CS-E4075 - Special Course in Machine Learning, Data Science and Artificial Intelligence D: Signed graphs: spectral theory and applications

Community detection

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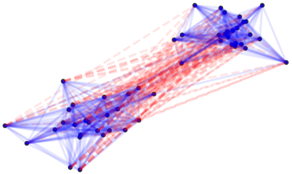
Aalto University 2021

- We have seen methods to detect “good” partitions.



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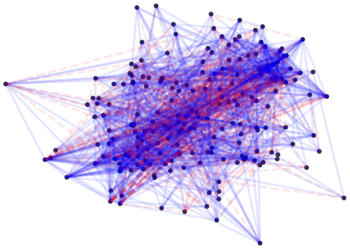
- But how about the case when a good partition is “hidden”?



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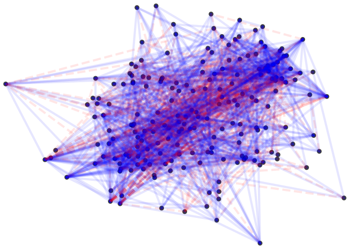

We want a method that:

- Finds a subgraph that can be partitioned well if there is one,
- tries to maximize the size of the subgraph,
- discards vertices that disagree with the parititon.

Recall:

Consider a correlation clustering instance $G = (V, E^-, E^+)$, and a partition $V = C_1 \cup C_2$.

Let A be the adjacency matrix of G.

Let x be the partition indicator vector, i.e.

$$x_i = \begin{cases} 1 & \text{if } v_i \in C_1 \\ -1 & \text{if } v_i \in C_2. \end{cases}$$

Then $x^T A x = \text{agreements} - \text{disagreements}$.

Maximizing $x^T A x$, $x \in \{-1, 1\}$ is equivalent to 2-correlation-clustering (in opt.).

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Alternative formulation:

Maximize $\frac{x^T A x}{x^T x}$, $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

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Some properties of this formulation:

- Upper-bound: $\lambda_{\max}(A)$.
- Generalizes Densest Subgraph.
- NP-hard.

Maximize $\frac{x^T A x}{x^T x}$, $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

Algorithm:

- Compute v, the leading eigenvector of A.
- For every possible threshold θ, build x_θ so that

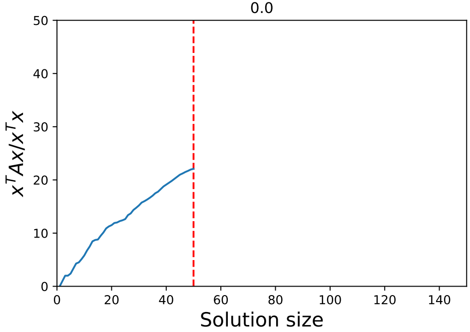
$$x_{\theta,i} = \begin{cases} \text{sgn}(v_i) & \text{if } |v_i| \geq \theta, \\ 0 & \text{otherwise.} \end{cases}$$

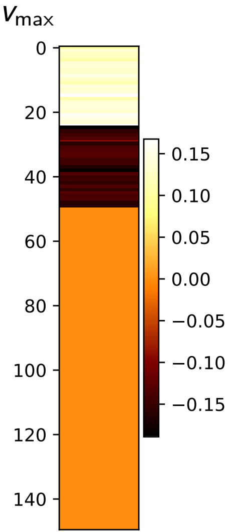
- Output x_θ that maximizes $\frac{x_\theta^T A x_\theta}{x_\theta^T x_\theta}$ over all θ.

We will do some tests with a modified stochastic block model.

- n_i: size of community i;
- η: size of the outlier set;
- p_{in}: probability an edge within communities has of existing;
- p_{out}: probability an edge between communities has of existing;
- p_{in}^-: probability an edge within communities has of being negative;
- p_{out}^-: probability an edge between communities has of being negative;
- p_n: probability an edge adjacent to an outlier has of existing;
- p_n^-: probability an edge adjacent to an outlier has of being negative;

$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 0/100,\ p_n^- = 0.25.$$





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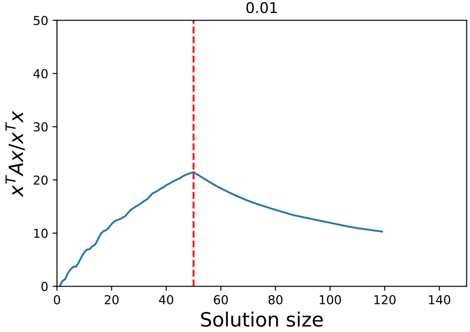
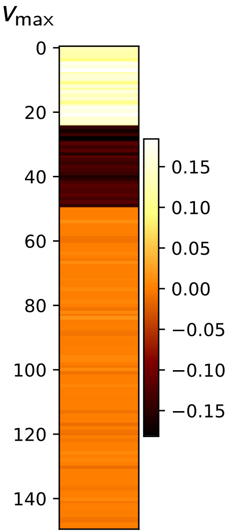
$$p_n = 1/100,\ p_n^{-} = 0.25.$$





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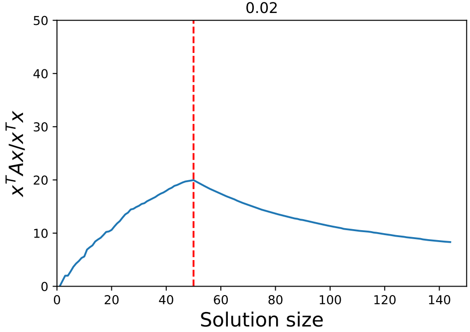

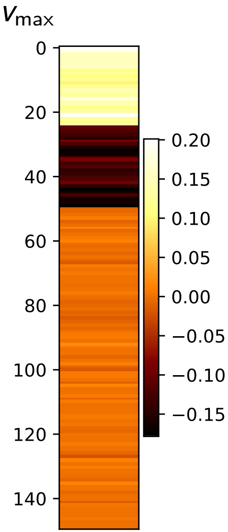

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$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$

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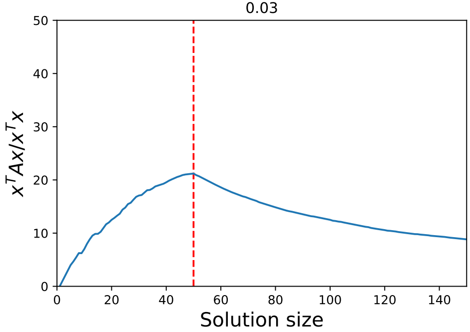
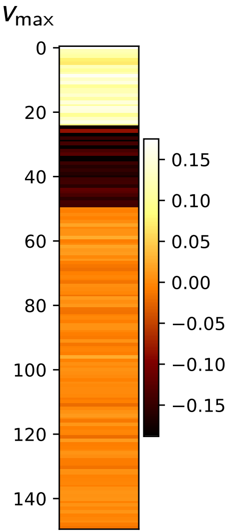
$$p_n = 3/100,\ p_n^{-} = 0.25.$$





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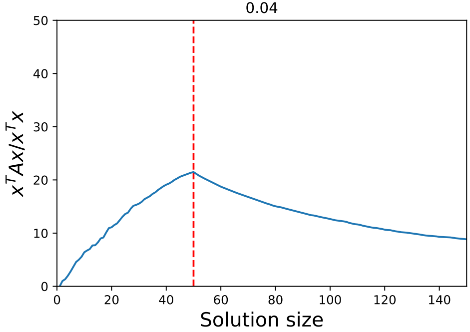

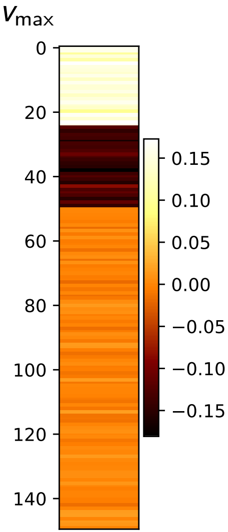

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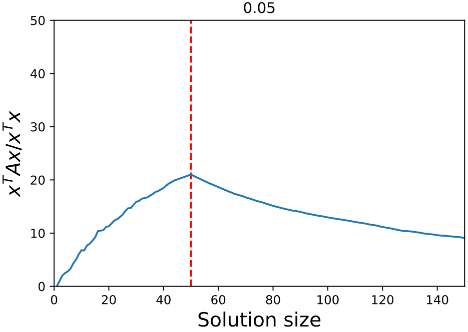

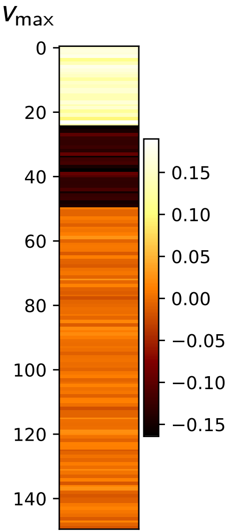

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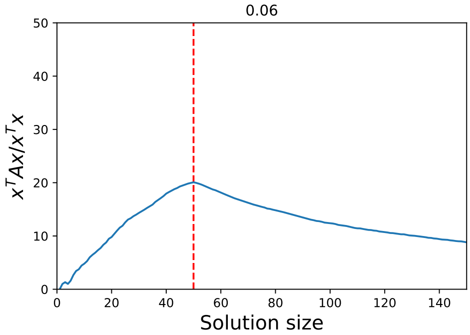

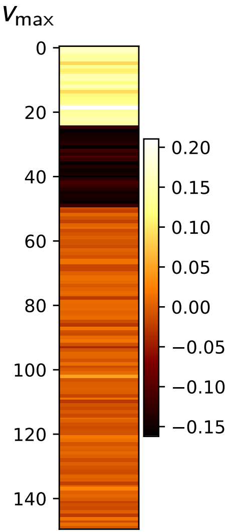

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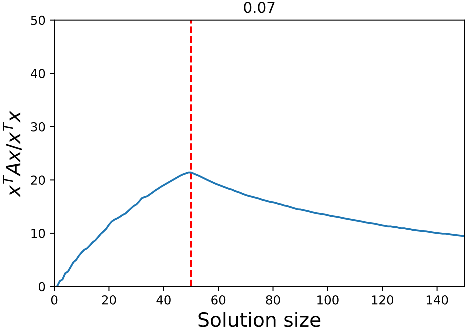

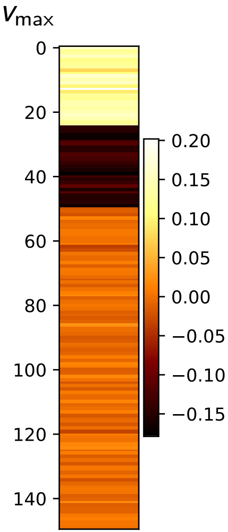

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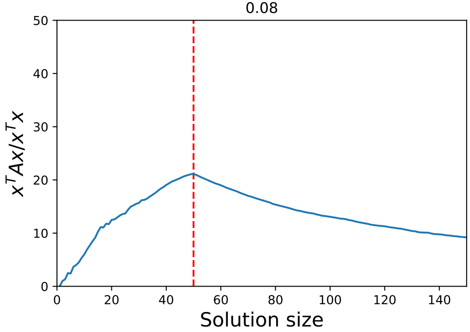

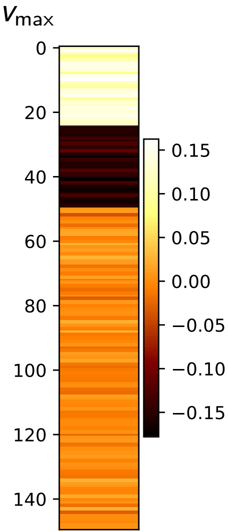

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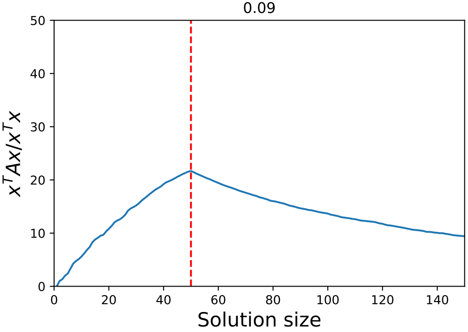

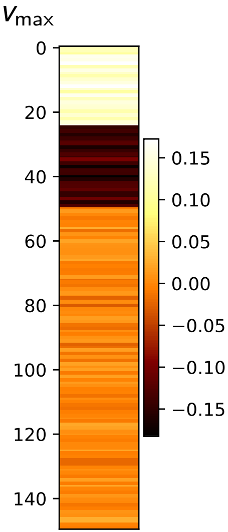

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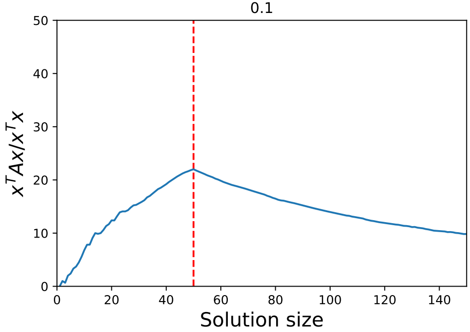

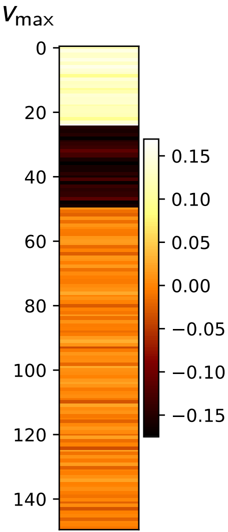

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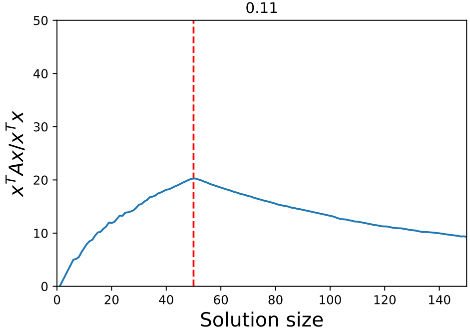

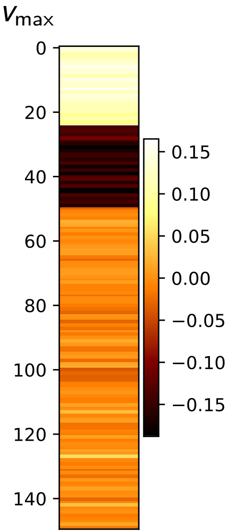

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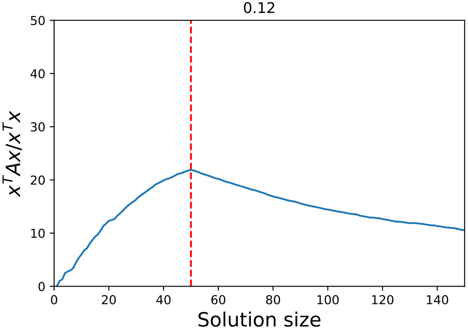
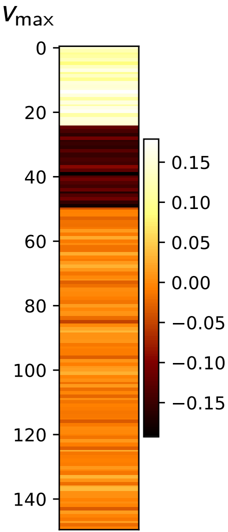
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 12/100,\ p_n^{-} = 0.25.$$





$n_1 = n_2 = 25, \eta = 100$
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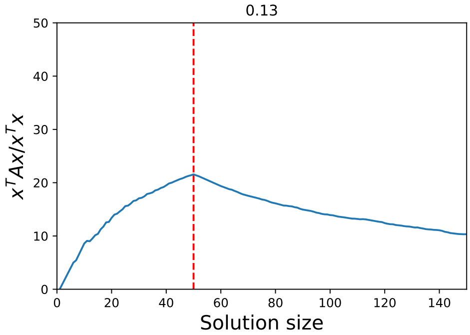
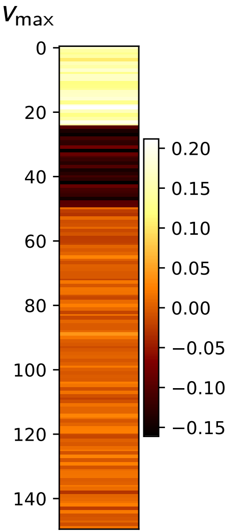
$p_n = 13/100$, $p_n^- = 0.25$.





