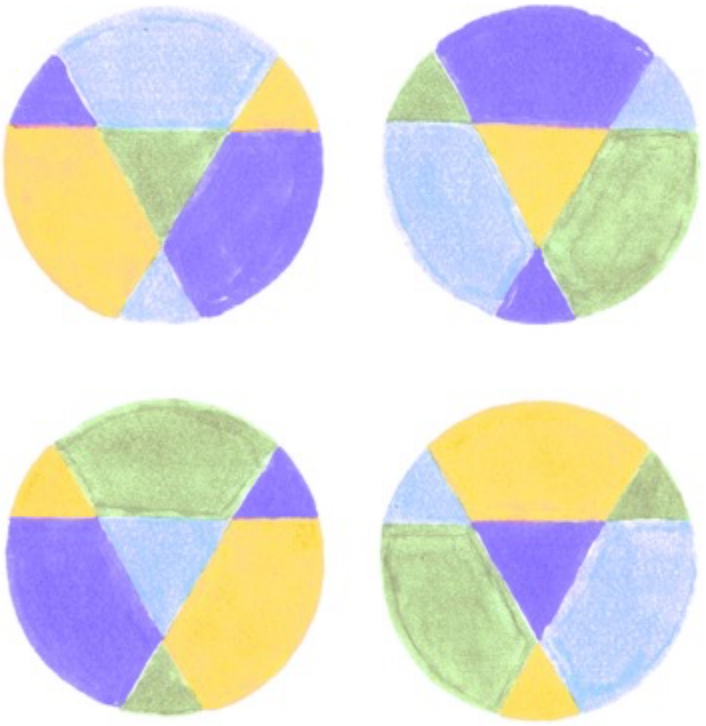
PROJECTIVE GEOMETRY
PART 7

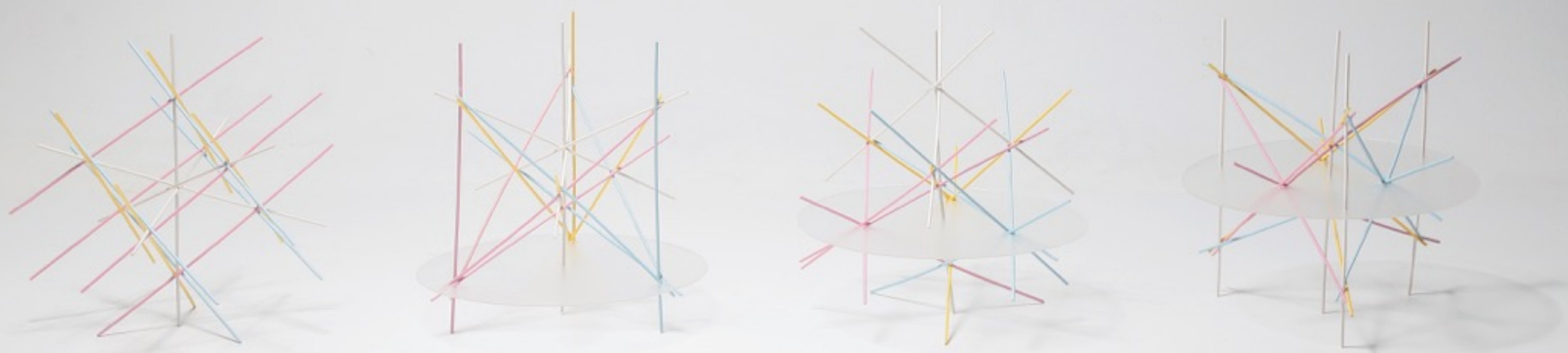


Taneli Luotoniemi

CRYSTAL FLOWER IN HALLS OF MIRRORS 2021

REYE'S CONFIGURATION
(CONTINUED)

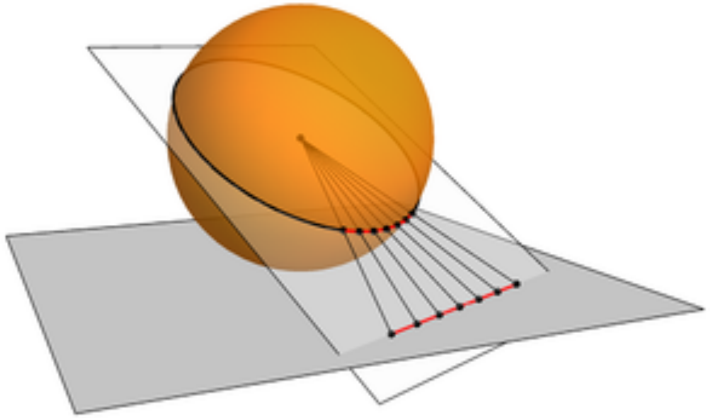
René Magritte: *Les promenades d' Euclide* (1955) https://youtu.be/d-Krgh_v2ds



