119    .0089722
             |
       _cons |   .0016858   .0003333     5.06   0.000     .0010326     .002339
-------------+----------------------------------------------------------------
D_ue_euro    |
dlnpind_euro |
         L1. |   1.173706   1.443759     0.81   0.416     -1.65601    4.003421
         L2. |  -.1060152   1.444838    -0.07   0.942    -2.937846    2.725816
             |
     ue_euro |
         LD. |   .2300881   .0640266     3.59   0.000     .1045982     .355578
        L2D. |   .4883861    .064123     7.62   0.000     .3627074    .6140648
             |
       _cons |   .0006486   .0064515     0.10   0.920    -.0119962    .0132933
------------------------------------------------------------------------------
```

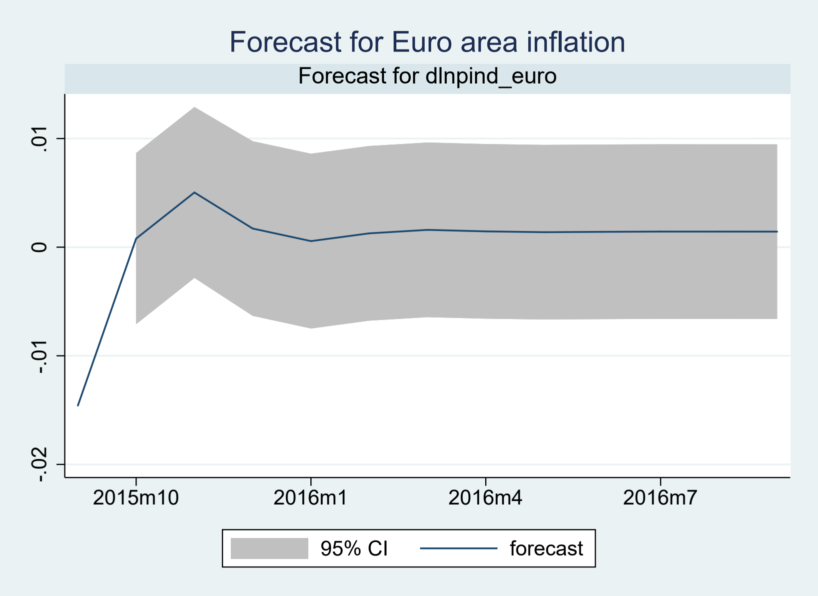
VAR estimation results

```
. var dlnpind_euro d.ue_euro, lags(1 2 3)

Vector autoregression

Sample:  2000m5 - 2015m9                          Number of obs     =       185
Log likelihood =   975.7829                       AIC               = -10.39765
FPE            =   1.05e-07                       HQIC              = -10.29889
Det(Sigma_ml)  =   8.99e-08                       SBIC              = -10.15395

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dlnpind_euro          7     .004039   0.0641   12.67015   0.0486
D_ue_euro             7     .077409   0.4029   124.8098   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dlnpind_euro |
dlnpind_euro |
         L1. |   .0393011   .0773214     0.51   0.611     -.112246    .1908483
         L2. |  -.2281727   .0747801    -3.05   0.002    -.3747389   -.0816064
         L3. |  -.0917675   .0771614    -1.19   0.234    -.2430011     .059466
             |
     ue_euro |
         LD. |  -.0015937   .0038003    -0.42   0.675    -.0090422    .0058548
        L2D. |   .0027843   .0034209     0.81   0.416    -.0039206    .0094892
        L3D. |  -.0011178   .0038034    -0.29   0.769    -.0085723    .0063366
             |
       _cons |   .0018605   .0003637     5.12   0.000     .0011478    .0025733
-------------+----------------------------------------------------------------
D_ue_euro    |
dlnpind_euro |
         L1. |   .6617554   1.481956     0.45   0.655    -2.242825    3.566336
         L2. |   .1285749   1.433248     0.09   0.929     -2.68054     2.93769
         L3. |  -.9287684   1.478889    -0.63   0.530    -3.827338    1.969801
             |
     ue_euro |
         LD. |   .1499964   .0728378     2.06   0.039     .0072368    .2927559
        L2D. |    .452871   .0655663     6.91   0.000     .3243634    .5813785
        L3D. |   .1552676   .0728961     2.13   0.033     .0123939    .2981414
             |
       _cons |   .0023387   .0069703     0.34   0.737    -.0113229    .0160004
------------------------------------------------------------------------------
```

VAR estimation results



VAR estimation results



Granger causality

- Do lagged values of X predict Y_t, or the other way round?
- $H0$: the coefficients on all the lagged values of X in the Y regression are jointly insignificant.
- Granger causality $\neq$ causality.
- Does it make sense to assume the future does not affect the past?

VAR on inflation and UE

Stata code

```
var dlnpind_euro d.ue_euro
vargranger
```

Testing Granger causality

```
. vargranger
```

Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
dlnpind_euro	D.ue_euro	.63417	2	0.728
dlnpind_euro	ALL	.63417	2	0.728
D_ue_euro	dlnpind_euro	.66253	2	0.718
D_ue_euro	ALL	.66253	2	0.718