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$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$

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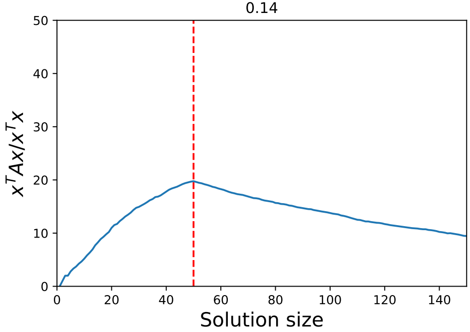
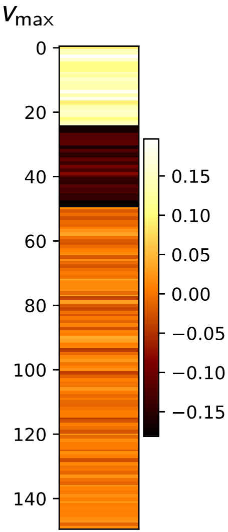
$$p_n = 14/100,\ p_n^{-} = 0.25.$$





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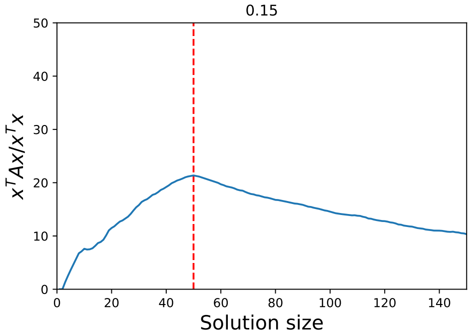
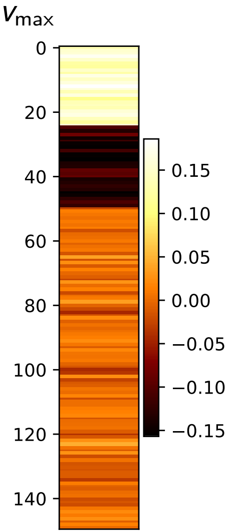
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 15/100,\ p^-_n = 0.25.$$





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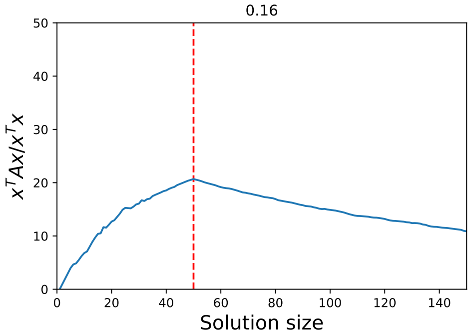
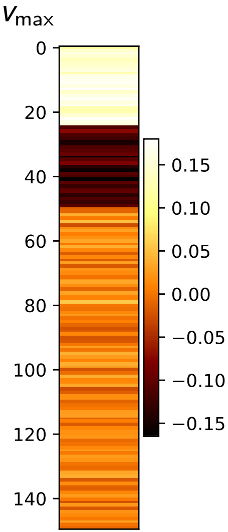
$$p_n = 16/100,\ p^-_n = 0.25.$$





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$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
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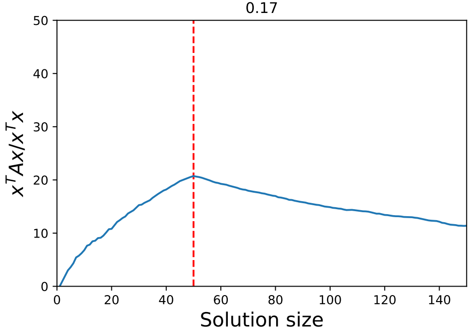
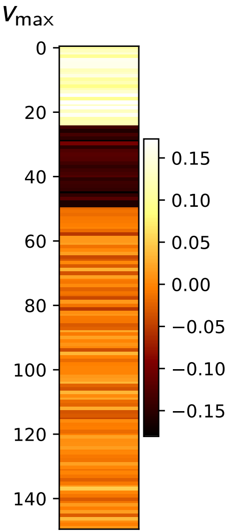
$$p_n = 17/100,\ p_n^{-} = 0.25.$$





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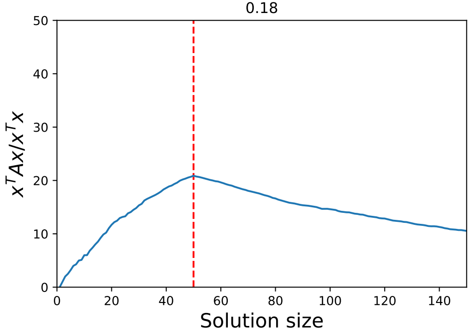
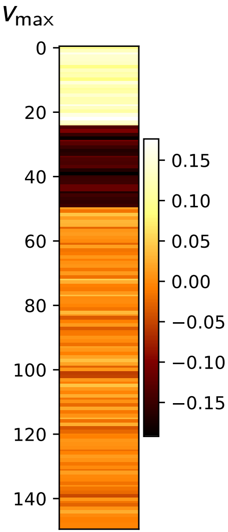
$$p_n = 18/100,\ p_n^{-} = 0.25.$$





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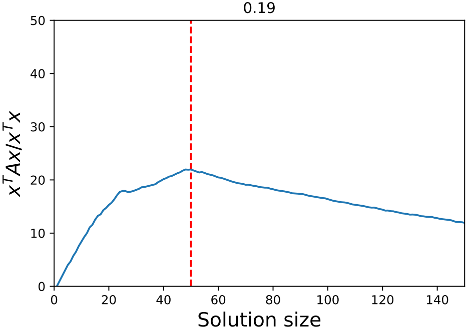
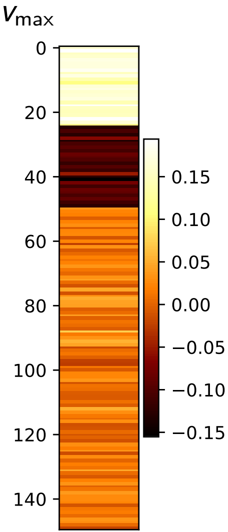
$$p_n = 19/100,\ p_n^{-} = 0.25.$$





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$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
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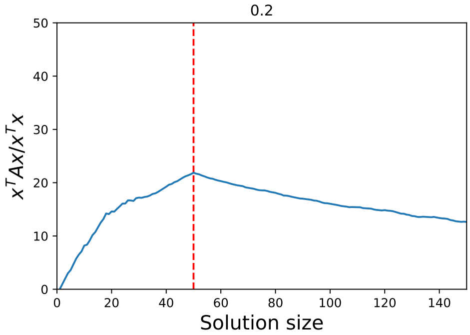
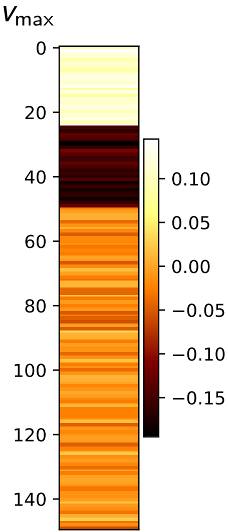
$$p_n = 20/100,\ p_n^- = 0.25.$$





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$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
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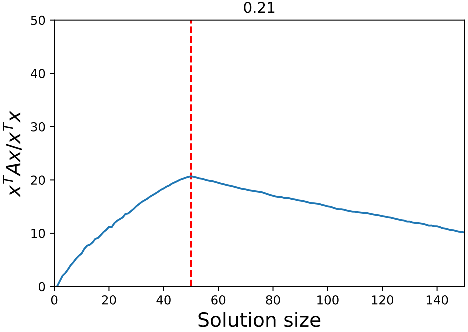
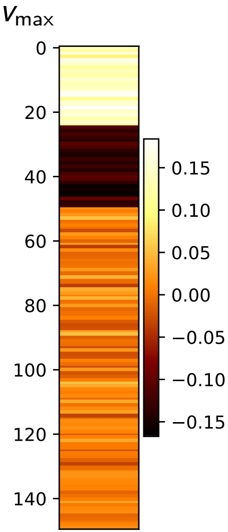
$$p_n = 21/100,\ p_n^{-} = 0.25.$$





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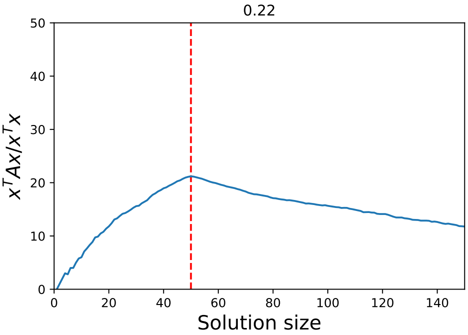
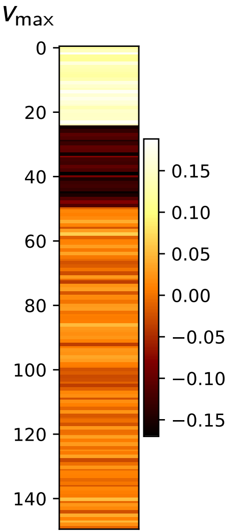
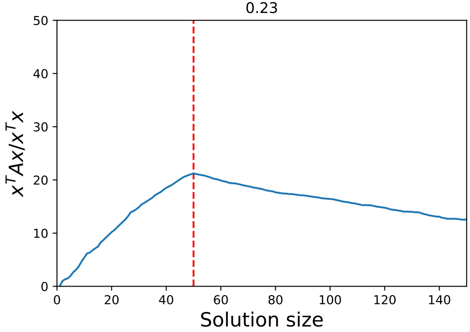
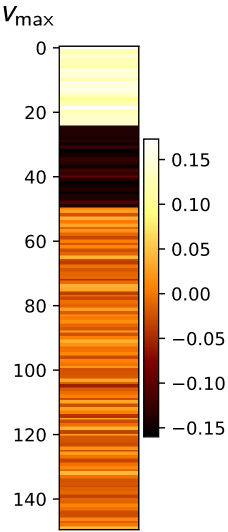
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 22/100,\ p_n^{-} = 0.25.$$





$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 23/100,\ p_n^- = 0.25.$$

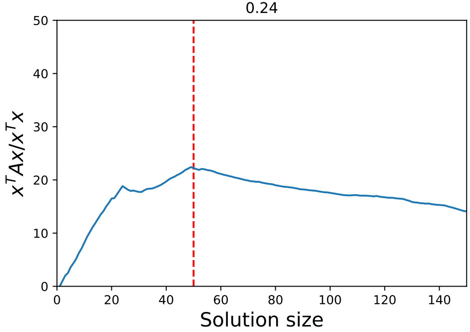


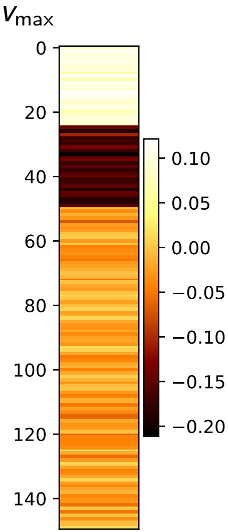


$$n_1 = n_2 = 25, \eta = 100$$

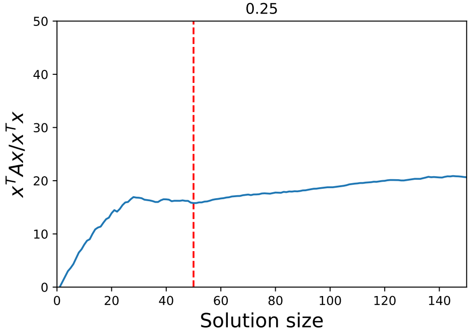
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,\ p^-_{\text{out}} = 0.9.$$

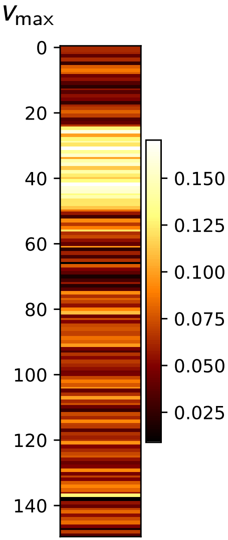
$$p_n = 24/100,\ p^-_n = 0.25.$$



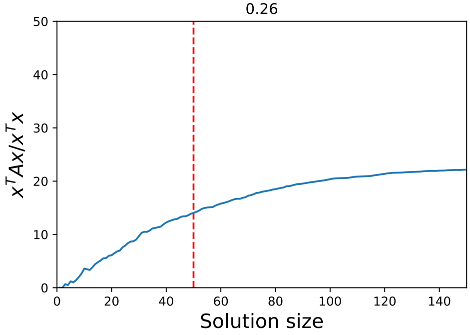


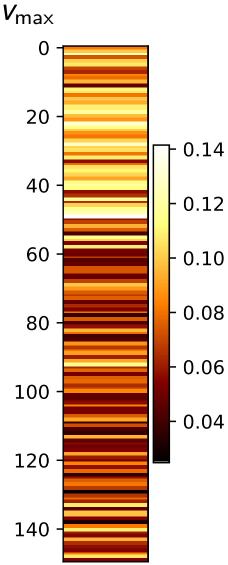
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 25/100,\ p_n^{-} = 0.25.$$



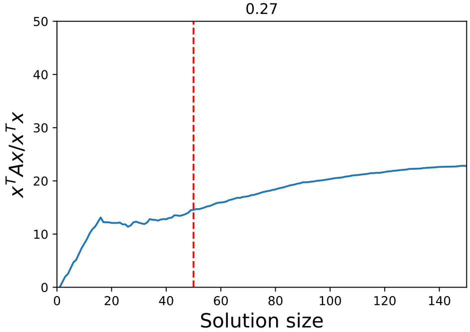


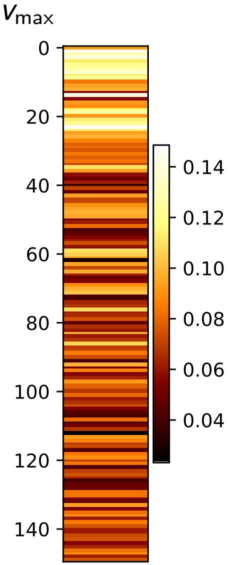
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 26/100,\ p_n^{-} = 0.25.$$