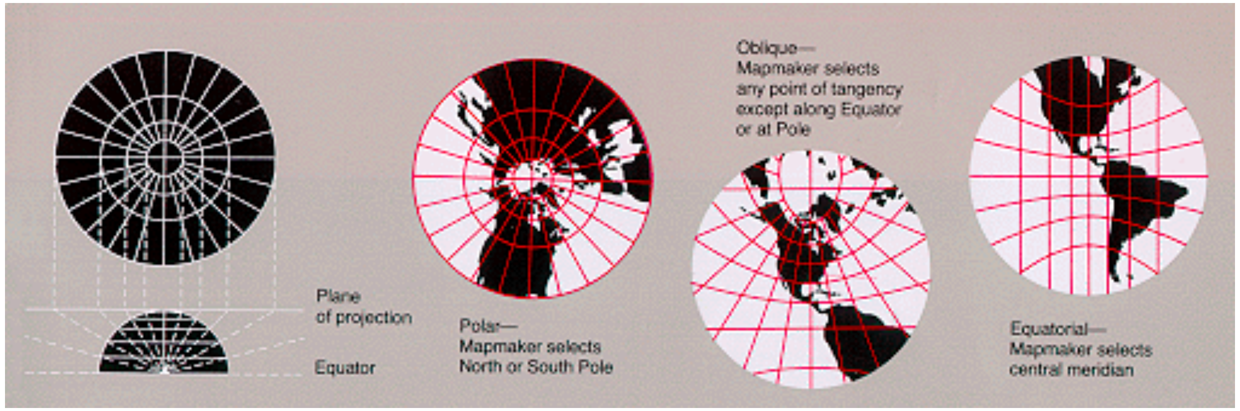
Gnomonic projection

From Wikipedia, the free encyclopedia

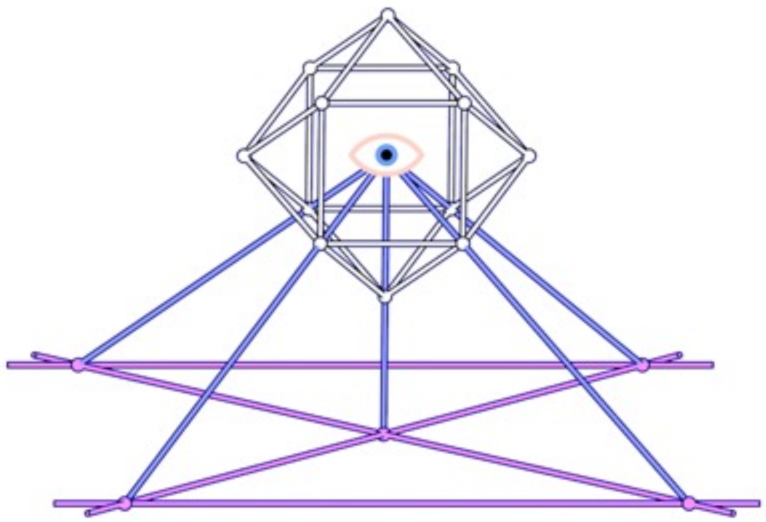
A **gnomonic map projection** displays all great circles as straight lines, resulting in any straight line segment on a gnomonic map showing a geodesic, the shortest route between the segment's two endpoints. This is achieved by casting surface points of the sphere onto a tangent plane, each landing where a ray from the center of the sphere passes through the point on the surface and then on to the plane. No distortion occurs at the tangent point, but distortion increases rapidly away from it. Less than half of the sphere can be projected onto a finite map. Consequently, a rectilinear photographic lens, which is based on the gnomonic principle, cannot image more than 180 degrees.

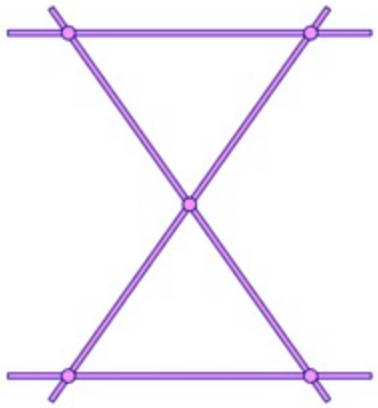


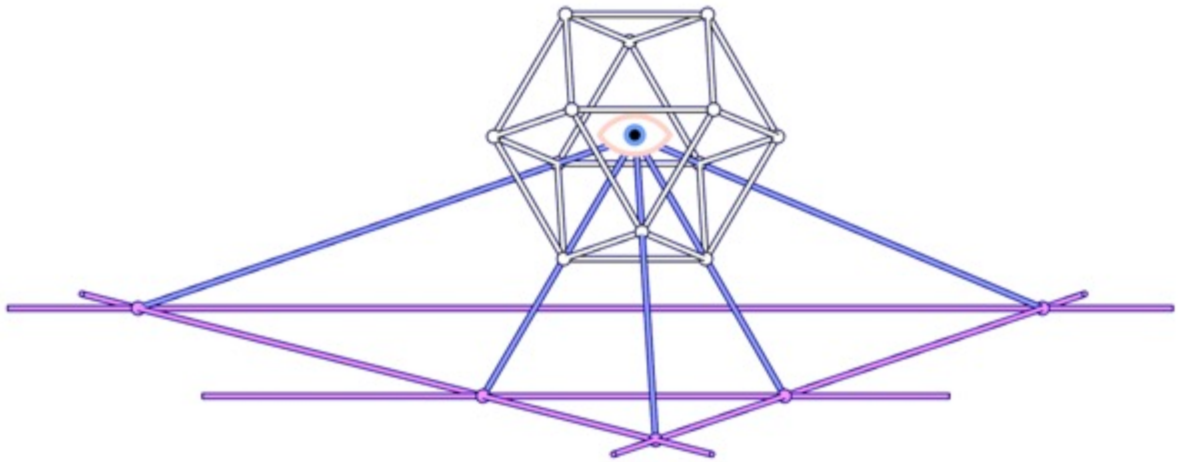
Great circles transform to straight lines via gnomonic projection

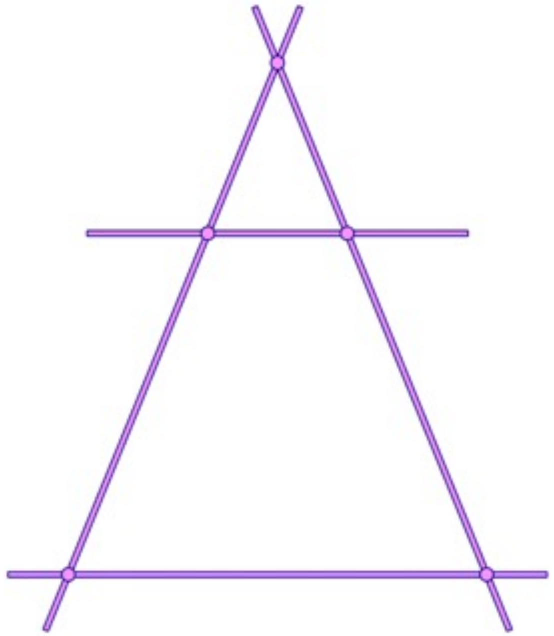


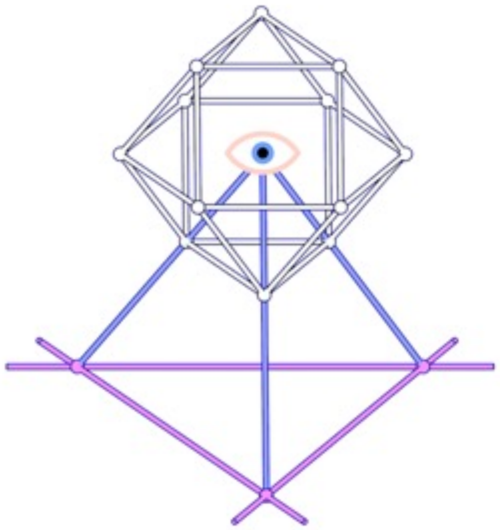
Examples of gnomonic projections

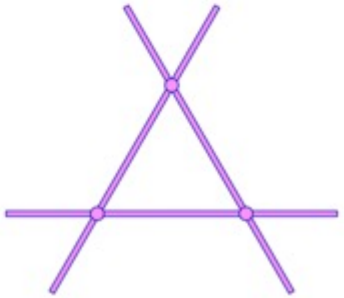


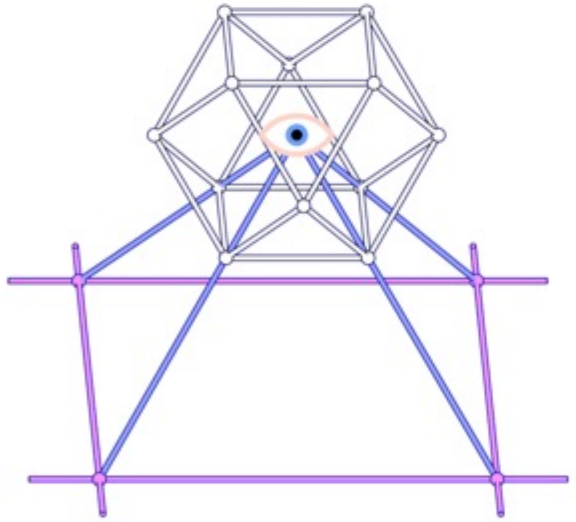


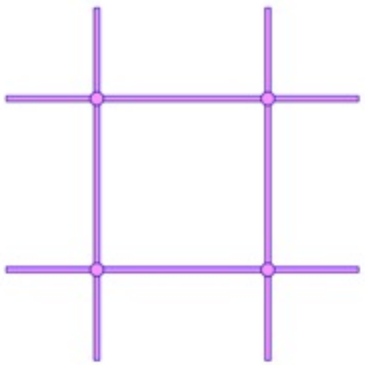


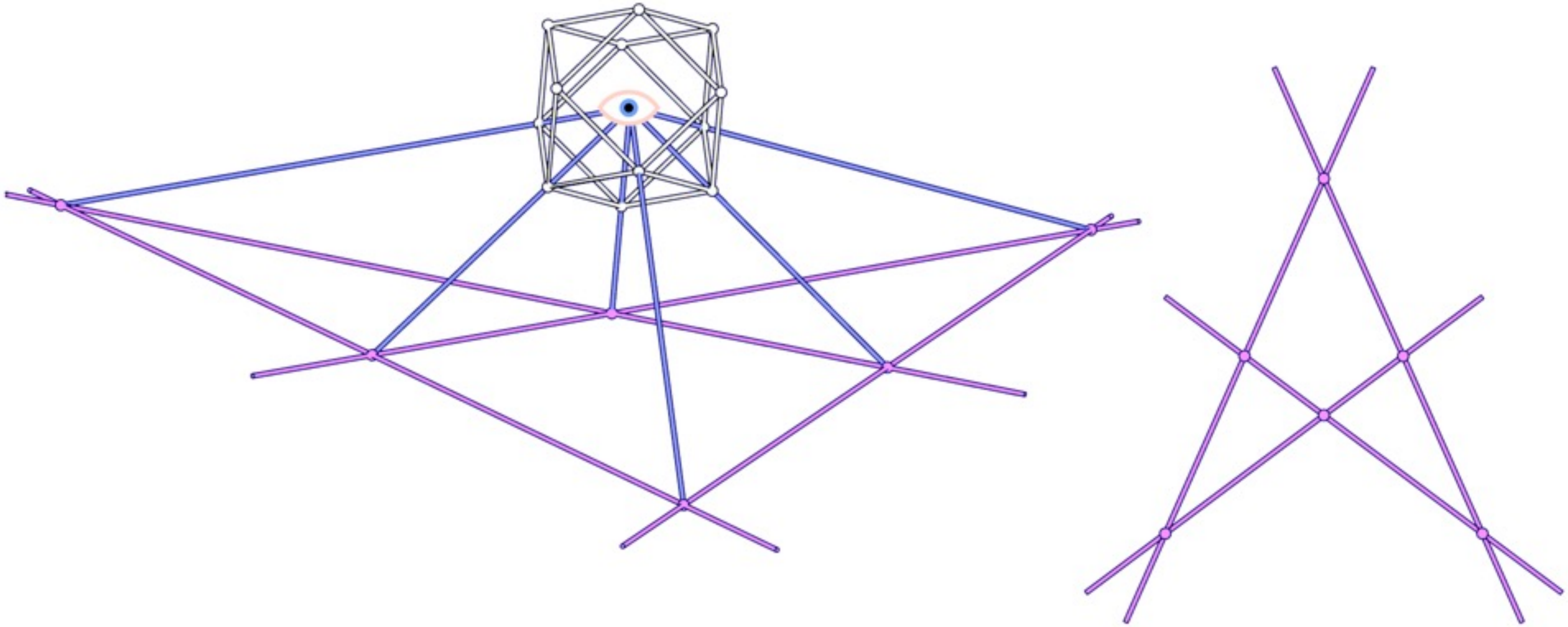


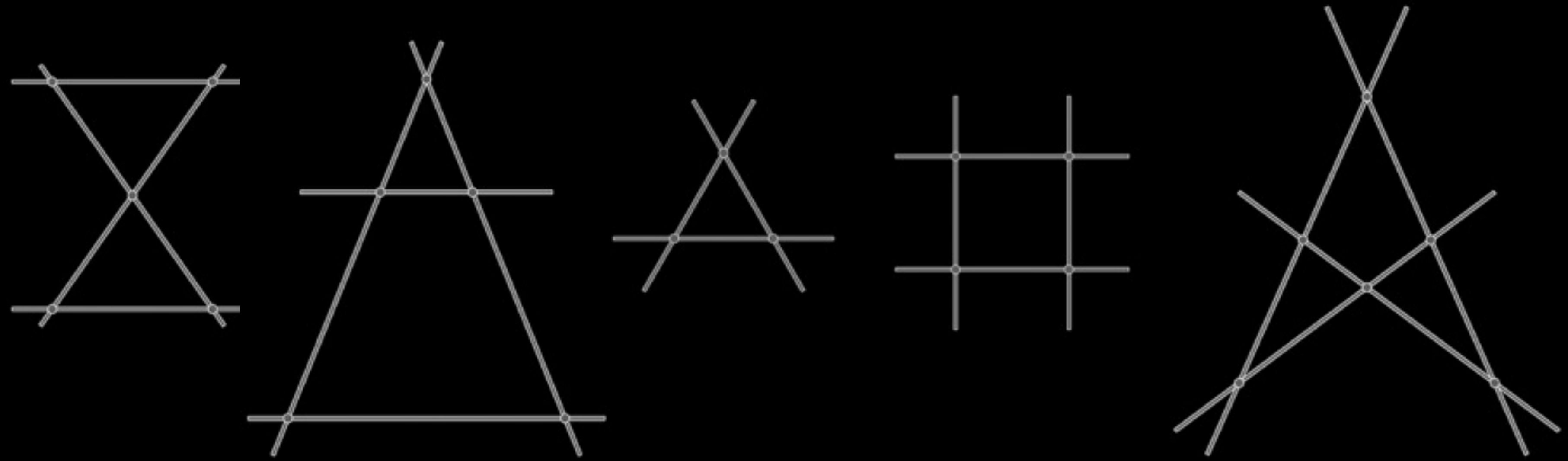


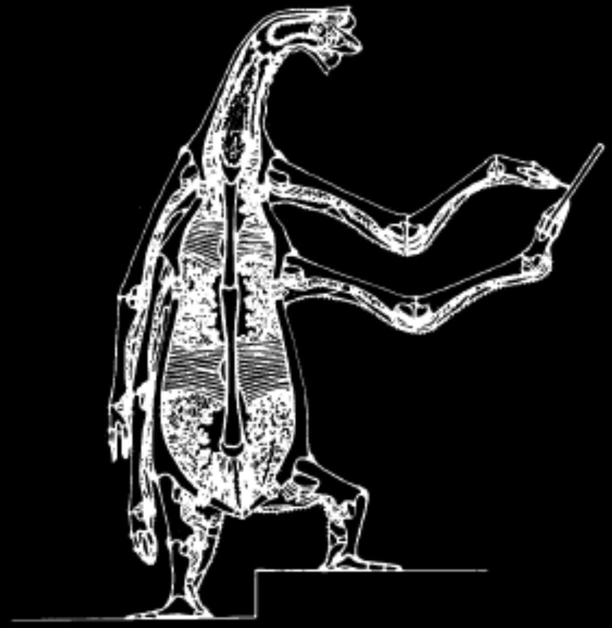










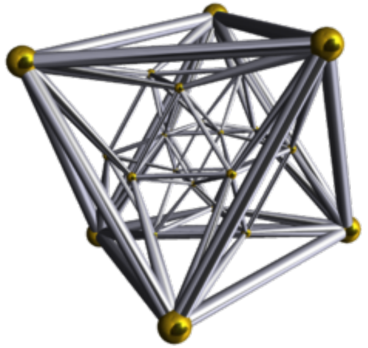
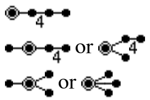


24-cell

From Wikipedia, the free encyclopedia

In geometry, the **24-cell** is the convex regular 4-polytope (four-dimensional analogue of a Platonic solid) with Schläfli symbol {3,4,3}. It is also called $\mathbf{C_{24}}$, **icositetrachoron**, **octaplex** (short for "octahedral complex"), **icosatetrahedroid**,[1] **octacube**, **hyper-diamond** or **polyoctahedron**, being constructed of octahedral cells.

The boundary of the 24-cell is composed of 24 octahedral cells with six meeting at each vertex, and three at each edge. Together they have 96 triangular faces, 96 edges, and 24 vertices. The vertex figure is a cube. The 24-cell is self-dual. In fact, the 24-cell is the unique convex self-dual regular Euclidean polytope which is neither a polygon nor a simplex. Due to this singular property, it does not have a good analogue in 3 dimensions.