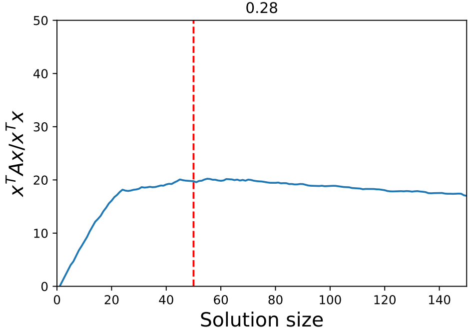


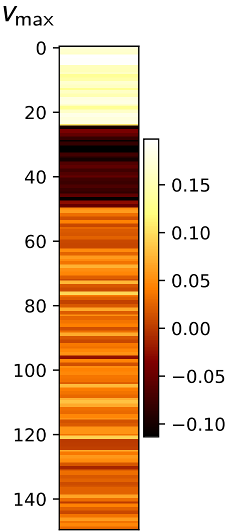
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 27/100,\ p^-_n = 0.25.$$



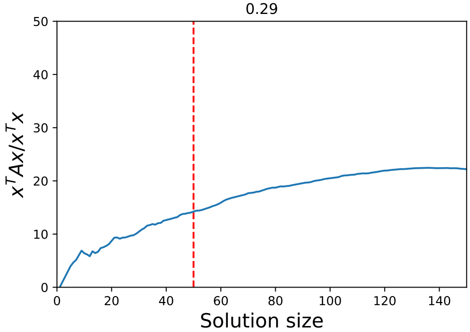


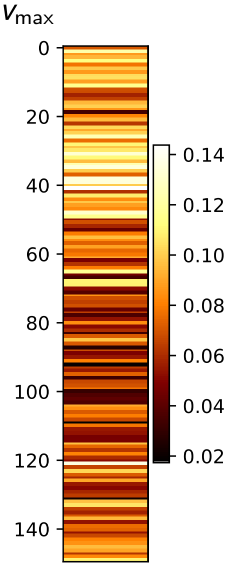
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 28/100,\ p^-_n = 0.25.$$



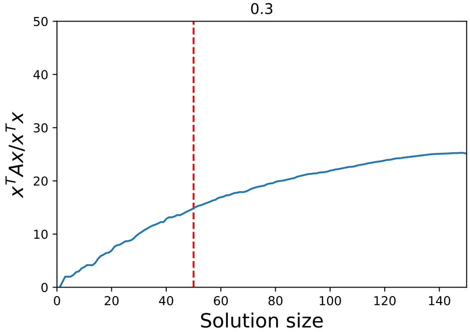


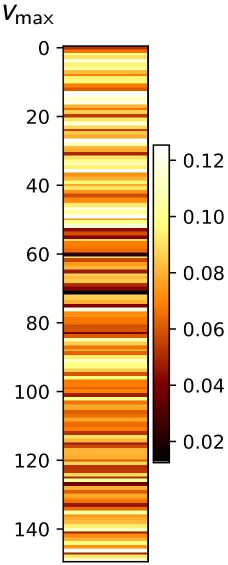
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 29/100,\ p_n^- = 0.25.$$



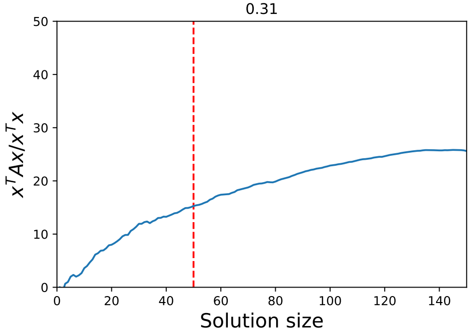


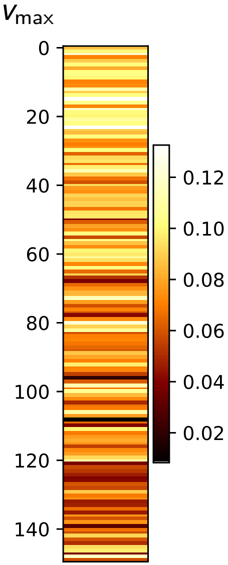
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 30/100,\ p_n^- = 0.25.$$





$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 31/100,\ p_n^- = 0.25.$$



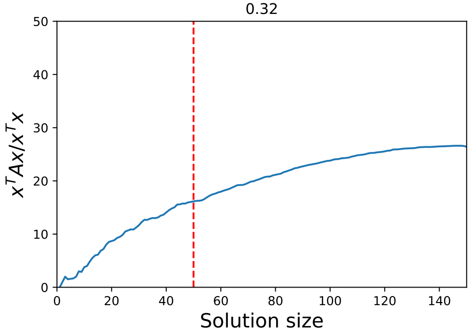


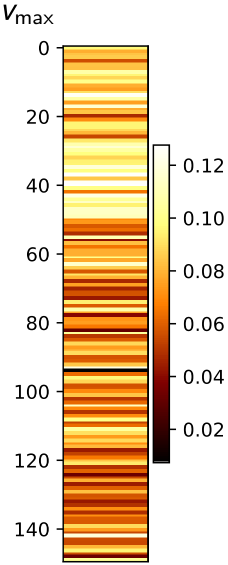
$$n_1 = n_2 = 25, \eta = 100$$

$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$

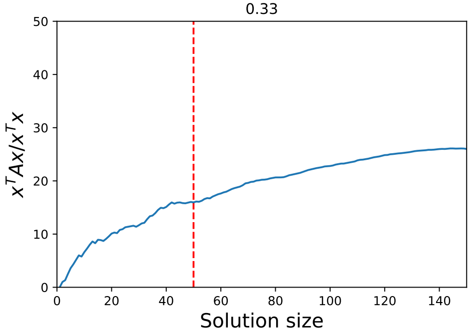
$$p_{\text{out}}^{-} = 0.9.$$

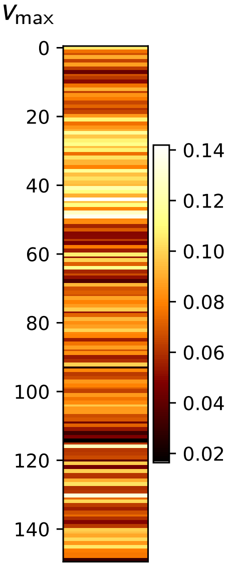
$$p_n = 32/100,\ p_n^{-} = 0.25.$$



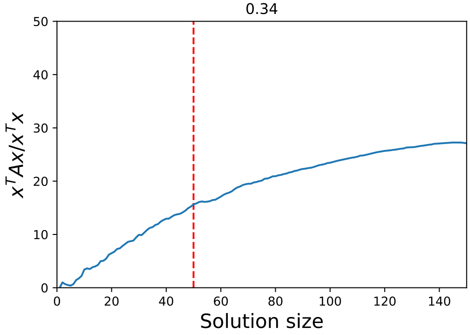


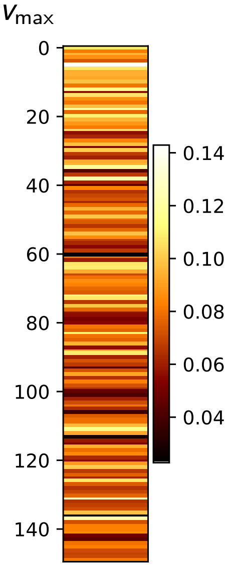
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 33/100,\ p_n^{-} = 0.25.$$



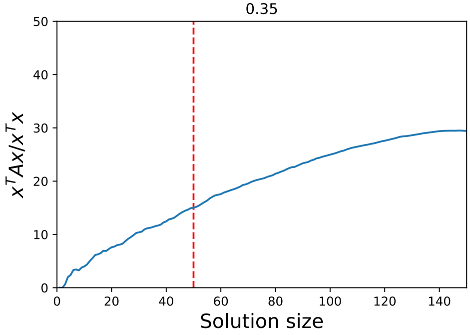


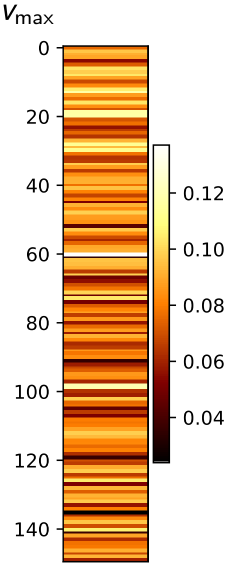
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 34/100,\ p_n^- = 0.25.$$



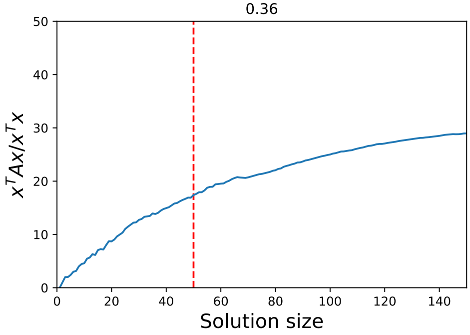


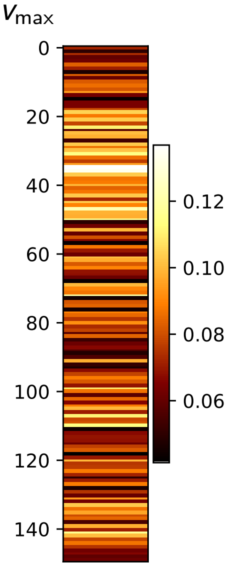
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 35/100,\ p_n^{-} = 0.25.$$





$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 36/100,\ p_n^- = 0.25.$$



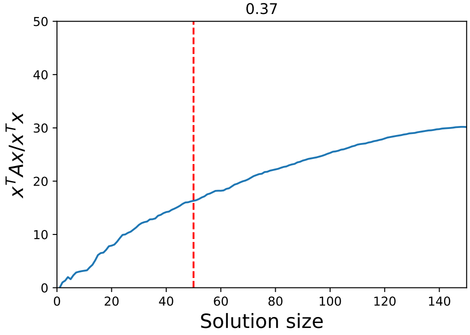


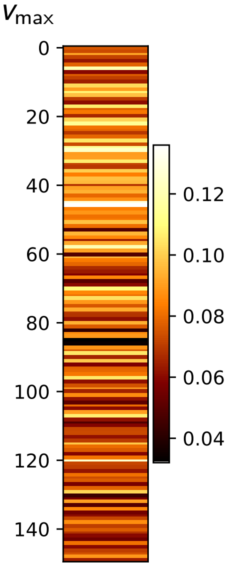
$$n_1 = n_2 = 25, \eta = 100$$

$$p_{in} = 0.8,\ p_{out} = 0.2,\ p_{in}^{-} = 0.05,$$

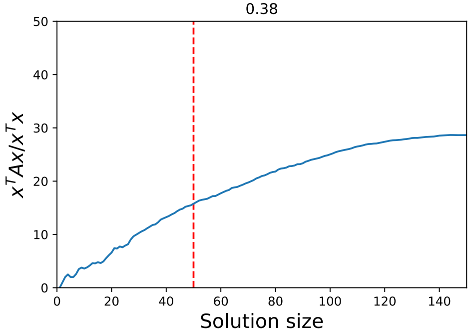
$$p_{out}^{-} = 0.9.$$

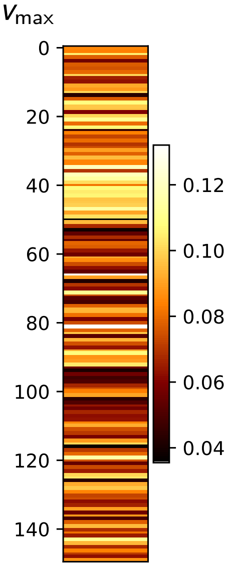
$$p_n = 37/100,\ p_n^{-} = 0.25.$$



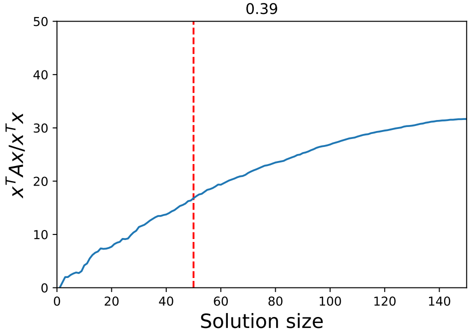


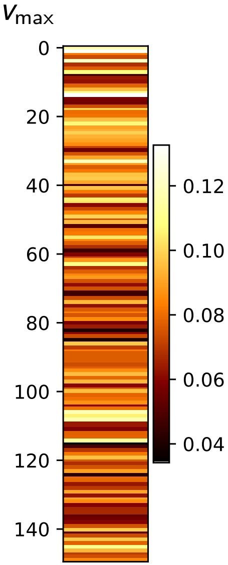
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 38/100,\ p_n^{-} = 0.25.$$



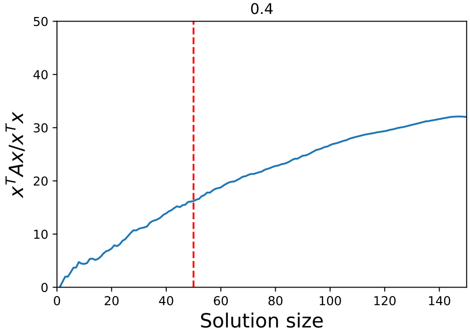


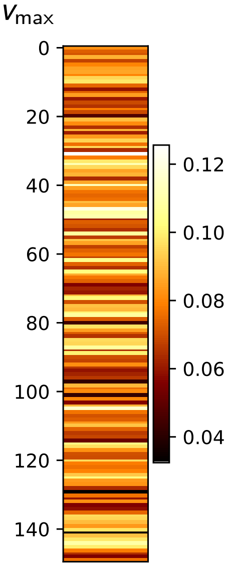
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 39/100,\ p_n^{-} = 0.25.$$



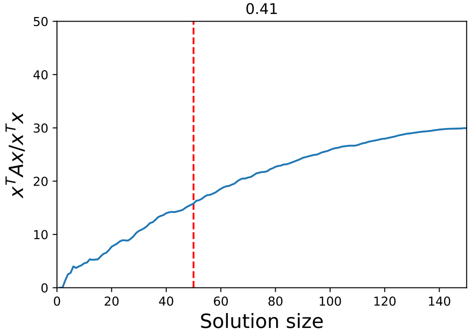


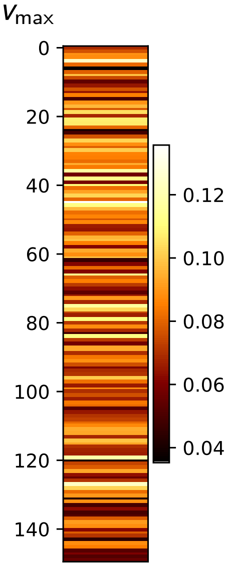
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 40/100,\ p^-_n = 0.25.$$