24-cell	
 Schlegel diagram (vertices and edges)	
Type	Convex regular 4-polytope
Schläfli symbol	{3,4,3} $r\{3,3,4\} = \left\{\begin{array}{l}3\\3,4\end{array}\right\}$ $\{3^{1,1,1}\} = \left\{\begin{array}{l}3\\3\\3\end{array}\right\}$
Coxeter diagram	
Cells	24 {3,4} 
Faces	96 {3}
Edges	96
Vertices	24
Vertex figure	Cube
Petrie polygon	dodecagon
Coxeter group	F_4, [3,4,3], order 1152 B_4, [4,3,3], order 384 D_4, $[3^{1,1,1}]$, order 192
Dual	Self-dual
Properties	convex, isogonal, isotoxal, isohedral
Uniform index	22

Contents

Constructions

A 24-cell is given as the convex hull of its vertices. The vertices of a 24-cell centered at the origin of 4-space, with edges of length 1, can be given as follows: 8 vertices obtained by permuting

$$(\pm 1, 0, 0, 0)$$

and 16 vertices of the form

$$(\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}).$$

The first 8 vertices are the vertices of a regular 16-cell and the other 16 are the vertices of the dual tesseract. This gives a construction equivalent to cutting a tesseract into 8 cubical pyramids, and then attaching them to the facets of a second tesseract. This is equivalent to the dual of a rectified 16-cell. The analogous construction

Edges	96
Vertices	24
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Petrie polygon	dodecagon
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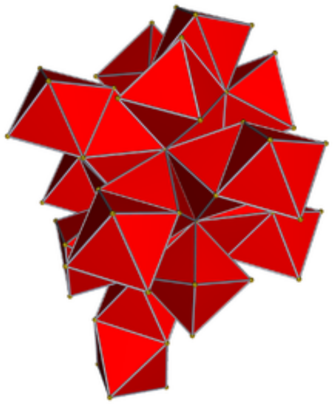


Net

We can further divide the last 16 vertices into two groups: those with an even number of minus (–) signs and those with an odd number. Each of groups of 8 vertices also define a regular 16-cell. The vertices of the 24-cell can then be grouped into three sets of eight with each set defining a regular 16-cell, and with the complement defining the dual tesseract.

The vertices of the dual 24-cell are given by all permutations of

$$(\pm 1, \pm 1, 0, 0).$$

The dual 24-cell has edges of length $\sqrt{2}$ and is inscribed in a 3-sphere of radius $\sqrt{2}$.

Another method of constructing the 24-cell is by the rectification of the 16-cell. The vertex figure of the 16-cell is the octahedron; thus, cutting the vertices of the 16-cell at the midpoint of its incident edges produce 8 octahedral cells. This process also rectifies the tetrahedral cells of the 16-cell which also become octahedra, thus forming the 24 octahedral cells of the 24-cell.