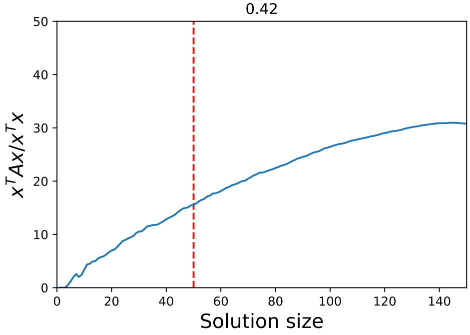


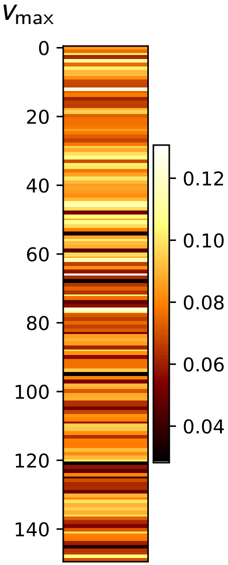
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 41/100,\ p^-_n = 0.25.$$



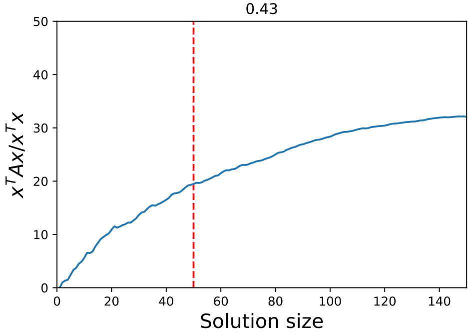


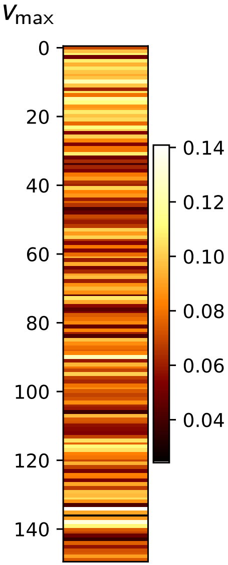
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 42/100,\ p_n^- = 0.25.$$



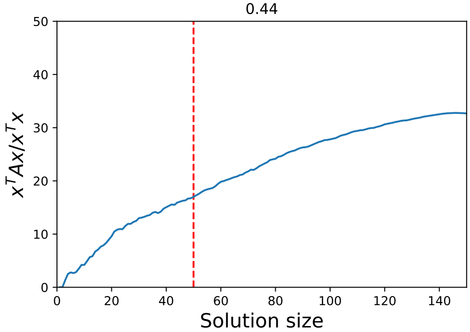


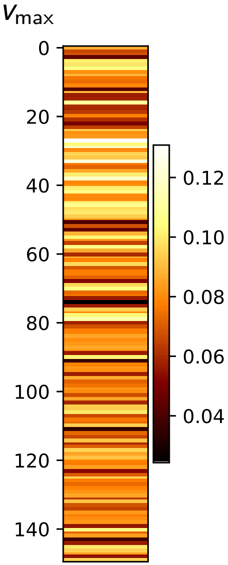
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 43/100,\ p_n^{-} = 0.25.$$



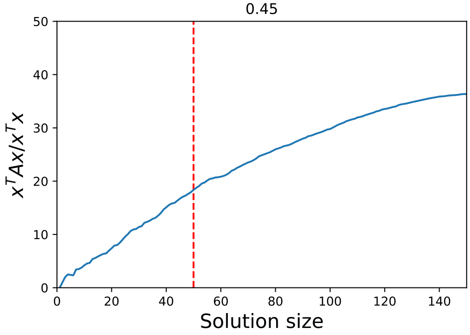


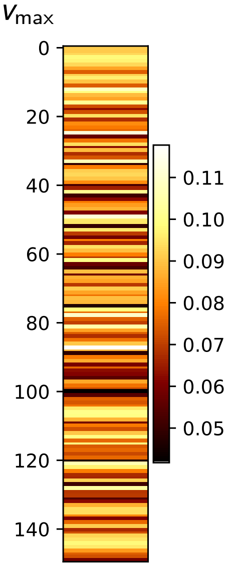
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{in} = 0.8,\ p_{out} = 0.2,\ p_{in}^- = 0.05,$$
$$p_{out}^- = 0.9.$$
$$p_n = 44/100,\ p_n^- = 0.25.$$



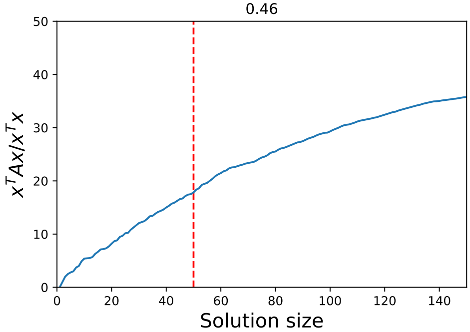


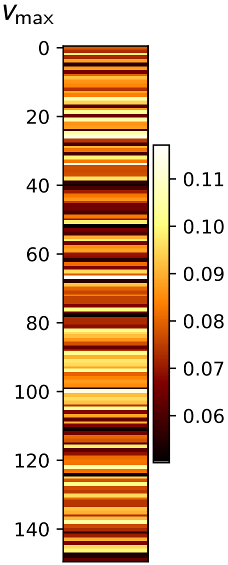
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 45/100,\ p^-_n = 0.25.$$



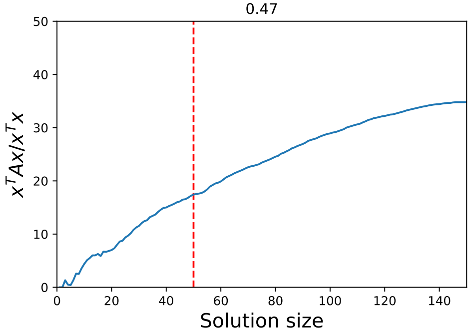


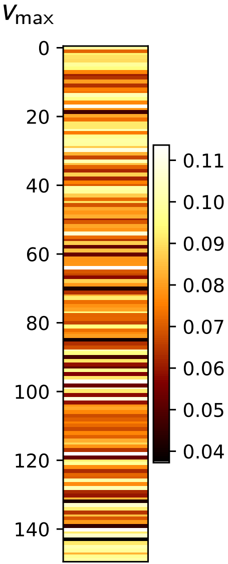
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 46/100,\ p_n^{-} = 0.25.$$



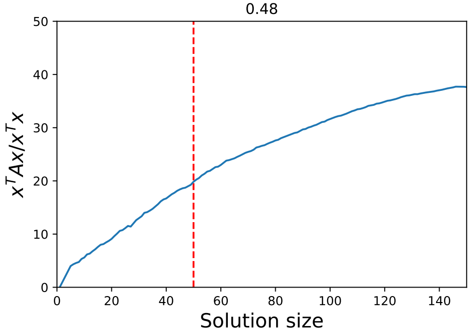


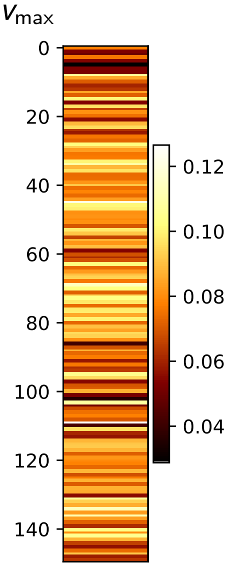
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 47/100,\ p_n^{-} = 0.25.$$



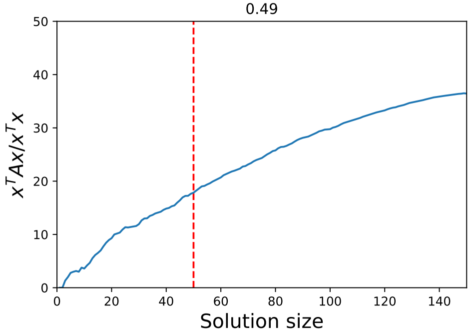


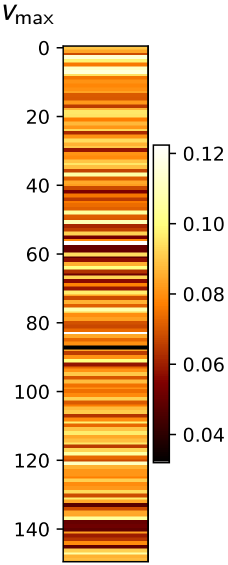
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 48/100,\ p^-_n = 0.25.$$



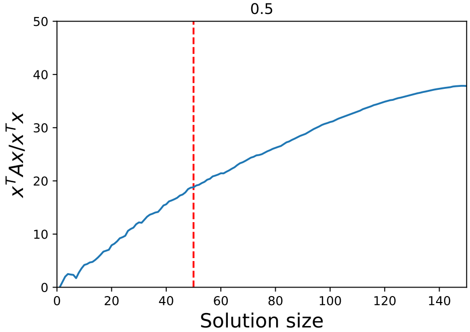


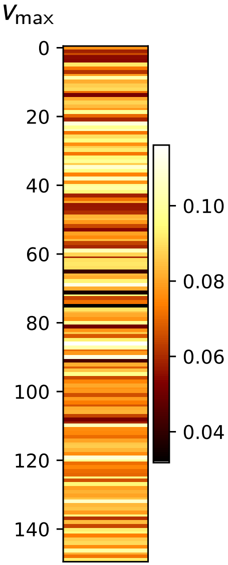
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 49/100,\ p_n^- = 0.25.$$



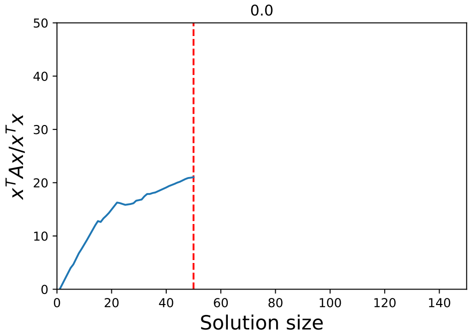


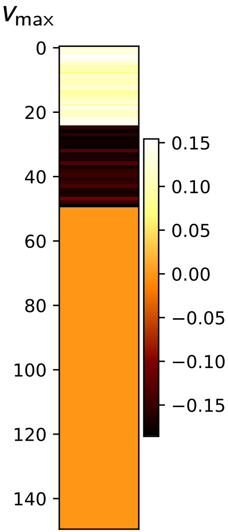
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 50/100,\ p_n^{-} = 0.25.$$



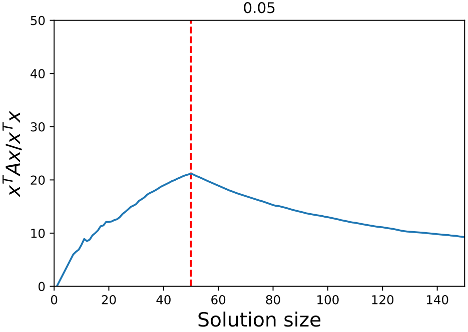


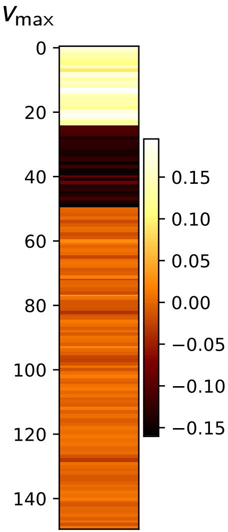
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 0/100,\ p_n^{-} = 0.5.$$



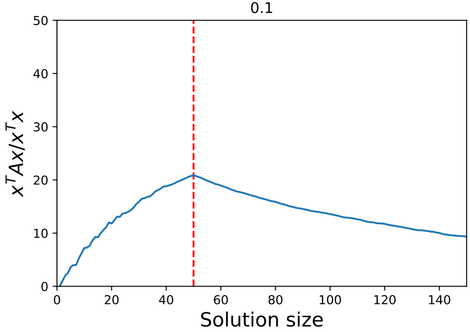


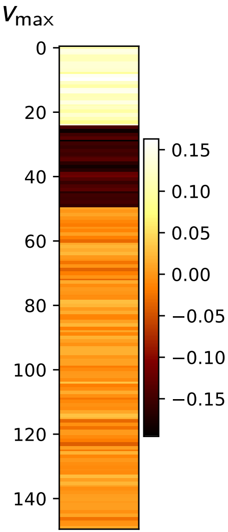
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 5/100,\ p_n^{-} = 0.5.$$



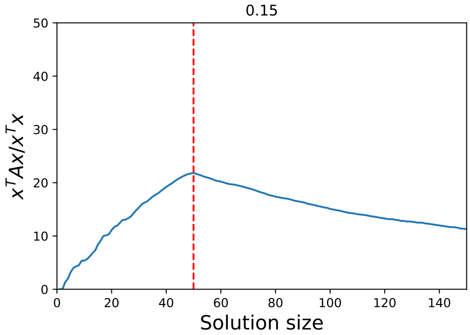


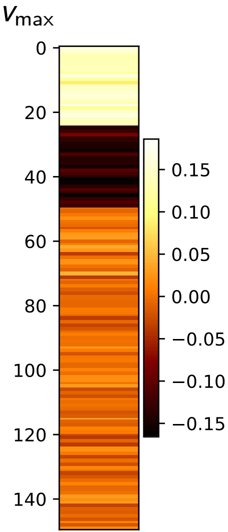
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 10/100,\ p_n^{-} = 0.5.$$



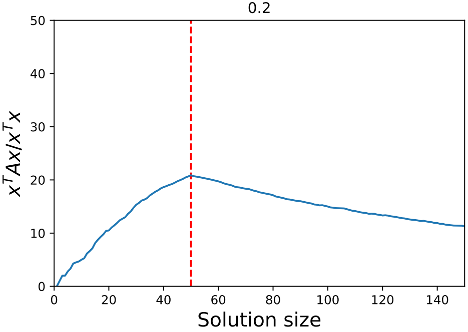


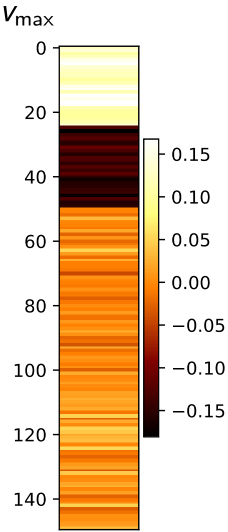
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{in} = 0.8,\ p_{out} = 0.2,\ p^-_{in} = 0.05,$$
$$p^-_{out} = 0.9.$$
$$p_n = 15/100,\ p^-_n = 0.5.$$