Tessellations

A regular tessellation of 4-dimensional Euclidean space exists with 24-cells, called an icositetrachoric honeycomb, with Schläfli symbol {3,4,3,3}. Hence, the dihedral angle of a 24-cell is 120°.[2] The regular dual tessellation, {3,3,4,3} has 16-cells. (See also List of regular polytopes which includes a third regular tessellation, the tesseractic honeycomb {4,3,3,4}.)

Symmetries, root systems, and tessellations

The 24 vertices of the 24-cell represent the root vectors of the simple Lie group D_4. The vertices can be seen in 3 hyperplanes, with the 6 vertices of an octahedron cell on each of the outer hyperplanes and 12 vertices of a cuboctahedron on a central hyperplane. These vertices, combined with the 8 vertices of the 16-cell, represent the 32 root vectors of the B_4 and C_4 simple Lie groups.

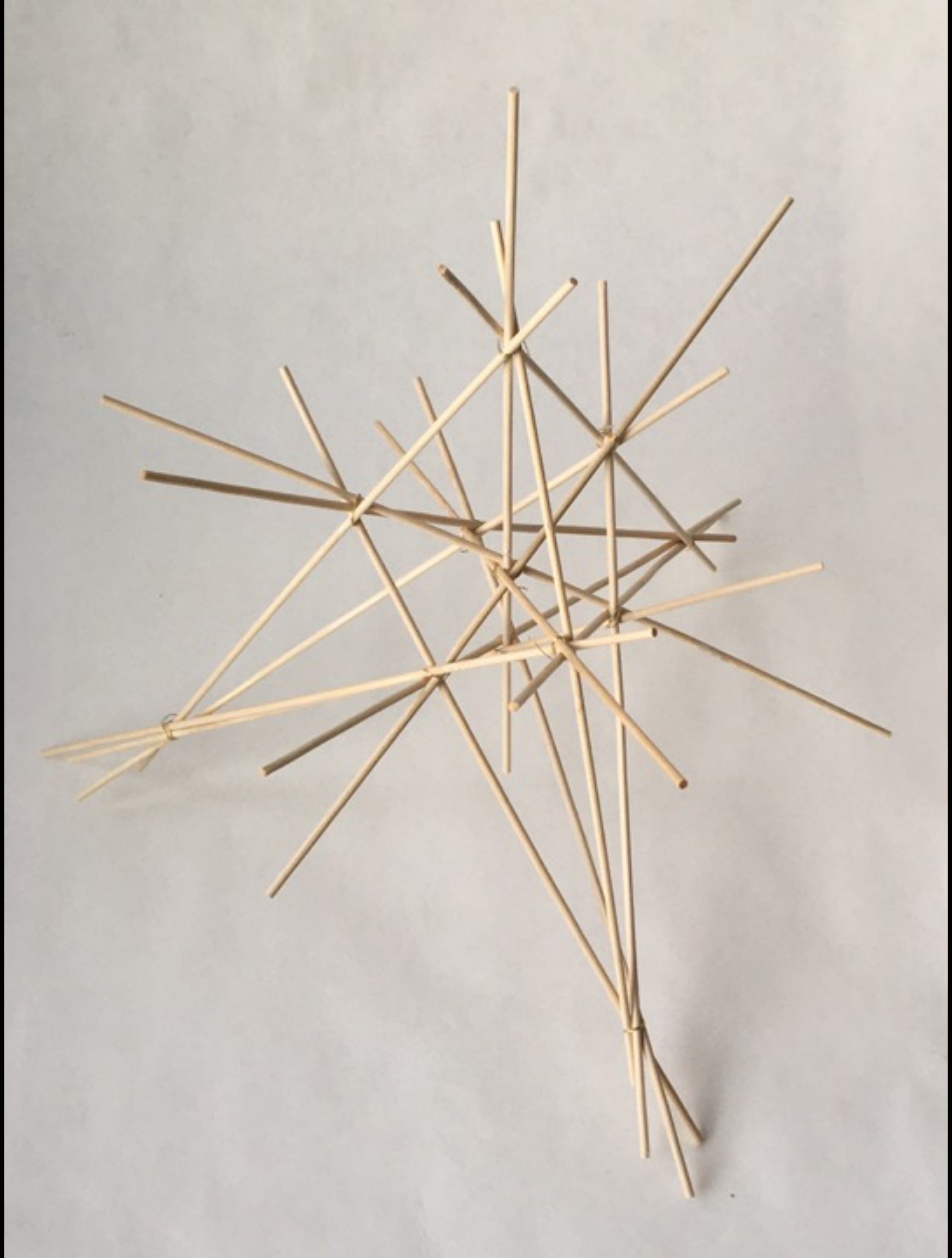
The 48 vertices (or strictly speaking their radius vectors) of the union of the 24-cell and its dual form the root system of type F_4. The 24 vertices of the original 24-cell form a root system of type D_4; its size has the ratio $\sqrt{2}$:1. This is likewise true for the 24 vertices of its dual. The full symmetry group of the 24-cell is the Weyl group of F_4, which is generated by reflections through the hyperplanes orthogonal to the F_4 roots. This is a solvable group of order 1152. The rotational

REYE'S CONFIGURATION

12 points
16 lines
12 planes

4 lines and 6 planes per point
3 points and 3 planes per line
6 points and 4 lines per plane (a complete quadrilateral)

Stick model building video (slightly blurry)
https://youtu.be/fby53U_n4o8



Space Hug at the West Bund Art Centre, Shanghai

https://www.aalto.fi/en/news/aalto-math-arts-in-shanghai-future-lab-exhibition

LARGE SCALE CONFIGURATION WORKSHOP

WORKSHOP SCHEDULE

PAINTING & ASSEMBLING

26th of April

DESARGUES' CONFIGURATION

27th of April

COMPLETE HEXAHEDRON

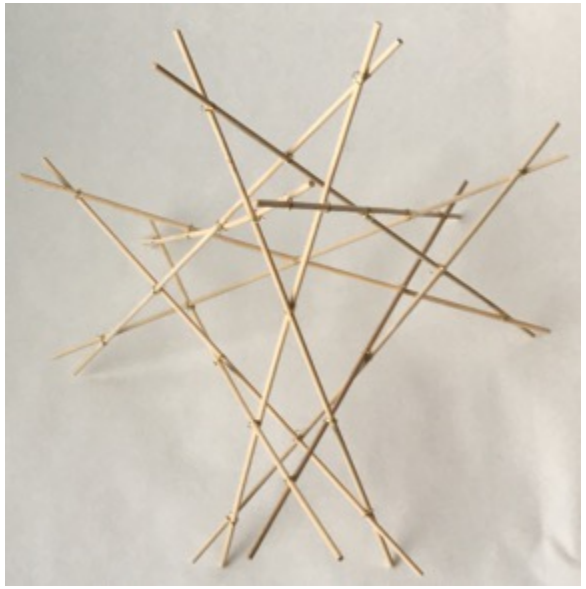
28th of April

COMPLETE HEXACHORON

29th of April

REYE'S CONFIGURATION

30th of April



SCHLÄFLI'S DOUBLE–SIX

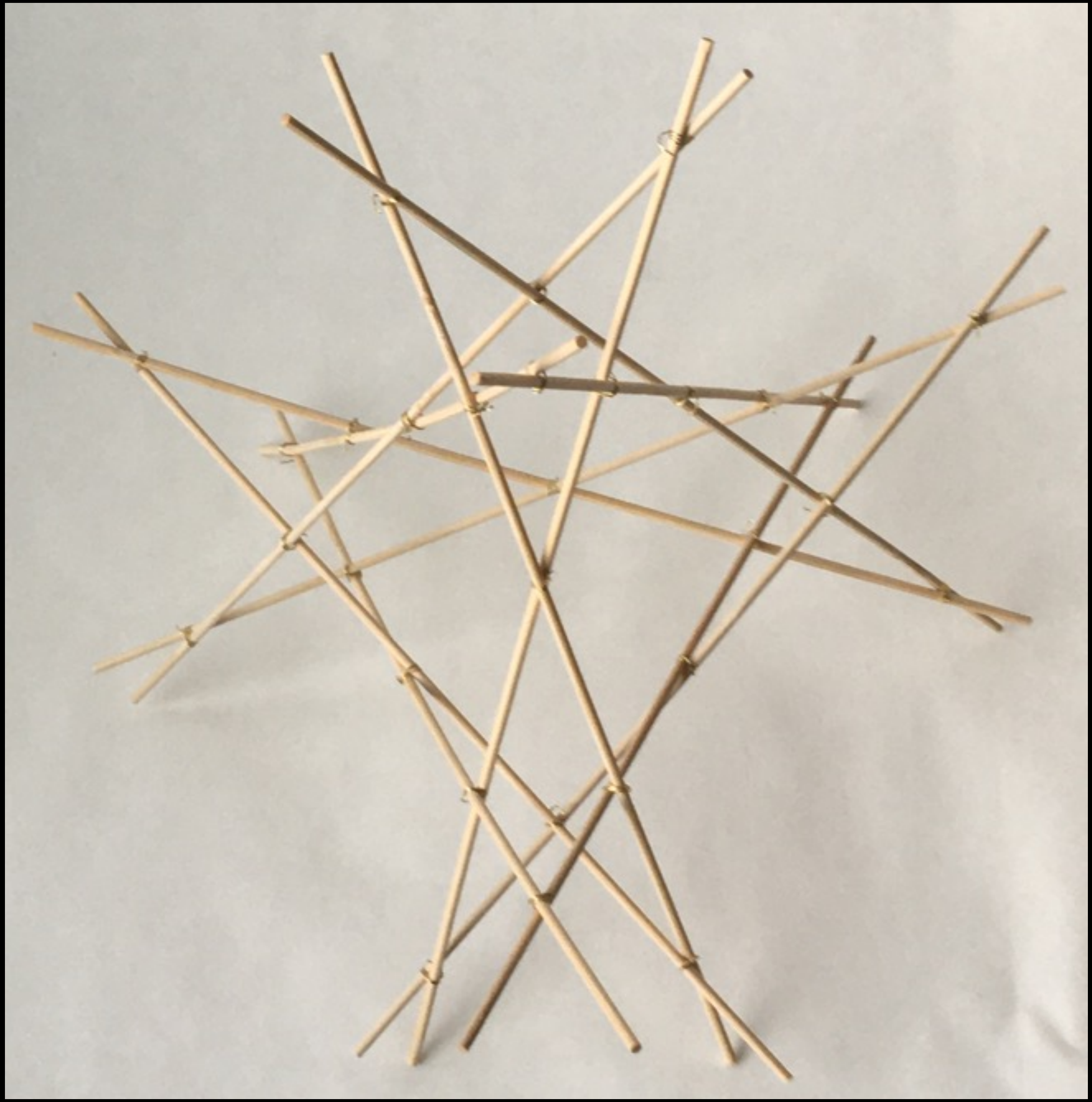
Each day the workshop will last from 10am - 6 pm / Location: Otakaari 1