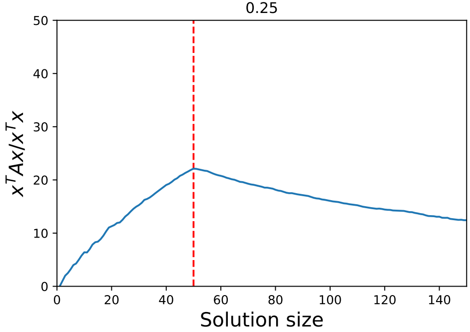


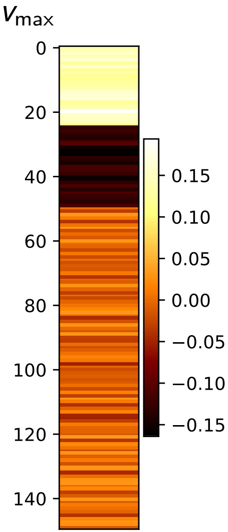
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 20/100,\ p_n^{-} = 0.5.$$



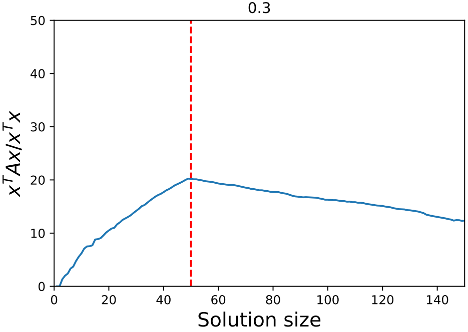


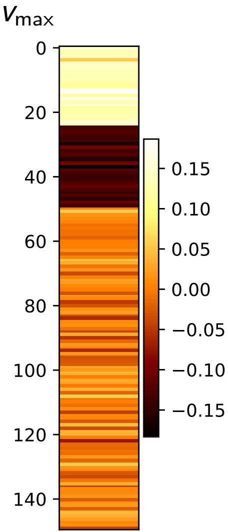
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 25/100,\ p_n^- = 0.5.$$



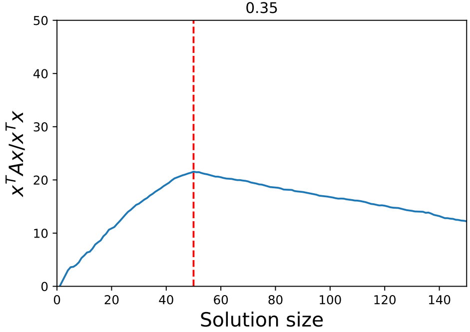


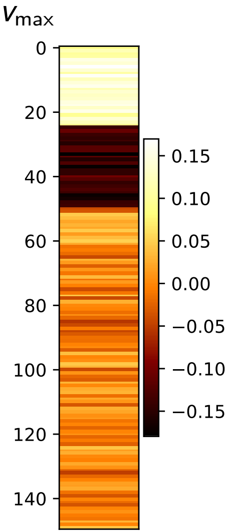
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 30/100,\ p_n^- = 0.5.$$



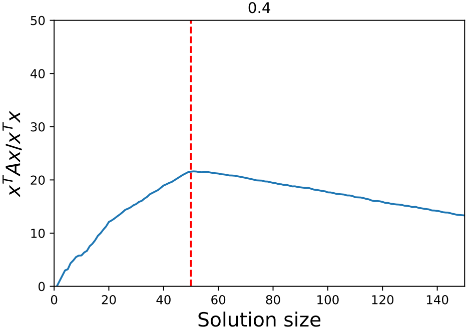


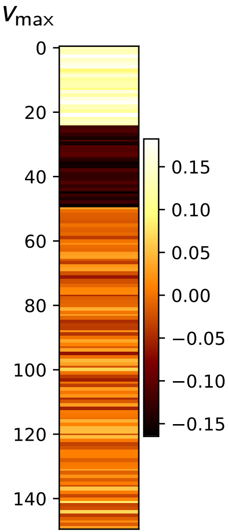
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 35/100,\ p_n^- = 0.5.$$



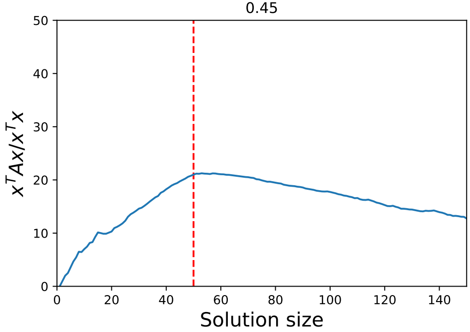


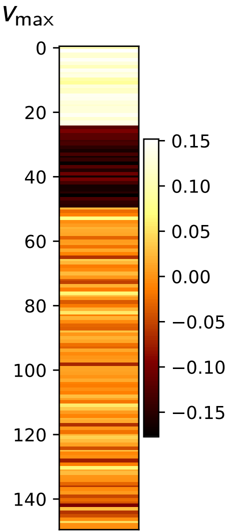
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 40/100,\ p_n^- = 0.5.$$



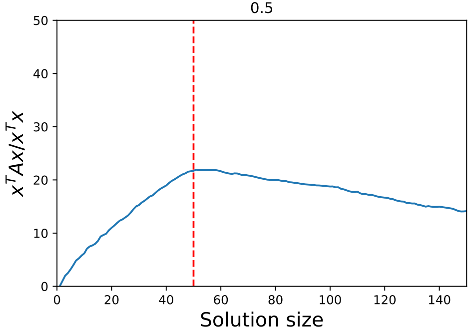


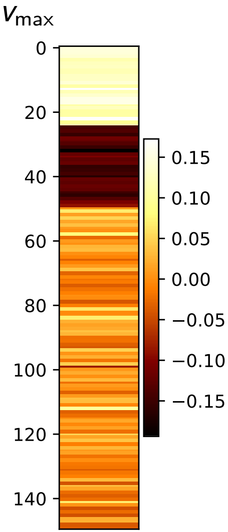
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 45/100,\ p_n^- = 0.5.$$





$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 50/100,\ p^-_n = 0.5.$$



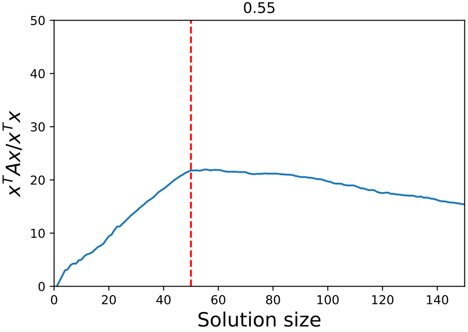


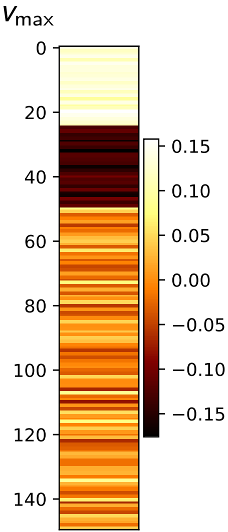
$n_1 = n_2 = 25, \eta = 100$

$p_{\text{in}} = 0.8$, $p_{\text{out}} = 0.2$, $p_{\text{in}}^- = 0.05$,

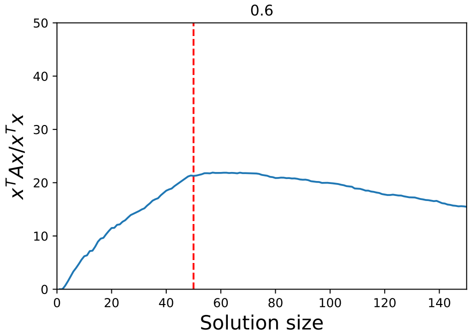
$p_{\text{out}}^- = 0.9$.

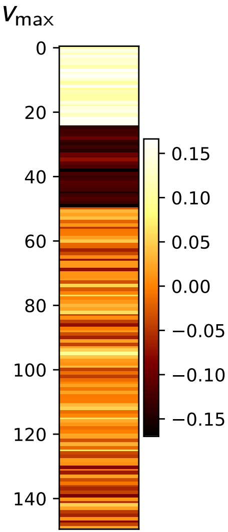
$p_n = 55/100$, $p_n^- = 0.5$.



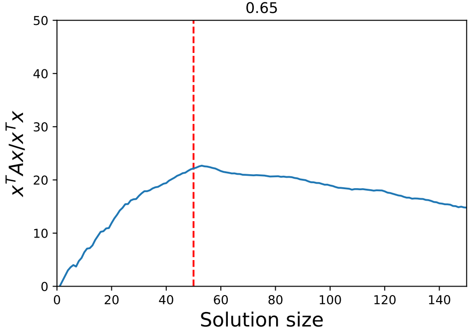


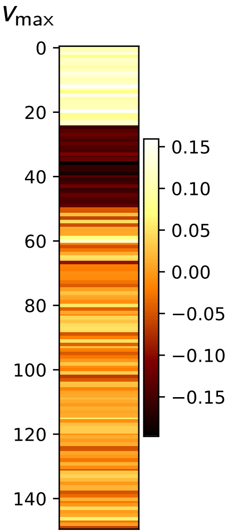
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 60/100,\ p_n^{-} = 0.5.$$



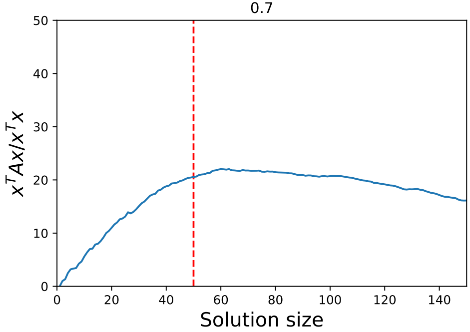


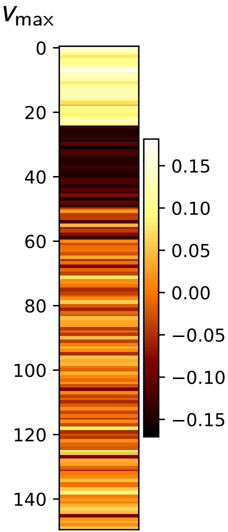
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 65/100,\ p_n^{-} = 0.5.$$



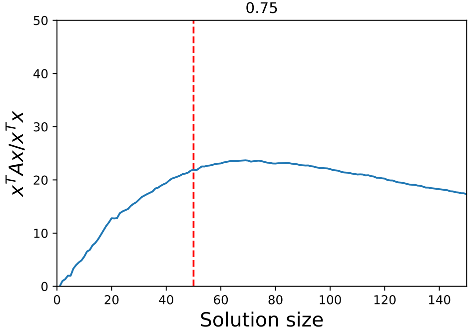


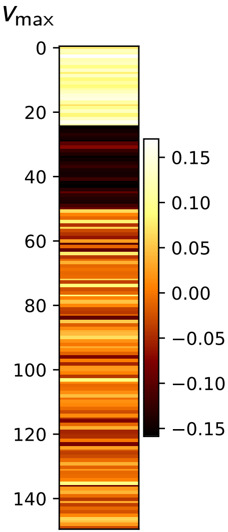
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 70/100,\ p^-_n = 0.5.$$



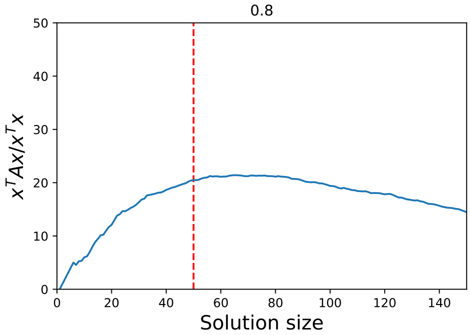


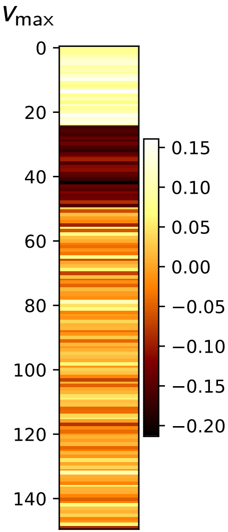
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 75/100,\ p_n^{-} = 0.5.$$



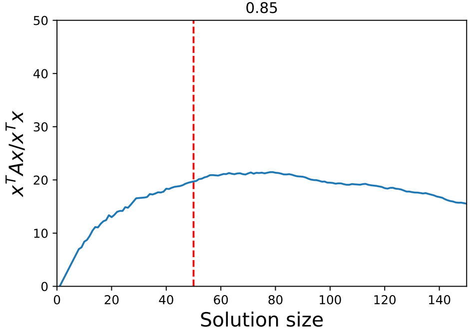


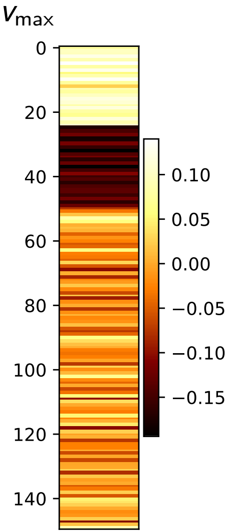
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 80/100,\ p_n^{-} = 0.5.$$



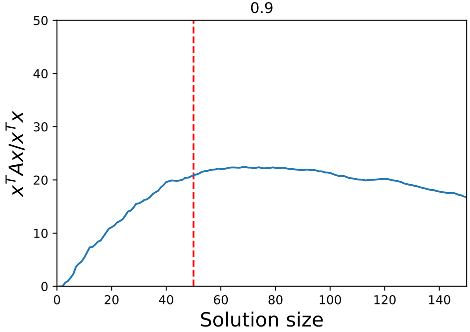


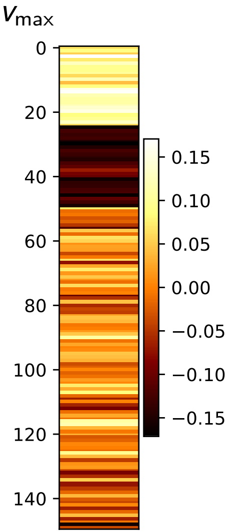
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,\ p^-_{\text{out}} = 0.9.$$
$$p_n = 85/100,\ p^-_n = 0.5.$$



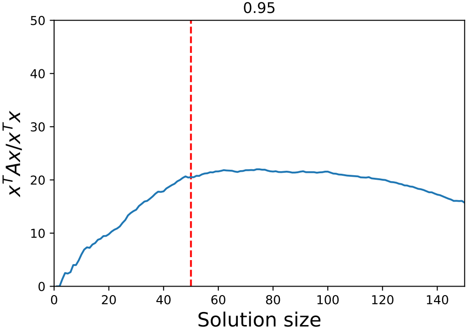


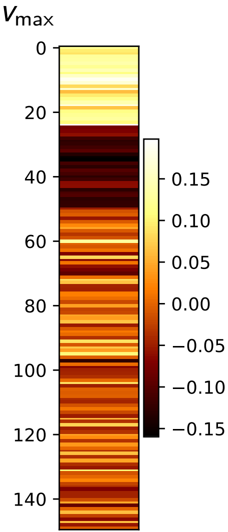
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 90/100,\ p_n^{-} = 0.5.$$



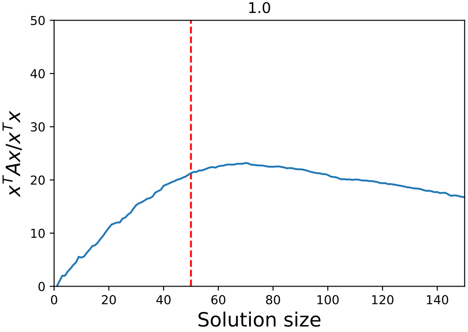


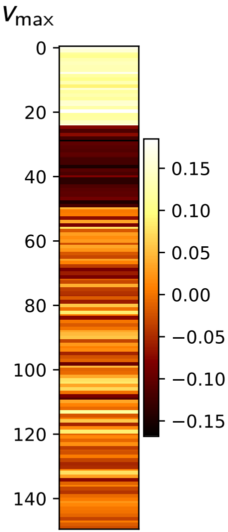
$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 95/100,\ p^-_n = 0.5.$$





$$n_1 = n_2 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 100/100,\ p^-_n = 0.5.$$





Maximize $\frac{x^T A x}{x^T x}$. $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

Randomized algorithm:

- Compute v, the leading eigenvector of A.
- Set

$$x_i = \begin{cases} \mathrm{sgn}(v_i) & \text{with probability } |v_i|, \\ 0 & \text{with probability } 1 - |v_i|. \end{cases}$$

Properties:

Maximize $\frac{x^T Ax}{x^T x}$. $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

Randomized algorithm:

- Compute v, the leading eigenvector of A.
- Set

$$x_i = \begin{cases} \text{sgn}(v_i) & \text{with probability } |v_i|, \\ 0 & \text{with probability } 1 - |v_i|. \end{cases}$$

Properties:

- $\mathbb{E}[x] = v_{\text{max}}$.