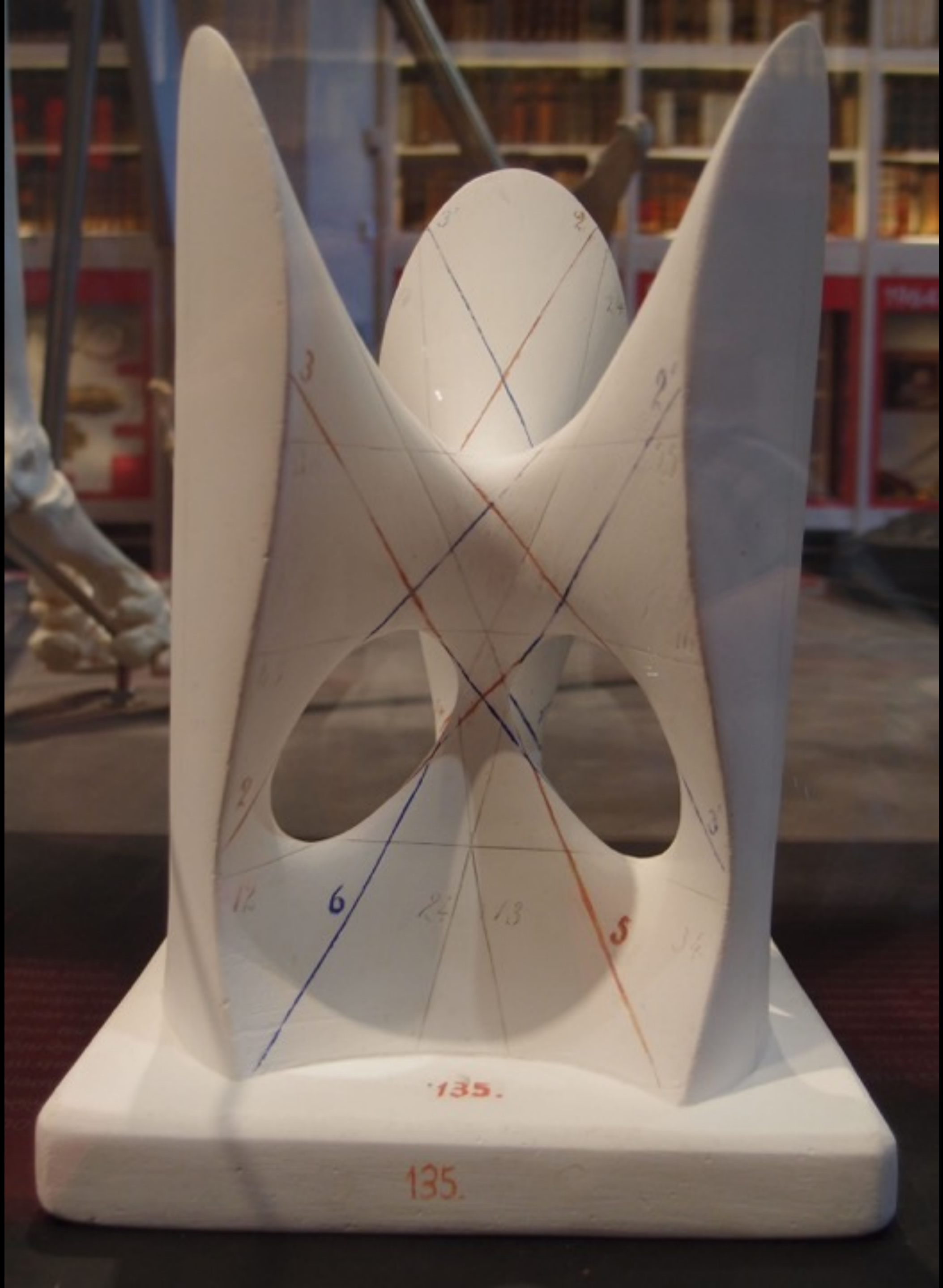
Please put 'X' under date which suits you the best.
https://docs.google.com/spreadsheets/d/1UvnS0dWmE-KsuagHXrfNHHl_o4OSCdoezlgDK3gbX7c/edit?usp=sharing

SCHLÄFLI'S DOUBLE–SIX

30 points
12 lines

2 lines per point
5 points per line





Clebsch surface