Maximize $\frac{x^T A x}{x^T x}$. $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

Randomized algorithm:

- Compute v, the leading eigenvector of A.
- Set

$$x_i = \begin{cases} \text{sgn}(v_i) & \text{with probability } |v_i|, \\ 0 & \text{with probability } 1 - |v_i|. \end{cases}$$

Properties:

- $\mathbb{E}[x] = v_{\max}$.
- $\mathbb{E}\left[\frac{x^T A x}{x^T x}\right] = \Omega(\lambda_{\max}/\sqrt{n})$.

Maximize $\frac{x^T A x}{x^T x}$. $x \in \{-1, 0, 1\}^n \backslash \{0\}^n$.

Randomized algorithm:

- Compute v, the leading eigenvector of A.
- Set

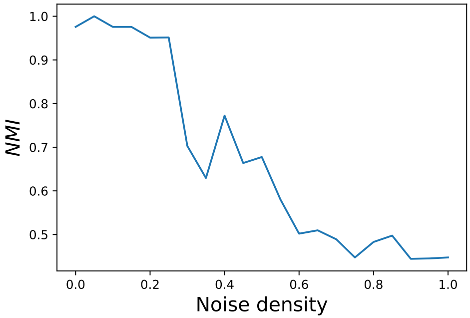
$$x_i = \begin{cases} \operatorname{sgn}(v_i) & \text{with probability } |v_i|, \\ 0 & \text{with probability } 1 - |v_i|. \end{cases}$$

Properties:

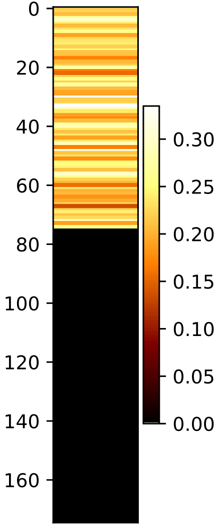
- $\mathbb{E}[x] = v_{\max}$.
- $\mathbb{E}\left[\frac{x^T A x}{x^T x}\right] = \Omega(\lambda_{\max}/\sqrt{n})$.
- There exist instances where $OPT = \mathcal{O}(\lambda_{\max}/\sqrt{n})$.

For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 0/100,\ p^-_n = 0.5.$$

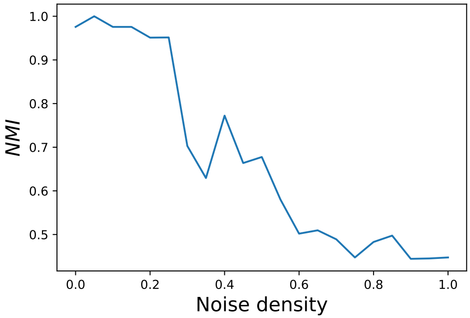


Row norms of matrix of leading k eigenvectors:

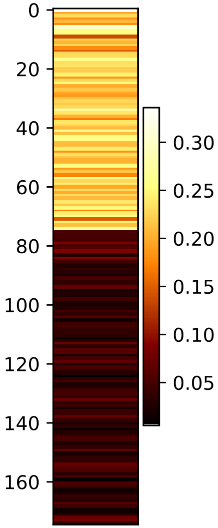


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 5/100,\ p_n^{-} = 0.5.$$

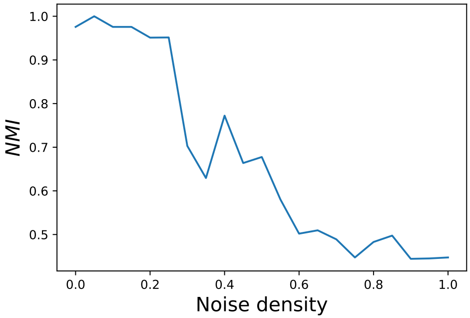


Row norms of matrix of leading k eigenvectors:

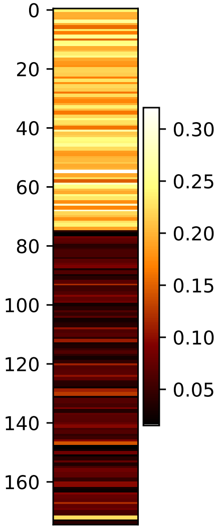


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 10/100,\ p^-_n = 0.5.$$

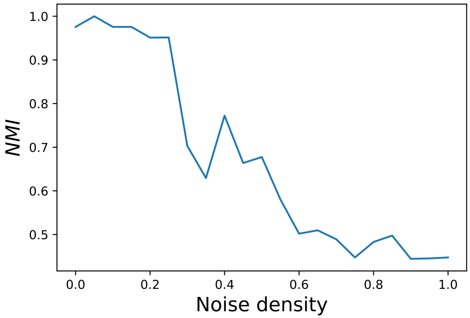


Row norms of matrix of leading k eigenvectors:

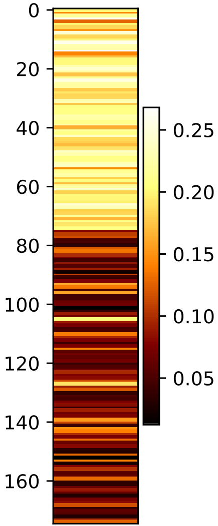


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 15/100,\ p_n^{-} = 0.5.$$

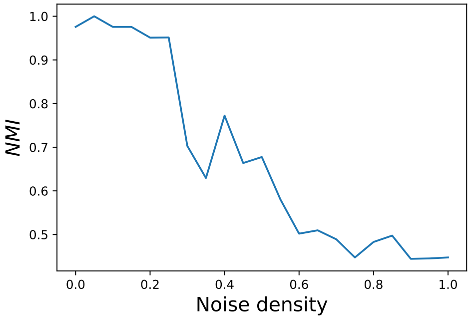


Row norms of matrix of leading k eigenvectors:

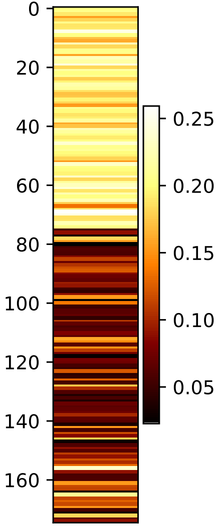


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 20/100,\ p_n^- = 0.5.$$

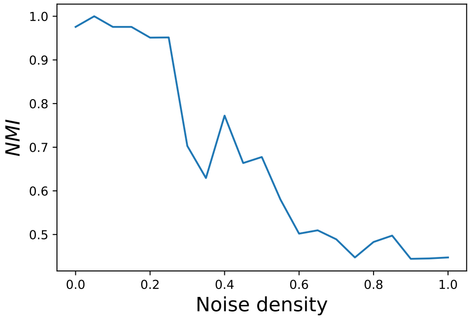


Row norms of matrix of leading k eigenvectors:

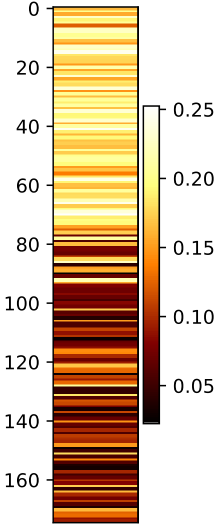


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 25/100,\ p^-_n = 0.5.$$

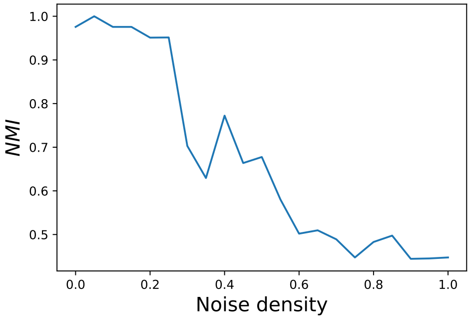


Row norms of matrix of leading k eigenvectors:

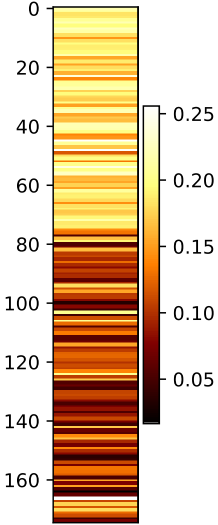


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 30/100,\ p_n^{-} = 0.5.$$

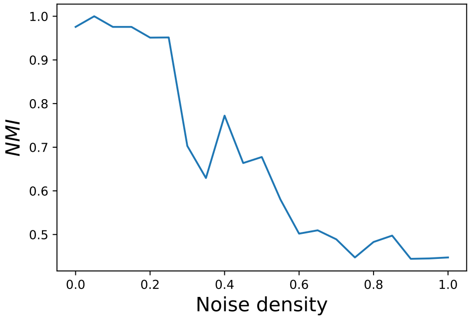


Row norms of matrix of leading k eigenvectors:

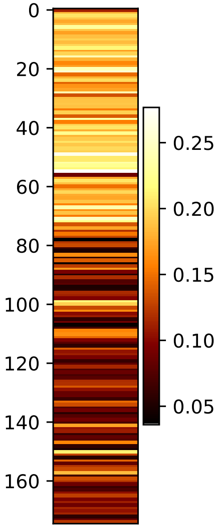


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 35/100,\ p_n^- = 0.5.$$

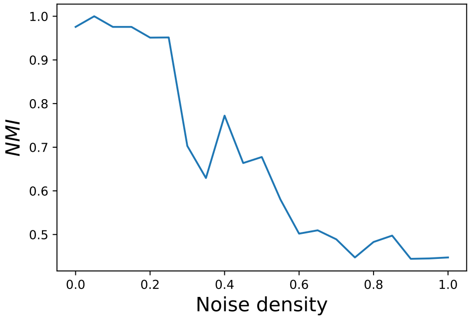


Row norms of matrix of leading k eigenvectors:

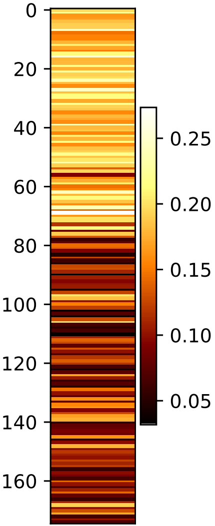


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 40/100,\ p_n^{-} = 0.5.$$

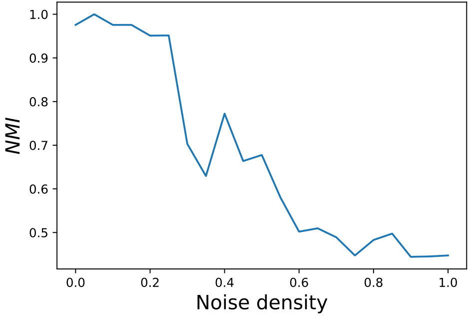


Row norms of matrix of leading k eigenvectors:

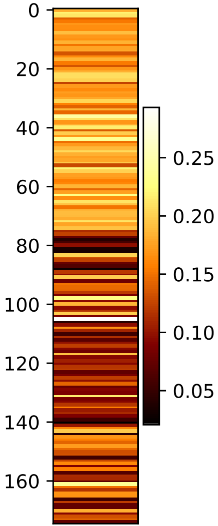


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{in} = 0.8,\ p_{out} = 0.2,\ p_{in}^- = 0.05,$$
$$p_{out}^- = 0.9.$$
$$p_n = 45/100,\ p_n^- = 0.5.$$

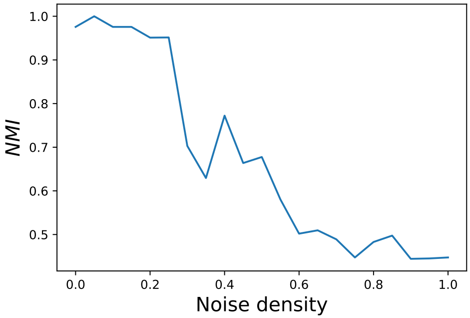


Row norms of matrix of leading k eigenvectors:

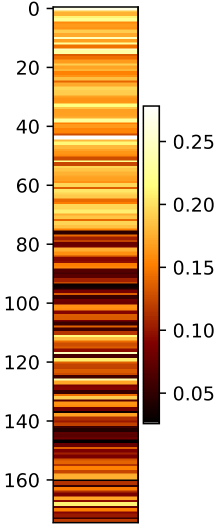


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{in} = 0.8, p_{out} = 0.2, p^-_{in} = 0.05,$$
$$p^-_{out} = 0.9.$$
$$p_n = 50/100, p^-_n = 0.5.$$

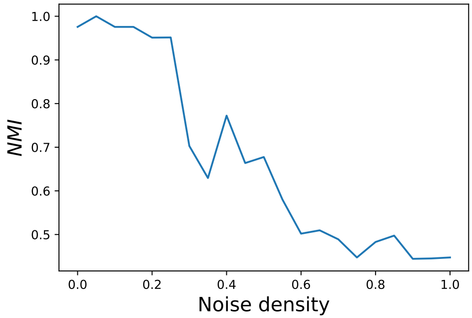


Row norms of matrix of leading k eigenvectors:

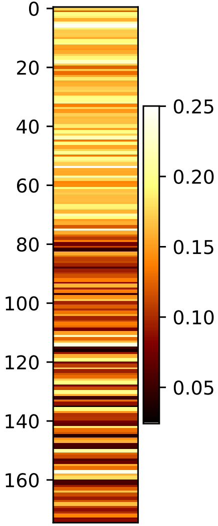


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 55/100,\ p^-_n = 0.5.$$

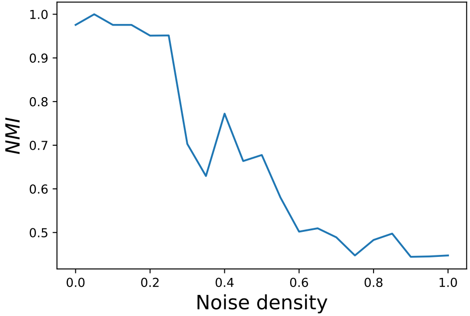


Row norms of matrix of leading k eigenvectors:

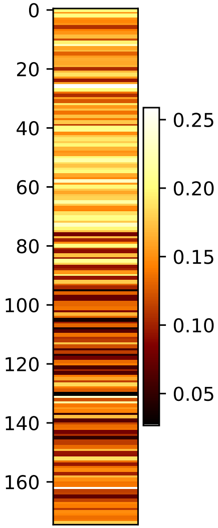


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 60/100,\ p^-_n = 0.5.$$

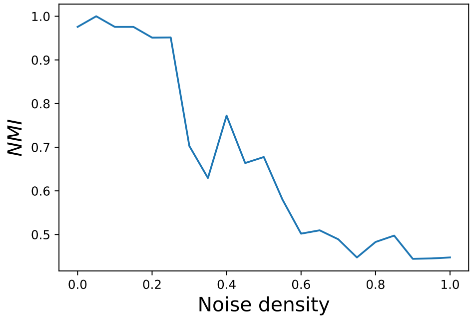


Row norms of matrix of leading k eigenvectors:

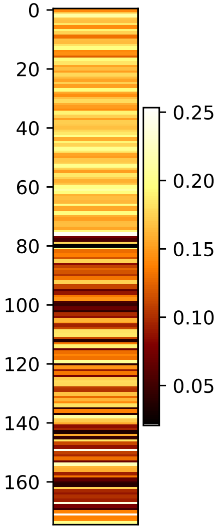


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 65/100,\ p_n^{-} = 0.5.$$

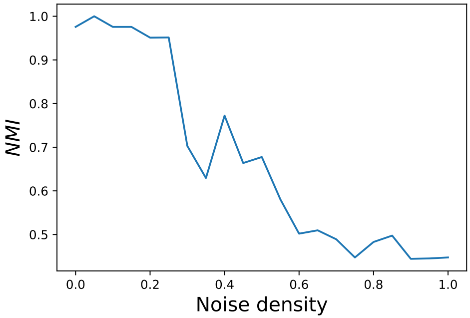


Row norms of matrix of leading k eigenvectors:

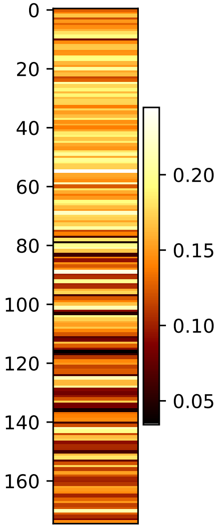


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 70/100,\ p_n^{-} = 0.5.$$

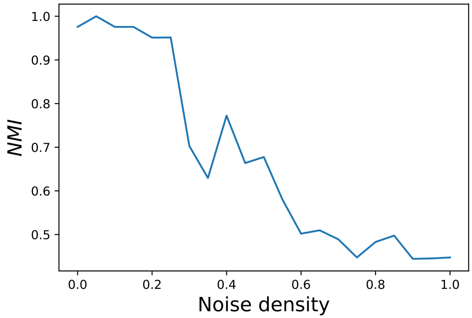


Row norms of matrix of leading k eigenvectors:

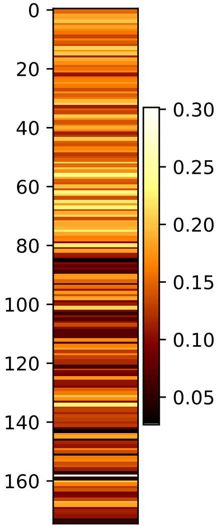


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 75/100,\ p_n^- = 0.5.$$

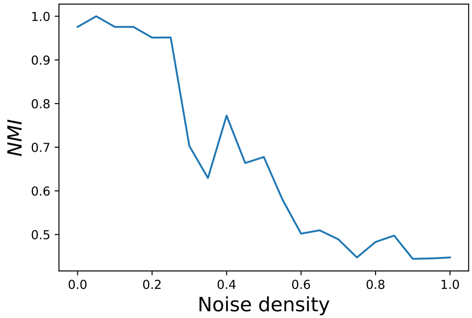


Row norms of matrix of leading k eigenvectors:

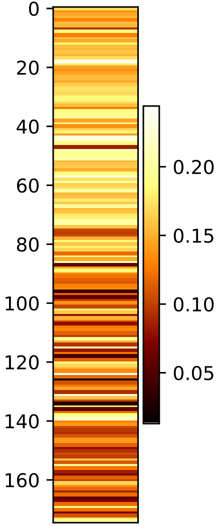


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 80/100,\ p_n^{-} = 0.5.$$

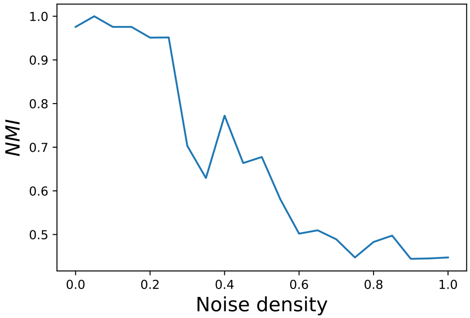


Row norms of matrix of leading k eigenvectors:

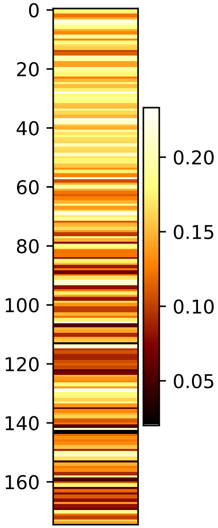


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 85/100,\ p_n^{-} = 0.5.$$

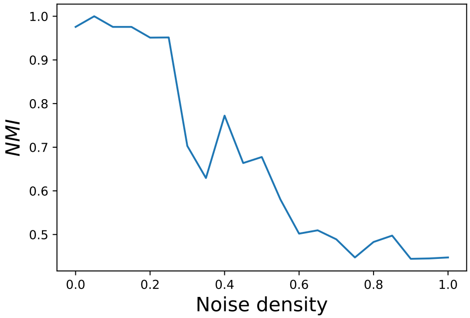


Row norms of matrix of leading k eigenvectors:

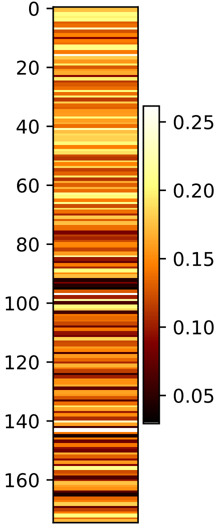


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^{-}_{\text{in}} = 0.05,$$
$$p^{-}_{\text{out}} = 0.9.$$
$$p_n = 90/100,\ p^{-}_n = 0.5.$$

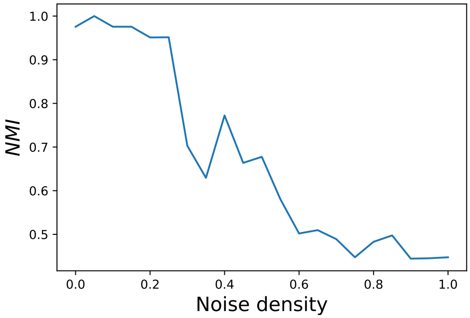


Row norms of matrix of leading k eigenvectors:

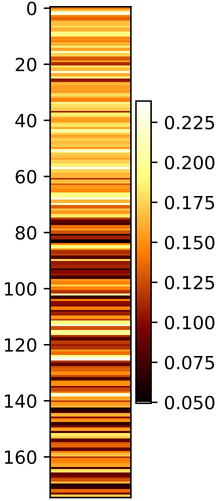


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 95/100,\ p^-_n = 0.5.$$

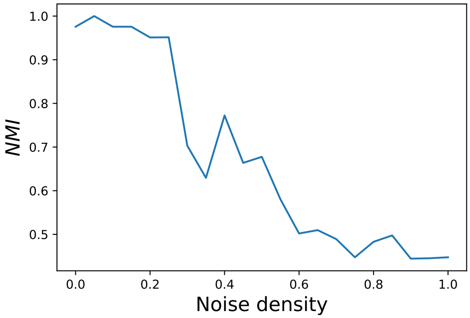


Row norms of matrix of leading k eigenvectors:

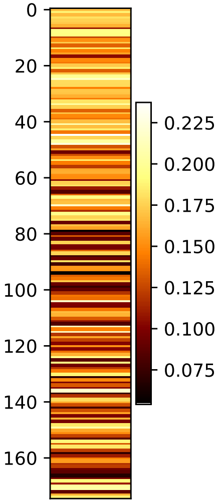


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 100/100,\ p_n^- = 0.5.$$

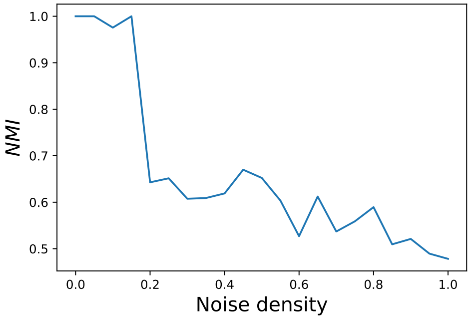


Row norms of matrix of leading k eigenvectors:

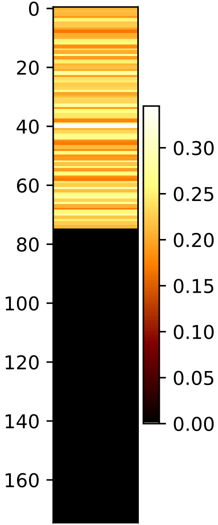


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 0/100,\ p_n^{-} = 0.25.$$

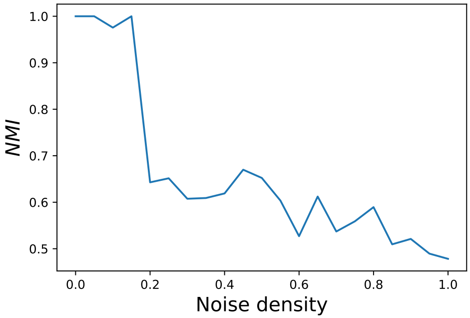


Row norms of matrix of leading k eigenvectors:

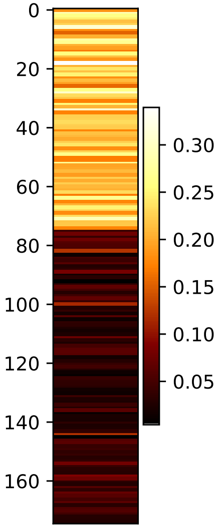


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 5/100,\ p_n^- = 0.25.$$

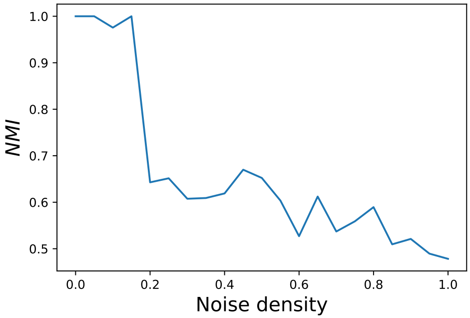


Row norms of matrix of leading k eigenvectors:

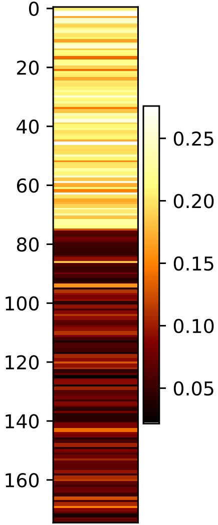


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 10/100,\ p_n^- = 0.25.$$

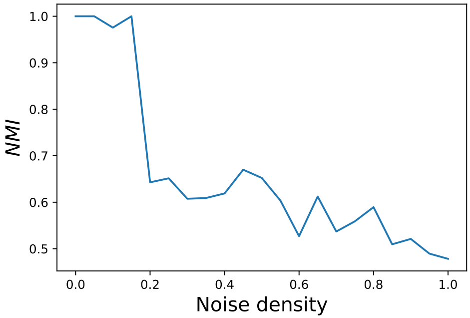


Row norms of matrix of leading k eigenvectors:

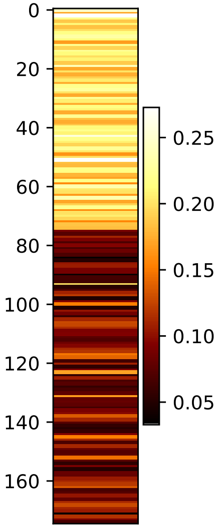


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 15/100,\ p_n^- = 0.25.$$

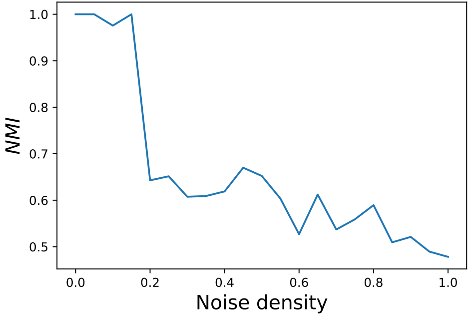


Row norms of matrix of leading k eigenvectors:

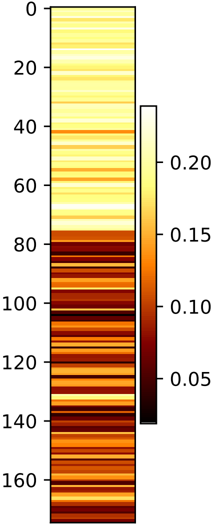


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 20/100,\ p_n^- = 0.25.$$

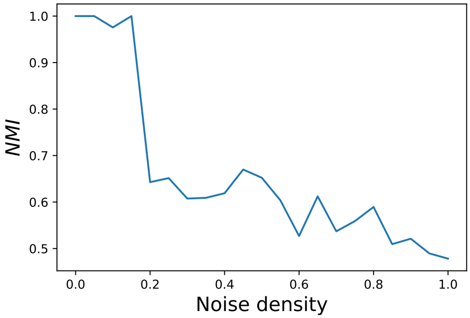


Row norms of matrix of leading k eigenvectors:

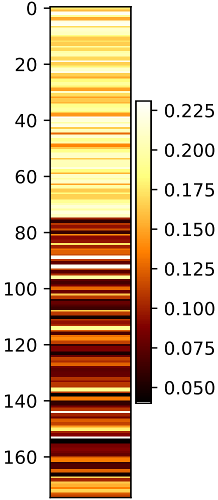


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 25/100,\ p_n^{-} = 0.25.$$

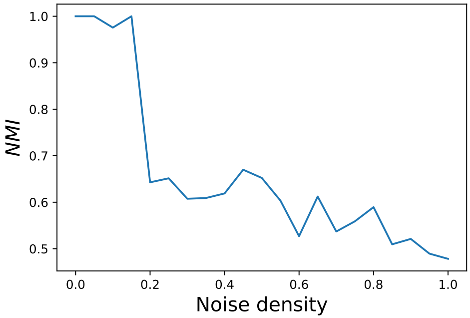


Row norms of matrix of leading k eigenvectors:

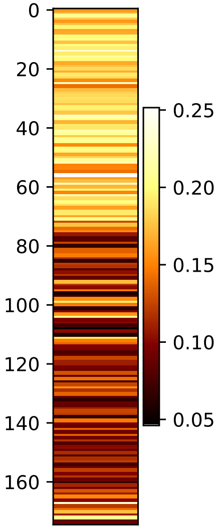


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 30/100,\ p^-_n = 0.25.$$

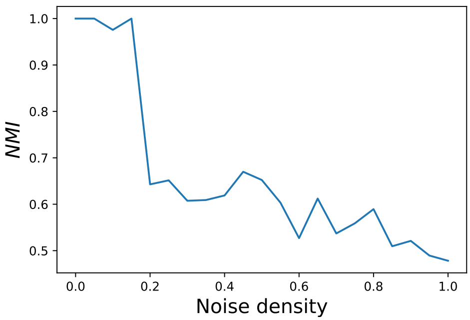


Row norms of matrix of leading k eigenvectors:

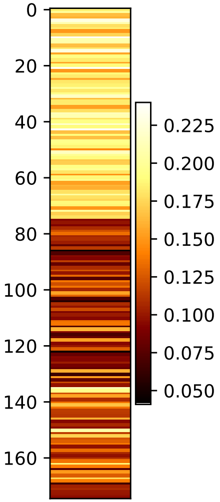


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{in} = 0.8,\ p_{out} = 0.2,\ p_{in}^{-} = 0.05,$$
$$p_{out}^{-} = 0.9.$$
$$p_n = 35/100,\ p_n^{-} = 0.25.$$

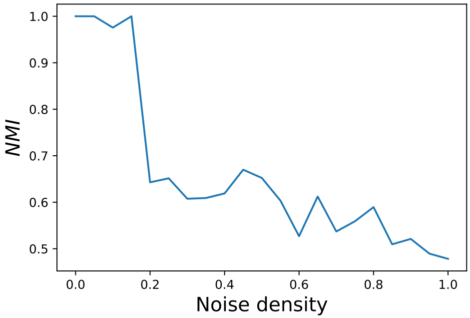


Row norms of matrix of leading k eigenvectors:

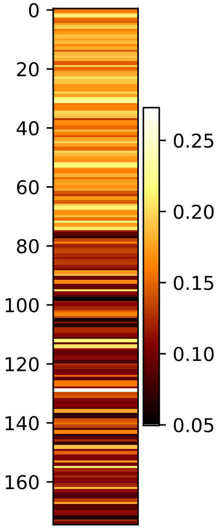


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 40/100,\ p_n^{-} = 0.25.$$

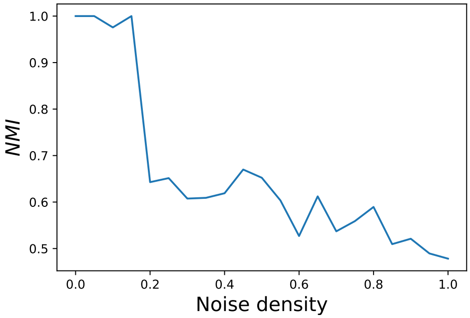


Row norms of matrix of leading k eigenvectors:

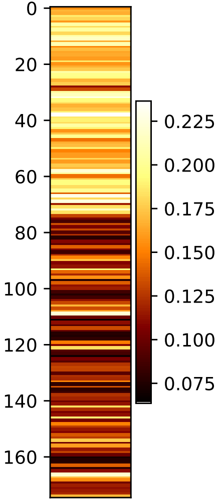


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 45/100,\ p_n^{-} = 0.25.$$

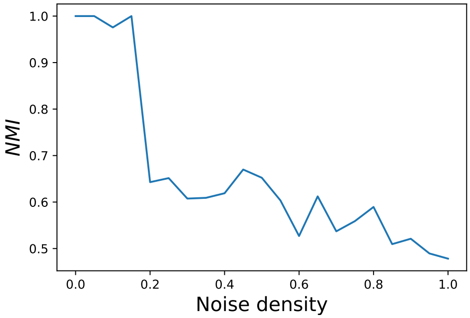


Row norms of matrix of leading k eigenvectors:

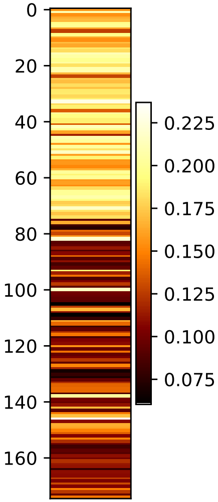


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^{-} = 0.05,$$
$$p_{\text{out}}^{-} = 0.9.$$
$$p_n = 50/100,\ p_n^{-} = 0.25.$$

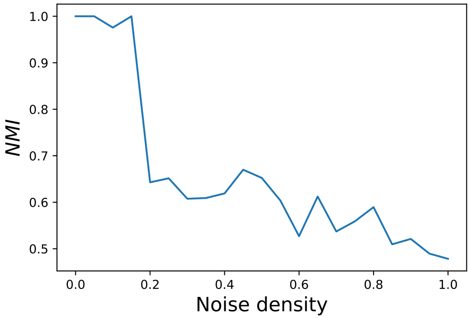


Row norms of matrix of leading k eigenvectors:

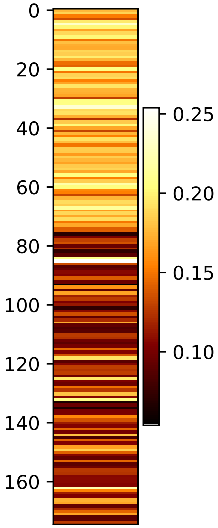


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 55/100,\ p^-_n = 0.25.$$

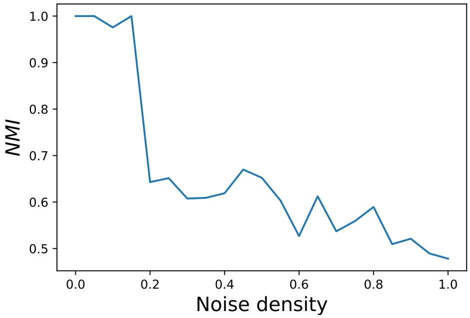


Row norms of matrix of leading k eigenvectors:

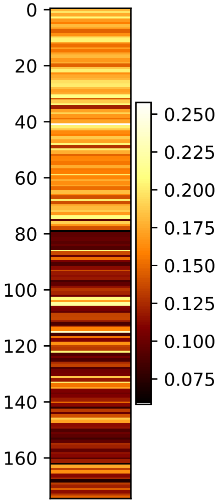


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 60/100,\ p^-_n = 0.25.$$

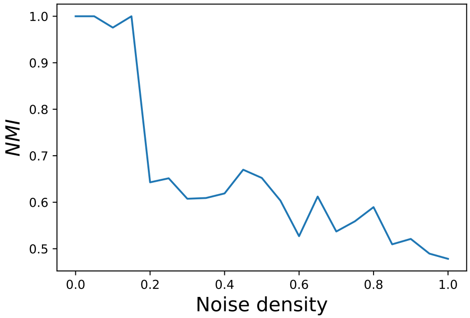


Row norms of matrix of leading k eigenvectors:

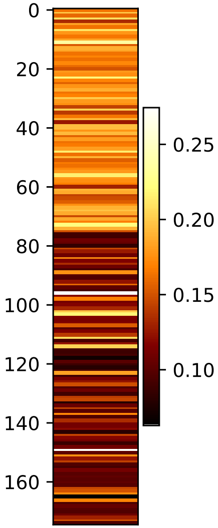


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 65/100,\ p^-_n = 0.25.$$

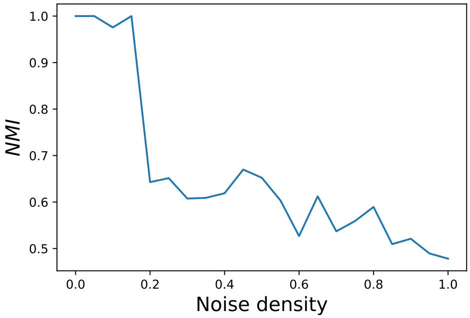


Row norms of matrix of leading k eigenvectors:

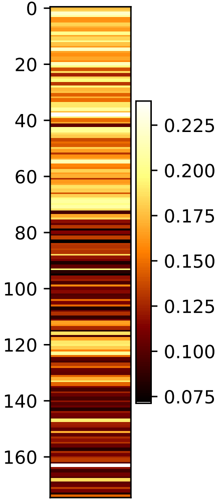


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 70/100,\ p^-_n = 0.25.$$

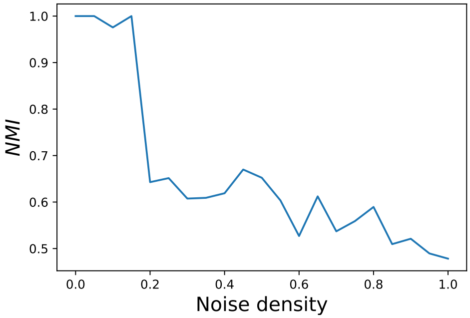


Row norms of matrix of leading k eigenvectors:

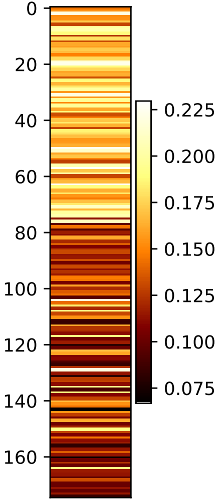


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 75/100,\ p^-_n = 0.25.$$

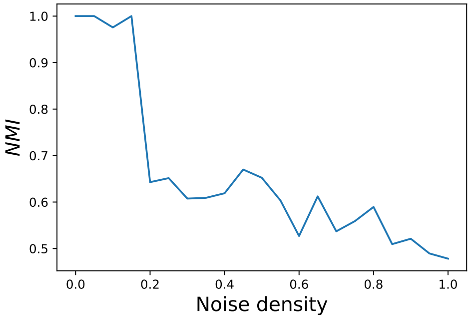


Row norms of matrix of leading k eigenvectors:

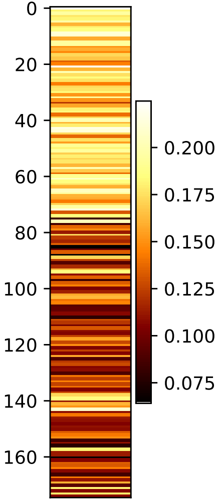


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 80/100,\ p^-_n = 0.25.$$

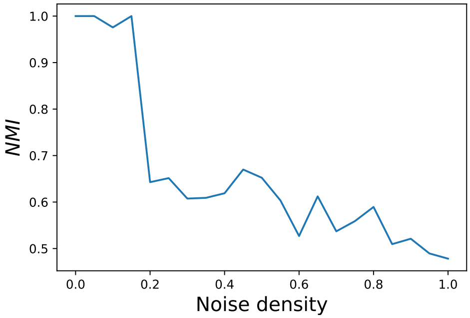


Row norms of matrix of leading k eigenvectors:

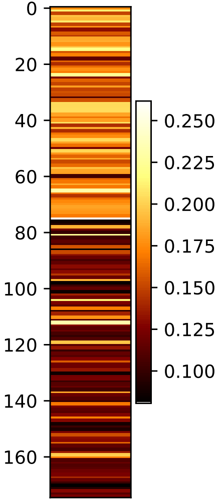


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 85/100,\ p^-_n = 0.25.$$

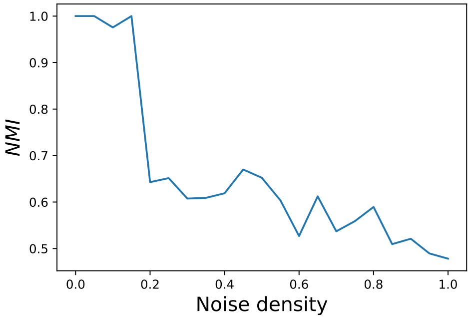


Row norms of matrix of leading k eigenvectors:

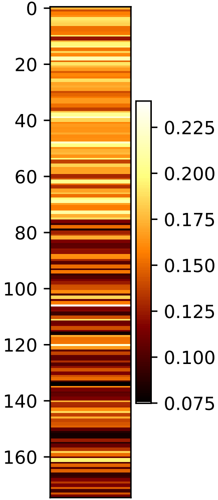


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 90/100,\ p^-_n = 0.25.$$

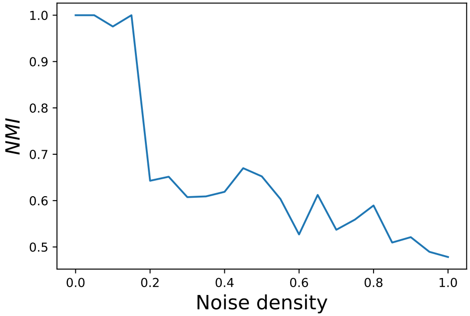


Row norms of matrix of leading k eigenvectors:

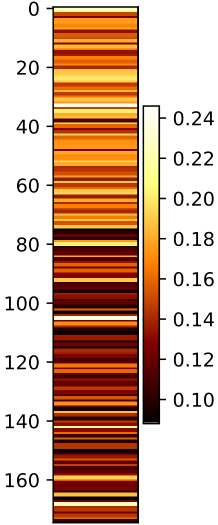


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p^-_{\text{in}} = 0.05,$$
$$p^-_{\text{out}} = 0.9.$$
$$p_n = 95/100,\ p^-_n = 0.25.$$

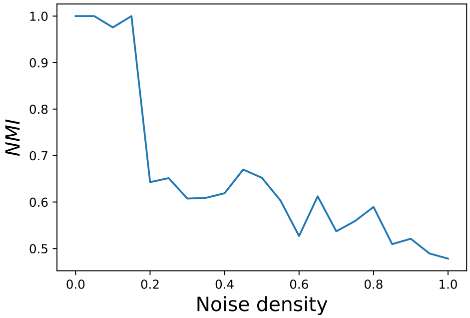


Row norms of matrix of leading k eigenvectors:

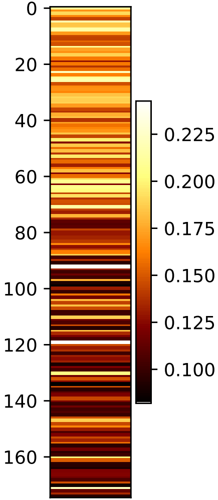


For $k > 2$ communities:

$$n_1 = n_2 = n_3 = 25, \eta = 100$$
$$p_{\text{in}} = 0.8,\ p_{\text{out}} = 0.2,\ p_{\text{in}}^- = 0.05,$$
$$p_{\text{out}}^- = 0.9.$$
$$p_n = 100/100,\ p_n^- = 0.25.$$



Row norms of matrix of leading k eigenvectors:



Take-aways from this lecture:

- Formulating the problem of community detection in signed graphs.
- Spectral algorithms.
- Quality guarantees and spectral gap.
- More than 2 communities.